12: Polyphase Filters

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- Summary
Heavy Lowpass filtering

Filter Specification:
Sample Rate: 20 kHz
Passband edge: 100 Hz \( (\omega_1 = 0.03) \)
Stopband edge: 300 Hz \( (\omega_2 = 0.09) \)
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Design with Remez-exchange algorithm
Order = 360
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**Symmetric FIR Filter:**
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Maximum Decimation Frequency

If a filter passband occupies only a small fraction of \([0, \pi]\), we can downsample then upsample without losing information.
Maximum Decimation Frequency

If a filter passband occupies only a small fraction of $[0, \pi]$, we can downsample then upsample without losing information.

\[ x[n] \xrightarrow{H(z)} 4:1 \quad 1:4 \xrightarrow{} y[n] \]

![Graph showing filter response](image-url)
Maximum Decimation Frequency

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$$x[n] \xrightarrow{H(z)} 4:1 \xrightarrow{1:4} y[n]$$

**Downsample**: aliased components at offsets of $\frac{2\pi}{K}$ are almost zero because of $H(z)$.
If a filter passband occupies only a small fraction of \([0, \pi]\), we can downsample then upsample without losing information.

\[
\begin{align*}
  x[n] &\xrightarrow{H(z)} \underline{4:1} \rightarrow \underline{1:4} \rightarrow y[n] \\
\end{align*}
\]

**Downsample:** aliased components at offsets of \(\frac{2\pi}{K}\) are almost zero because of \(H(z)\)

**Upsample:** Images spaced at \(\frac{2\pi}{K}\) can be removed using another low pass filter
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To avoid aliasing in the passband, we need

\[
\frac{2\pi}{K} - \omega_2 \geq \omega_1 \quad \Rightarrow \quad K \leq \frac{2\pi}{\omega_1 + \omega_2}
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Centre of transition band must be \( \leq \) intermediate Nyquist freq, $\frac{\pi}{K}$

\[
\begin{align*}
\text{|H| (dB)} & \quad \omega_1 & \quad \omega_2 & \quad \omega \\
0 & \quad \omega_1 & \quad 1 & \quad 2 & \quad 3 & \quad 4 \\
-60 & \quad -20 & \quad 0 & \quad -20 & \quad -60
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If a filter passband occupies only a small fraction of \([0, \pi]\), we can downsample then upsample without losing information.

\[
x[n] \xrightarrow{H(z)} 4:1 \xrightarrow{1:4} \frac{y[n]}{4}
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We must add a **lowpass filter** to remove the images:
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We must add a lowpass filter to remove the images:

\[
H(z) \xrightarrow{7:1} 1:7 \xrightarrow{\text{LPF}}
\]

Passband noise = noise floor at output of \(H(z)\) plus \(10 \log_{10} (K - 1)\) dB.
Polyphase decomposition

For our filter: original Nyquist frequency = 10 kHz and transition band centre is at 200 Hz so we can use $K = 50$. 
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Example: $M = 399$, $K = 50$
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Example: $M = 399, K = 50 \Rightarrow R = \frac{M+1}{K} = 8$
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$$= \sum_{m=0}^{K-1} h[m]z^{-m} + \sum_{m=0}^{K-1} h[m+K]z^{-(m+K)} + \cdots \quad [R \text{ terms}]$$
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where $h_m[r] = h[m + Kr]$

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$h_3[r] = [h[3], h[53], \ldots, h[303], h[353]]$
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This is a polyphase implementation of the filter $H(z)$.
$H(z)$ is low pass so we downsample its output by $K$ without aliasing.
**Downsampled Polyphase Filter**

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Using the Noble identities, we can move the resampling back through the adders and filters. \( H_m(z^K) \) turns into \( H_m(z) \) at a lower sample rate.
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We still perform 400 multiplications but now only once for every $K$ input samples.
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Multiplications per input sample = 8 (down by a factor of 50 😊) but \( v[n] \) has the wrong sample rate (😢).
Polyphase Upsampler

To restore sample rate: upsample and then lowpass filter to remove images.

\[ v[i] \xrightarrow{1:K} \text{LPF} \xrightarrow{} v[n] \]
Polyphase Upsampler

To restore sample rate: upsample and then lowpass filter to remove images

We can use the same lowpass filter, $H(z)$, in polyphase form:

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\sum_{m=0}^{K-1} z^{-m} \sum_{r=0}^{R-1} h_m[r] z^{-Kr}
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The overall system implements:

\[ x[n] \rightarrow H(z) \rightarrow K:1 \rightarrow v[i] \rightarrow 1:K \rightarrow H(z) \rightarrow K \rightarrow y[n] \]
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$H(e^{j\omega})$ reaches $-10$ dB at the downsampler Nyquist frequency of $\frac{\pi}{K}$.
Complete Filter

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For $V(e^{j\omega})$, passband gain (blue curve) follows the same curve as $X(e^{j\omega})$. 
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For $V(e^{j\omega})$, passband gain (blue curve) follows the same curve as $X(e^{j\omega})$. Noise arises from $K$ aliased spectral intervals.

Unit white noise in $X(e^{j\omega})$ gives passband noise floor at $-69$ dB (red curve) even though stop band ripple is below $-83$ dB (due to $K - 1$ aliased stopband copies).
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We can also share the gains and adders between all 50 filters and use commutators to switch the coefficients.
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We now need 7 delays, 7 adders and 8 gains for the entire filter.
We can again use a commutator. The outputs from all 50 filters are added together to form $v[i]$. 
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Downsampler Implementation

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We can sum the outputs of the gain elements using an accumulator which sums blocks of \( K \) samples.

\[
\sum_{r=0}^{K-1} u[K_i - r]
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Now we can share all the components and use commutators to switch the gain coefficients.
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We need 7 delays, 7 adders, 8 gains and 8 accumulators in total.
Filtering should be performed at the lowest possible sample rate
- reduce filter computation by $K$
- actual saving is only $\frac{K}{2}$ because you need a second filter
- downsampled Nyquist frequency $\geq \max (\omega_{\text{passband}}) + \frac{\Delta \omega}{2}$
Filtering should be performed at the **lowest possible sample rate**
- reduce filter computation by $K$
- actual saving is only $\frac{K}{2}$ because you need a second filter
- downsampling Nyquist frequency $\geq \max(\omega_{\text{passband}}) + \frac{\Delta\omega}{2}$

**Polyphase decomposition:** split $H(z)$ as $\sum_{m=0}^{K-1} z^{-m} H_m(z^K)$
- each $H_m(z^K)$ can operate on subsampled data
- combine the filtering and down/up sampling
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  - actual saving is only $\frac{K}{2}$ because you need a second filter
  - downsampling Nyquist frequency $\geq \max(\omega_{passband}) + \frac{\Delta \omega}{2}$

Polyphase decomposition: split $H(z)$ as $\sum_{m=0}^{K-1} z^{-m} H_m(z^K)$
  - each $H_m(z^K)$ can operate on subsampled data
  - combine the filtering and down/up sampling

Noise floor is higher because it arises from $K$ spectral intervals that are aliased together by the downsampling.
Filtering should be performed at the lowest possible sample rate
  - reduce filter computation by $K$
  - actual saving is only $\frac{K}{2}$ because you need a second filter
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Share components between the $K$ filters
  - multiplier gain coefficients switch at the original sampling rate
  - need a new component: accumulator/downsampler ($K : \Sigma$)
Summary

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For further details see Harris 5.