Two Results for Adaptive Output Feedback Stabilization of Nonlinear Systems ⋆

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Abstract

The problem of (adaptive) stabilization by means of output feedback of a class of nonlinear systems is addressed and solved. The proposed method relies on the asymptotic reconstruction of a stabilizing state feedback control law, does not require stable zero dynamics nor the construction of a Lyapunov function for the closed loop system, and treats in a unified way unknown parameters and unmeasured states. The applicability of the proposed method is discussed via theoretical examples. Finally, it is shown that the proposed method yields a solution to the problem of output feedback regulation for a DC-to-DC power converter and the efficacy of the resulting controller is verified via experiments.

Keywords: adaptive control; nonlinear systems; output feedback; power converters.

1 Introduction

The problem of asymptotic stabilization by means of output feedback of general nonlinear systems has been widely studied in the last years. Following the fundamental existence result presented in [19], several attempts have been made to construct explicit output feedback algorithms for classes of nonlinear systems. In particular, the class of systems that are linear in the unmeasured states has received special attention, see *e.g.* [14,4,5,8,7] and references therein. In [14] the class of systems in so-called parametric output feedback form has been studied and a class of stabilizing control laws has been proposed. In [4,5] a dynamic output feedback stabilization method has been proposed on the basis of the solution of two subproblems: a state feedback problem and an output injection problem. The solutions of such subproblems are then combined to construct an output feedback stabilizer. It must be noted that the results in [4,5] are not applicable in the presence of unknown parameters and rely strongly on the use of Lyapunov functions which are quadratic in the unmeasured states. In [8], see also [7, Chapter 7], a very special class of systems has been studied, namely systems described by equations of the form [1]

$$
\begin{aligned}
\dot{\eta} &= F(y)\eta + G(y) \\
\dot{\zeta} &= f(\zeta) + g(\zeta)\xi_1 \\
\dot{\xi}_1 &= \xi_2 + \phi_1(\xi_1, \zeta)\eta \\
&\vdots \\
\dot{\xi}_n &= \phi_0(y) + \phi_n(y)\eta + \phi_u(y)u \\
y &= \mathrm{col}(\zeta, \xi_1, \cdots, \xi_n)
\end{aligned}
\tag{1}
$$

and it is further assumed that the η subsystem is (uniformly) stable when $G(\cdot) = 0$, and (uniformly) bounded-input bounded-state stable if $G(\cdot)$ is regarded as an input. For this class of systems, and under some additional assumptions on the ζ subsystem, it is shown that global asymptotic output tracking with global boundedness can be obtained by means of a dynamic output feedback control law. The construction of this control law is based on arguments similar to those exploited in the adaptive stabilization schemes studied in detail in [12] and related works [2]. In [18], see also [10] for an example, a special subclass of the class of systems described by

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[1] Despite the use of a different notation, the class of systems studied in [8] coincides with the systems described by equations (1).

[2] In particular, it is obvious that the η subsystem of the system (1) may contain equations of the form $\dot{\eta}_i = 0$, *i.e.*

equations (1), but without the stability assumption on the η subsystem, has been studied. Therein it is shown that a dynamic output feedback control law achieving global asymptotic stability can be constructed using an iterative procedure similar to the one used in [8], and relying on the construction of an exponential observer. It is worth noting that in [18] the matrix $F(\cdot)$ has a special structure, namely it is the sum of a constant matrix in companion form and with all zero eigenvalues and a rank one matrix resulting from an output injection [3]. Finally, in the papers [15] and [11], the output feedback stabilization problem has been studied for a subclass of systems described by equations (1), where the matrix $F(\cdot)$ has a special triangular form.

Goal of this paper is to (partly) extend the results in [8] and [18]. In particular, we do not assume that the η subsystem of the system (1) is (uniformly) stable when $G(\cdot) = 0$, and (uniformly) bounded-input bounded-state stable if $G(\cdot)$ is regarded as an input, and we do not require the matrix $F(\cdot)$ to have a special form. However, a uniform stabilizability condition (see Assumption (A2) in Proposition 1) will be required. Note that this condition, which may seem restrictive, is a necessary condition for the existence of an output feedback stabilizer if the system is linear [4]. As a result, such an assumption may be regarded as a natural nonlinear counterpart of the necessary condition in the linear case. It must be noted that, while most of the results in the considered references rely on the construction of a Lyapunov function for the system in closed loop with the dynamic controller, the result presented in the paper does not require the knowledge of such a Lyapunov function. In fact, as it will be apparent from the proof of the main proposition in Section 3, in the present context, the closed loop controlled system can be written, using a special coordinates transformation, as the cascaded interconnection of two stable systems possessing specific input-output and asymptotic properties [5]. In this respect, this paper is inspired by the adaptive stabilization results recently presented in [3,2], see also [1] for an application. In turn, the results therein are based on ideas borrowed from the theory of the nonlinear regulator problem and the notions of system immersion and manifold invariance.

The rest of the paper is organized as follows. In Section 2 we introduce the class of systems studied in the paper, we briefly discuss some of their properties, and we formally pose the control problem addressed. In Section 3 we state and prove our main result, and we apply it to an academic example that cannot be dealt with with the results in [7]. In Section 4 we discuss possible extensions and derive a solution to the classical problem of a magnetic levitation system with position measurement. In Section 5 we show how the general theory can be used to design an adaptive output feedback stabilizing controller for a DC-to-DC boost converter and we test the proposed controller by experiments. Finally, in Section 6 we provide some conclusions.

2 Problem formulation

In what follows, to avoid unnecessary complex formulae and for clarity, we restrict (mainly) our interest to systems described by equations of the form

$$\begin{aligned} \dot{\eta} &= A(y,u)\eta + B(y,u) \\ \dot{y} &= \psi_0(y,u) + \psi_1(y,u)\eta, \end{aligned} \tag{2}$$

with state $(\eta, y) \in I\!R^n \times I\!R^p$, output y and control input $u \in I\!R^m$. Note that, with a proper definition of functions and mappings, the above system can be rewritten in the form of the system (1). However, unlike the system (1), the system (2) does not need to be affine in the input and its η subsystem may depend upon the (measurable) input signal u. To underline the difference from the stabilization results in [8,18] it is worth discussing briefly the properties of the zero dynamics of the underlying systems. A simple analysis (see also the discussion in [8]) reveals that the system (1) is weakly minimum phase, and the systems studied in [18] are (exponentially) minimum phase. On the contrary, systems described by equations of the form (2) are not necessarily minimum phase. Moreover, because of the presence of u in the η subsystem, the properties of the zero dynamics are not (in general) related to the properties of the matrix $A(\cdot)$, whereas for the system (1) the properties of the zero dynamics are related to the properties of the matrix $F(\cdot)$.

Along with the system (2) we consider a performance output ρ, defined as

$$\rho = h(y, \eta) \tag{3}$$

for some mapping $h(\cdot)$, and we pose the following regulation problem.

Problem (Output feedback regulation problem) *Consider the system (2) and the performance variable ρ defined in (3). Find (if possible) a dynamic output feedback control law described by equations of the form*

$$\dot{\hat{\eta}} = \pi(y, \hat{\eta}), \qquad u = \alpha(y, \hat{\eta}) \tag{4}$$

such that all trajectories of the closed loop system (2)-(4) are bounded and

$$\lim_{t\to\infty} \rho(t) = 0. \tag{5}$$

equations describing unknown parameters, hence some of the adaptive (state feedback) stabilization problems studied in [12] are special instances of the problem addressed in [8].

[3] Note that, whereas in the framework described in [8] the variable η may include both unmeasured states and unknown (constant) parameters, in the approach in [18] the variable η may not include unknown parameters.

[4] See [9] where this statement has been proved.

[5] Note, however, that the stability properties of each subsystem may be established using Lyapunov functions.

Remark 1 In general $\rho = 0$ does not imply $y = 0$ and $\eta = 0$. In fact, the performance variable ρ has been introduced because it is in general not possible to regulate the whole state (η, y) to zero (or to a desired equilibrium). This is the case, for example, if the vector η contains unknown parameters.

3 Main result

In this section we present the main result of the paper, namely we show how the problem of dynamic output feedback regulation for the system (2) can be solved.

Proposition 1 *Consider a system described by equations of the form (2) and a performance variable ρ defined as in (3). Suppose the following assumptions hold.*

(A1) There exists a full information control law

$$u^\star = \alpha(y, \eta) \tag{6}$$

such that all trajectories of the closed loop system (2)-(6) are bounded and are such that condition (5) holds. Moreover, the system (2) with $u = \alpha(y, \eta + d(t))$ is globally bounded-input bounded-state stable with respect to the input $d(t)$.

(A2) There exists a mapping $\beta(y)$ such that the system

$$\dot z = \left(A(y, u) - \frac{\partial \beta}{\partial y}\psi_1(y, u) \right) z \tag{7}$$

is uniformly globally stable for any y and u, and $z(t)$ is such that, for any fixed y and η,

$$\lim_{t\to\infty} [\alpha(y, \eta + z(t))] = \alpha(y, \eta). \tag{8}$$

Then there exists a dynamic output feedback control law, described by equations of the form (4), solving the output feedback regulation problem.

Remark 2 Proposition 1 states that it is possible to solve the considered output feedback regulation problem provided that two subproblems are solvable. The former is a *robust* full information stabilization problem, the latter is a problem of *robust* stabilization by output injection with a constraint on the Ω-limit set of the state z. Note, moreover, that the output injection gain has to possess a special form, *i.e.* it is the Jacobian matrix of the mapping $\beta(y)$.

Proof: To begin with, consider the system (2) and the dynamic output feedback controller

$$\dot{\hat\eta} = w, \qquad u = \alpha(y, M\hat\eta + \beta(y)), \tag{9}$$

where $\alpha(\cdot)$ is as in equation (6), M is an invertible matrix, $\beta(y) : \mathbb{R}^p \to \mathbb{R}^n$ is as in Assumption (A2) and w is a new control signal. Let

$$z = M\hat\eta - \eta + \beta(y) \tag{10}$$

and note that the closed loop system (2)-(9) can be written in the η, y and z coordinates as

$$\begin{aligned} \dot\eta &= A(y, \alpha(y, \eta+z))\eta + B(y, \alpha(y, \eta+z)) \\ \dot y &= \psi_0(y, \alpha(y, \eta+z)) + \psi_1(y, \alpha(y, \eta+z))\eta, \\ \dot z &= Mw - A(y, \alpha(y, \eta+z))\big(-z + M\hat\eta + \beta(y)\big) \\ &\quad - B(y, \alpha(y, \eta+z)) + \frac{\partial\beta}{\partial y}\Big[\psi_0(y, \alpha(y, \eta+z)) \\ &\quad + \psi_1(y, \alpha(y, \eta+z))\big(-z + M\hat\eta + \beta(y)\big)\Big]. \end{aligned} \tag{11}$$

It must be noted that, with the exception of the terms $A(y, \alpha(y, \eta+z))z$ and $\frac{\partial\beta}{\partial y}\psi_1(y, \alpha(y, \eta+z))z$, all terms in the $\dot z$ equation are measurable. Therefore, by invertibility of the matrix M, it is possible to choose a w yielding

$$\begin{aligned} \dot\eta &= A(y, \alpha(y, \eta+z))\eta + B(y, \alpha(y, \eta+z)) \\ \dot y &= \psi_0(y, \alpha(y, \eta+z)) + \psi_1(y, \alpha(y, \eta+z))\eta, \\ \dot z &= \left[A(y, \alpha(y, \eta+z)) - \frac{\partial\beta}{\partial y}\psi_1(y, \alpha(y, \eta+z))\right] z. \end{aligned} \tag{12}$$

As a result, by Assumption (A2), the variable z remains bounded for all t, and it is such that equation (8) holds. Hence, by Assumption (A1), y and η are bounded for all t and condition (5) holds, which proves the claim. □

Remark 3 Assumption (A2) can be replaced by the following stronger condition.

(A2′) There exists a mapping $\beta(y)$ such that the system (7) is uniformly globally stable for any y and u, and $z(t)$ is such that, for any y and u, $\lim_{t\to\infty} z(t) = 0$.

If assumption (A2′) holds, then $\hat\eta$ can be used to construct an asymptotic estimate of the unmeasured states η. However, the *reconstruction* of η is not necessary to achieve the desired control goal.

Remark 4 As should be apparent from the proof of Proposition 1, condition (A2) implies that it is possible to asymptotically reconstruct the full information control $u^\star$ achieving global regulation. Note also that the dynamic output feedback control law resulting from the application of Proposition 1 is not based on the certainty equivalence principle. In fact, the unmeasured state η in the full information control law is replaced in the output feedback control law by the term $M\hat\eta + \beta(y)$.

Remark 5 Although the result in Proposition 1 may seem conceptually similar to the result in [5], *i.e.* an output feedback controller is designed using the solutions of two subproblems, there are important differences. Firstly, in [5] the state feedback and output injection problems have to be solved by means of Lyapunov techniques, as the output feedback controller makes use of the Lyapunov functions associated with the above problems. Moreover, the Lyapunov functions have to be quadratic in the unmeasured states and have to satisfy some non-trivial coupling conditions. In the present context, it is not necessary to use Lyapunov techniques to solve the subproblems described in Assumptions (A1) and (A2). Moreover, even if one makes use of Lyapunov functions to establish the facts in Assumptions (A1) and (A2), such Lyapunov functions are not employed (or exploited) in the design of the output feedback stabilizers, hence do not have to possess special properties or to be related. Secondly, the output injection problem in [4] can be solved with any injection term, whereas the output injection problem in Assumption (A2) restricts the injection gain to be a Jacobian matrix.

Remark 6 The result in [18] is a special case of the result in Proposition 1. In fact, using a simple backstepping construction it is possible to design a full information controller such that Assumption (A1) holds. Moreover, Assumption (A2) can be easily satisfied, as shown also in [18], *i.e.* there exists a mapping $\beta(y)$ such that the matrix $A(y) - \frac{\partial\beta}{\partial y}\psi_1(y)$ is constant and Hurwitz [6].

We conclude this section with a simple example, which has been adapted from [7, Chapter 7].

Example 1 Consider a three-dimensional system described by equations of the form

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= x_3 + x_2^2 \qquad\qquad y = \mathrm{col}(x_2, x_3) \\ \dot{x}_3 &= x_1 x_3^2 + x_1 + u \end{aligned} \tag{13}$$

Note that the zero dynamics of the system (13) are described by $\dot{x}_1 = x_1$, hence the system is not minimum phase and cannot be stabilized using the method in [7]. On the contrary, it is now shown that using the result in Proposition 1 it is possible to achieve asymptotic stabilization by dynamic output feedback [7]. For, note that a simple application of a backstepping procedure shows that there exists a full information control law such that Assumption (A1) holds. To verify Assumption (A2), note that the matrix $A(y,u) - \frac{\partial\beta}{\partial y}\psi_1(y,u)$ in equation (7) reduces to $1 - \frac{\partial\beta}{\partial x_3}(1+x_3^2)$. Hence, the selection $\beta(y) = k\arctan x_3$ with $k > 1$ is such that the system (7) in Assumption (A2) reduces to $\dot{z} = -(k-1)z$, which is uniformly, globally, asymptotically stable.

4 Further results and examples

In this section we discuss a possible way of extending the applicability of the proposed method to a more general class of systems, namely to systems that are not necessarily linear in the unmeasured states. To this end, and to avoid unnecessary cumbersome notation, we restrict our attention to systems described by equations of the form

$$\begin{aligned} \dot{\eta} &= A(y,u)\eta + B(y,u) \\ \dot{y} &= \psi_0(y,u) + \psi_1(y,u)\eta + \psi_2(y,u)\eta^2 \end{aligned} \tag{14}$$

[6] In [18] $A(\cdot)$ and $\psi_1(\cdot)$ do not depend on u.

[7] In this case $\rho = \mathrm{col}(x_1, x_2, x_3)$.

with $\eta \in I\!R$, *i.e.* there is only one unmeasured state, $y \in I\!R^p$ and $u \in I\!R^m$. Similar considerations can be carried out for the case $\eta \in I\!R^n$. The basic idea is to make use of overparameterization to overcome the nonlinear parameterization with respect to η. This approach has been widely used in adaptive control and has been recently used, in the framework of control design for robotic manipulators, in [13].

Proposition 2 *Consider a system described by equations of the form (14) and a performance variable ρ defined as in (3). Suppose the following assumptions hold.*

(B1) There exists a full information control law

$$u^\star = \alpha(y, \eta, \eta^2) \tag{15}$$

such that all trajectories of the closed loop system (14)-(15) are bounded and are such that condition (5) holds. Moreover, the system (2) with $u = \alpha(y, \eta + d_1(t), \eta^2 + d_2(t))$ is globally bounded-input bounded-state stable with respect to the inputs $d_1(t)$ and $d_2(t)$.

(B2) There exists a mapping $\beta(y)$ such that the system

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \left(\begin{bmatrix} A(y,u) & 0 \\ 2B(y,u) & 2A(y,u) \end{bmatrix} - \frac{\partial\beta}{\partial y}\begin{bmatrix} \psi_1(y,u) & \psi_2(y,u) \end{bmatrix} \right) z \tag{16}$$

is uniformly globally stable for any y and u, and $z(t)$ is such that, for any fixed y and η,

$$\lim_{t\to\infty} \left[\alpha(y, \eta + z_1(t), \eta^2 + z_2(t))\right] = \alpha(y, \eta, \eta^2). \tag{17}$$

Then there exists a dynamic output feedback control law, described by equations of the form (4), solving the output feedback regulation problem.

The proof of Proposition 2 is similar to the one of Proposition 1, hence it is omitted. To illustrate the application of the proposition, we discuss two examples. The first is a modification of Example 1, while the second refers to the classical control problem of the magnetically levitated ball with position measurement.

Example 2 If the second equation of the system (13) is replaced with $\dot{x}_2 = x_3 + x_1^2$, then the system becomes quadratic in the unmeasured state x_1, hence Proposition 1 is not applicable. However, asymptotic stabilization can be achieved by means of Proposition 2. For, note that Assumption (B1) can be satisfied using backstepping. To verify Assumption (B2), note that the system (16) reduces to

$$\dot{z} = \begin{bmatrix} 1 - \dfrac{\partial\beta_1}{\partial x_3}(1+x_3^2) & -\dfrac{\partial\beta_1}{\partial x_2} \\ 2x_2 - \dfrac{\partial\beta_2}{\partial x_3}(1+x_3^2) & 2 - \dfrac{\partial\beta_2}{\partial x_2} \end{bmatrix} z, \tag{18}$$

where $\beta_1(y)$ (respectively, $\beta_2(y)$) is the first (respectively, second) component of the vector $\beta(y)$. Hence, the selection $\beta_1(y) = x_2^2 + k \arctan x_3$ and $\beta_2(y) = 2kx_2$ with $k > 1$ yields

$$\dot{z} = -\begin{bmatrix} k-1 & 2x_2 \\ -2x_2 & 2(k-1) \end{bmatrix} z, \tag{19}$$

which is uniformly, globally, asymptotically stable.

Example 3 Consider a magnetic levitation system consisting of an iron ball in a vertical magnetic field created by a single electromagnet. We assume that the electromagnet has inductance L, resistance R and is supplied by a voltage source $u(t)$. The problem is to regulate the ball to a nominal position by controlling the voltage. The model of the system is given by the equations

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{\phi(x_1)}{m} x_3^2 - g, \quad \dot{x}_3 = -\frac{R}{L} x_3 + \frac{1}{L} u, \tag{20}$$

where x_1 is the (upward) displacement of the ball from the nominal position, x_2 is the velocity of the ball, x_3 is the current through the electromagnet, m is the mass of the ball, g is the acceleration of gravity, and $\phi(x_1)$ is a known strictly positive function with well-defined integral. We assume that only the state x_1 is available for measurement, *i.e.* $y = x_1$. Following the idea of Proposition 2, we define the vector $\eta = \mathrm{col}(x_2, x_3, x_3^2)$ and rewrite the system (20) as

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{\phi(y)}{m} \\ 0 & -\frac{R}{L} & 0 \\ 0 & \frac{2}{L}u & -\frac{2R}{L} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} + \begin{bmatrix} -g \\ \frac{u}{L} \\ 0 \end{bmatrix}, \quad \dot{y} = \eta_1. \tag{21}$$

Using one of several methods[8], it can be shown that there exists a full information control law such that Assumption (A1) of Proposition 1 holds. To verify Assumption (A2), let $\beta(y) = [\ \lambda y,\ 0,\ \frac{1}{m}\int \phi(y) dy]^T$, where λ is a positive constant. Then equation (7) is written as

$$\dot{z} = \begin{bmatrix} -\lambda & 0 & \frac{\phi(y)}{m} \\ 0 & -\frac{R}{L} & 0 \\ -\frac{\phi(y)}{m} & \frac{2}{L}u & -\frac{2R}{L} \end{bmatrix} z. \tag{22}$$

Assume that u is uniformly bounded, *i.e.* there exists a positive constant k such that $|u(t)| < k$, for all t. Then the system (22) is uniformly, globally, asymptotically stable with the Lyapunov function $\frac{1}{2}z_1^2 + \frac{k^2}{2R^2}z_2^2 + \frac{1}{2}z_3^2$. We summarize these results in the following proposition.

Proposition 3 *Consider the magnetic levitation system described by equations (20) and the performance variable $\rho = x_1$. Suppose that there exists a control law $\alpha(x_1, x_2, x_3, x_3^2)$ satisfying Assumption (A1) of Proposition 1 and such that $|\alpha(\cdot)| \leq k$, for some $k > 0$. Then there exists a dynamic output feedback control law, described by equations of the form (4), solving the considered output feedback regulation problem.*

[8] See [20] for a comparative study.

5 Adaptive output feedback control of a boost converter

In this section we consider the problem of stabilization of a DC-to-DC boost converter composed of the interconnection of an inductance, a switch, a capacitor and a constant load, see [16] for further detail. The averaged model of the system is described by equations of the form

$$\dot{x}_1 = -\frac{1}{L} u x_2 + \frac{E}{L}, \qquad \dot{x}_2 = -\frac{1}{RC} x_2 + \frac{1}{C} u x_1, \tag{23}$$

in which x_1 represents the input (inductor) current, x_2 represents the output (capacitor) voltage, u is a continuous control signal representing the slew rate of the PWM circuit controlling the switch, and C, L, R and E are positive constants. We assume that only the state x_2 is available for feedback, the constant E is unknown, and $u \in [\epsilon, 1]$, with $0 < \epsilon < 1$. We also assume that the state vector lives in the positive orthant, *i.e.* $x_i(t) \geq 0$, for all t. The control problem is to regulate the voltage across the capacitor to a positive value $x_2^\star$ using an output feedback control law.

Following the methodology proposed in Sections 2 and 3, we rewrite the system as

$$\begin{aligned} \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L} u y \end{bmatrix} \\ \dot{y} &= -\frac{1}{RC} y + \frac{1}{C} u \eta_2, \end{aligned} \tag{24}$$

where η_1 denotes the unknown parameter E, η_2 the unmeasured state x_1, and y the measurable output x_2, and define the performance variable $\rho = x_2 - x_2^\star$. The η subsystem is not stable, hence the results in [7] are not applicable, however the general result in Proposition 1 applies, as illustrated in the following statement.

Proposition 4 *Consider the DC-to-DC boost converter described by equations (24) and the performance variable $\rho = y - x_2^\star$. Then for any $0 < \epsilon < 1$ and any $x_2^\star$ such that*

$$\epsilon \leq \frac{E}{x_2^\star} \leq 1, \tag{25}$$

there exists a dynamic output feedback control law, described by equations of the form (4), solving the output feedback regulation problem. Moreover, along all trajectories of the closed loop system, $u(t) \in [\epsilon, 1]$ for all t.

Proof: To begin with, note that the constraint (25) is a consequence of the control constraint $u \in [\epsilon, 1]$, *i.e.* if $u \in [\epsilon, 1]$ then all achievable equilibria of the system (24) are such that (25) holds. Consider now the full information control law [9]

$$u^\star = \mathrm{sat}_{[\epsilon,1]}\left(\frac{\eta_1}{x_2^\star}\right). \tag{26}$$

It has been shown in [16] that the considered full information control law yields asymptotic regulation. Moreover, using the Lyapunov function $L\eta_2^2 + Cy^2$ it is easy to show that, for any $u \in [\epsilon, 1]$, the trajectories of the system (24) are bounded. Hence, Assumption (A1) of Proposition 1 holds. Consider now Assumption (A2). Let $\beta(y) = [\,\lambda_1 y,\ \lambda_2 y\,]^T$ with $\lambda_1 > 0$ and $\lambda_2 > 0$. Following the construction of the proof of Proposition 1, we define the dynamic output feedback controller

$$\begin{aligned}
\dot{\hat\eta}_1 &= -\frac{\lambda_1}{C}\left(u\left(\hat\eta_2 + \lambda_2 y\right) - \frac{1}{R}y\right)\\
\dot{\hat\eta}_2 &= -\frac{\lambda_2}{C}\left(u\left(\hat\eta_2 + \lambda_2 y\right) - \frac{1}{R}y\right) + \frac{1}{L}\left(\hat\eta_1 + \lambda_1 y - uy\right)\\
u &= \mathrm{sat}_{[\epsilon,1]}\left(\frac{\hat\eta_1 + \lambda_1 y}{x_2^\star}\right).
\end{aligned} \tag{27}$$

Note that the system (7) reduces to

$$\dot z = \begin{bmatrix} 0 & -\dfrac{\lambda_1}{C}u \\ \dfrac{1}{L} & -\dfrac{\lambda_2}{C}u \end{bmatrix} z \tag{28}$$

with $u \in [\epsilon, 1]$. To complete the proof, it is necessary to show that, for any $0 < \epsilon < 1$, it is possible to select $\lambda_1 > 0$ and $\lambda_2 > 0$ such that the system (28) is uniformly asymptotically stable. To this end, let $\bar u = \frac{1+\epsilon}{2}$ and rewrite the system (28) as

$$\dot z = \begin{bmatrix} 0 & -\dfrac{\lambda_1}{C}\bar u \\ \dfrac{1}{L} & -\dfrac{\lambda_2}{C}\bar u \end{bmatrix} z + \begin{bmatrix} \dfrac{\lambda_1}{C}\bar u \\ \dfrac{\lambda_2}{C}\bar u \end{bmatrix} w \tag{29}$$

with $w = \frac{\delta u}{\bar u} r$, $\delta u = \bar u - u$, and $r = [\,0,\ 1\,]z$, *i.e.* as a linear system, which is asymptotically stable (since $\lambda_1 > 0$ and $\lambda_2 > 0$), in closed loop with a time-varying gain $\delta u/\bar u$, which is such that $|\frac{\delta u}{\bar u}| \le \frac{1-\epsilon}{1+\epsilon}$. Note now that the H_∞ norm of the system (29) with output r is given by

$$\gamma(\mu) = \frac{1}{\sqrt{1 + 2\mu^2\sqrt{1 + 2/\mu} - 2\mu^2 - 2\mu}}, \tag{30}$$

where $\mu = \frac{\lambda_1}{\lambda_2^2}\frac{C}{L\bar u}$. Hence, a simple application of the small gain theorem [10] shows that the system (28) is asymptotically stable provided $\gamma(\mu)\frac{1-\epsilon}{1+\epsilon} < 1$. Since $\gamma(\mu) \ge 1$ for any $\mu \ge 0$, this condition is equivalent to

$$\epsilon > \frac{\gamma(\mu) - 1}{\gamma(\mu) + 1}. \tag{31}$$

Note now that the function on the right-hand side of (31) is zero for $\mu = 0$ and it is monotonically increasing and tends to one for $\mu \to \infty$, hence for any $\epsilon > 0$ it is always possible to select $\mu > 0$, thus $\lambda_1 > 0$ and $\lambda_2 > 0$, such that condition (31) holds. This concludes the proof. $\square$

Remark 7 Condition (31) can be read in two directions. Given $\lambda_1 > 0$ and $\lambda_2 > 0$ it is possible to compute a set of values for ϵ for which the system (28) is uniformly asymptotically stable. Given $\epsilon > 0$ it is possible to compute $\lambda_1 > 0$ and $\lambda_2 > 0$ such that the system (28) is uniformly asymptotically stable. Note that for any fixed ϵ there are infinite pairs $\{\lambda_1, \lambda_2\}$ such that condition (31) holds, and these can be easily parameterized. This degree of freedom can be exploited to improve performance.

Remark 8 As observed in Remark 3, as the system (28) is uniformly, globally, asymptotically stable, it is not only possible to achieve regulation by output feedback for the boost converter (23), but it is also possible to estimate the state x_1 and the unknown parameter E.

Laboratory experiments have been carried out on the DC-to-DC boost converter described in detail in [6]. The physical parameters of the converter are: $C = 94\mu$F, $L = 1.36m$H and $R = 120\Omega$. The switching frequency of the PWM is $20k$Hz. The desired output voltage is $x_2^\star = 15$V. The control lower bound ϵ has been fixed to 0.1 and $\lambda_1 = 0.2$ and $\lambda_2 = 1$ have been set to satisfy condition (31). The first experiment comprises a change in the input voltage E from 10V to 7V, at $t = 0.02$s. Figure 1 shows the inductor current x_1, together with its estimate $\hat x_1$, the output voltage x_2, the control signal u and the input voltage estimate $\hat E$. We see that the output voltage tracks the desired value, despite partial

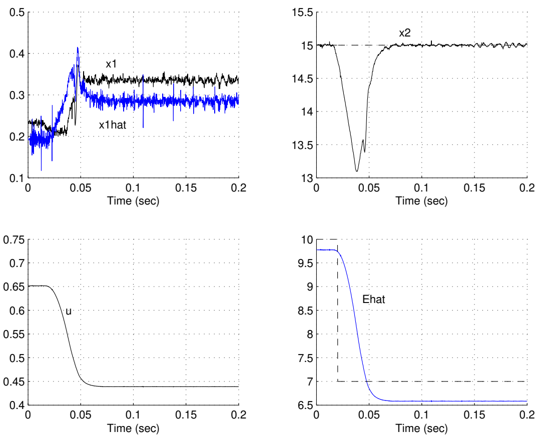

Fig. 1. State and control histories for a change in the input voltage from $E = 10$V to $E = 7$V.

[9] The saturation function is not required in the definition of $u^\star$ as, by condition (25), the argument of the saturation is always in the interval $[\epsilon, 1]$, however it is required in the construction of the output feedback controller.

[10] See [17, Proposition 3.4.7].

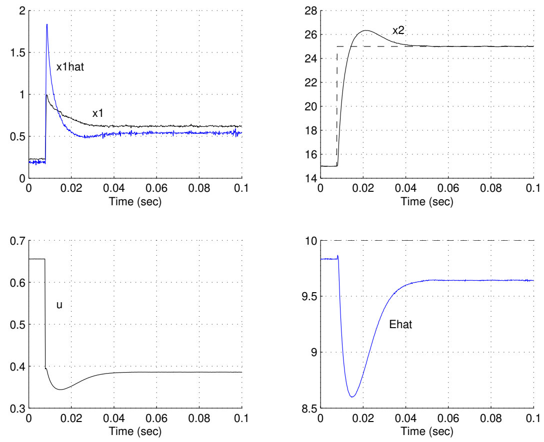

Fig. 2. State and control histories for a change in the output reference from $x_2^\star = 15$V to $x_2^\star = 25$V.

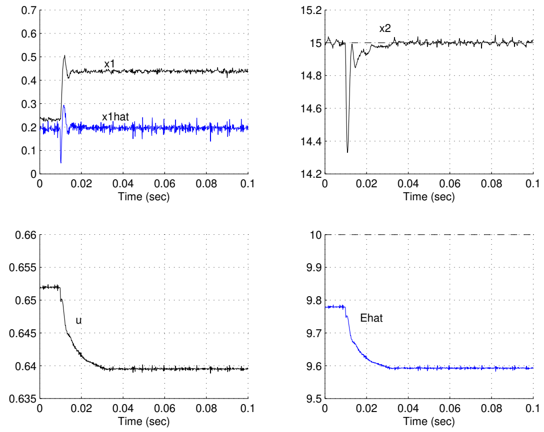

Fig. 3. State and control histories for a change in the load from $R = 120\Omega$ to $R = 60\Omega$.

state measurement and the change in the input voltage. In addition, the inductor current and the input voltage estimates approach the true values with a static error, which is due to unmodeled parasitic elements. Note that, even in the presence of this error, the (static) error in the output voltage is zero. This very useful property is due to the inherent integral action in the controller (27) [11]. Figure 2 shows the response of the system to a change in the output voltage reference $x_2^\star$ from 15V to 25V, at $t = 0.01$s, when $E = 10$V. We see that the output voltage tracks the desired reference value with a small overshoot.

In contrast with the full information control law (26), the controller (27) requires knowledge of the parameter R. This is a potential drawback, since in most applications the load is not known precisely. Surprisingly, it can be shown that it is not necessary to know the precise value of R, but any constant estimate (within a certain range) can be used in the implementation of the control law (27), *i.e.* output voltage tracking is (locally) achieved despite the uncertainty in the parameter R. To prove this fact, consider again the system (24) in closed loop with the adaptive output feedback controller (27), where R is replaced by a constant estimate $\hat{R}$. The system (7) is now written as

$$\dot{z} = \begin{bmatrix} 0 & -\dfrac{\lambda_1}{C}u \\ \dfrac{1}{L} & -\dfrac{\lambda_2}{C}u \end{bmatrix} z + \begin{bmatrix} \dfrac{\lambda_1}{C}\tilde{G} \\ \dfrac{\lambda_2}{C}\tilde{G} \end{bmatrix} x_2, \qquad (32)$$

where $\tilde{G} = 1/\hat{R} - 1/R$. A straightforward computation shows that, for sufficiently small $\tilde{G}$, the linearization of the closed loop system (23)-(32) at the equilibrium $(x_1, x_2, z_1, z_2) = (\frac{x_2^{\star 2}}{RE}, x_2^\star, 0, \frac{\tilde{G}x_2^{\star 2}}{E})$ is asymptotically stable. For example, assuming $\hat{R} = 120\Omega$ and substituting the initial values for E, L, C, $x_2^\star$, λ_1, λ_2, we find that the eigenvalues of the Jacobian matrix remain strictly in the left half-plane for any R that satisfies $0.5\Omega \leq R \leq 8686\Omega$. Hence, for any load R in this range, the closed loop system is locally asymptotically stable. Figure 3 shows the response of the system to a step change in the load R from 120Ω to 60Ω, at $t = 0.01$s, for $E = 10$V and $x_2^\star = 15$V. We observe that the estimate of E is not affected significantly; as a result, the output voltage is almost insensitive to load changes.

[11] Note that from (27), ignoring the saturation, we obtain $\dot{\hat{\eta}}_1 - \dfrac{\lambda_1}{\lambda_2}\dot{\hat{\eta}}_2 = \dfrac{\lambda_1(\hat{\eta}_1 + \lambda_1 y)}{\lambda_2 L x_2^\star}(y - x_2^\star)$.

6 Conclusions and outlook

The problem of adaptive output feedback stabilization for a class of nonlinear systems has been discussed. It has been shown that, with proper definitions of the *estimation error* and of the *parameter error*, it is possible to relax some of the assumptions in [8] and to recover, as a particular case, the results in [18]. The proposed method, which relies on the recently developed adaptive control tools described in [2,3], does not require the knowledge of a Lyapunov function for the closed loop system and is not restricted to minimum phase systems. The method has been used to solve the robust tracking problem for a DC-to-DC boost converter with partial parameter and state information. The theoretical results have been verified via experiments.

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