6: Window Filter Design

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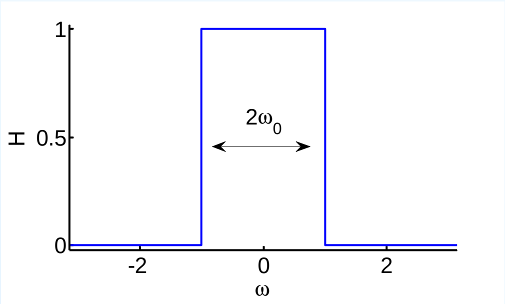

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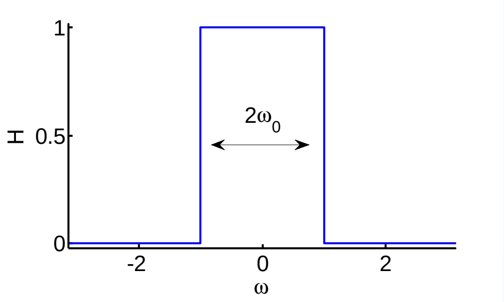

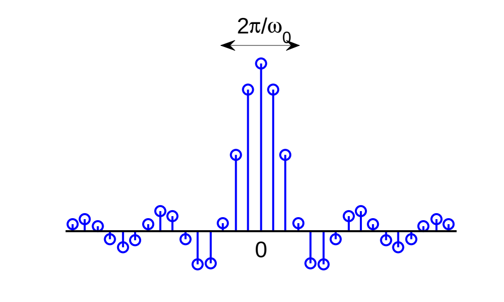

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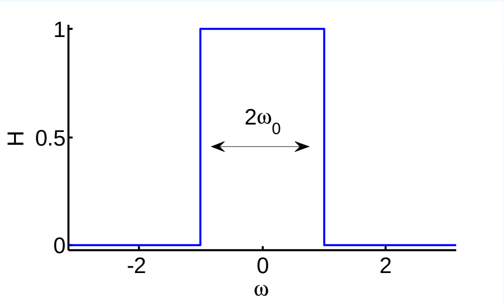

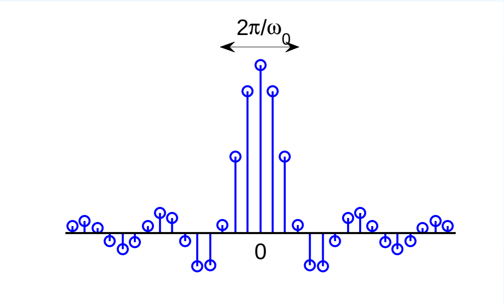

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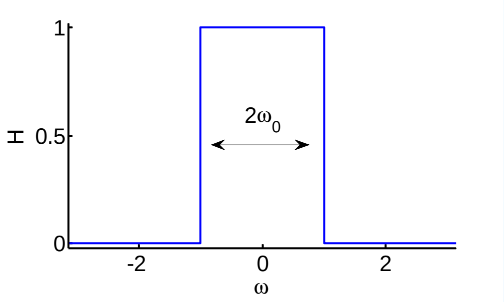

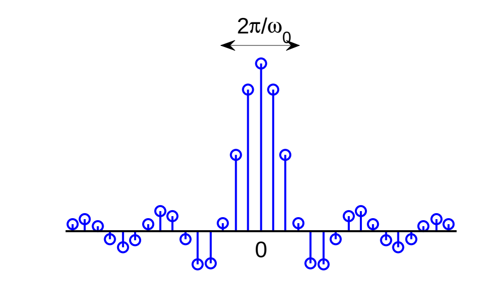

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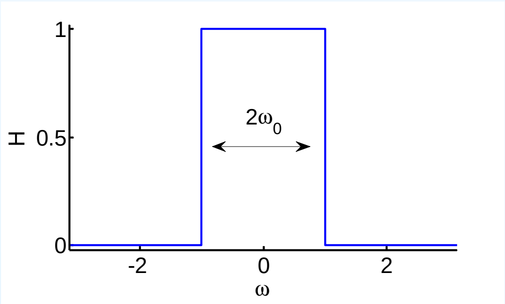

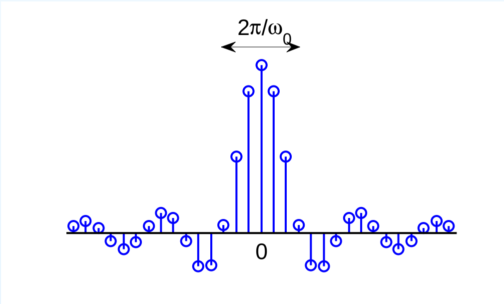

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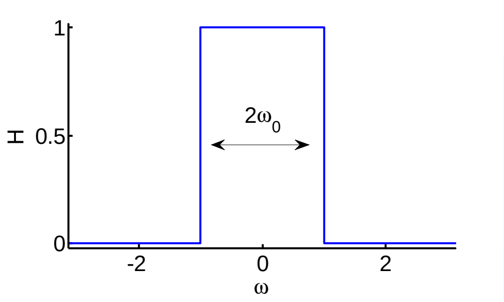

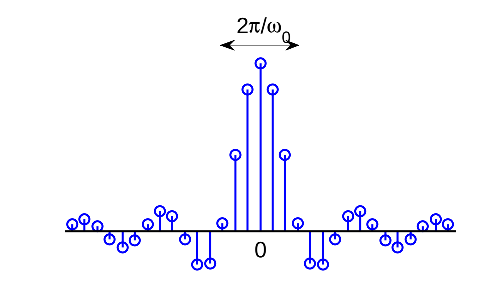

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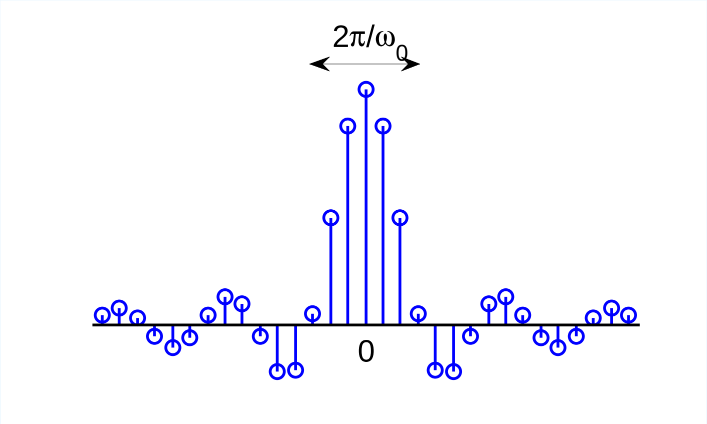
Sadly $h[n]$ is infinite and non-causal. Solution: multiply $h[n]$ by a window

Rectangular window

Truncate to $\pm\frac{M}{2}$ to make finite



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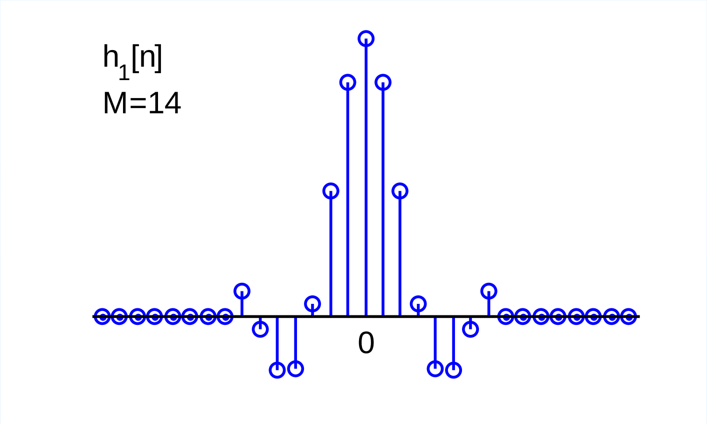

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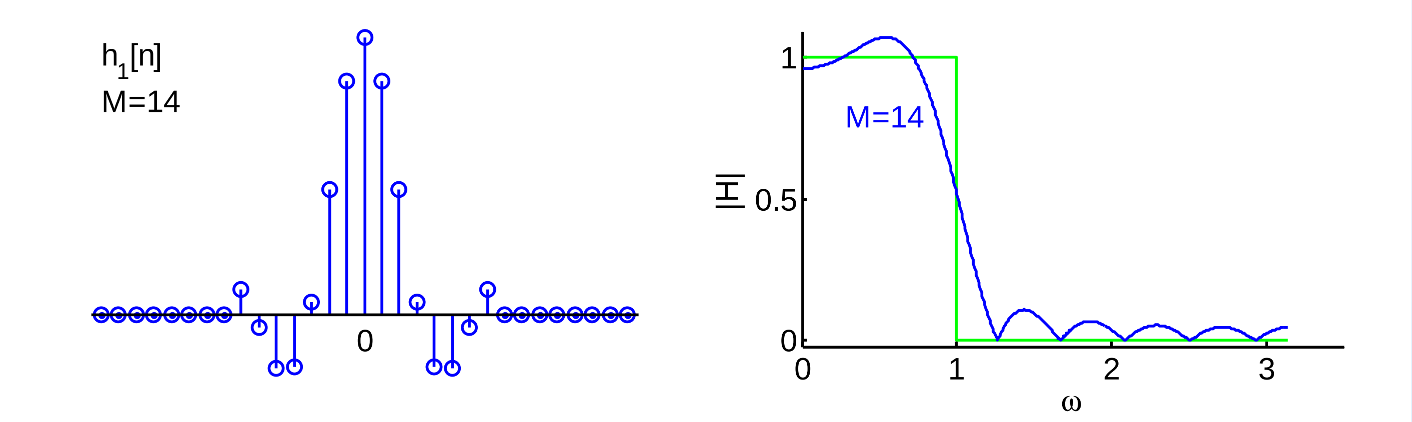
Define mean square error (MSE) in frequency domain

$$E = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega}) - H_1(e^{j\omega})\right|^2 d\omega$$



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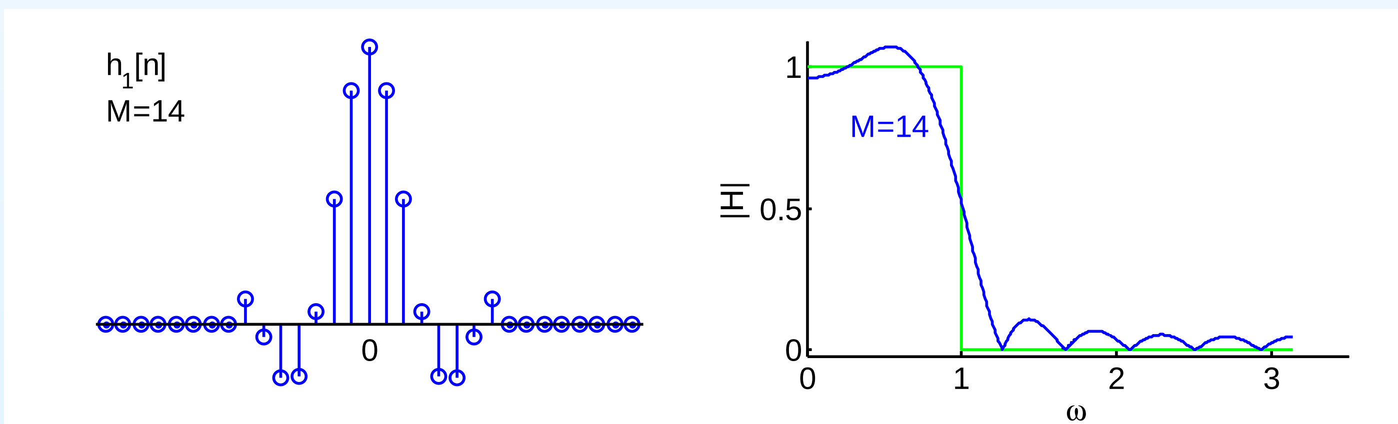

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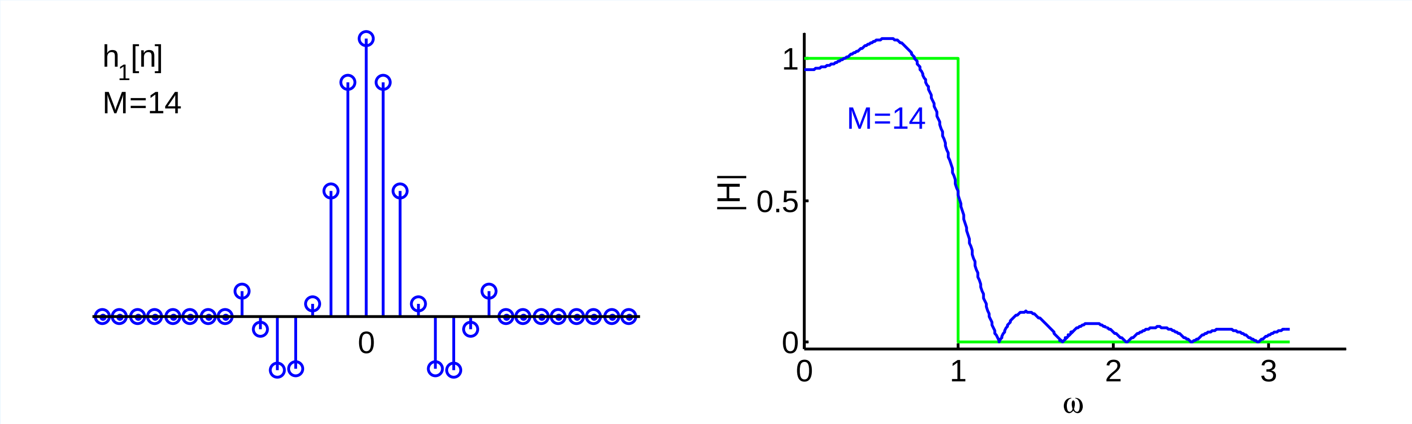

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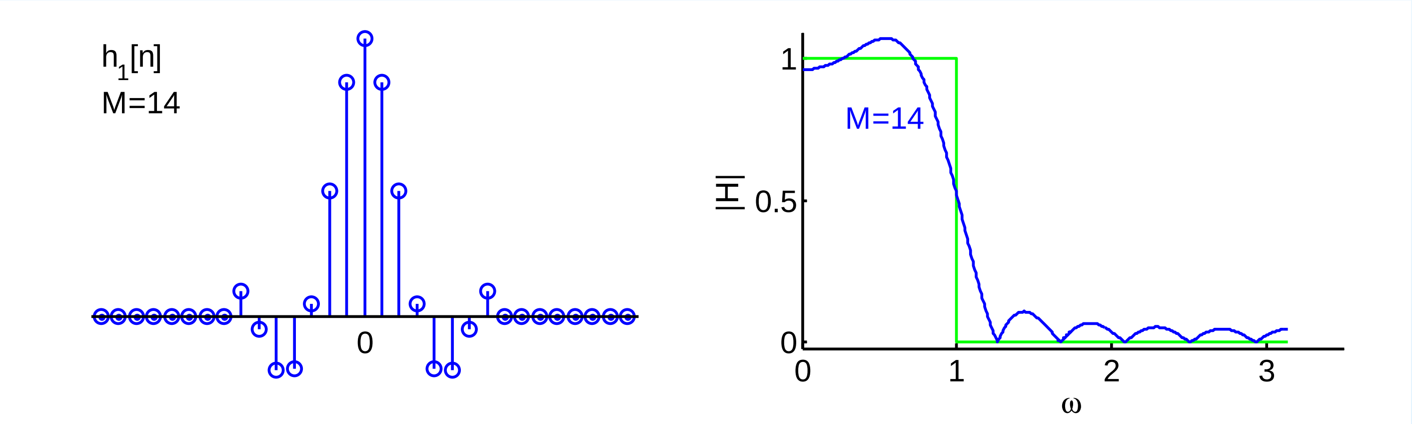

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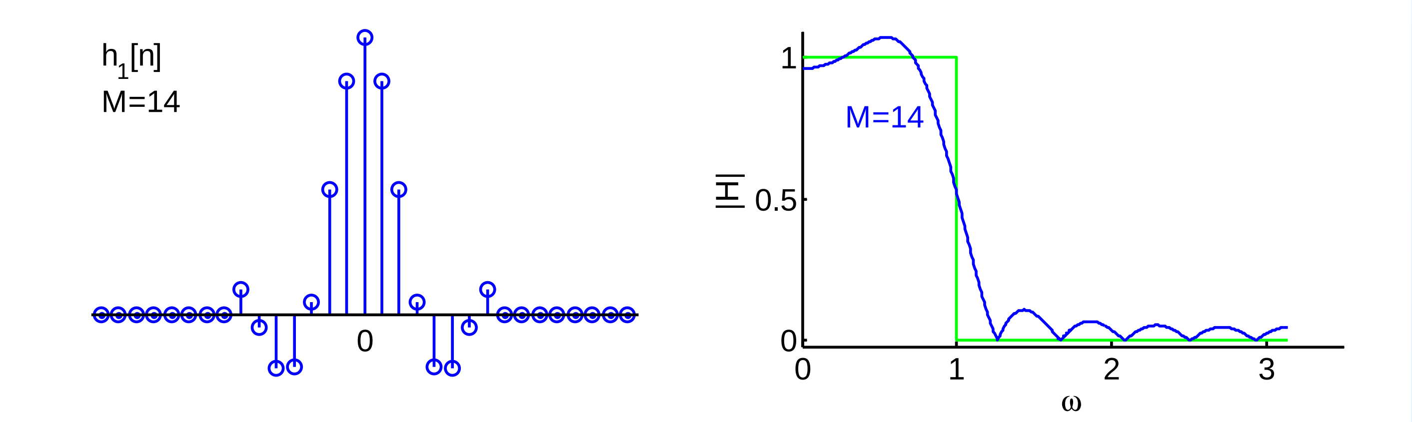

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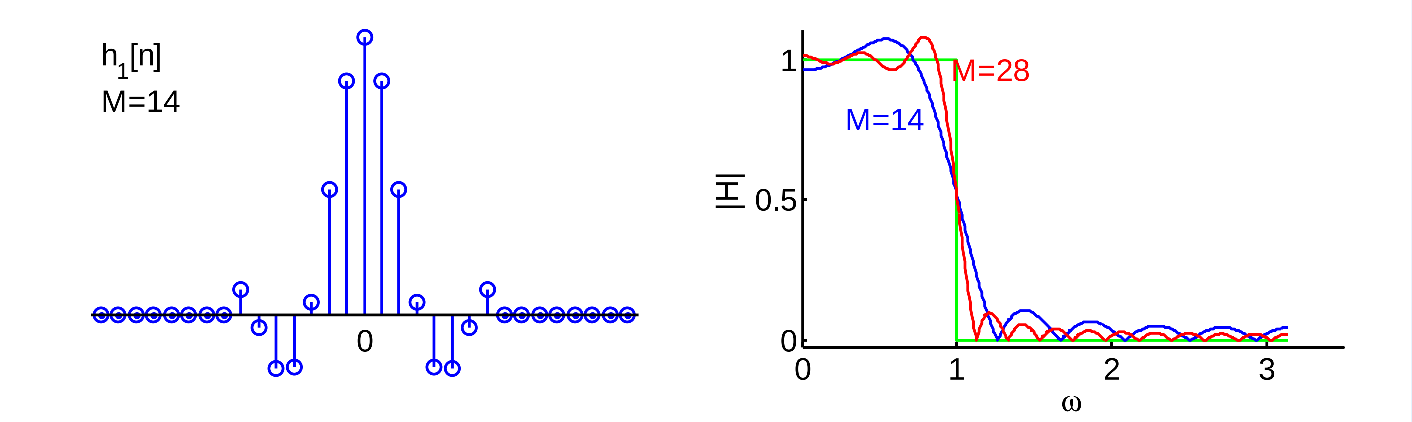

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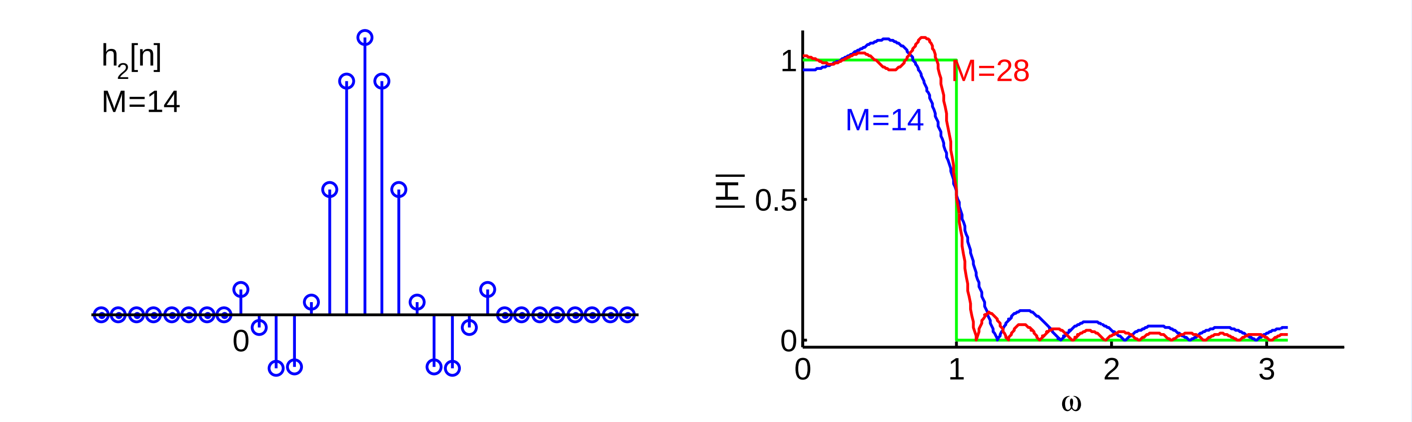

Normal to delay by $\frac{M}{2}$ to make causal. Multiplies $H(e^{j\omega})$ by $e^{-j\frac{M}{2}\omega}$.

Dirichlet Kernel

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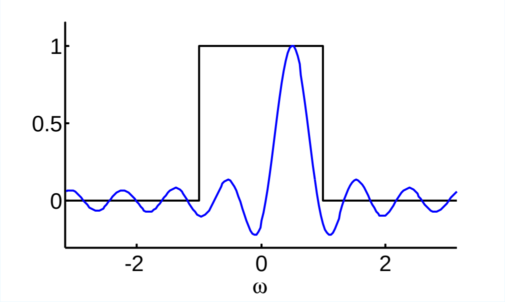

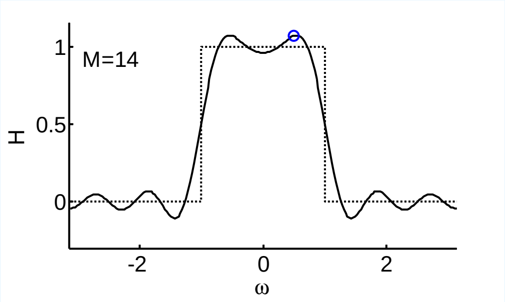

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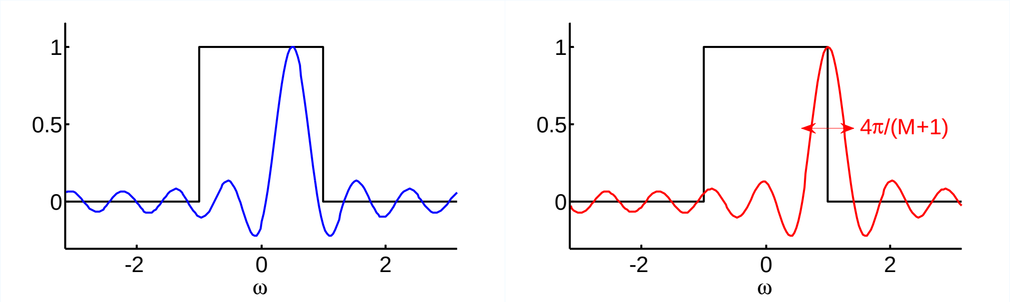

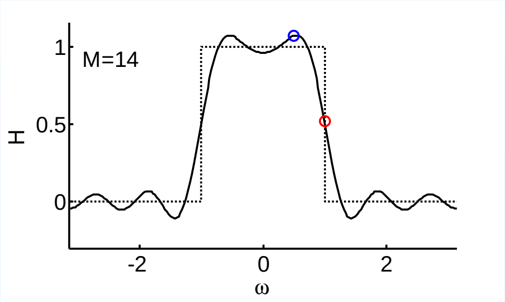

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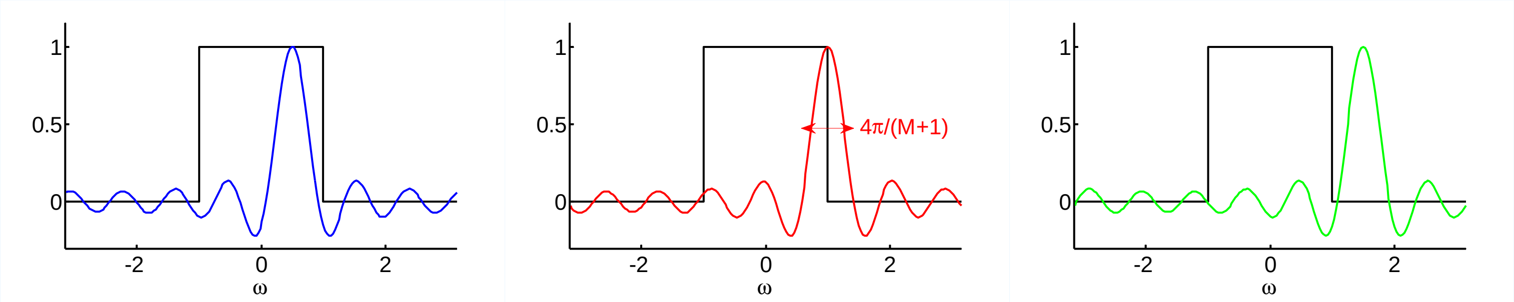

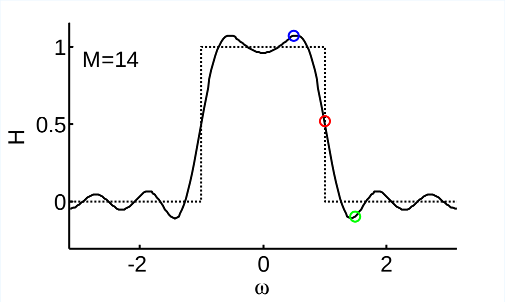

Dirichlet Kernel

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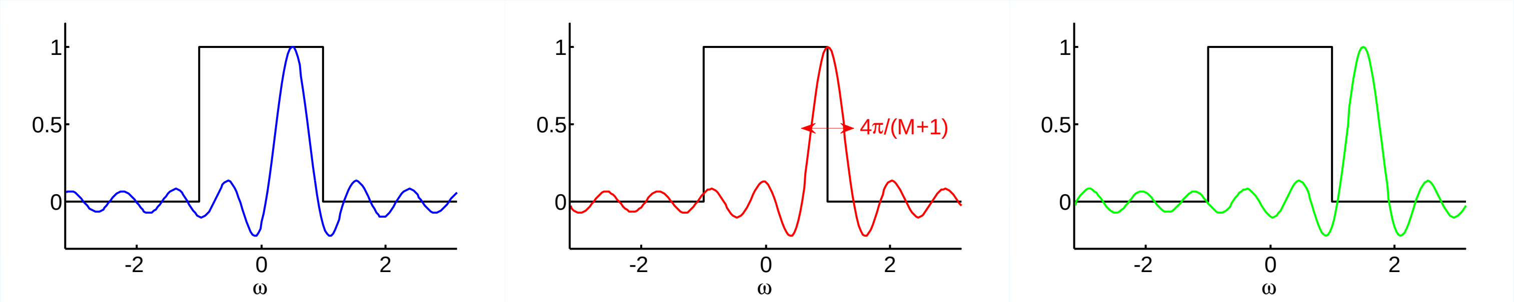

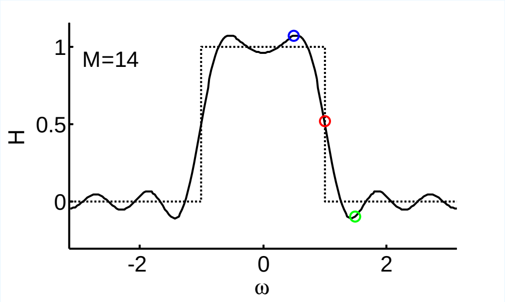

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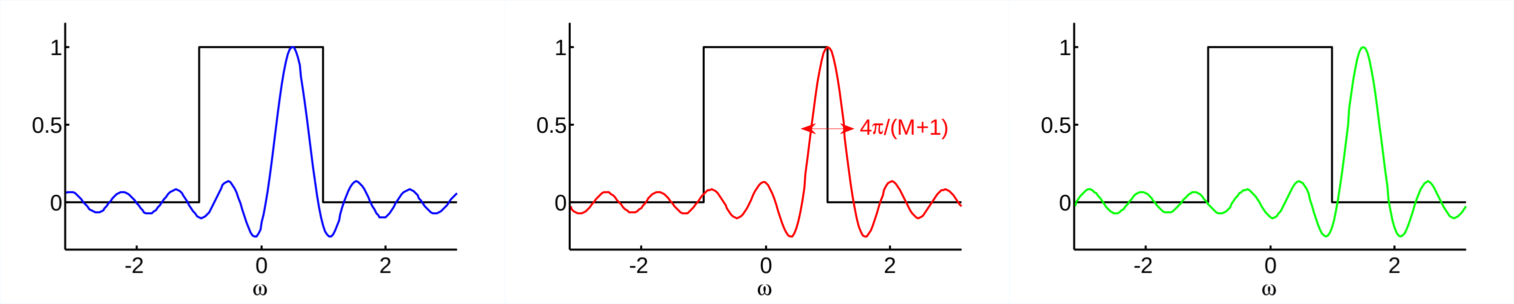

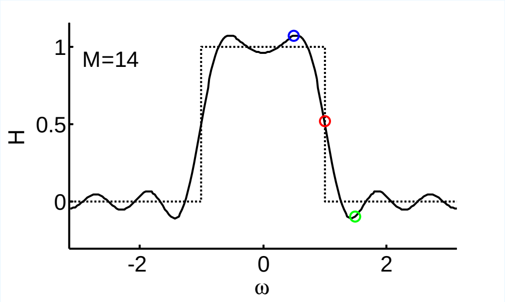

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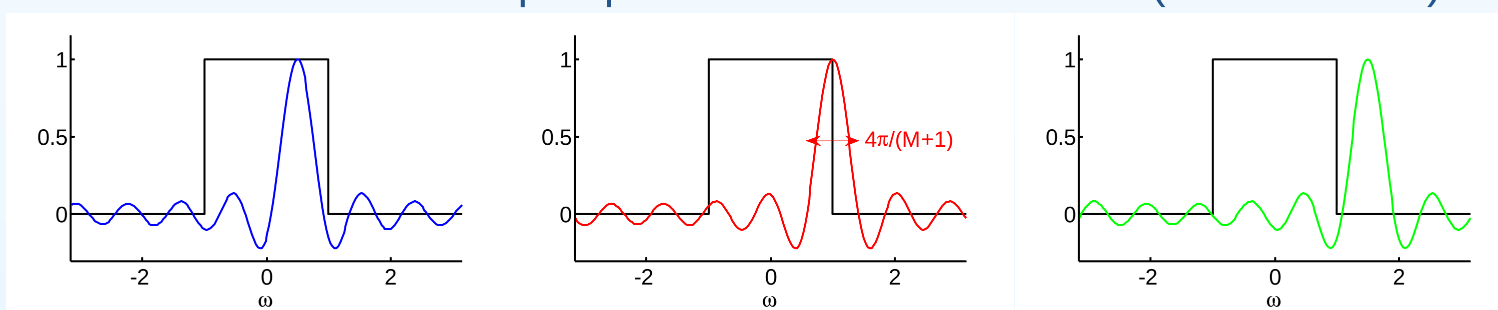

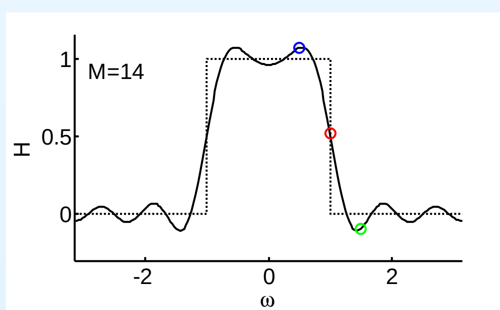

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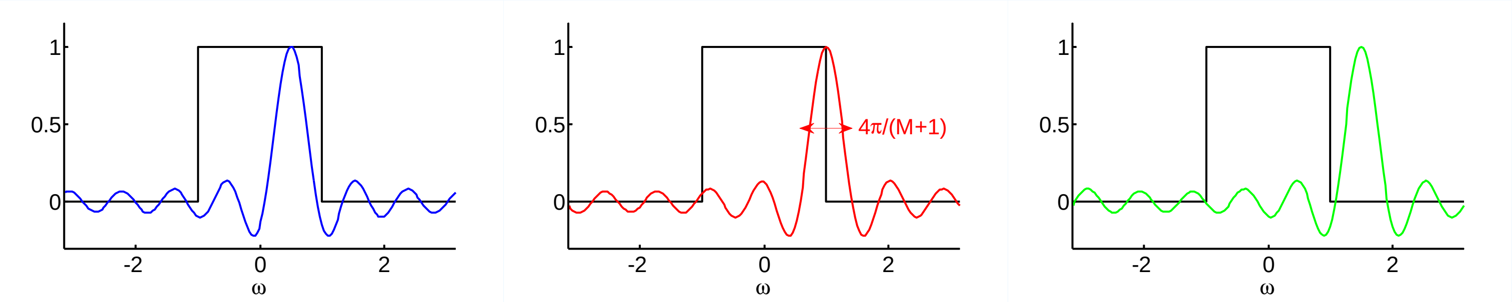

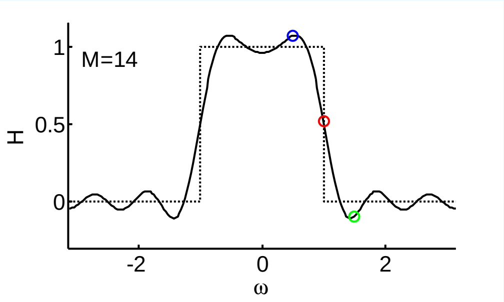

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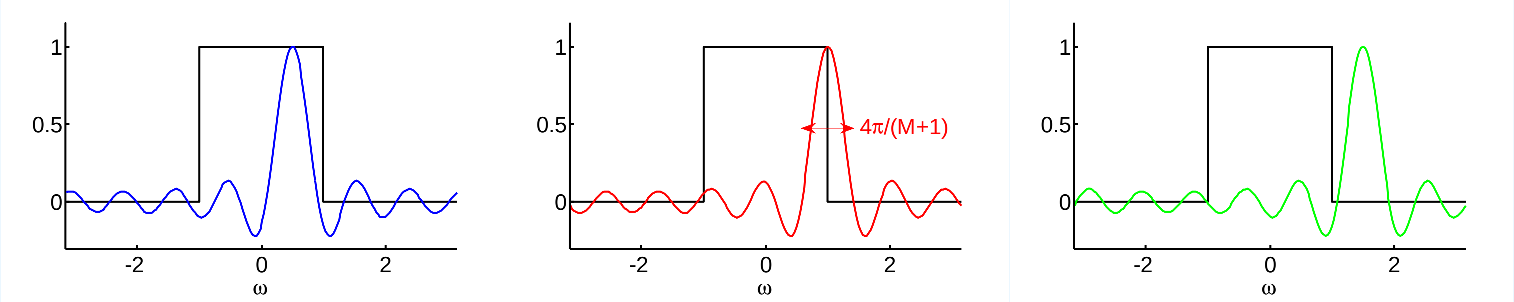

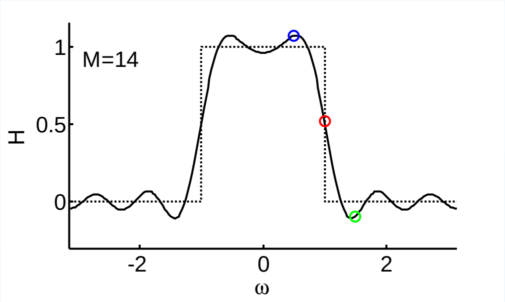

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Window relationships

When you multiply an impulse response by a window $M+1$ long

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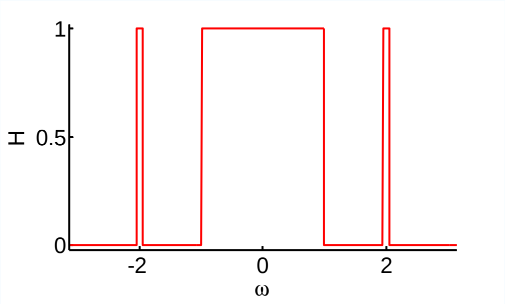

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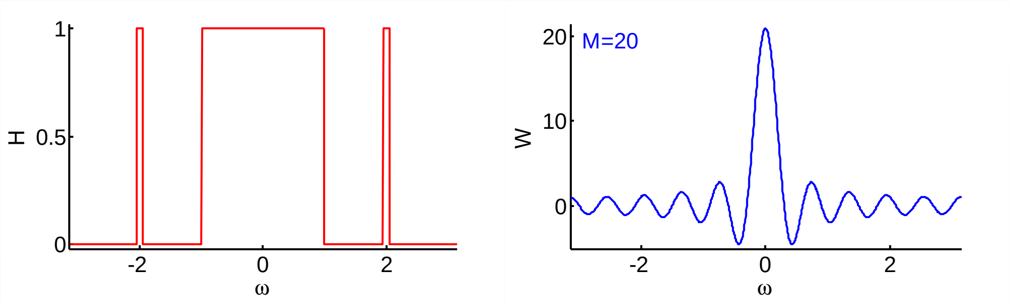

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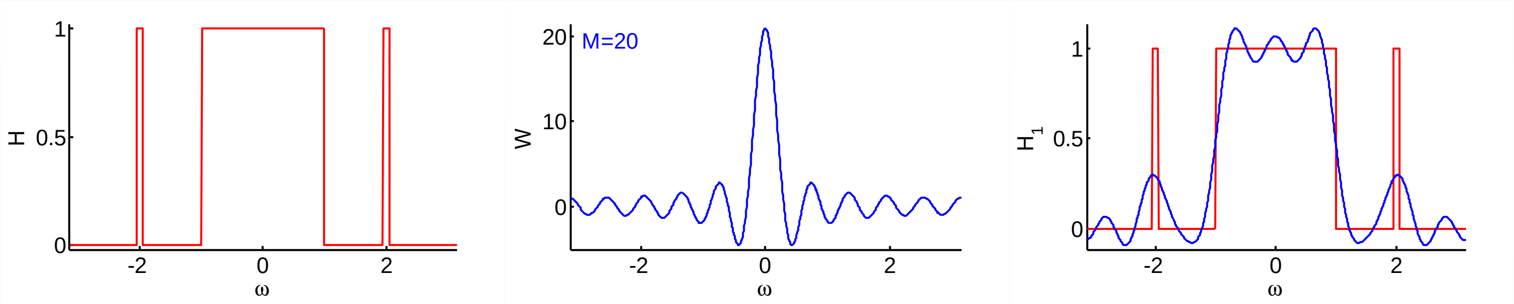

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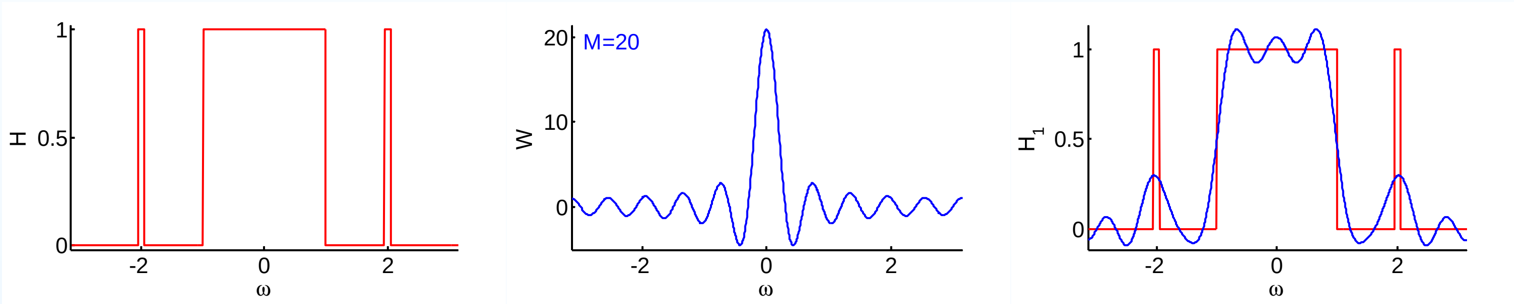

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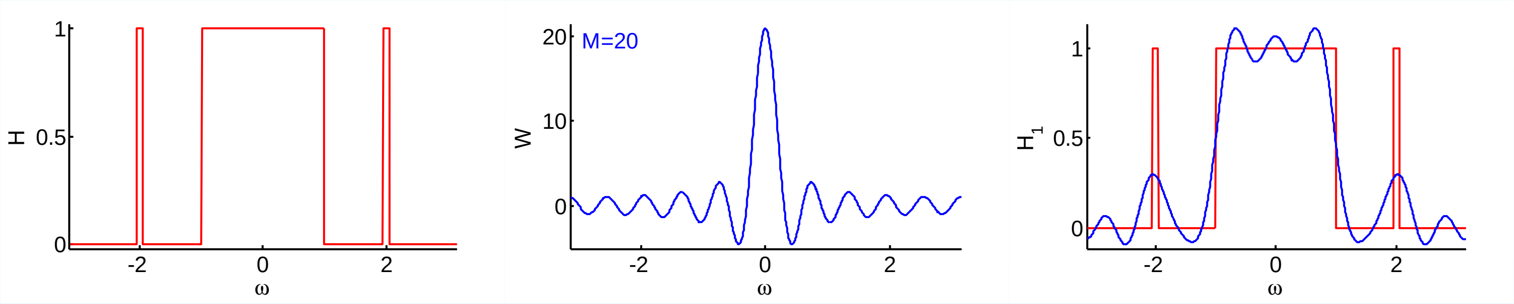

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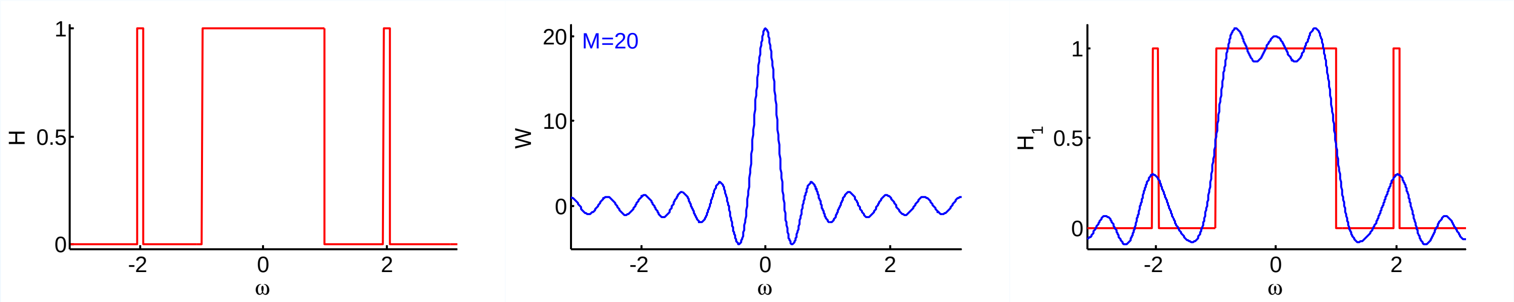

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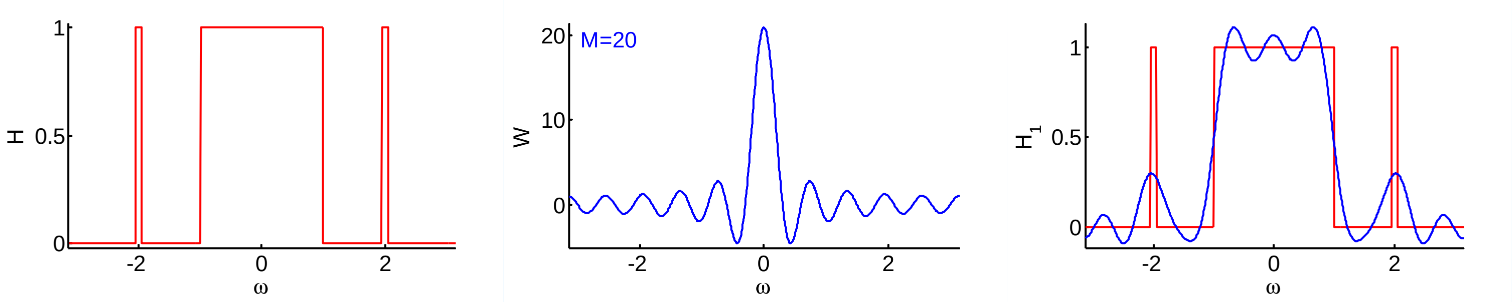

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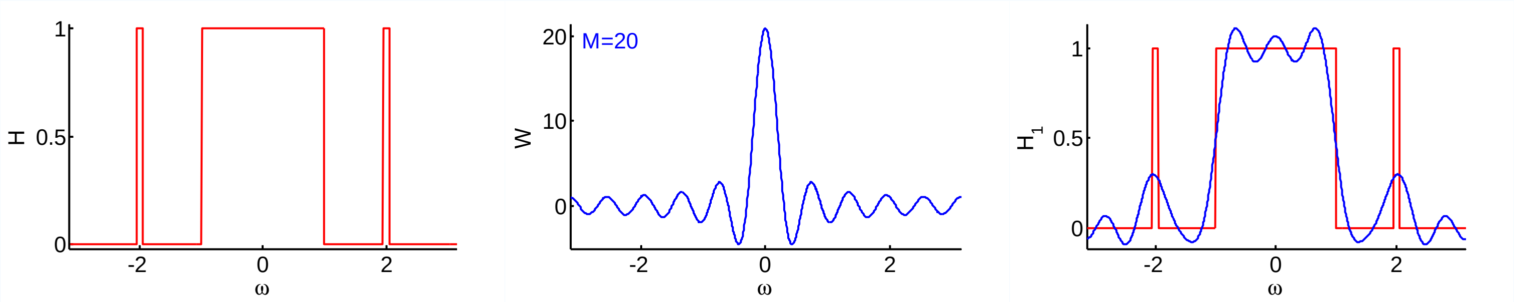

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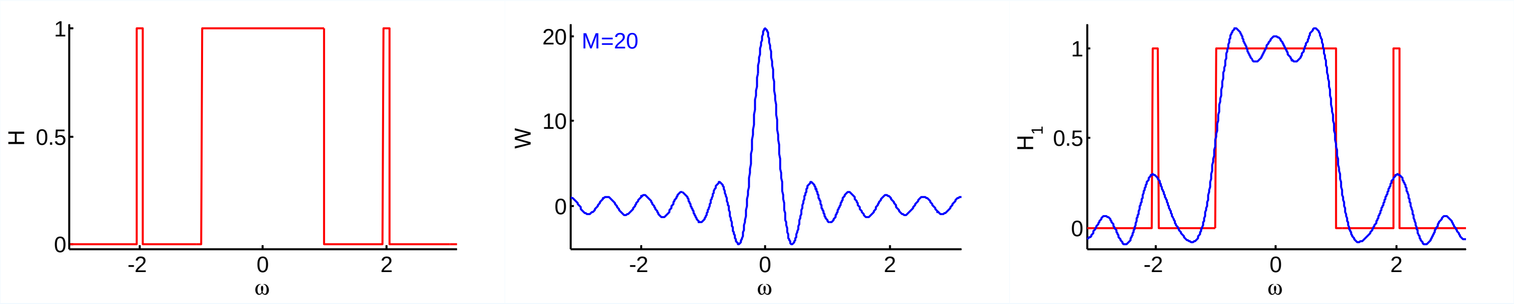

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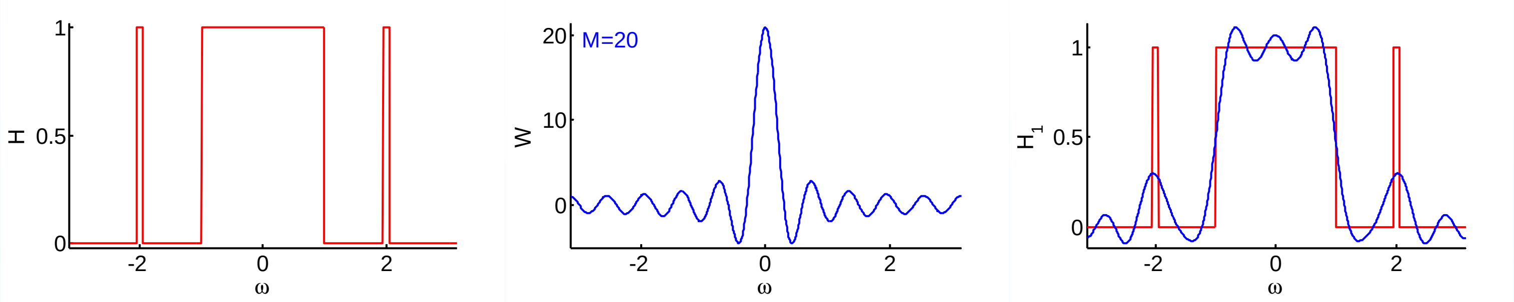

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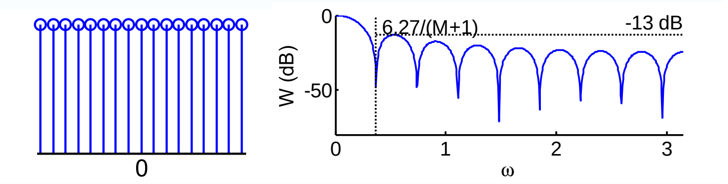
Common Windows

Rectangular: $w[n] \equiv 1$



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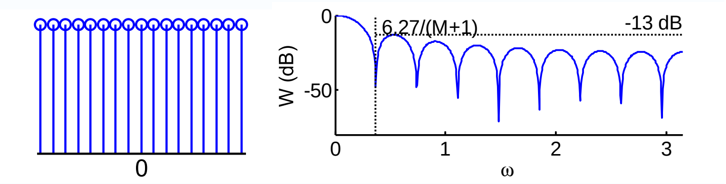

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Rectangular: $w[n] \equiv 1$

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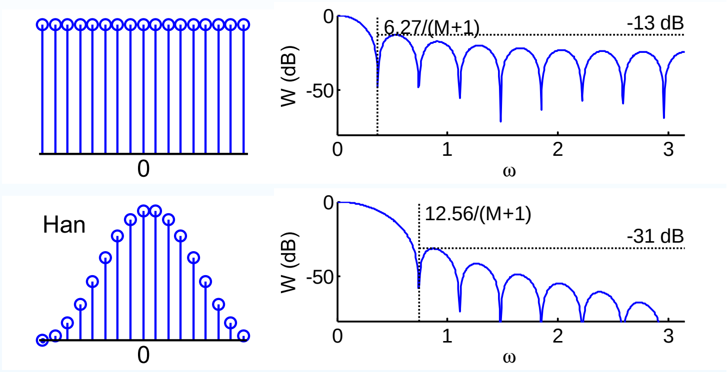
Hanning: $0.5 + 0.5c_1$

$c_k = \cos \frac{2\pi kn}{M+1}$



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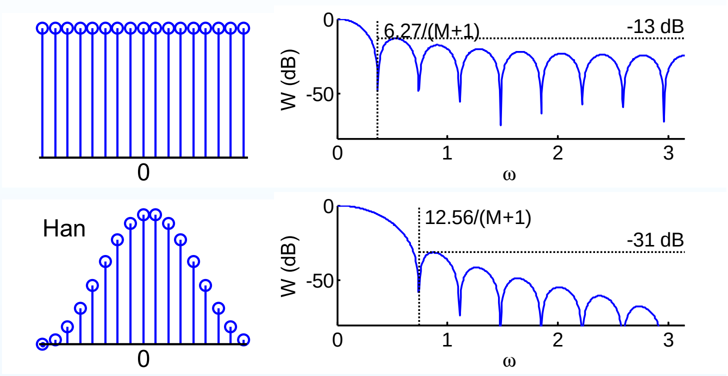

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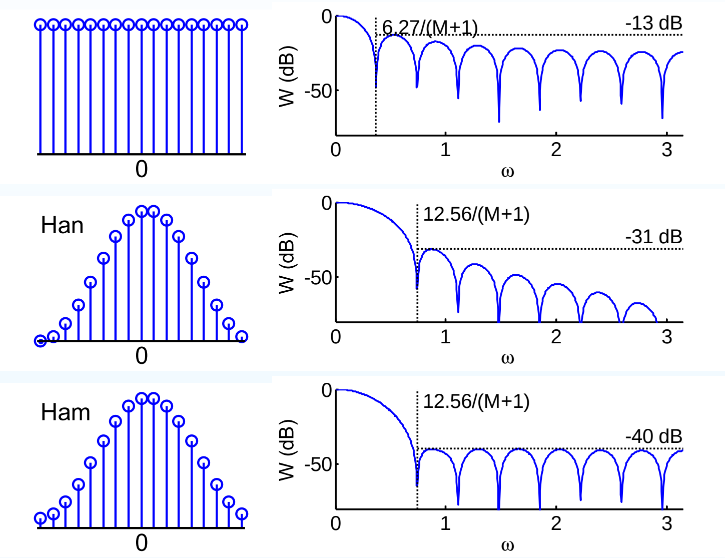

6: Window Filter Design
- Inverse DTFT
- Rectangular window
- Dirichlet Kernel +
- Window relationships
- Common Windows
- Order Estimation
- Example Design
- Frequency sampling
- Summary
- MATLAB routines

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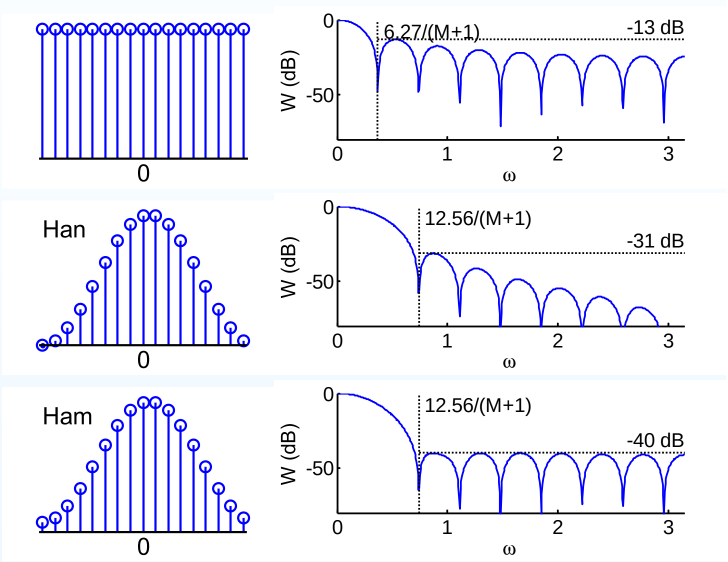
best peak sidelobe



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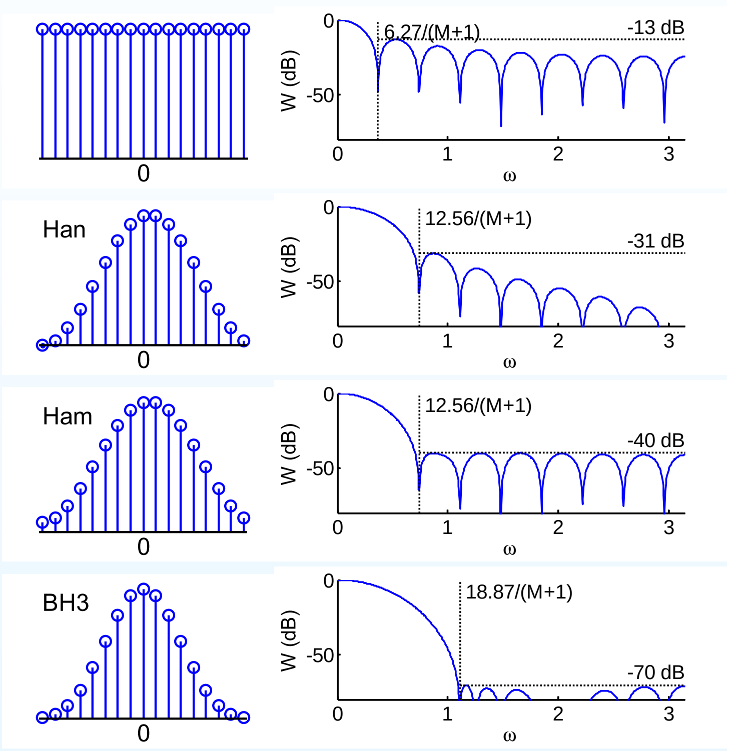

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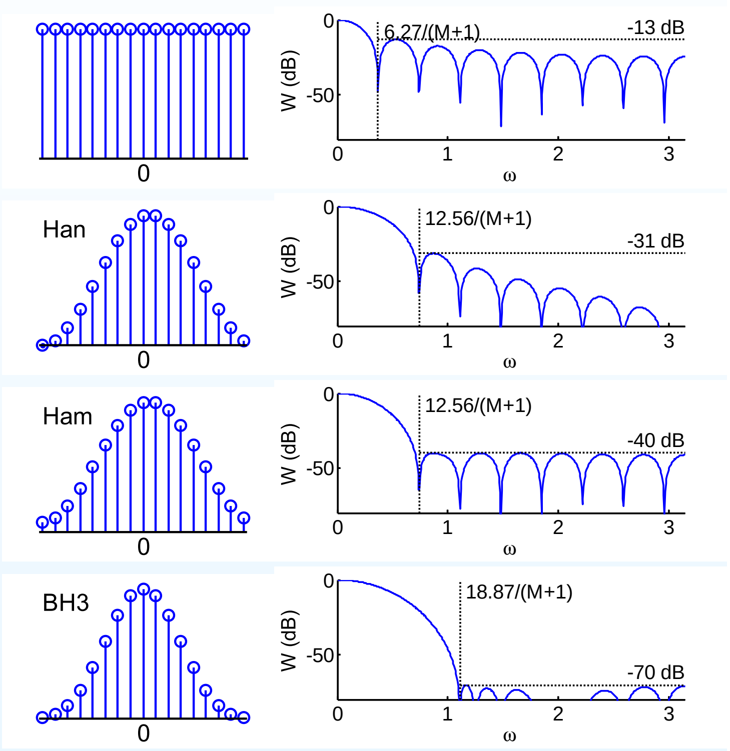

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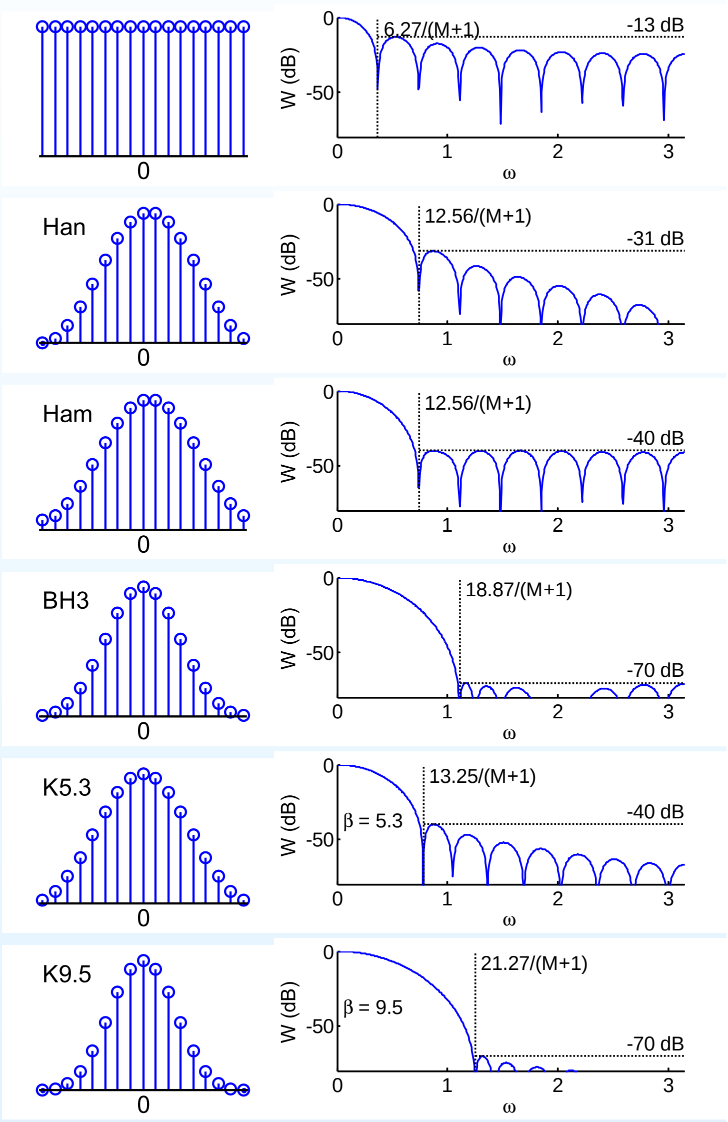

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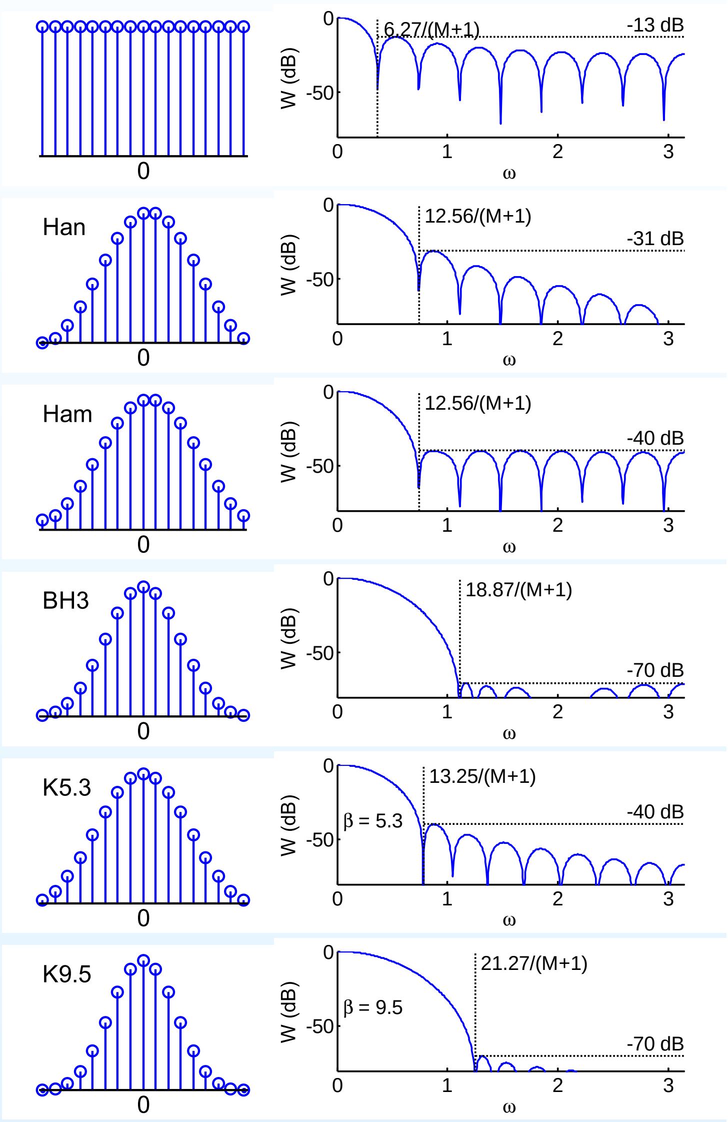
Good compromise:

Width v sidelobe v decay



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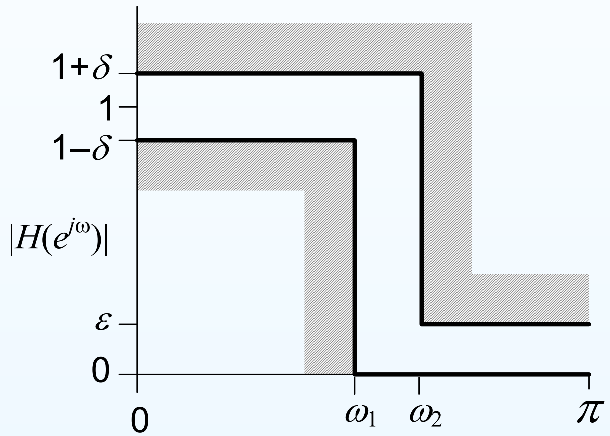
E.g. for lowpass filter



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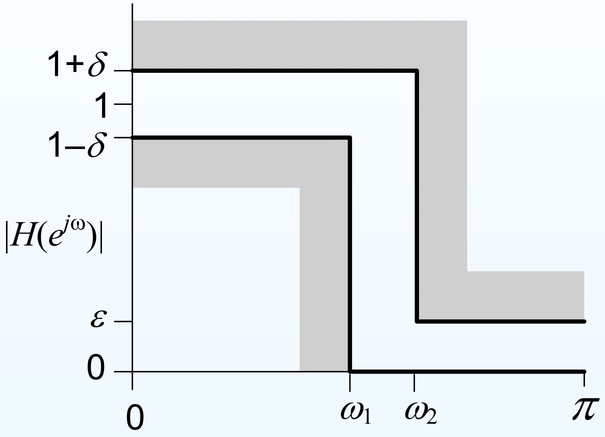

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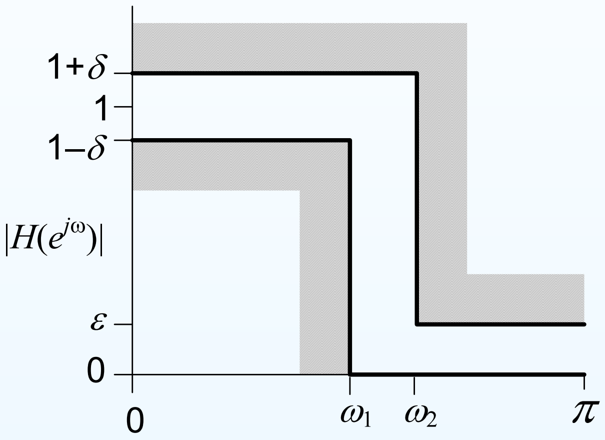

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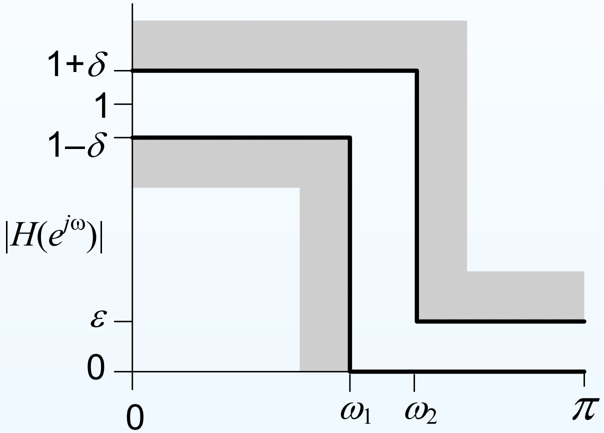

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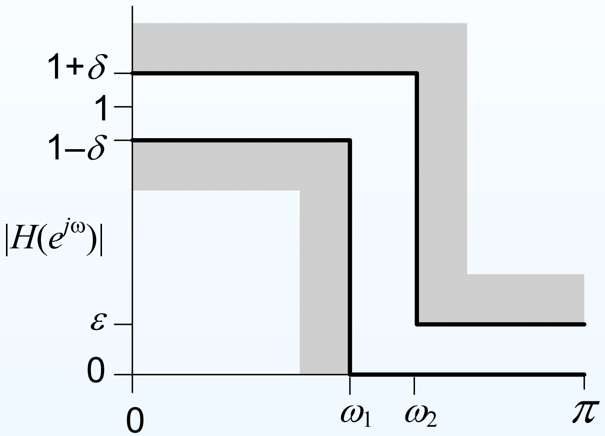

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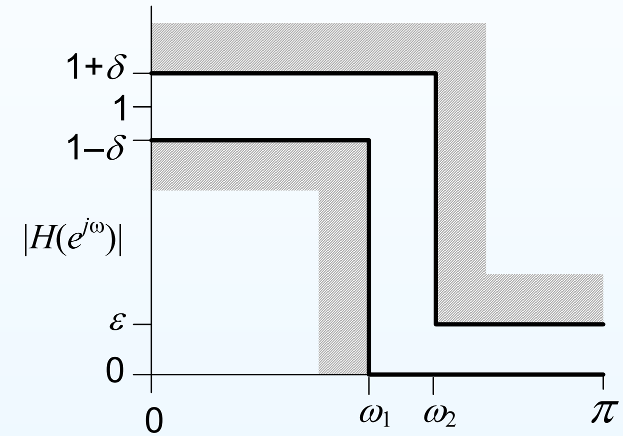

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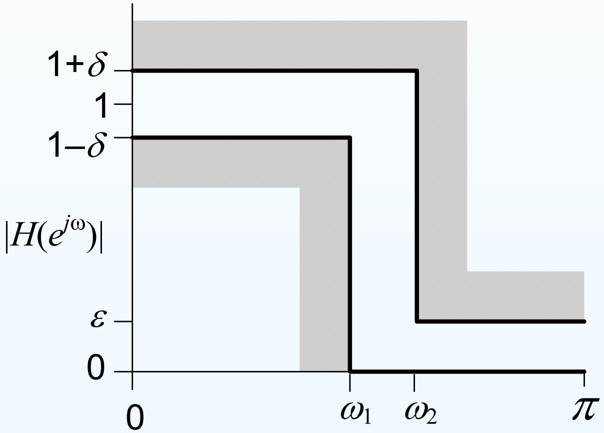

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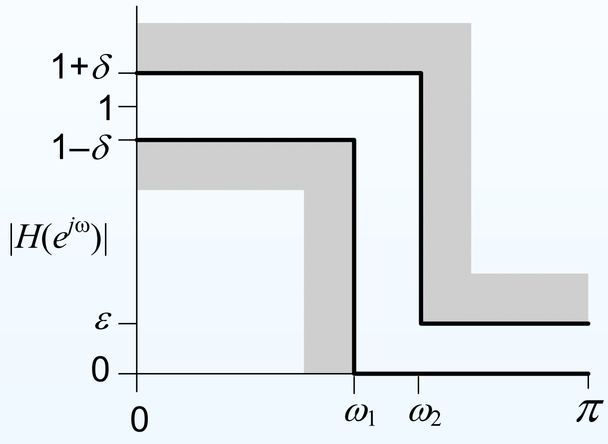

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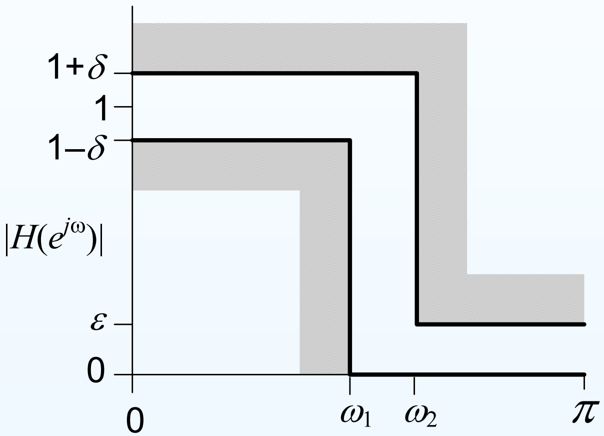

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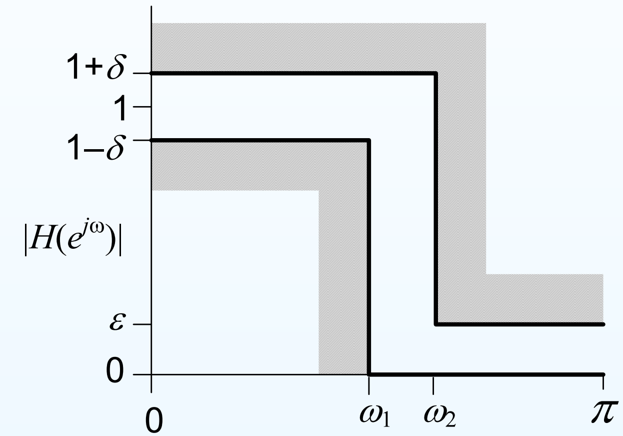

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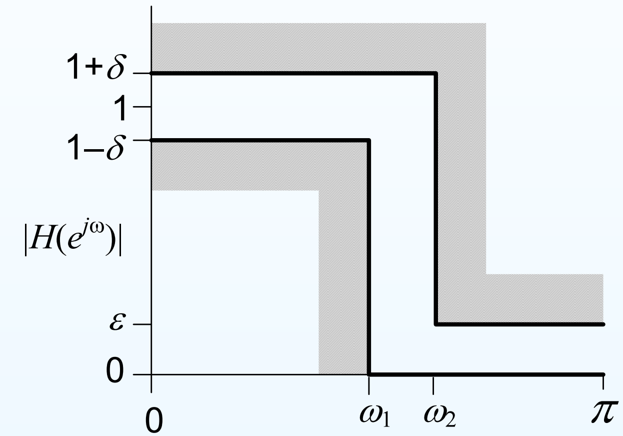

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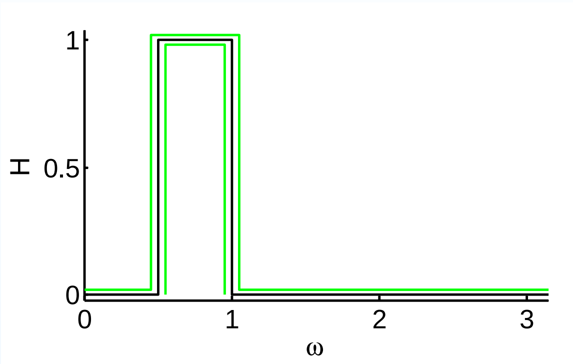

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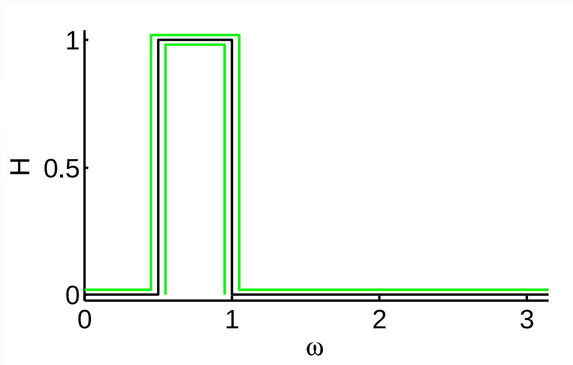

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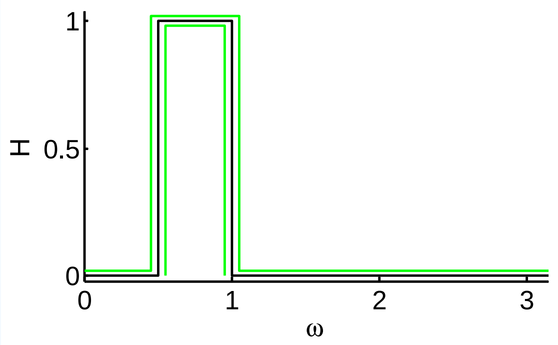

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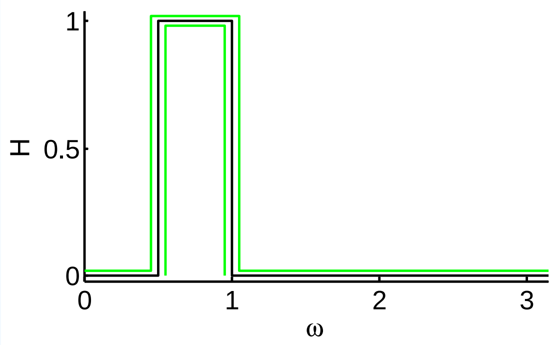

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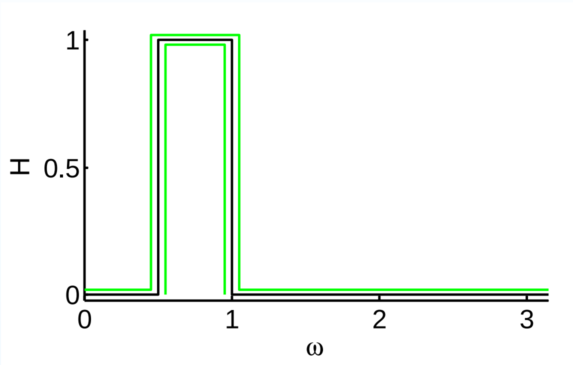

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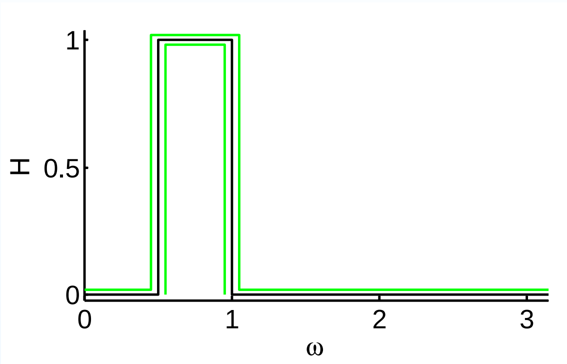

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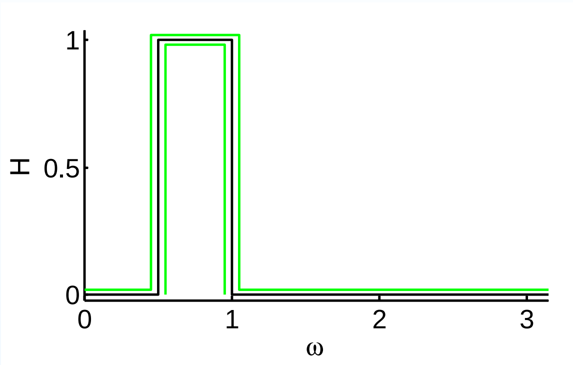

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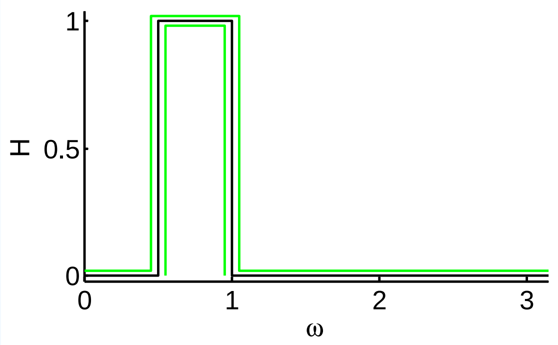

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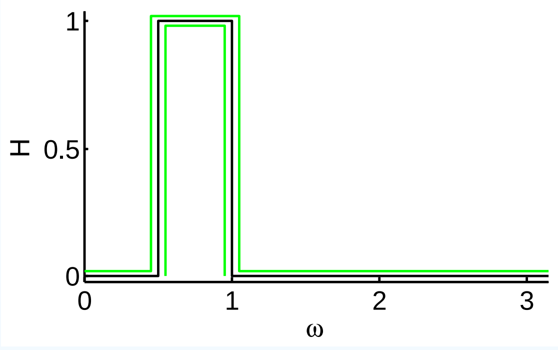

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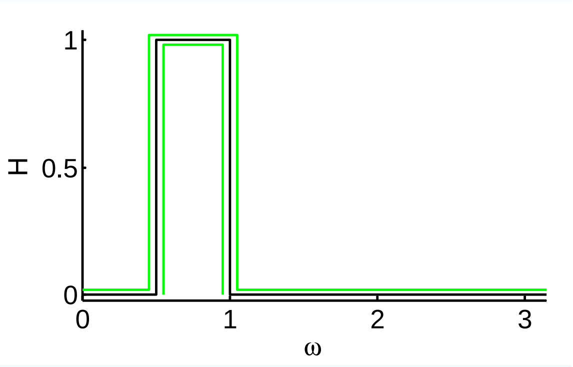

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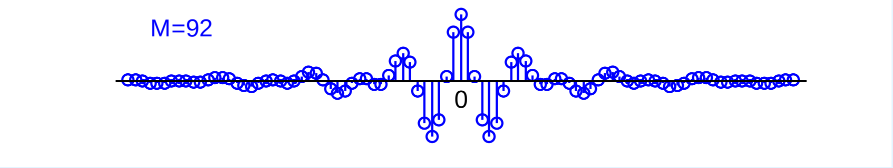

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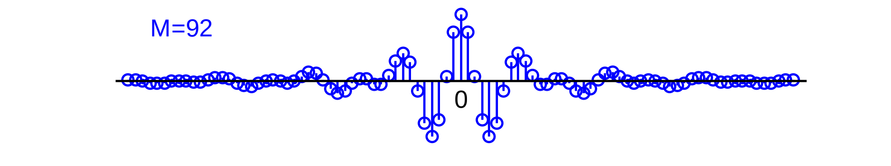

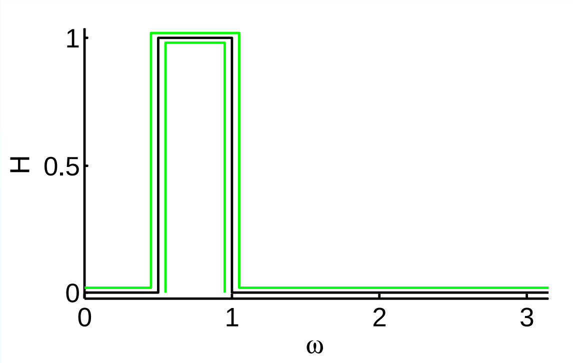

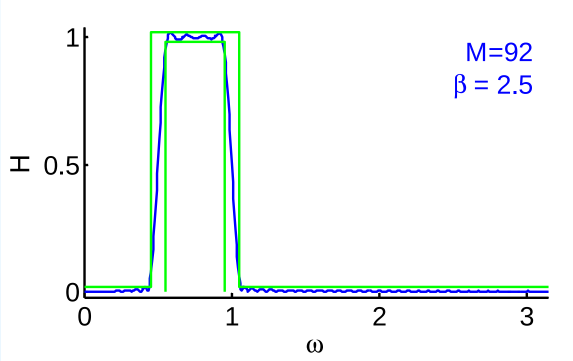

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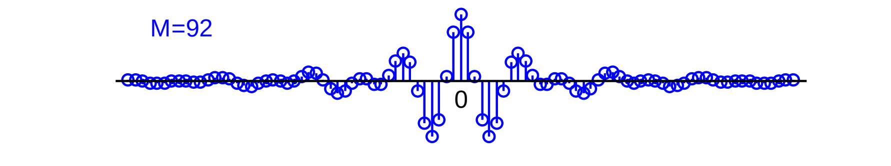

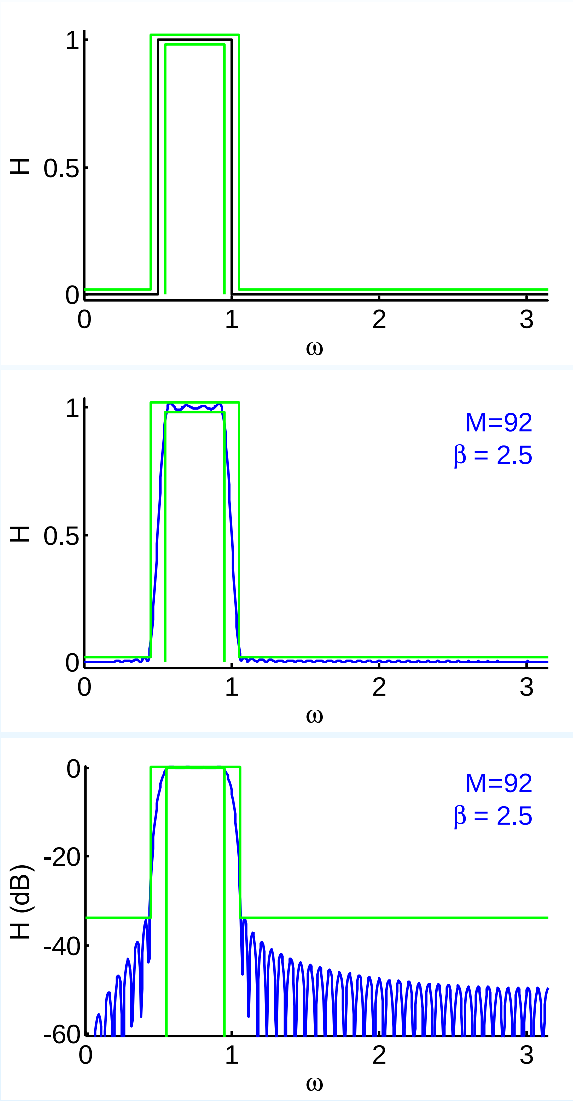

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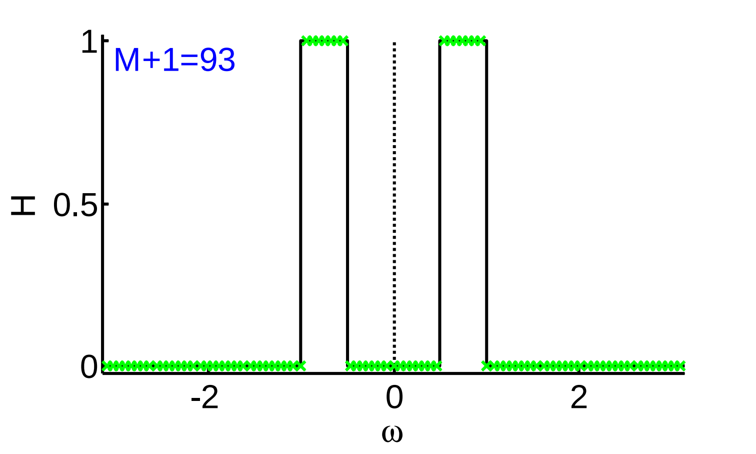

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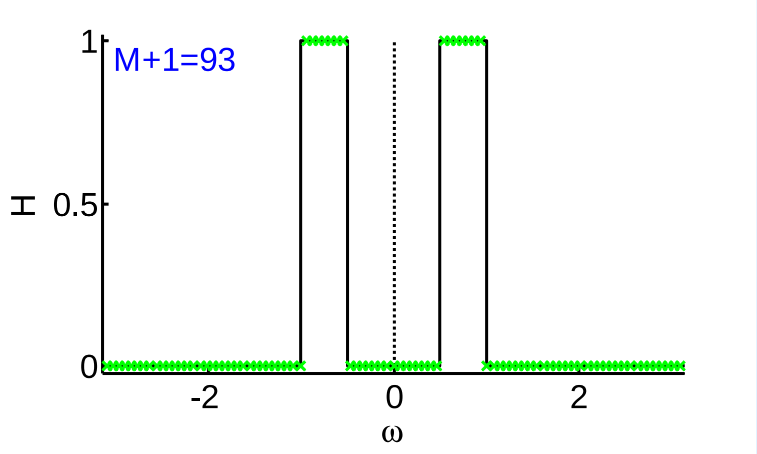

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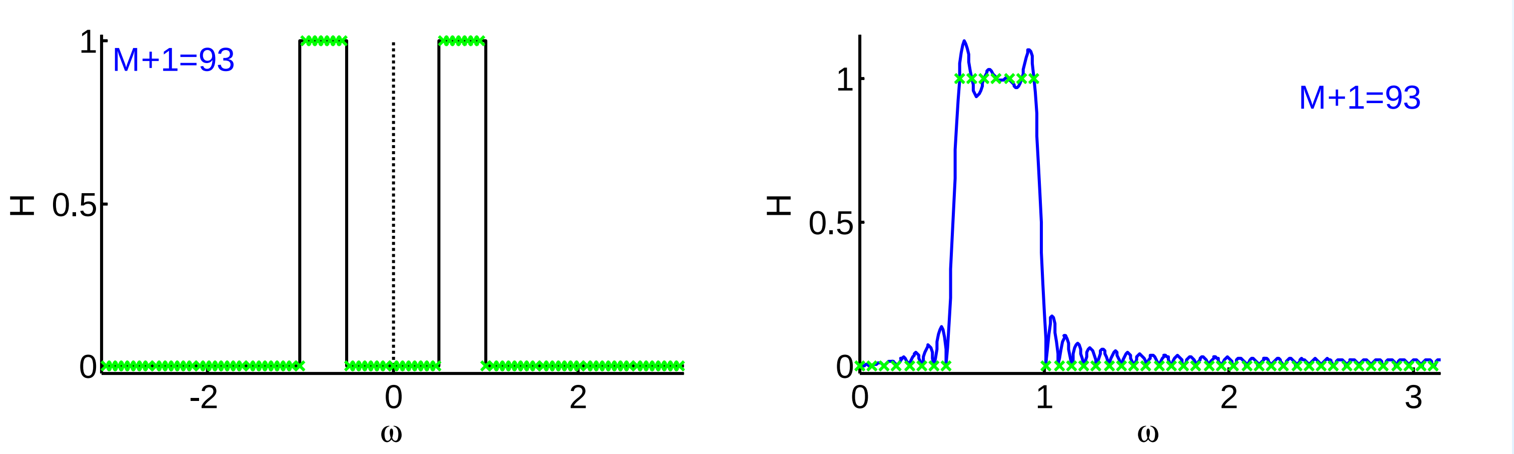

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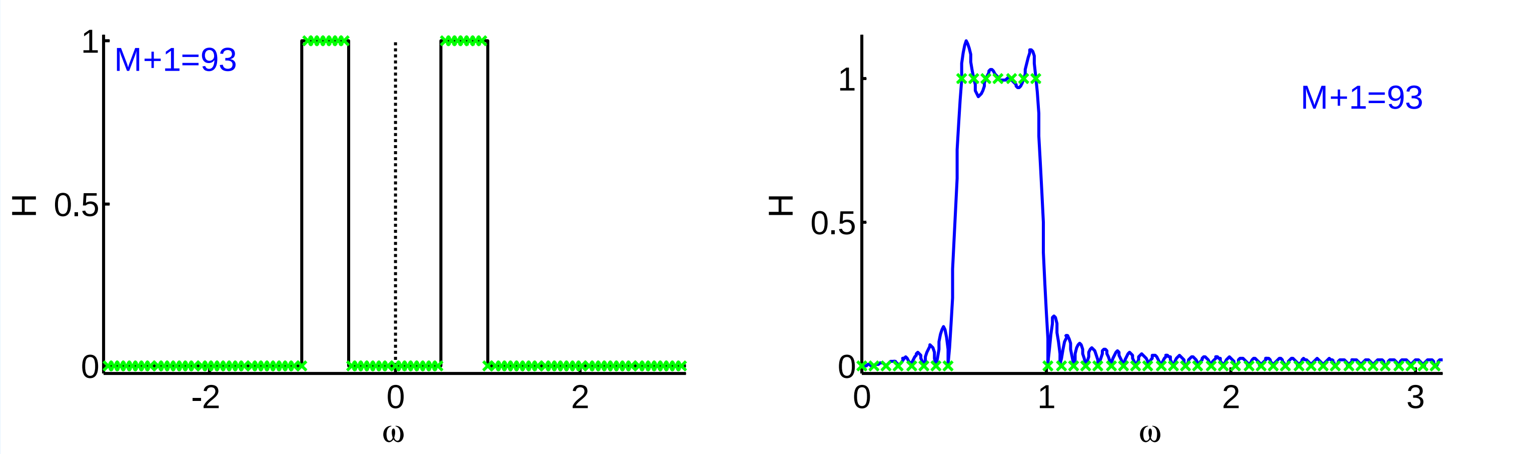

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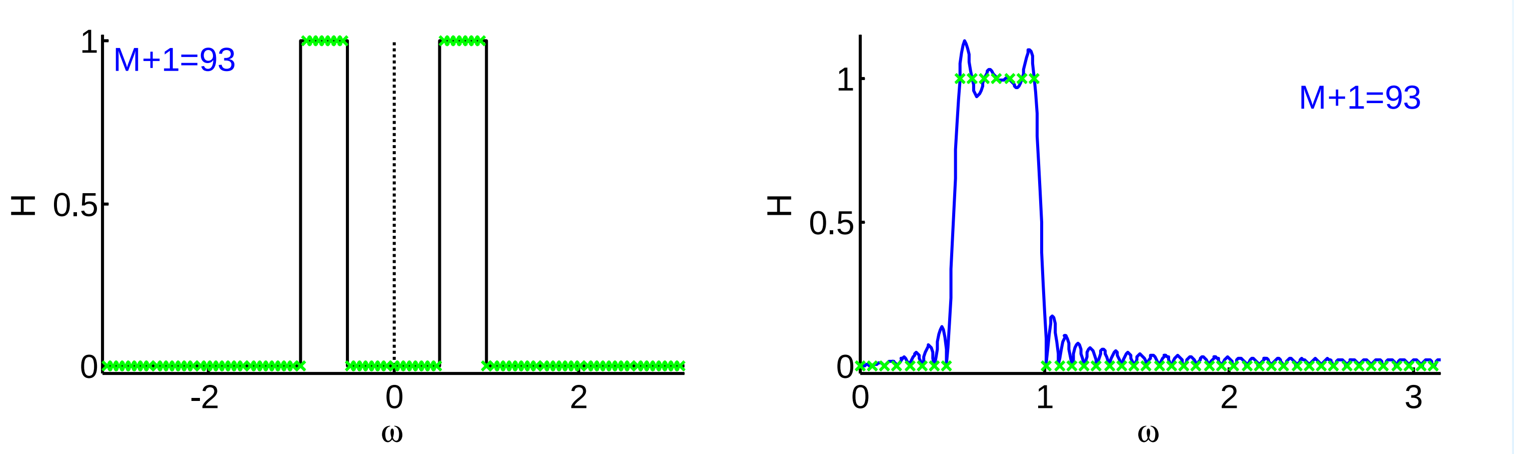

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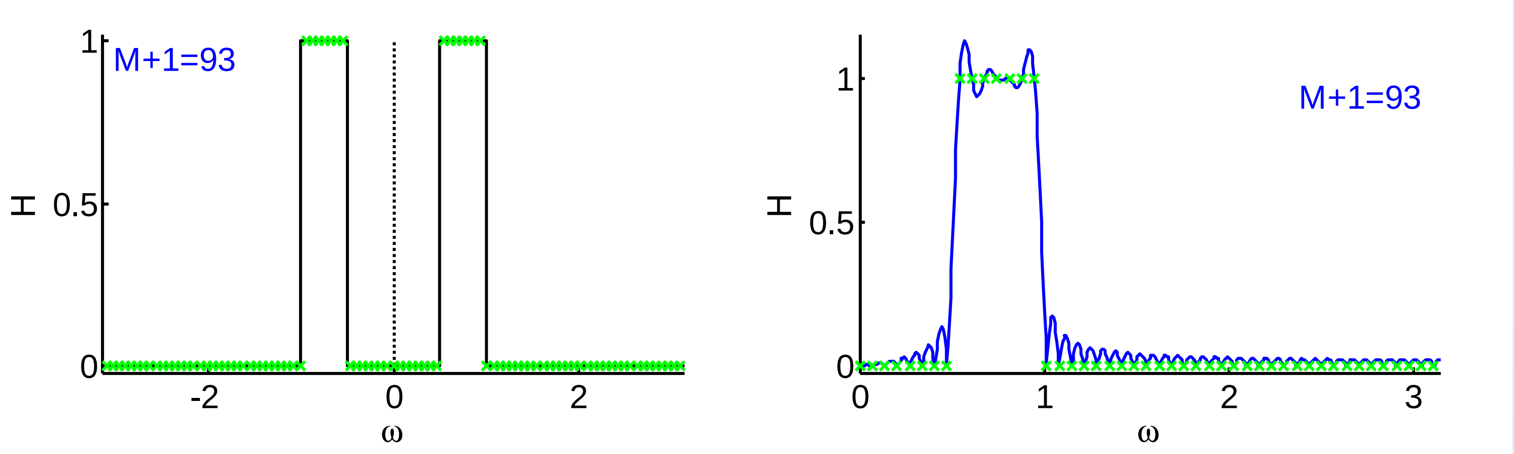

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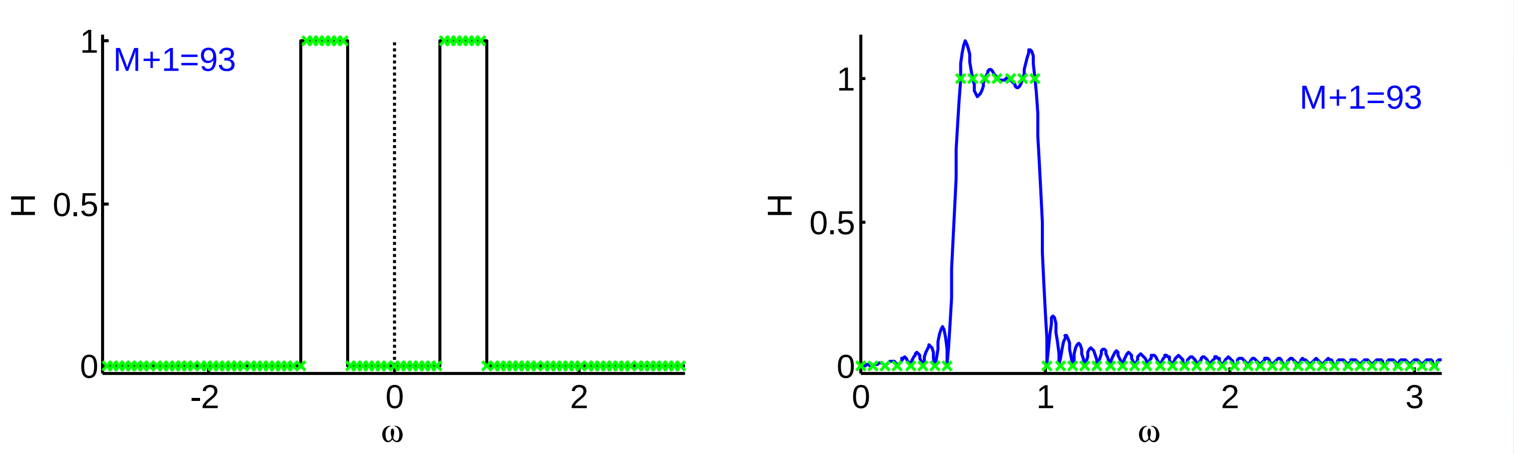

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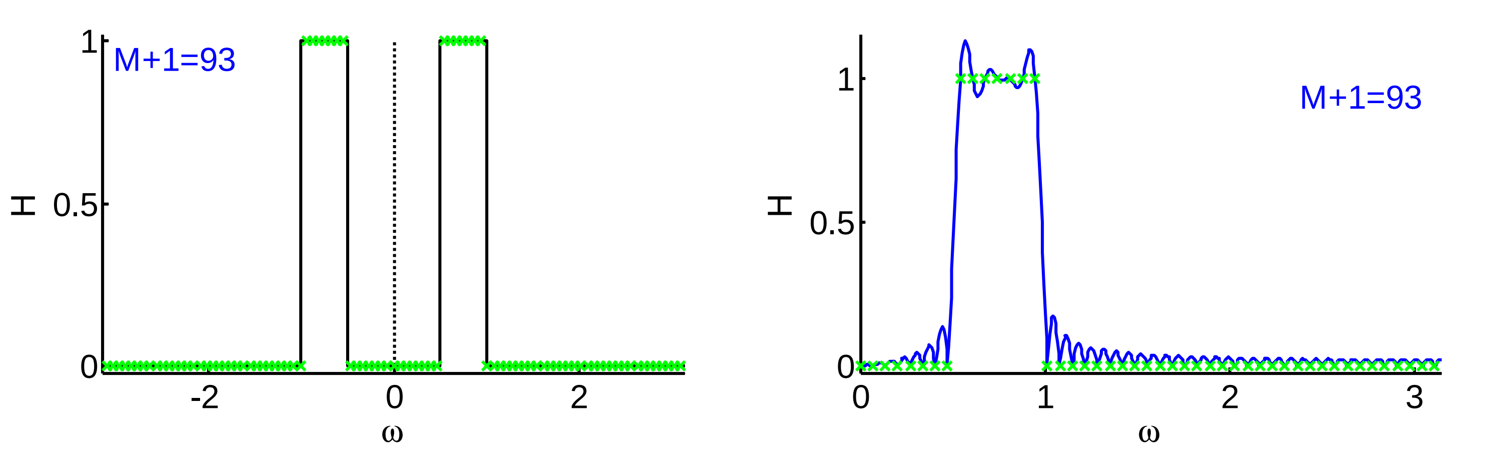
(3) use non-uniform points with more near transition (can't use IDFT)



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  - Ideal lowpass has $h[n] = \frac{\sin \omega_0 n}{\pi n}$
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For further details see Mitra: 7, 10.

MATLAB routines

diric(x,n)	Dirichlet kernel: $\frac{\sin 0.5nx}{\sin 0.5x}$
hanning hamming kaiser	Window functions (Note ’periodic’ option)
kaiserord	Estimate required filter order and β