Local robust regulation of chained systems

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Abstract

In this work a discontinuous control law for (nonholonomic) chained systems yielding global asymptotic (exponential) regulation and local robustness against the effect of (small) external disturbances is presented.

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1 Introduction

The problem of asymptotic stabilization or regulation of nonholonomic systems has been widely studied in recent years. A possible stabilization *tool* has been introduced in [2], where discontinuous[1] control laws for chained systems have been designed using the so-called σ-process [1]. A similar idea has been developed independently in [26] and [19]. This methodology has been further used to cope with robustness issues and with more general classes of systems. In particular, the extension of this approach to high-order chained forms has been reported in [17] and some robustness considerations have been studied in [29]. Finally, the same ideas have been used in [15] to develop discontinuous control laws for a large class of nonlinear systems. It must be noted that the approach pursued in this paper is strongly related to the work in [20, 27, 28], where the stabilization problems for general chained systems and for an under-actuated spacecraft in the presence of control saturation have been addressed.

Despite the substantial research effort, some fundamental problems still remain open. In particular, all the control laws presented in the above mentioned papers (see also references therein) are non-robust against (small) external disturbances. As a matter of fact, as noted in [4], the presence of arbitrarily small exogenous disturbances may generate unbounded trajectories. Finally, the control laws presented in these papers do not yield Lyapunov stability, but only exponential attractivity, and not all initial conditions are *driven* to the origin.

In the present work we partially address these issues. More precisely, we propose a simple modification of the control law considered in [2] which yields global asymptotic (exponential) regulation, and local robustness against exogenous disturbances[2].

We focus our attention on n-dimensional chained systems with two controls (see [22] for detail), *i.e.* systems described by equations of the form

$$\begin{aligned} \dot{x}_1 &= u_1, \\ \dot{x}_2 &= u_2, \\ \dot{x}_3 &= x_2 u_1, \\ &\vdots \\ \dot{x}_n &= x_{n-1} u_1. \end{aligned} \tag{1}$$

As observed in several research papers, system (1) is a classical example of a system which is completely controllable but not asymptotically stabilizable by smooth (or continuous or even a class of discontinuous) static or dynamic feedback laws. For this reason the problem of designing *stabilizing* control laws has attracted a lot of interest, see [2–7, 9, 10, 12, 14–16, 18, 21–25, 29] and references therein for further detail.

[1] The term discontinuous is used here in a special sense, see [2] for detail.

[2] See Section 5 for further detail.

2 Preliminary results

In this section we discuss a preliminary fact which is instrumental to prove the main results of the paper. Consider the system (1), the control law[3]

$$
\begin{aligned}
u_1 = u_{1A}(x) = &\quad -x_1 \\
u_2 = u_{2A}(x) = &\quad p_2 x_2 + p_3 \frac{x_3}{x_1} + p_4 \frac{x_4}{x_1^2} + \cdots + p_n \frac{x_n}{x_1^{n-2}},
\end{aligned} \tag{2}
$$

and the matrix

$$
A = \left[\begin{array}{ccccccc}
p_2 & p_3 & p_4 & p_5 & \cdots & p_{n-1} & p_n \\
-1 & 1 & 0 & 0 & \cdots & 0 & 0 \\
0 & -1 & 2 & 0 & \cdots & 0 & 0 \\
0 & 0 & -1 & 3 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & -1 & n-2
\end{array} \right]. \tag{3}
$$

It has been proved in [2] that the closed loop system (1)-(2) admits a unique forward solution for any initial condition $x(0)$ such that $x_1(0) \neq 0$, and that if[4]

$$\sigma(A) \subset \mathbb{C}^- \tag{4}$$

such a solution converges *exponentially* to zero. However, more structure can be exposed as detailed in the following simple fact, the proof of which is simple, hence omitted.

Lemma 1 *Consider the system (1) with initial condition $x(0)$ such that $x_1(0) \neq 0$ and the control law (2). Let p_i be such that*

$$\sigma(A+I) \subset \mathbb{C}^-, \tag{5}$$

$P = P^T > 0$ *such that*

$$(A+I)^T P + P(A+I) < 0,$$

and

$$z = z(x) = \eta^T P \eta \tag{6}$$

with

$$\eta = \left[\begin{array}{cccc} \frac{x_2}{x_1} & \frac{x_3}{x_1^2} & \cdots & \frac{x_n}{x_1^{n-1}} \end{array} \right]^T. \tag{7}$$

Then, $z(x) \geq 0$ for all x and, along the trajectories of the closed loop system with $x_1(0) \neq 0$, one has

$$\dot{z} \leq 0.$$

Moreover, if $x(0)$ is such that $z(x(0)) \leq k$ then, for any $t > 0$, $z(x(t)) < k$, i.e. *the region $z(x) \leq k$ is positively invariant.*

[3]The control law (2) is not defined for $x_1 = 0$, however this control law, which has been widely studied in [2], is not *implemented*, see Section 3 for further detail.

[4]For a square matrix F, $\sigma(F)$ denotes the spectrum of F.

Remark 1 In the control law (2) it could be possible to add an extra design parameter in the definition of u_1, *i.e.* it is possible to define $u_1 = -\lambda x_1$ for some $\lambda > 0$.

Remark 2 It is possible to obtain simple estimates for the region $z \leq k$. In fact, this region is contained in the set

$$\Omega_M = \{x \in \mathbb{R}^n \mid \tilde{z} \leq \frac{k}{\sigma_{min}(P)}\}$$

and contains the set

$$\Omega_m = \{x \in \mathbb{R}^n \mid \tilde{z} \leq \frac{k}{\sigma_{max}(P)}\},$$

where

$$\tilde{z} = \tilde{z}(x) = \left\| \begin{bmatrix} \frac{x_2}{x_1} & \frac{x_3}{x_1^2} & \cdots & \frac{x_n}{x_1^{n-1}} \end{bmatrix}^T \right\|^2 .$$

Note moreover that, for any $\kappa > 0$,

$$\tilde{z} \leq \kappa^2 \Rightarrow \left| \frac{x_i}{x_1^{i-1}} \right| \leq \kappa$$

for all $i = 2, \cdots, n$, hence

$$\Omega_M \cap \{x \in \mathbb{R}^n \mid x_1 = 0\} = \Omega_m \cap \{x \in \mathbb{R}^n \mid x_1 = 0\} = \{x \in \mathbb{R}^n \mid x_1 = x_2 = \cdots = x_n = 0\},$$

and that any point such that $x_1 = 0$ and $\|x\| > \epsilon$ possesses a (sufficiently small) neighborhood B_ϵ such that $B_\epsilon \cap \Omega_M = B_\epsilon \cap \Omega_m = \emptyset$. This means that the sets Ω_M and Ω_m, hence the set $z \leq k$, cannot get arbitrarily close to points such that $x_1 = 0$ outside any neighborhood of the origin.

3 A new discontinuous controller

One of the main results of the paper is summarized in the following statements, which provide a simple and natural modification of the control law presented in [2]. It must be noted that, despite the simplicity of the modification, this new control law possesses very appealing properties, as detailed in the next sections.

Proposition 1 *Consider the system (1) and the control law*

$$\begin{array}{l} u_1 = \left\{ \begin{array}{lll} u_{1A}(x) & = -x_1 & 0 \leq z \leq k \\ u_{1B}(x) & = sign(x_1) & z > k \end{array} \right. \\ u_2 = \left\{ \begin{array}{lll} u_{2A}(x) & = p_2 x_2 + p_3 \frac{x_3}{x_1} + p_4 \frac{x_4}{x_1^2} + \cdots + p_n \frac{x_n}{x_1^{n-2}} & 0 \leq z \leq k \\ u_{2B}(x) & = -\mu x_2 & z > k \end{array} \right. \end{array} \tag{8}$$

with $k > 0$, the p_i such that condition (5) holds, $\mu \geq 0$, z defined as in (6), and[5]

$$sign(x_1) = \left\{ \begin{array}{ll} 1 & x_1 \geq 0 \\ -1 & x_1 < 0 \end{array} \right. . \tag{9}$$

[5] Note the somewhat non-standard definition of the sign function.

Then the closed loop system (1)-(8) admits a unique forward solution for any initial condition $x(0)$ and such a solution converges exponentially *to zero.*

Proof. The proof can be arranged in two cases: i) $0 \leq z(0) \leq k$ and ii) $z(0) > k$. In the first case the control law (2) is used for all $t \geq 0$; hence, as discussed in Section 2 the state converges exponentially to zero. In the second case, the control law

$$\begin{aligned} u_1 &= u_{1B} = sign(x_1) \\ u_2 &= u_{2B} = -\mu x_2 \end{aligned} \tag{10}$$

is initially active. Therefore, a direct integration of the closed loop system (1)-(10) yields, if $\mu > 0$[6],

$$\begin{cases} x_1(t) &= x_{10} + t, \\ x_2(t) &= x_{20} e^{-\mu t}, \\ x_3(t) &= x_{30} + \dfrac{x_{20}}{\mu}(1 - e^{-\mu t}), \\ \vdots & \\ x_n(t) &= \sum_{j=3}^{n} \dfrac{x_{j0}\, t^{n-j}}{(n-j)!} + \sum_{j=4}^{n} \dfrac{(-1)^{j-4}\, x_{20}\, t^{n-j+1}}{\mu^{j-3}\,(n-j+1)!} + \dfrac{(-1)^{n-3}\, x_{20}}{\mu^{n-2}}(1 - e^{-\mu t}), \end{cases} \tag{11}$$

when $x_1(0) \geq 0$, and

$$\begin{cases} x_1(t) &= x_{10} - t, \\ x_2(t) &= x_{20} e^{-\mu t}, \\ x_3(t) &= x_{30} - \dfrac{x_{20}}{\mu}(1 - e^{-\mu t}), \\ \vdots & \\ x_n(t) &= \sum_{j=3}^{n} \dfrac{(-1)^{n-j}\, x_{j0}\, t^{n-j}}{(n-j)!} + \sum_{j=4}^{n} \dfrac{(-1)^{n-j}\, x_{20}\, t^{n-j+1}}{\mu^{j-3}\,(n-j+1)!} - \dfrac{x_{20}}{\mu^{n-2}}(1 - e^{-\mu t}), \end{cases} \tag{12}$$

when $x_1(0) < 0$.

It is simple to verify that in both cases $\lim_{t \to \infty} z(t) = 0$, so there exists a finite time $t^* \geq 0$, minimally defined, such that $z(t^*) = k$. Therefore, at $t = t^*$ there will be a switch from the controller (10) to the controller (2). For the properties of $z(t)$, the control law (2) will be active for all $t \geq t^*$, which allows to conclude, also in this second case, exponential convergence. $\triangleleft$

Remark 3 The control law (8) is obviously discontinuous. However, the nature of the discontinuity is different from the nature of the discontinuity of the control law in [2], because the control law (8) provides a bounded control action for any bounded x, and this is not the case for the control law in [2]. Despite this substantial difference the control

[6] A similar analysis can be carried out and the same conclusions can be drawn for $\mu = 0$. The details are omitted for brevity.

law discussed in this work inherits some of the properties of the control laws in [2]. In particular, the closed loop trajectories are *efficient*, and no oscillatory behavior is observed.

Remark 4 It is important to stress that the control law (8) does not belong to the class of discontinuous control laws discussed in [10, Section 2], hence Proposition 1 does not contradict [10, Theorem 6]. Moreover, the control law (8) is not an upper semi-continuous map, in the sense of [24] (see also [8, Chapter 2]), hence Proposition 1 is not in contrast with the conclusions of [24, Theorem 1].

Remark 5 It is worth noting that if $x_1(0) \neq 0$ and $x_i(0) = 0$ for $i = 2, 3, \cdots, n$ then $x_1(t) = x_1(0) \exp^{-t}$ and $x_i(t) = 0$ for $i = 2, \cdots, n$ and for all $t \geq 0$, *i.e.* the x_1-axis is invariant and internally stable

4 An interesting property

In this section we show that if the design parameters p_i, μ and the positive definite matrix P are properly selected then the closed loop system (1)-(8) possesses, for any $k > 0$, a property which is instrumental to derive the robustness result of Section 5.

Proposition 2 *Consider the closed loop system (1)-(8) with $\mu = 0$ and $k > 0$.*

Suppose the design parameters p_i are such that condition (5) holds. Suppose moreover that there exists a matrix $P = P^T > 0$ such that

$$(A + I)^T P + P(A + I) < 0 \tag{13}$$

and

$$F^T P + PF < 0, \tag{14}$$

where

$$F = \begin{bmatrix} -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -(n-1) \end{bmatrix}.$$

Then there exists a function $V(x)$, everywhere continuous and differentiable almost everywhere, with $V(0) = 0$ and $V(x) > 0$ for every nonzero x, such that along the trajectories of the closed loop system one has (almost everywhere)

$$\dot{V} = -(x_1^2 + x_2^2 + \cdots + x_n^2) = -||x||^2 < 0,$$

for all $x \neq 0$.

Proof. It has been proved in Proposition 1 that all trajectories of the closed loop system converge exponentially to zero. Moreover, let $x_{\bar{x}}(t)$ be the trajectory of the closed loop system (1)-(8) starting at $x(0) = \bar{x}$, and note that $x_{\bar{x}}(t)$ is a continuous function of t and that the function

$$V(\bar{x}) = \int_0^\infty \|x_{\bar{x}}(t)\|^2 dt$$

is well defined (*i.e.* always finite), and it is such that $V(0) = 0$ and $V(x) > 0$, for all $x \neq 0$. Moreover, by construction one has (almost everywhere) $\dot{V} = -\|x\|^2$. To complete the proof we need to show that the function $V(\bar{x})$ is a continuous function. To this end note that the only sets where the function $V(\bar{x})$ can be discontinuous are

$$S_1 = \{x \in \mathbb{R}^n \mid x_1 = 0\},$$

as for any $\epsilon > 0$ trajectories with initial conditions

$$x_0^+ = (\epsilon, x_{20}, \cdots, x_{n0})$$

and

$$x_0^- = (-\epsilon, x_{20}, \cdots, x_{n0})$$

have qualitatively different behavior, *i.e.* the former stays in the *positive* x_1 *half-space*, and the latter in the *negative* x_1 *half-space*, and

$$S_2 = \{x \in \mathbb{R}^n \mid z(x) = k\}.$$

As far as S_1 is concerned note that the closed loop vector field is symmetric with respect to the $x_1 = 0$ hyperplane, hence despite the different behavior one has

$$\lim_{\epsilon \to 0} (V(x_0^+) - V(x_0^-)) = 0,$$

which proves the claim.

Consider now trajectories starting *close* to S_2. If the initial condition is such that $z(x(0)) \leq k$ then the control law (u_{1A}, u_{2A}) is always active, *i.e.* it is active for all $t \geq 0$. If the initial condition is such that $z(x(0)) = k + \epsilon$, for some $\epsilon > 0$, then the control law (u_{1B}, u_{2B}) is active for $t \in [0, t^\star)$, for some $t^\star > 0$ and (u_{1A}, u_{2A}) is active for $t \geq t^\star$. Note that $t^\star$ is a function of ϵ, hence to complete the proof we need to show that $t^\star$ goes to zero when ϵ goes to zero. To this end, consider the closed loop system (1)-(8) in the set[7] $x_1 > 0$ and $z(x) > k$. In such a region consider the coordinates defined in equation (7) and note that[8]

$$\dot{\eta} = \frac{1}{x_1(0) + t} F\eta.$$

As a result

$$\dot{z} = \frac{1}{x_1(0) + t} \eta^T (F^T P + PF)\eta$$

[7] A similar analysis applies if $x_1 < 0$.

[8] As $z(x(0)) = k + \epsilon < +\infty$ then $x_1(0) \neq 0$.

which is negative for any finite $t \geq 0$ and any nonzero η (in particular for any η such that $z \neq 0$). As a result, the function $z(x(t))$ is strictly decreasing for all nonzero η and all (finite) $t \geq 0$, and this shows that all trajectories starting close to S_2 reach S_2 in finite time and this time tends to zero if $x(0)$ approaches S_2. We conclude that the function $V(\cdot)$ is continuous in S_2, which completes the proof. $\triangleleft$

Remark 6 The procedure used to construct the function $V(\bar{x})$ in the proof of Proposition 2 is classical. It has been introduced by Malkin, see [13], and has been recently used to establish converse Lyapunov results for switched systems in [11]. It must be noted that, because of lack of regularity, the function $V(\bar{x})$ is not a Lyapunov function, hence it is not possible to infer (local) Lyapunov stability of the closed loop system (1)-(8). Nevertheless, the property expressed in Proposition 2 is useful to establish robustness properties of the closed loop system, as detailed in the following. Note, moreover that the property summarized in Proposition 2 is not possessed by the control laws presented in [2, 4, 7, 15, 19, 29], which are therefore potentially less robust to model errors, measurement noise and exogenous disturbances.

Remark 7 The key condition in Proposition 2 is the existence of a common Lyapunov function for the (linear) systems

$$\dot{\xi} = (A + I)\xi \qquad\qquad \dot{\xi} = F\xi.$$

In general, it is not possible to guarantee existence of such a common Lyapunov function, and the existence may depends upon an appropriate selection of the parameters p_i. We simply note that this existence problem can be posed as a simple LMI feasibility problem.

Example 1 A simple computation shows that for $n = 3$, *i.e.* for three dimensional chained systems the conditions in Proposition 2 hold, for example, selecting $p_2 = -15$, $p_3 = 30$ and

$$P = \begin{bmatrix} 1 & -2 \\ -2 & 6 \end{bmatrix}.$$

5 A robustness result

The results established in Section 3 can be exploited to prove an interesting robustness property. In particular, we study robustness of the closed loop system against external disturbances.

Proposition 3 *Consider the system*

$$\begin{aligned} \dot{x}_1 &= u_1 + d_1, \\ \dot{x}_2 &= u_2 + d_2, \\ \dot{x}_3 &= x_2 u_1 + d_3, \\ &\vdots \\ \dot{x}_n &= x_{n-1} u_1 + d_n \end{aligned} \tag{15}$$

in closed loop with the control laws (8) with the p_i such that conditions (5), (13) and (14) hold, $\mu = 0$ and $k > 0$.

Then there exists a neighborhood of the origin $\Omega \subset \mathbb{R}^n$ such that for any neighborhood of the origin $\Omega_1 \subset \Omega$ one can find a positive number K_{Ω,Ω_1} such that the condition

$$||d||_\infty < K_{\Omega,\Omega_1}$$

implies that any trajectory of the closed loop system (15)-(8) starting in Ω enters Ω_1 in some finite time T and stays in Ω_1 for all $t \geq T$.

Proof. Along the trajectories of the closed loop system one has

$$\dot{V} = V_x\left(g(x)u(x) + d\right) = V_x g(x)u(x) + V_x d \leq -\|x\|^2 + \varphi(x)\|d\|_\infty, \tag{16}$$

for some bounded function $\varphi(x)$. Let now

$$M = \max_{x \in \Omega} \varphi(x)$$

and

$$M_1 = \min_{x \in \Omega \setminus \Omega_1} \parallel x \parallel^2 .$$

Then, in $\Omega \setminus \Omega_1$ one has $\dot{V} \leq -M_1 + \|d\|_\infty M$, hence the claim. ◁

Remark 8 It is worth stressing that the control law (8) is able to (locally) counteract (small) exogenous disturbances. As a result, the effect of small constant disturbances remains small. Note that, as detailed in [4, 29], most of the existing control laws for chained systems are unable to yield this property.

6 Simulations

For illustrative purposes some simple simulations, for a three dimensional chained system, are included. In all simulations we set the parameters p_2 and p_3 as discussed in Example 1. Finally, we select $\mu = 0$ and $k = 1/2$. Figure 1 displays the state histories, the control signal u_2[9], and the signal z (in log-scale) for the nominal chained system (1) in closed loop with the control law (8) from the initial condition $(x_1(0), x_2(0), x_3(0)) = (0, 0, 1)$. Note that the variable z is monotonically decreasing. Figure 2 displays the state histories, the control signal u_2, and the signal z (in log-scale) for the perturbed chained system (15) in closed loop with the control law (8) from the initial condition $(x_1(0), x_2(0), x_3(0)) = (0, 0, 1)$ and with $d_1 = 0$, $d_2 = 0$ and $d_3 = 0.1$. Note that, as discussed in detail in [4], all trajectories of this perturbed system in closed loop with $u_i = u_{iA}(\cdot)$ are unbounded.

[9] As $u_1 = -x_1$ we only display u_2.

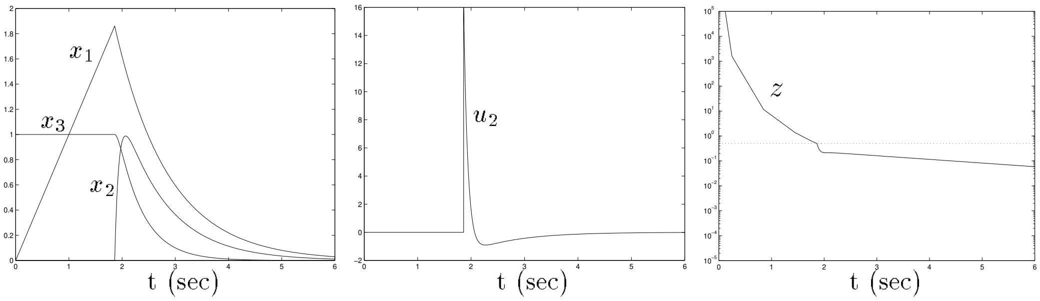

Figure 1: State histories, control signals and the variable z for a nominal simulation.

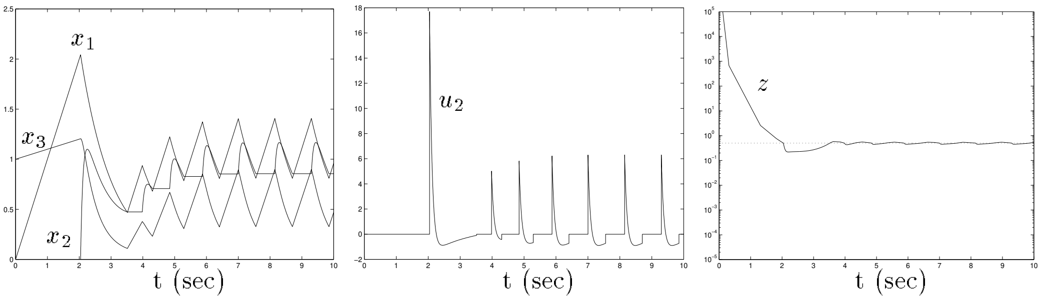

Figure 2: State histories, control signals and the variable z for a simulation with exogenous disturbances.

7 Conclusion

A novel discontinuous state feedback control law for chained systems has been proposed and the properties of the resulting closed loop system have been discussed in detail. It is shown that the proposed control law yields global exponential regulation and local robustness against (small) external disturbances. Simulations results complement the theoretical developments.

Further study is in progress to design control laws yielding global asymptotic stability in the sense of Lyapunov and to verify the robustness of the proposed control law in the presence of sensor noise, model errors and unmodeled dynamics.

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