13: Resampling Filters

Resampling

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e.g. Audio 44.1 kHz$\leftrightarrow 48$ kHz$\leftrightarrow 96$ kHz

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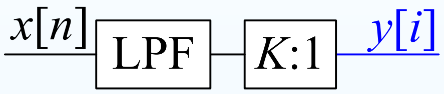

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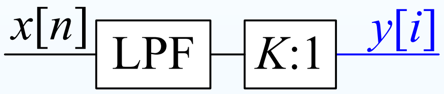

Upsample:

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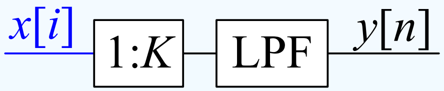

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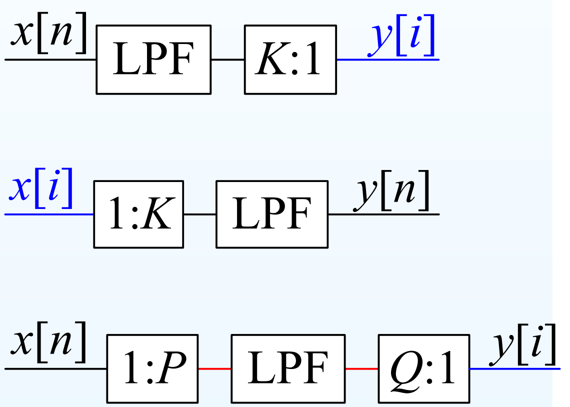
Rational ratio: $f_s \times \frac{P}{Q}$

LPF to lower of old and new Nyquist bandwidths: $\omega_0 = \frac{\pi}{\max(P,Q)}$



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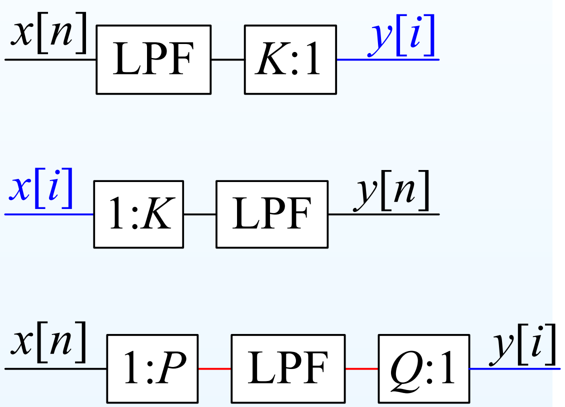

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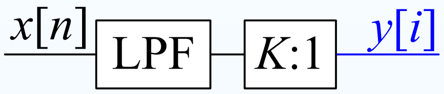

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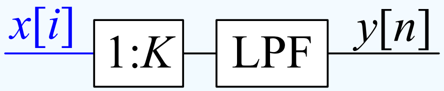

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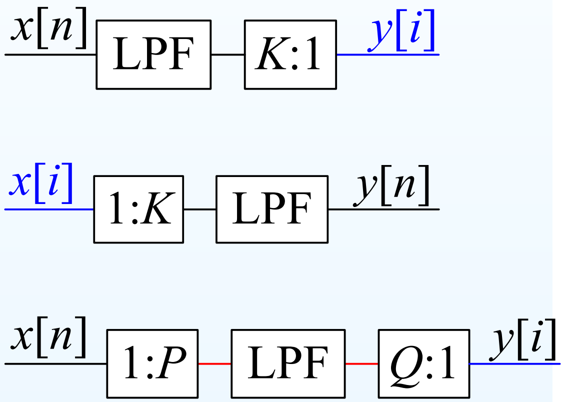

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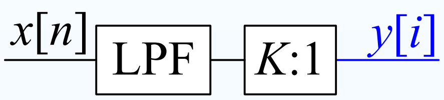

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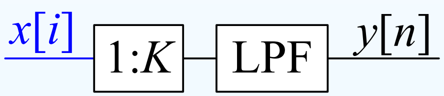

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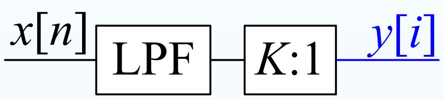

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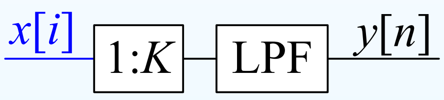

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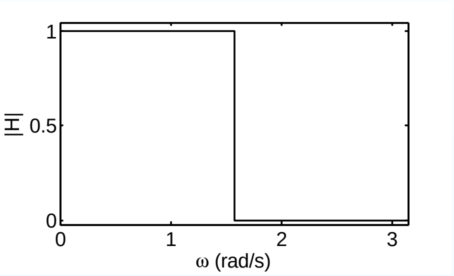
- Fractional semi-Transition bandwidth, $\alpha = \frac{\Delta\omega}{2\omega_0}$, is typically fixed.
  e.g. $\alpha = 0.05 \quad \Rightarrow \quad M \approx \frac{dK}{7\pi\alpha} = 0.9dK$ (where $\omega_0 = \frac{\pi}{K}$)

Halfband Filters

If $K = 2$ then the new Nyquist frequency is $\omega_0 = \frac{\pi}{2}$.



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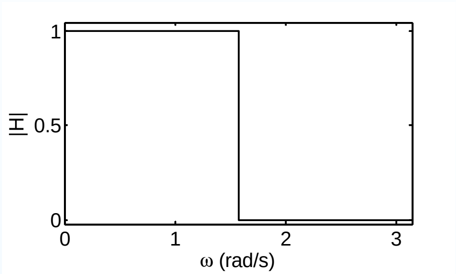
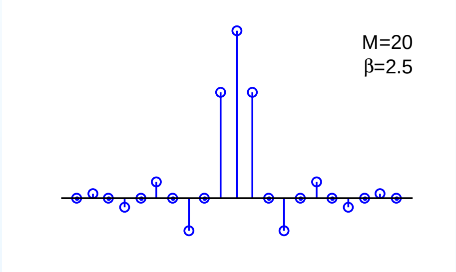
We multiply ideal response $\frac{\sin \omega_0 n}{\pi n}$ by a Kaiser window.





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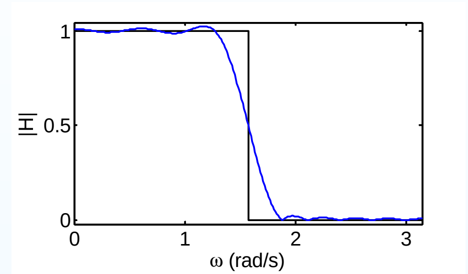

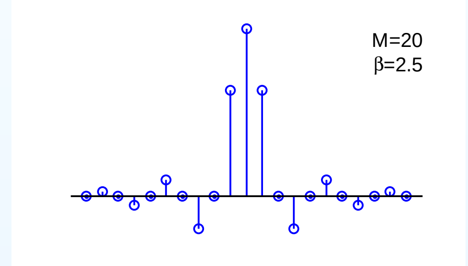

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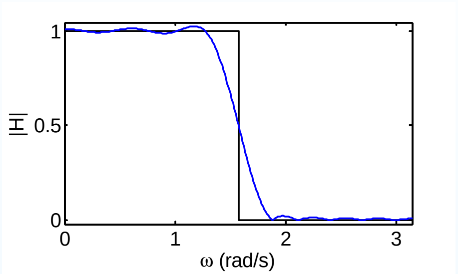

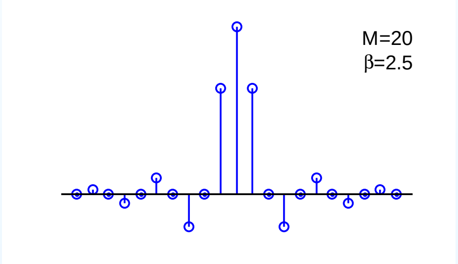

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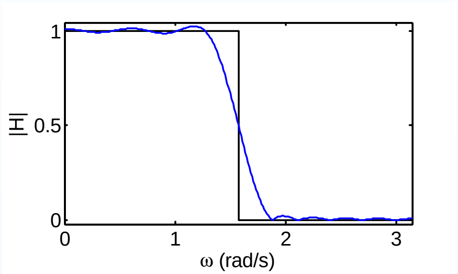
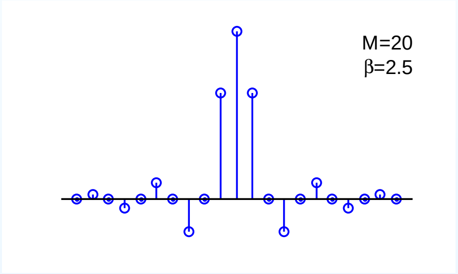
where $h_1[r] = h[2r + 1 - \frac{M}{2}]$





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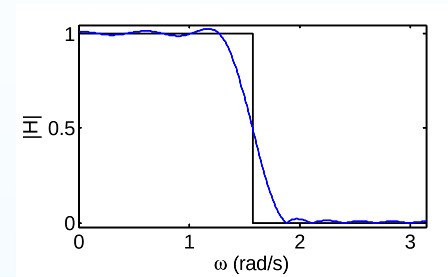
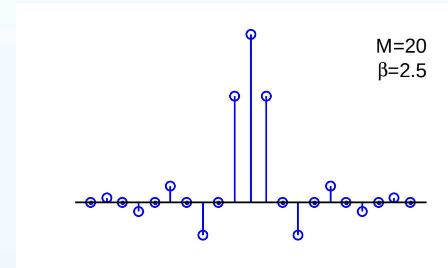
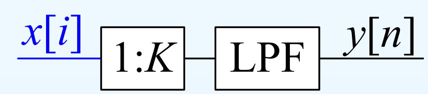
Half-band upsampler:







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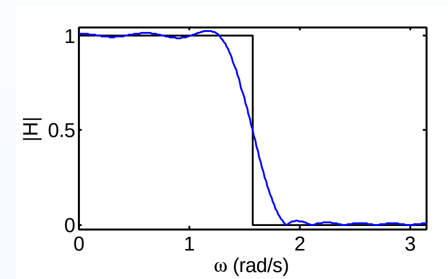

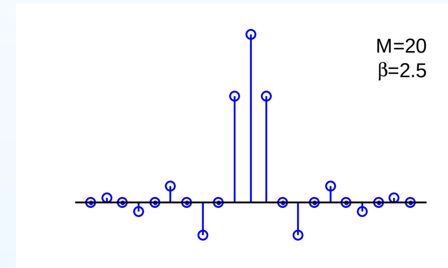

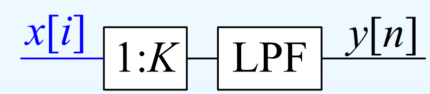

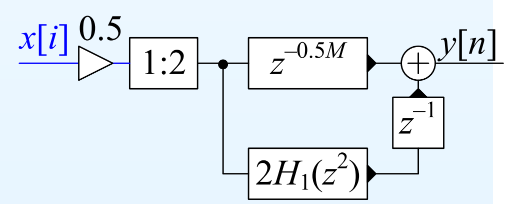

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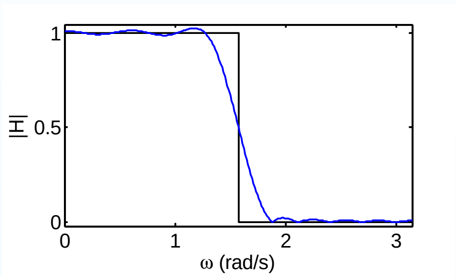

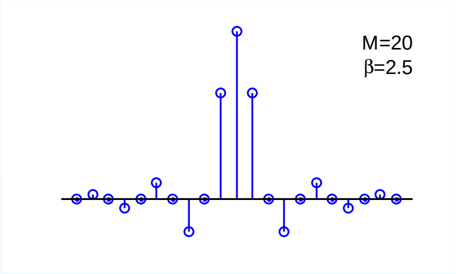

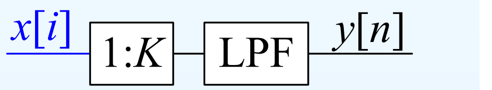

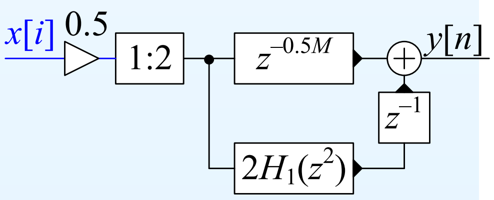

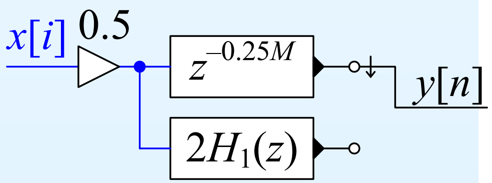

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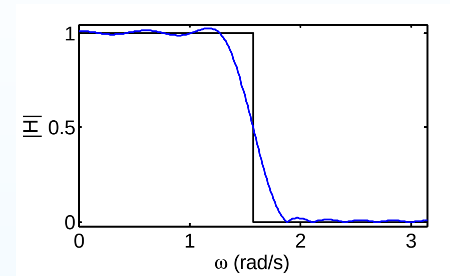

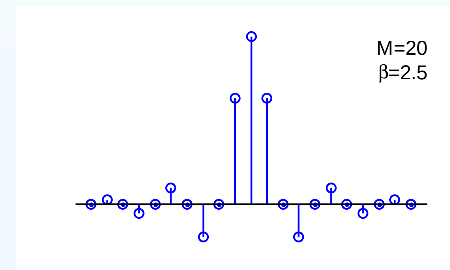

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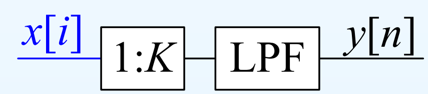

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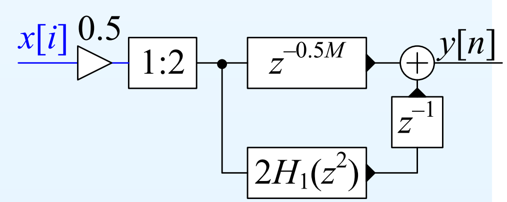
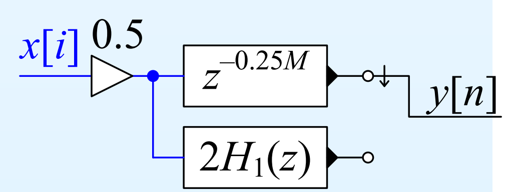


$H_1(z)$ is symmetrical with $\frac{M}{2}$ coefficients so we need $\frac{M}{4}$ multipliers in total (input gain of 0.5 can usually be absorbed elsewhere).



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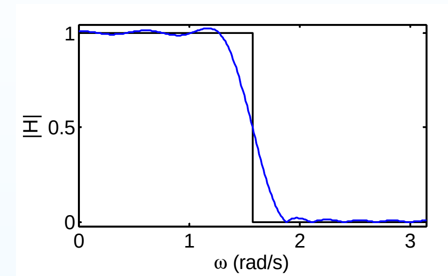
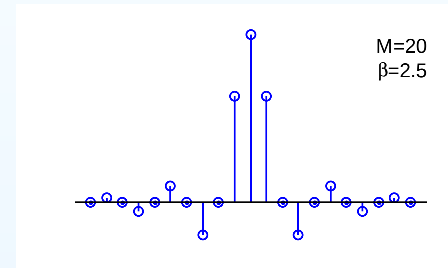
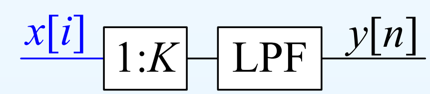
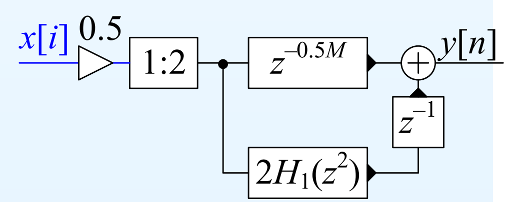
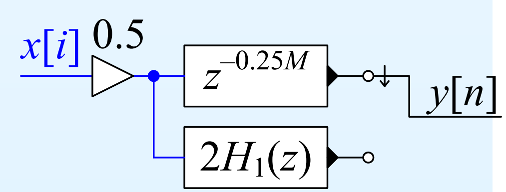
Computation: $\frac{M}{4}$ multiplies per input sample



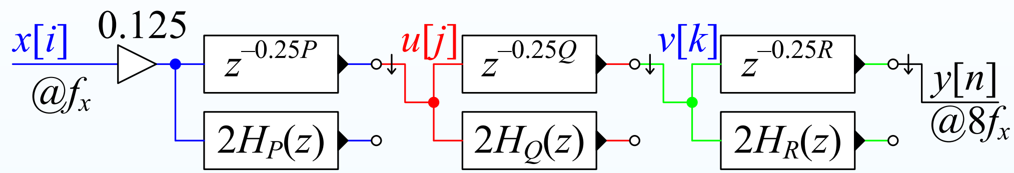


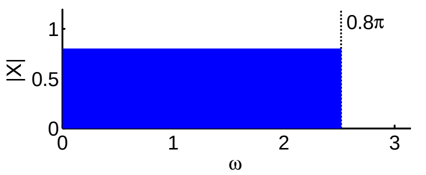






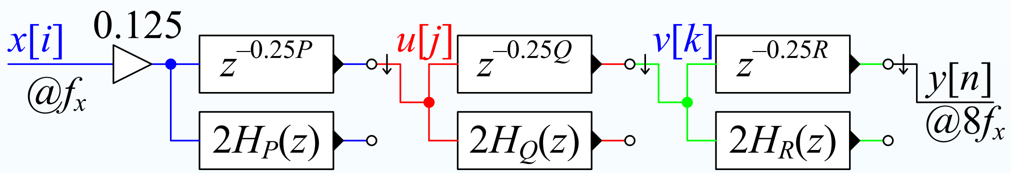
Dyadic 1:8 Upsampler

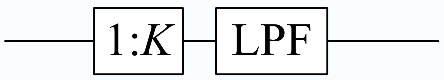


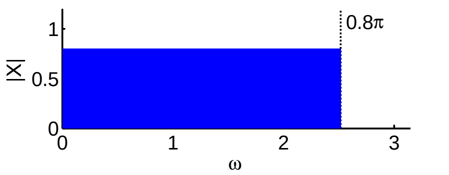


Suppose $X(z)$: BW = $0.8\pi \Leftrightarrow \alpha = 0.2$

Dyadic 1:8 Upsampler







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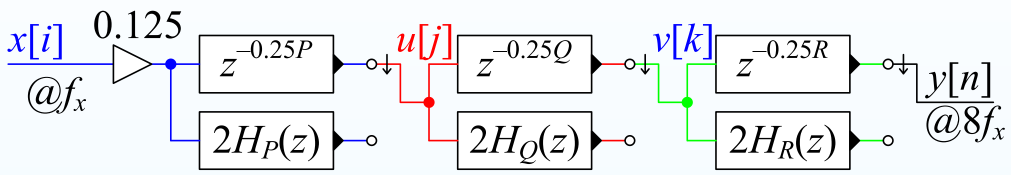
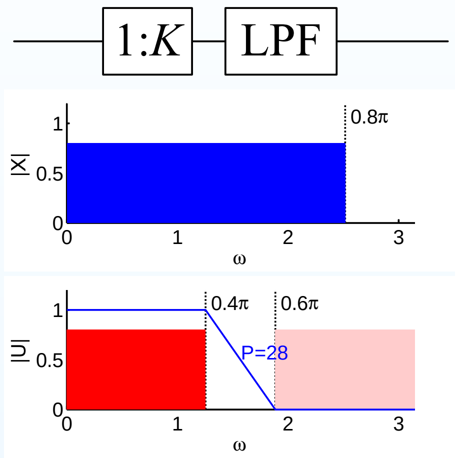
Upsample 1:2 $\rightarrow U(z)$:

Dyadic 1:8 Upsampler



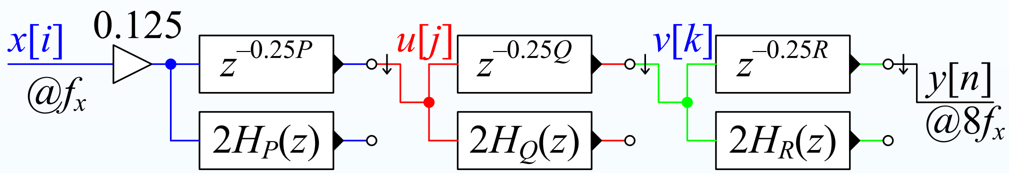


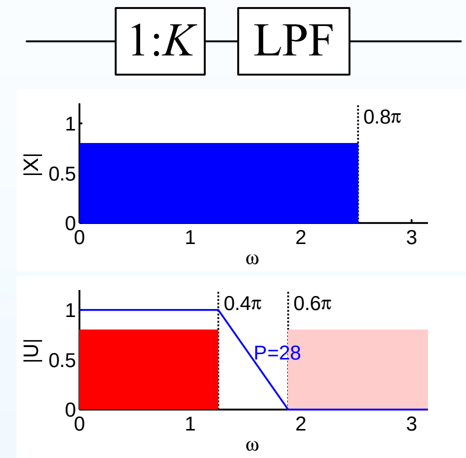
Suppose $X(z)$: BW = $0.8\pi \Leftrightarrow \alpha = 0.2$

Upsample 1:2 $\rightarrow U(z)$:

Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

Dyadic 1:8 Upsampler





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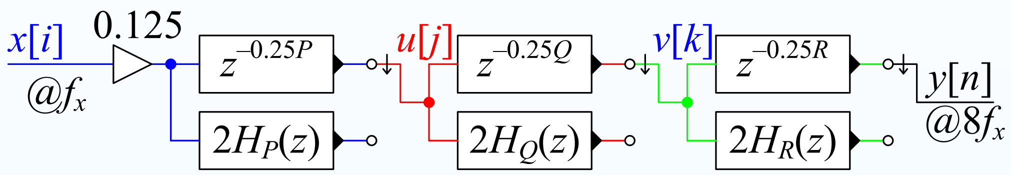
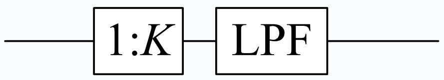
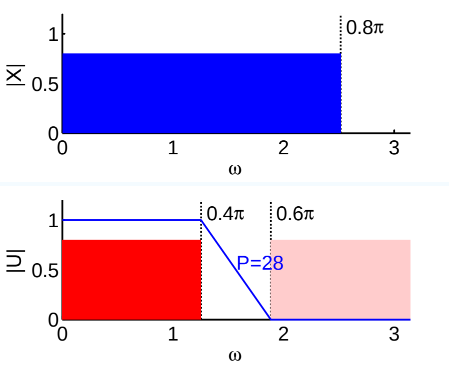
Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

For attenuation = 60 dB, $P \approx \frac{60}{3.5\Delta\omega} = 27.3$

Dyadic 1:8 Upsampler







Suppose $X(z)$: BW = $0.8\pi \Leftrightarrow \alpha = 0.2$

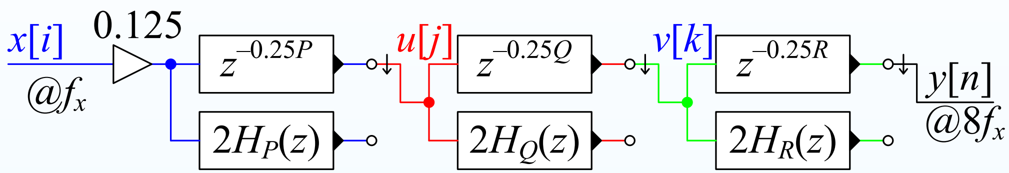
Upsample 1:2 $\rightarrow U(z)$:

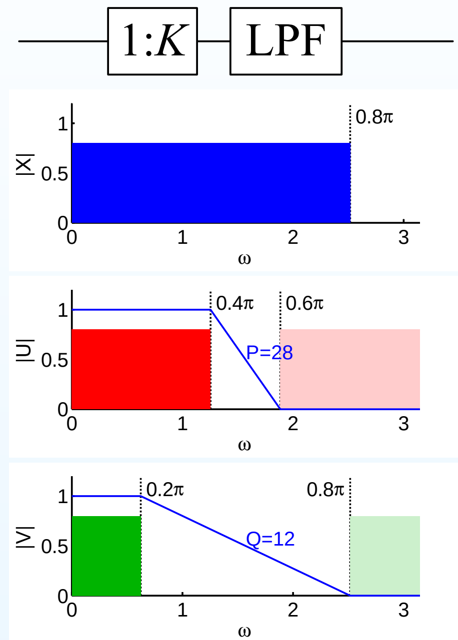
Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

For attenuation = 60 dB, $P \approx \frac{60}{3.5\Delta\omega} = 27.3$

Round up to a multiple of 4: $P = 28$

Dyadic 1:8 Upsampler





Suppose $X(z)$: BW = $0.8\pi \Leftrightarrow \alpha = 0.2$

Upsample 1:2 $\rightarrow U(z)$:

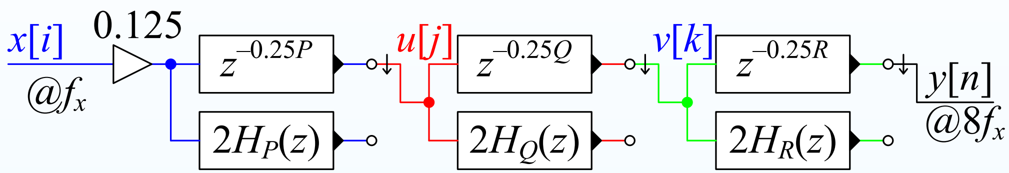
Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

For attenuation = 60 dB, $P \approx \frac{60}{3.5\Delta\omega} = 27.3$

Round up to a multiple of 4: $P = 28$

Upsample 1:2 $\rightarrow V(z)$: $\Delta\omega = 0.6\pi \Rightarrow Q = 12$

Dyadic 1:8 Upsampler



Suppose $X(z)$: BW = $0.8\pi \Leftrightarrow \alpha = 0.2$

Upsample 1:2 $\rightarrow U(z)$:

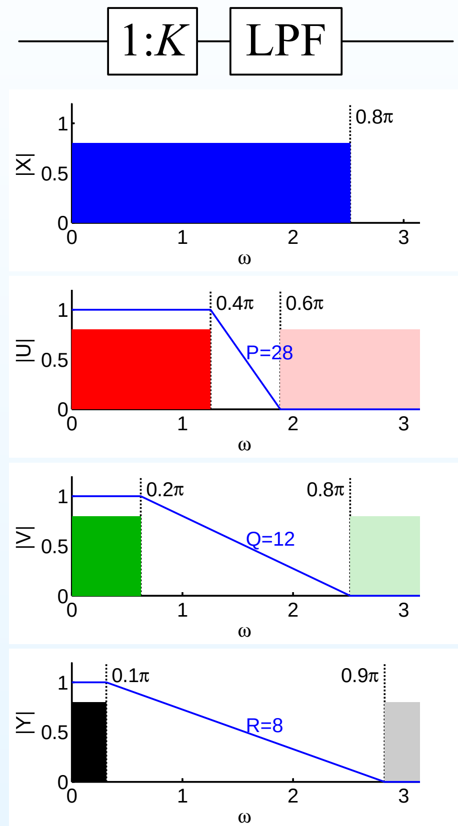
Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

For attenuation = 60 dB, $P \approx \frac{60}{3.5\Delta\omega} = 27.3$

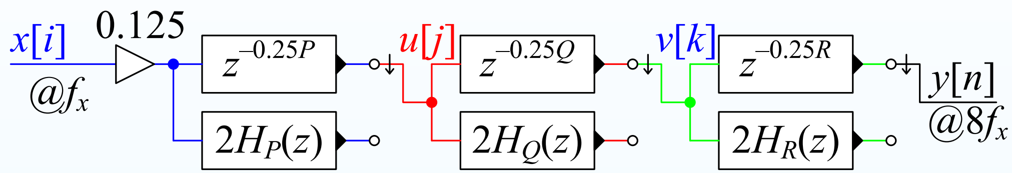
Round up to a multiple of 4: $P = 28$

Upsample 1:2 $\rightarrow V(z)$: $\Delta\omega = 0.6\pi \Rightarrow Q = 12$

Upsample 1:2 $\rightarrow Y(z)$: $\Delta\omega = 0.8\pi \Rightarrow R = 8$



Dyadic 1:8 Upsampler



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Upsample 1:2 $\rightarrow U(z)$:

Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

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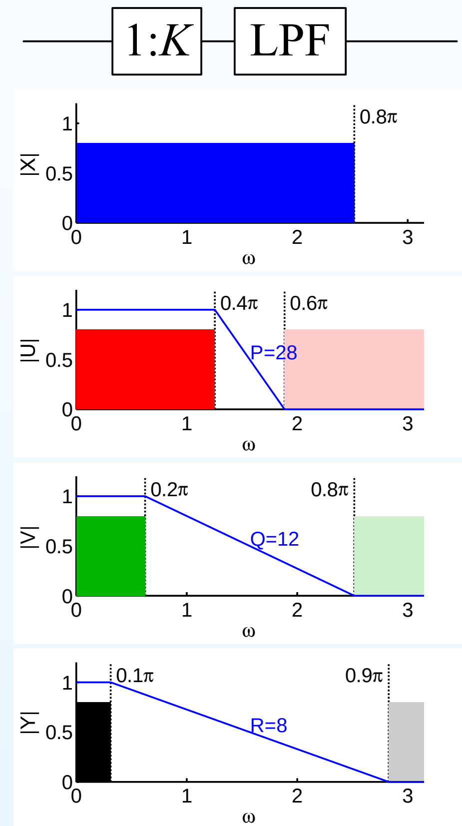
Round up to a multiple of 4: $P = 28$

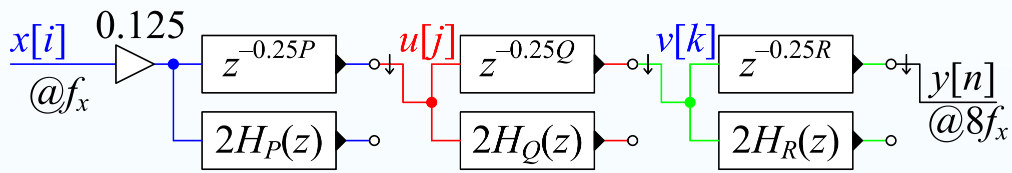
Upsample 1:2 $\rightarrow V(z)$: $\Delta\omega = 0.6\pi \Rightarrow Q = 12$

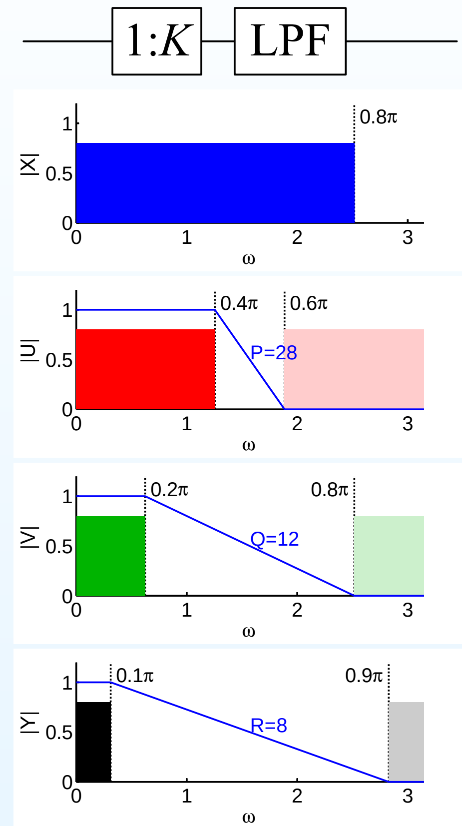
Upsample 1:2 $\rightarrow Y(z)$: $\Delta\omega = 0.8\pi \Rightarrow R = 8$

[diminishing returns + higher sample rate]



Dyadic 1:8 Upsampler





Suppose $X(z)$: BW = $0.8\pi \Leftrightarrow \alpha = 0.2$

Upsample 1:2 $\to U(z)$:

Filter $H_P(z)$ must remove image: $\Delta\omega = 0.2\pi$

For attenuation = 60 dB, $P \approx \frac{60}{3.5\Delta\omega} = 27.3$

Round up to a multiple of 4: $P = 28$

Upsample 1:2 $\to V(z)$: $\Delta\omega = 0.6\pi \Rightarrow Q = 12$

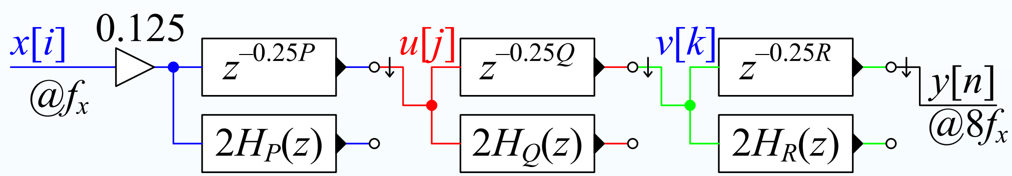
Upsample 1:2 $\to Y(z)$: $\Delta\omega = 0.8\pi \Rightarrow R = 8$

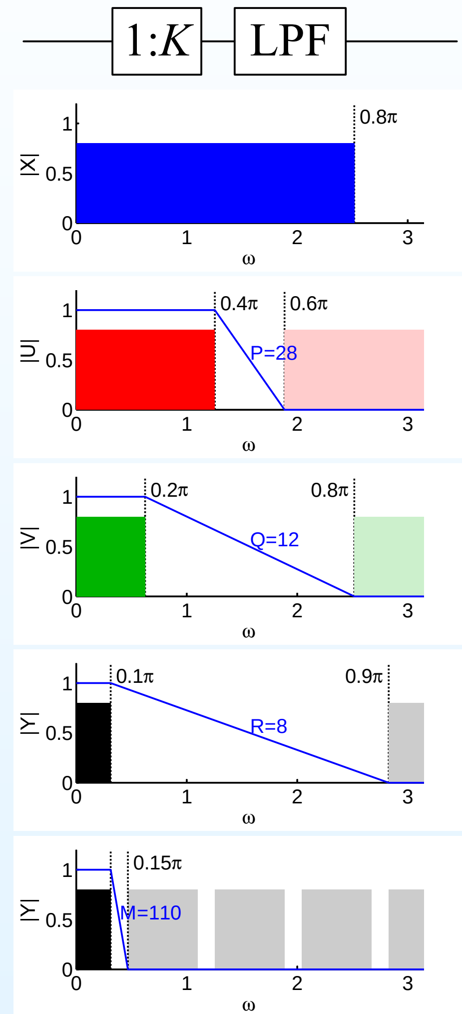
[diminishing returns + higher sample rate]

Multiplication Count:

$$\left(1 + \frac{P}{4}\right) \times f_x + \frac{Q}{4} \times 2f_x + \frac{R}{4} \times 4f_x = 22f_x$$

Dyadic 1:8 Upsampler





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Upsample 1:2 $\rightarrow V(z)$: $\Delta\omega = 0.6\pi \Rightarrow Q = 12$

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[diminishing returns + higher sample rate]

Multiplication Count:

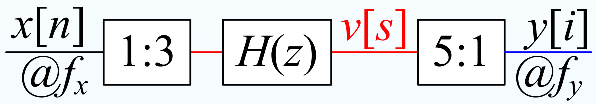
$$\left(1 + \frac{P}{4}\right) \times f_x + \frac{Q}{4} \times 2f_x + \frac{R}{4} \times 4f_x = 22f_x$$

Alternative approach using direct 1:8 upsampling:

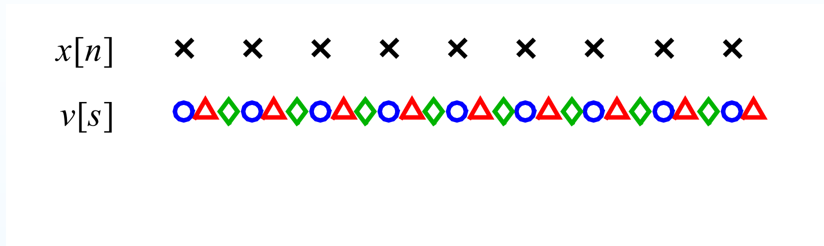
$\Delta\omega = 0.05\pi \Rightarrow M = 110 \Rightarrow 111f_x$ multiplications (using polyphase)

Rational Resampling

To resample by $\frac{P}{Q}$ do 1:P then LPF, then Q:1.

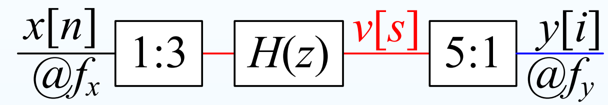


Rational Resampling

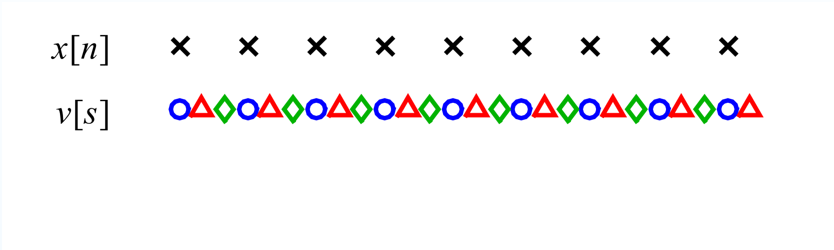


Resample by $\frac{P}{Q} \Rightarrow \omega_0 = \frac{\pi}{\max(P,Q)}$

To resample by $\frac{P}{Q}$ do 1:P then LPF, then Q:1.



Rational Resampling



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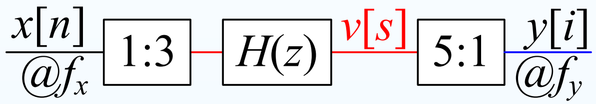

Resample by $\frac{P}{Q} \Rightarrow \omega_0 = \frac{\pi}{\max(P,Q)}$

$$\Delta\omega \triangleq 2\alpha\omega_0 = \frac{2\alpha\pi}{\max(P,Q)}$$

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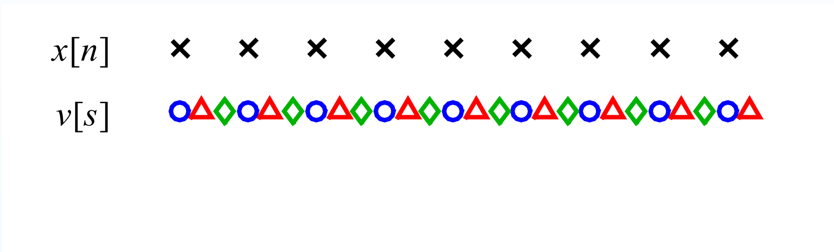

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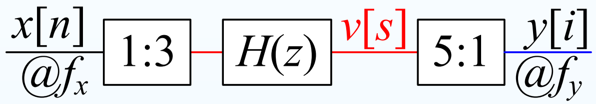

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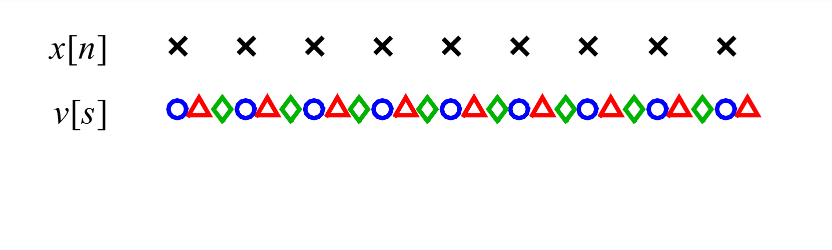
$$\Delta\omega \triangleq 2\alpha\omega_0 = \frac{2\alpha\pi}{\max(P,Q)}$$

Polyphase: $H(z) = \sum_{p=0}^{P-1} z^{-p} H_p(z^P)$

Rational Resampling



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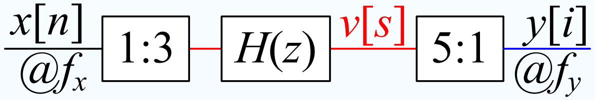

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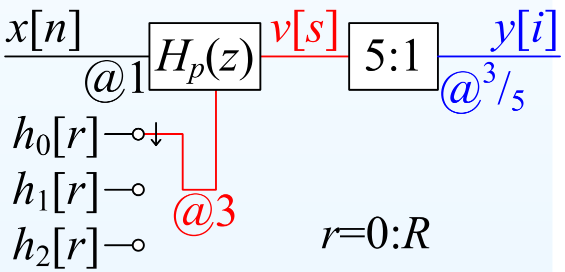

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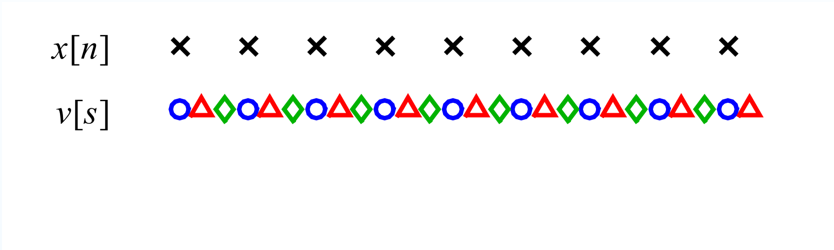

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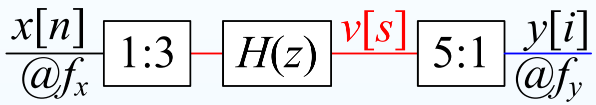

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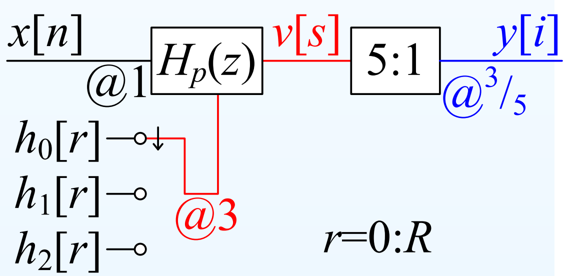
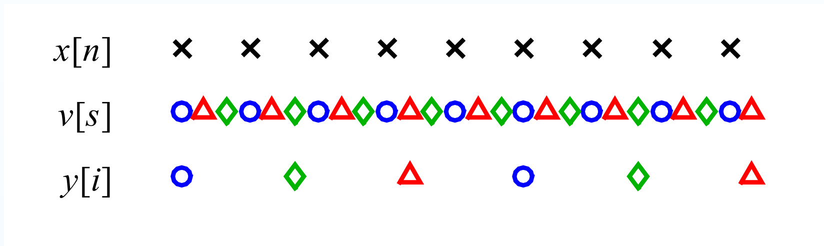
Commutate coefficients:

$v[s]$ uses $H_p(z)$ with $p = s \bmod P$



Rational Resampling



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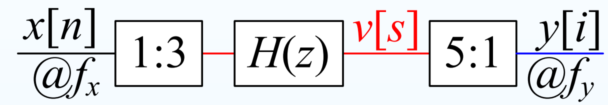
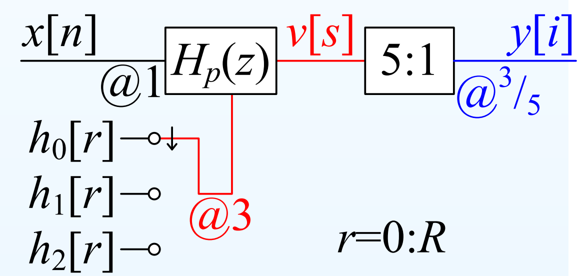
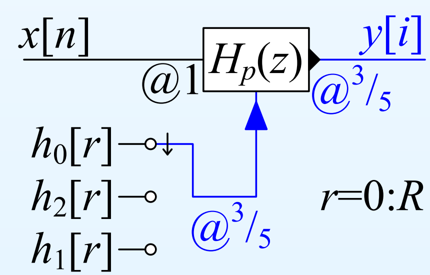
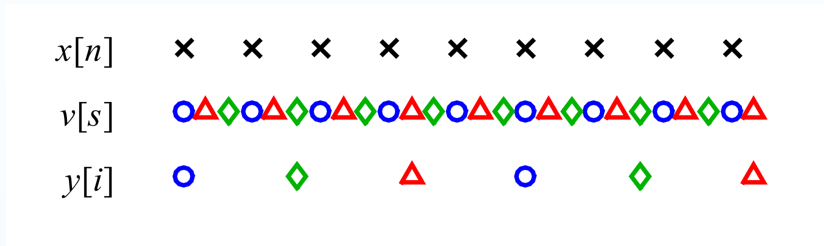






Rational Resampling



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Keep only every Q^{th} output:

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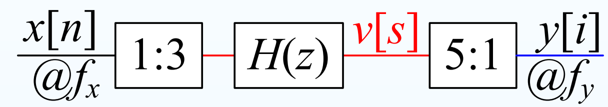
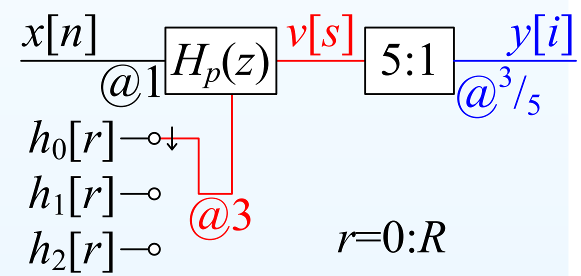
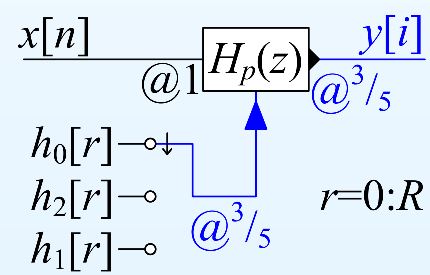
To resample by $\frac{P}{Q}$ do 1:P then LPF, then Q:1.







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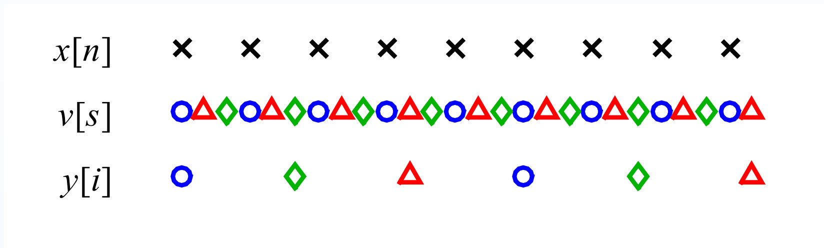

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Keep only every Q^{th} output:

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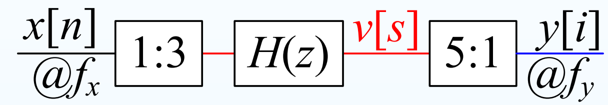
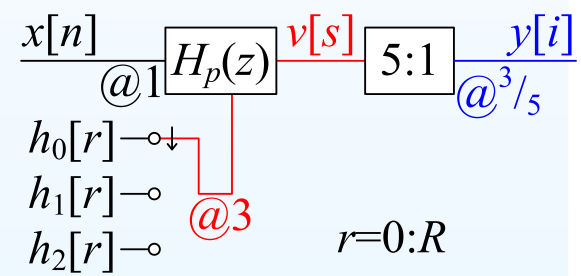
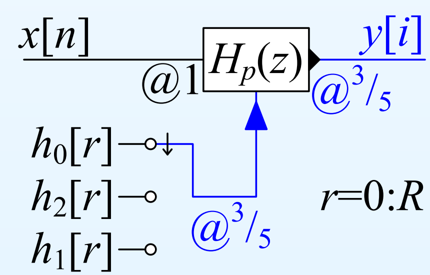
Multiplication Count:

$$H(z)\text{: } M+1 \approx \frac{60\text{ [dB]}}{3.5\Delta\omega} = \frac{2.7\max(P,Q)}{\alpha}$$

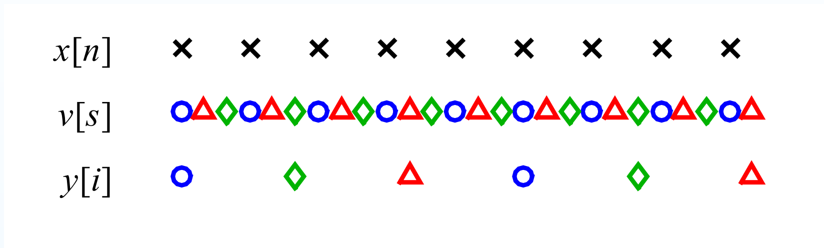
To resample by $\frac{P}{Q}$ do 1:P then LPF, then Q:1.







Rational Resampling



Resample by $\frac{P}{Q} \Rightarrow \omega_0 = \frac{\pi}{\max(P,Q)}$

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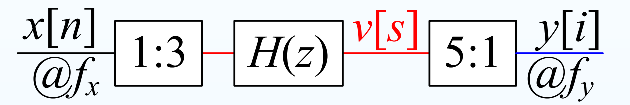
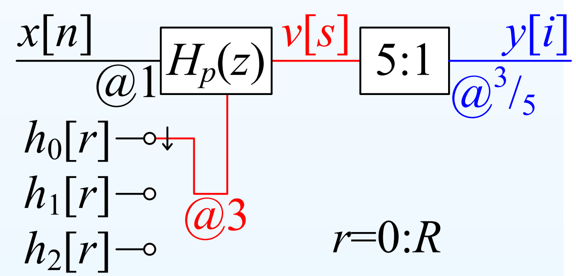
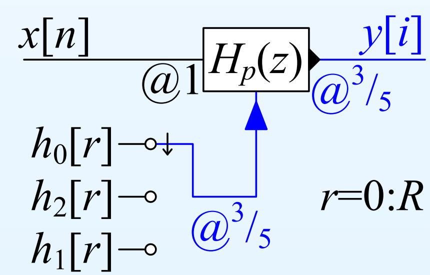
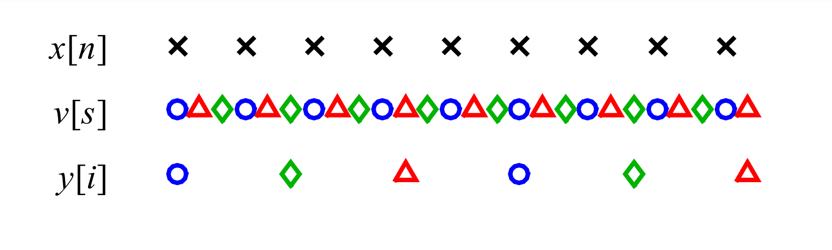






$M+1$ coeficients in all

Rational Resampling



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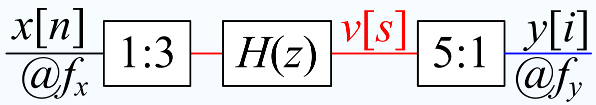

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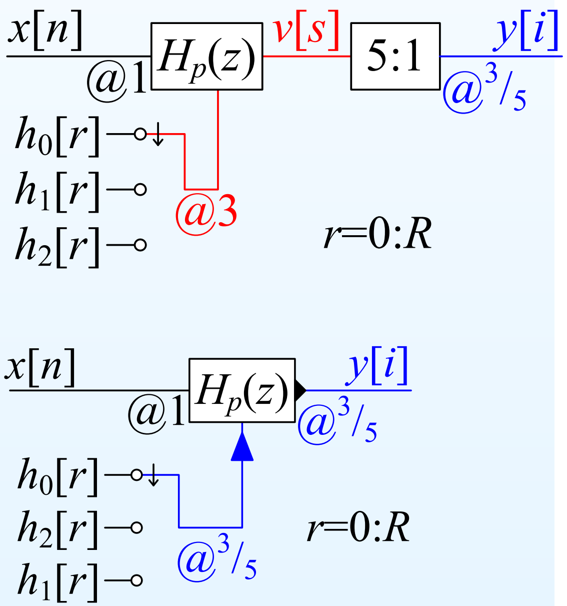

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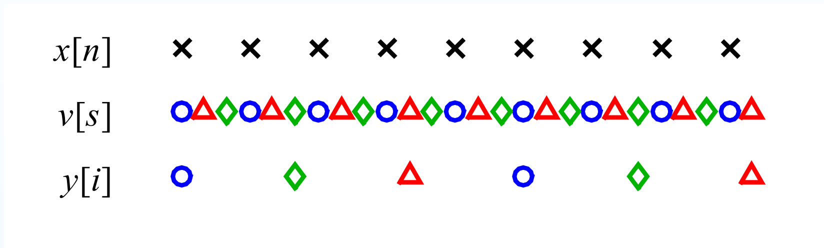
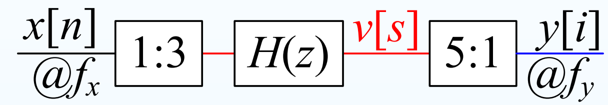
$M+1$ coeficients in all

Rational Resampling



To resample by $\frac{P}{Q}$ do 1:P then LPF, then Q:1.



Resample by $\frac{P}{Q} \Rightarrow \omega_0 = \frac{\pi}{\max(P,Q)}$

$\Delta\omega \triangleq 2\alpha\omega_0 = \frac{2\alpha\pi}{\max(P,Q)}$

Polyphase: $H(z) = \sum_{p=0}^{P-1} z^{-p} H_p(z^P)$

Commutate coefficients:

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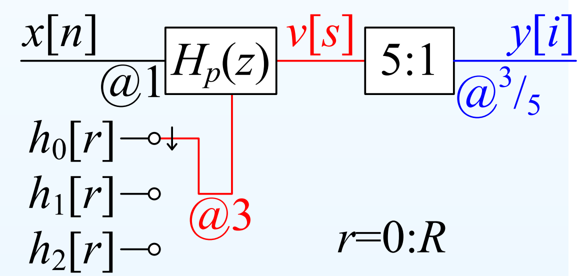

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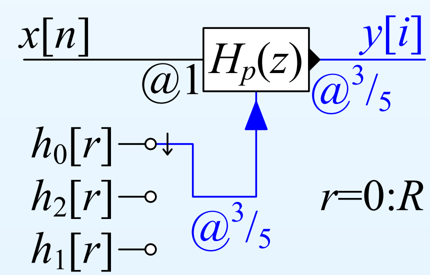

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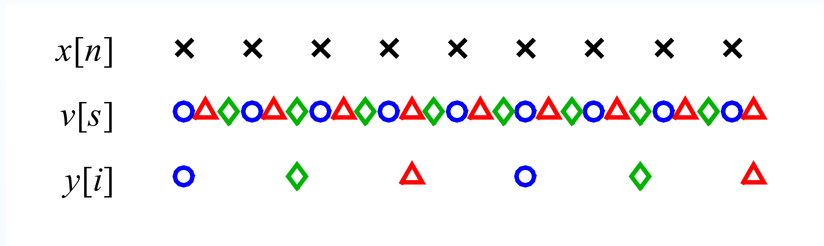

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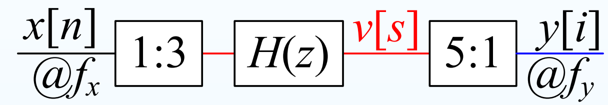

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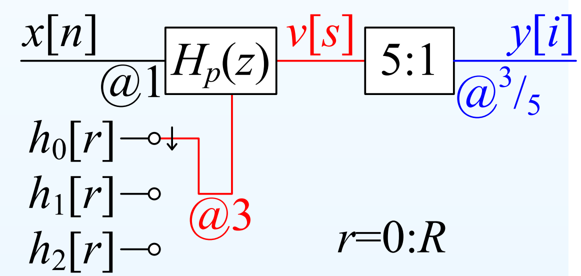

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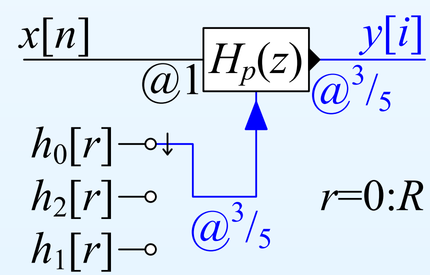

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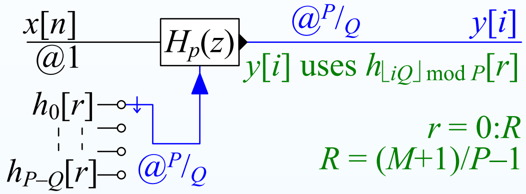

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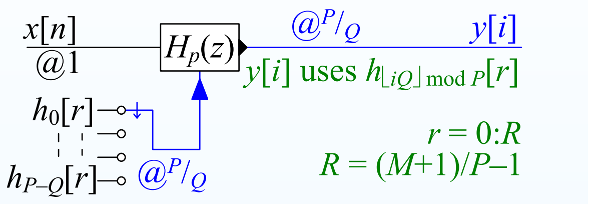

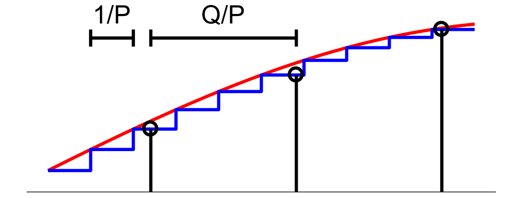

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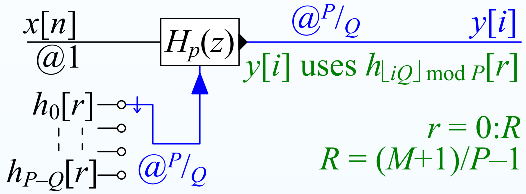

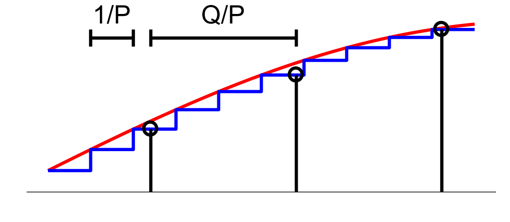

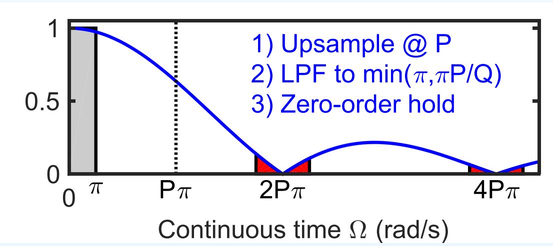

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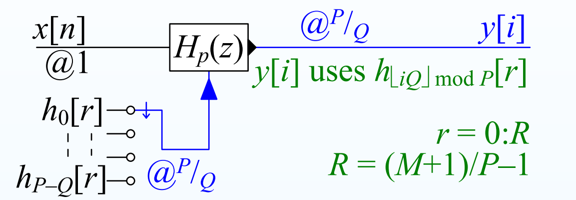

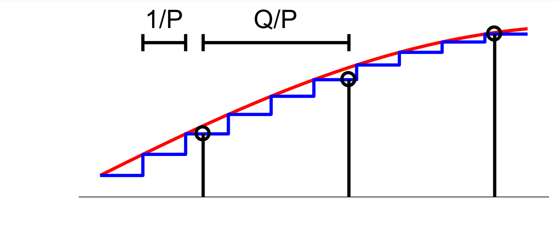

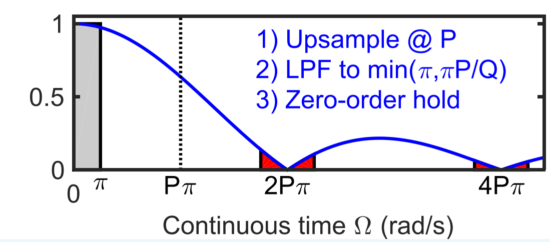

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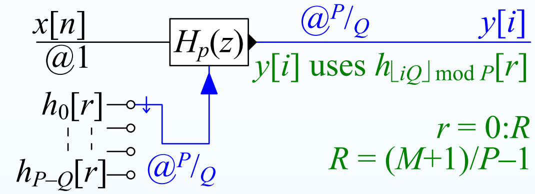

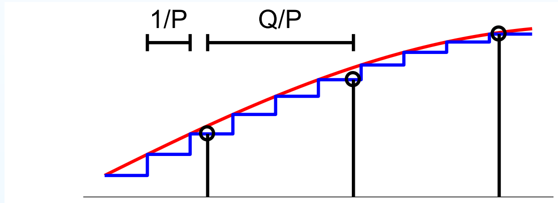

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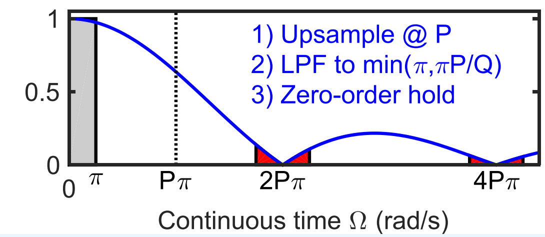

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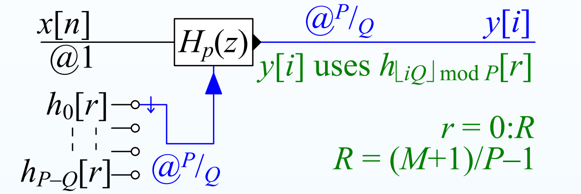

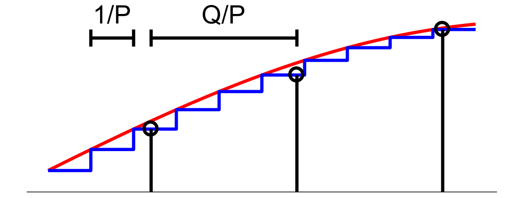

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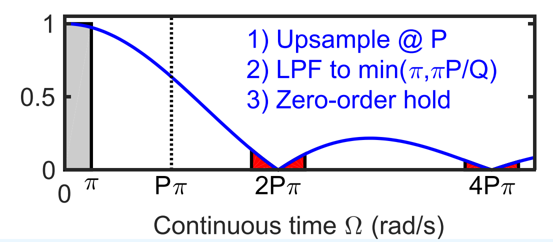

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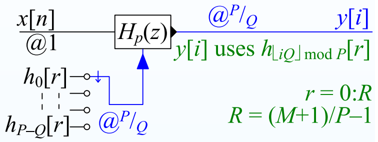
For worst case, $\Omega_1 = \pi$, need $P = 906$ to get -60 dB ☹

Polynomial Approximation

Suppose $P = 50$ and $H(z)$ has order $M = 249$
$H(z)$ is lowpass filter with $\omega_0 \approx \frac{\pi}{50}$



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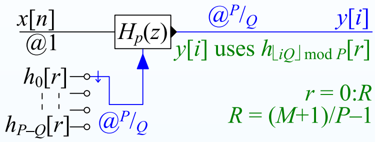

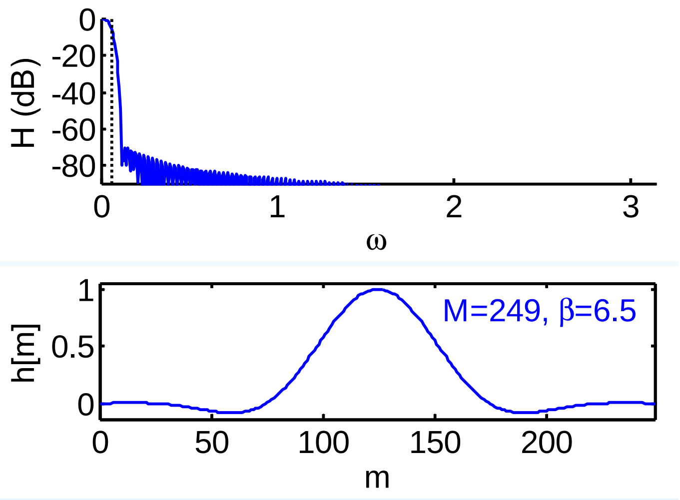

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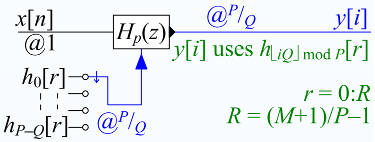
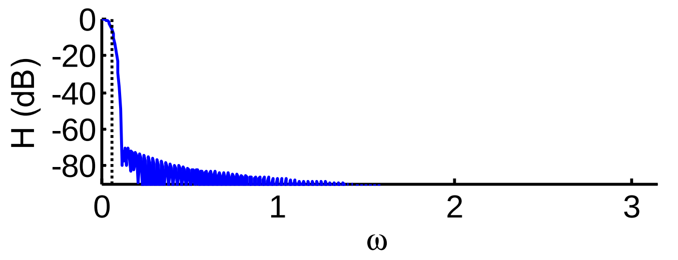
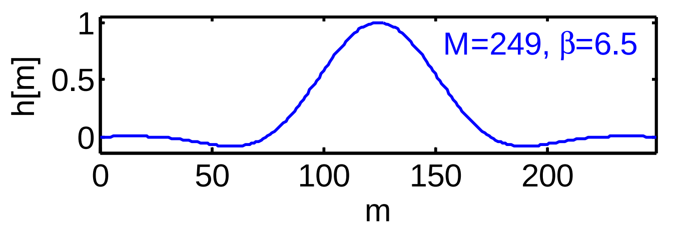
Split into 50 filters of length $R+1 = \frac{M+1}{P} = 5$:







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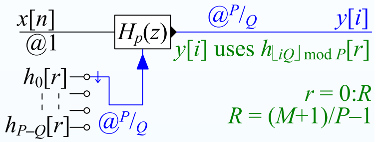

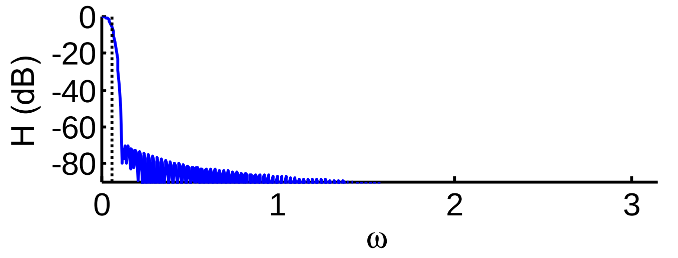

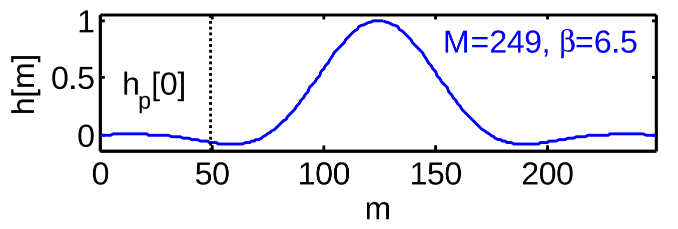

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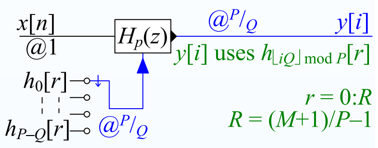

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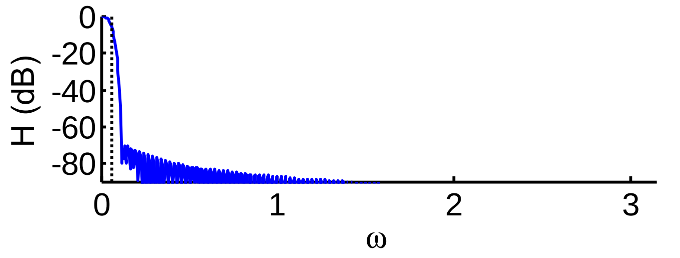

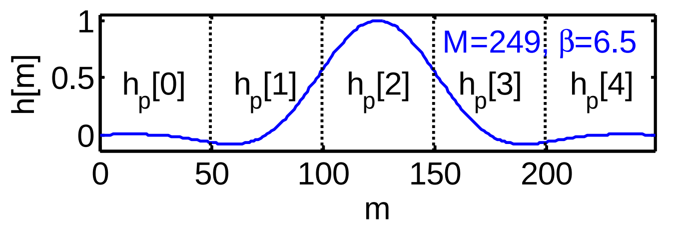

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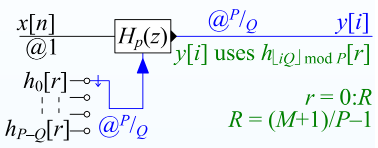

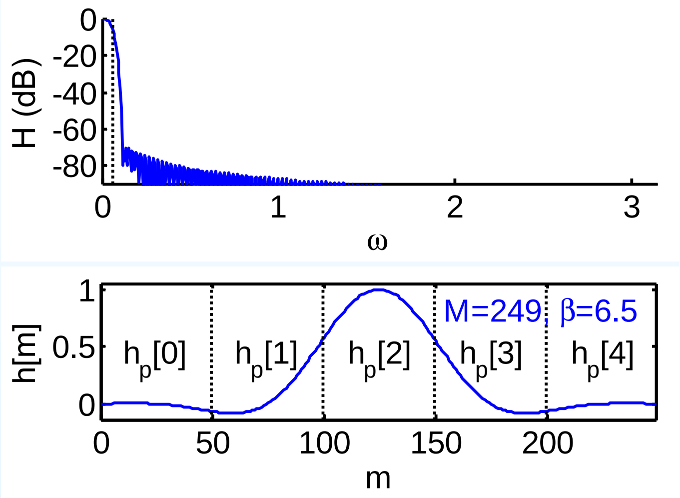

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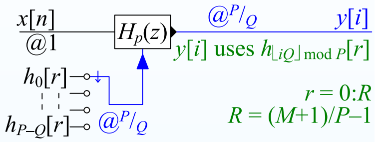

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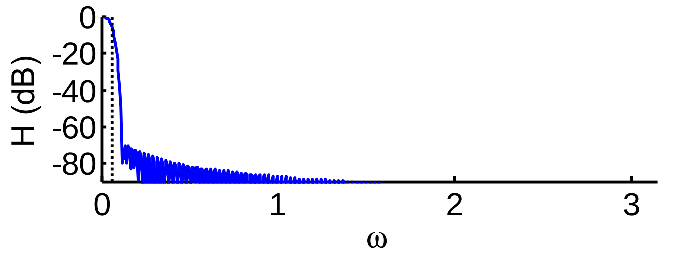
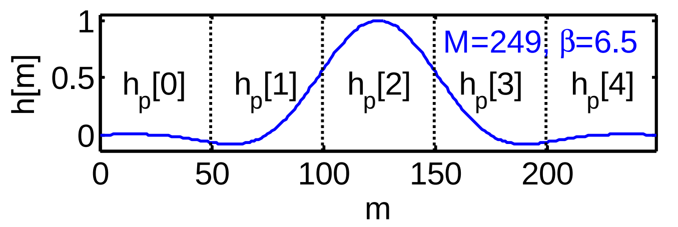


Use a polynomial of order L to approximate each segment:

$h_p[r] \approx f_r(\frac{p}{P})$ with $0 \le \frac{p}{P} < 1$



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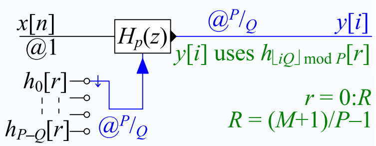

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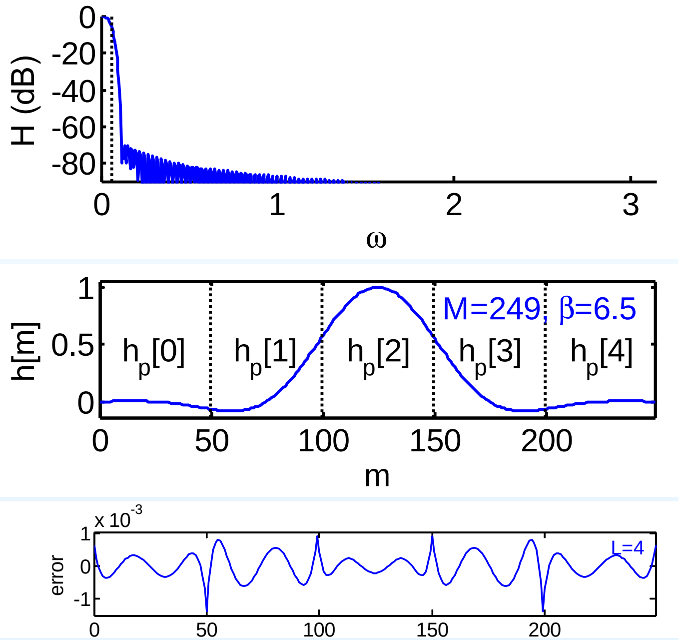
E.g. error $< 10^{-3}$ for $L = 4$



Polynomial Approximation

Suppose $P = 50$ and $H(z)$ has order $M = 249$

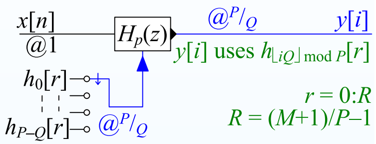
$H(z)$ is lowpass filter with $\omega_0 \approx \frac{\pi}{50}$

Split into 50 filters of length $R+1 = \frac{M+1}{P} = 5$:

$h_p[0]$ is the first P samples of $h[m]$

$h_p[1]$ is the next P samples, etc.

$h_p[r] = h[p + rP]$



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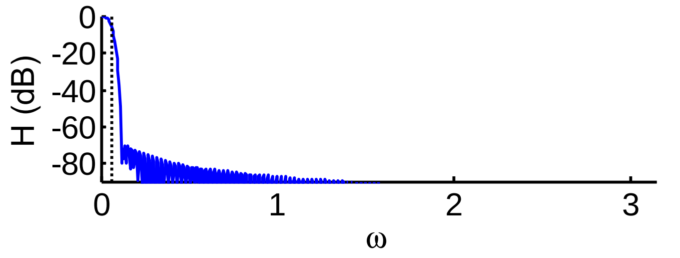

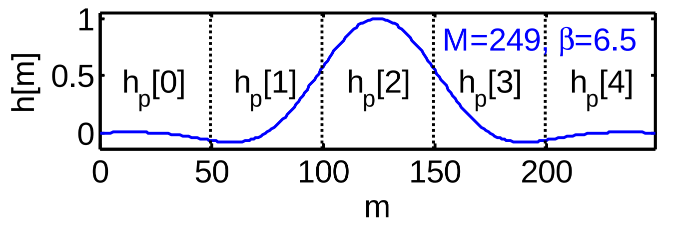

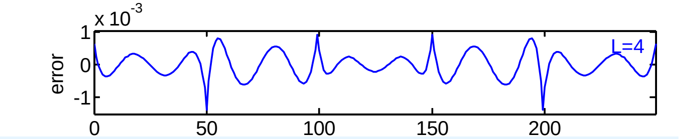

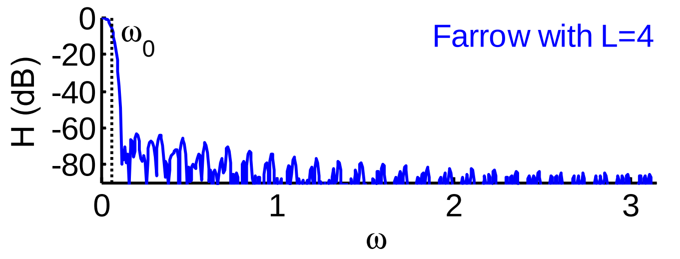

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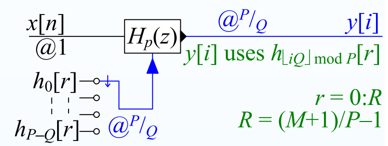

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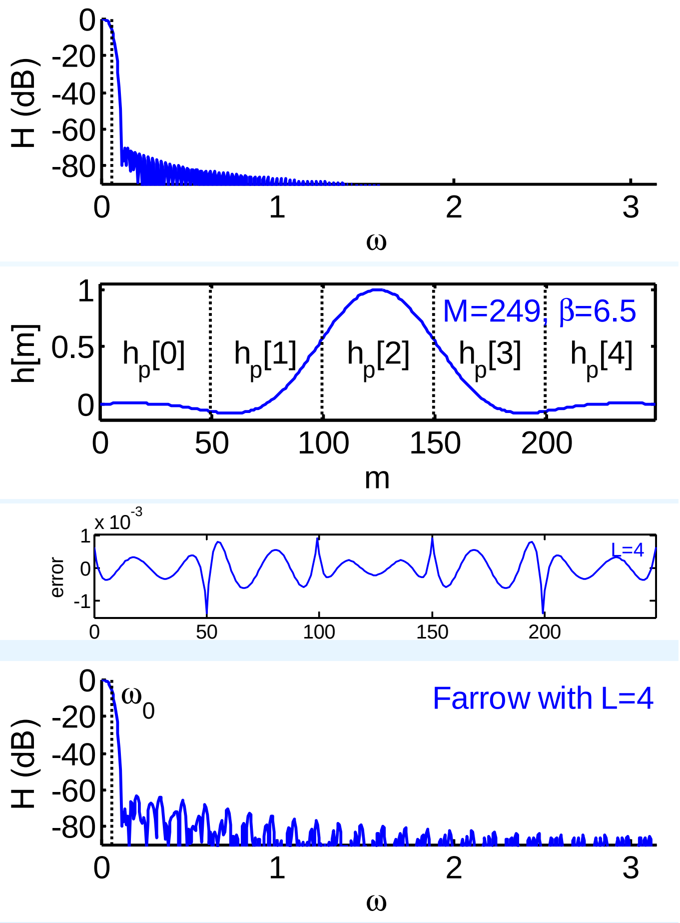

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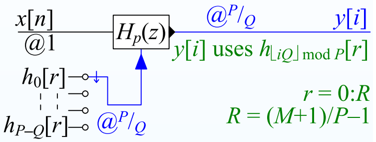

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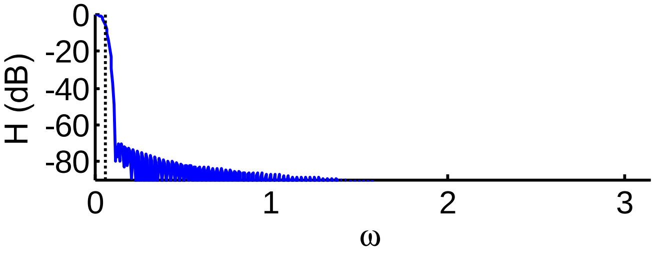

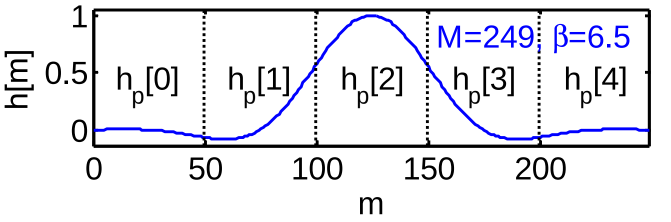

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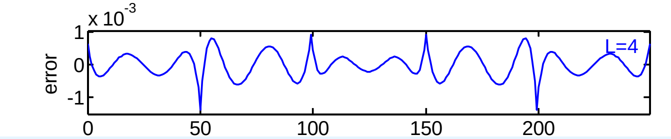

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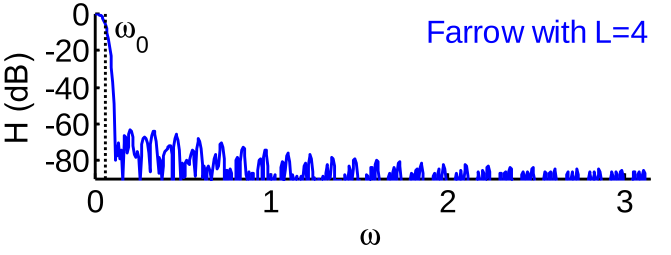

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Filter coefficients depend on fractional part of $i\frac{Q}{P}$:

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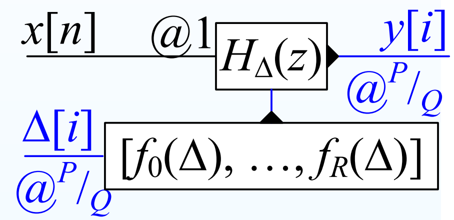

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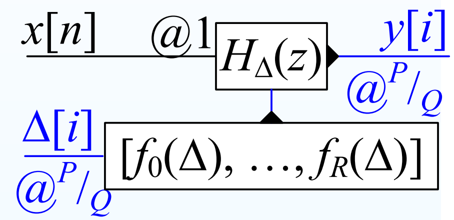

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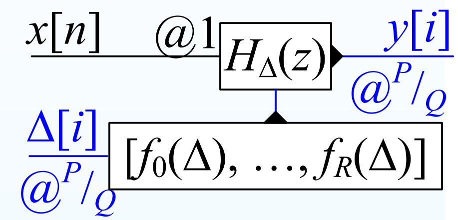

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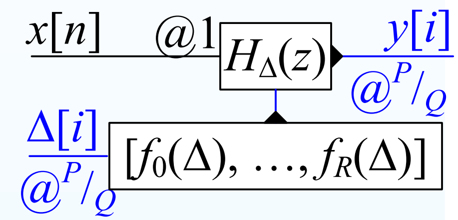

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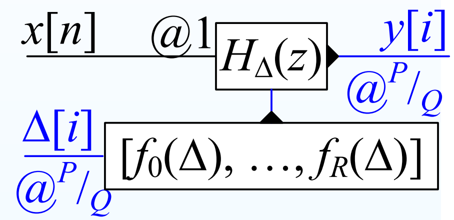

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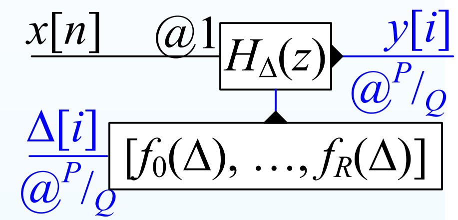

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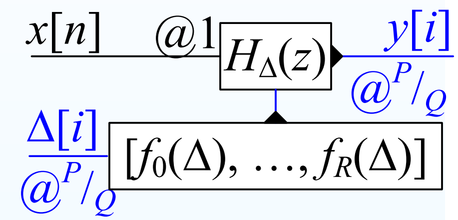

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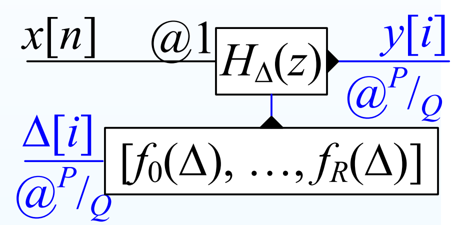

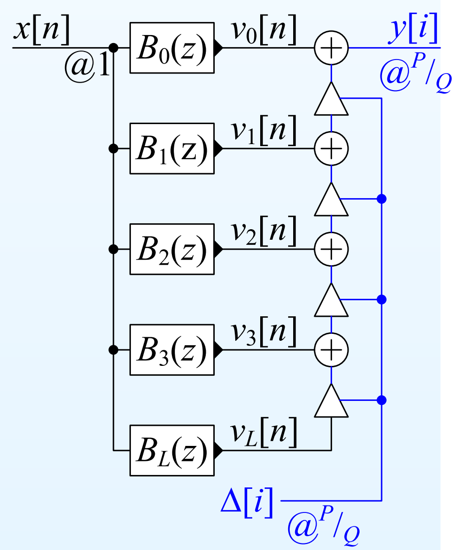

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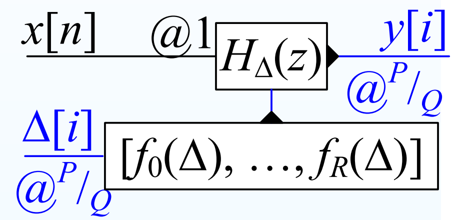

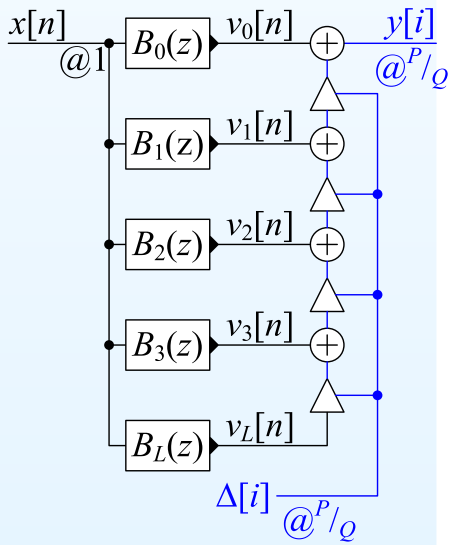

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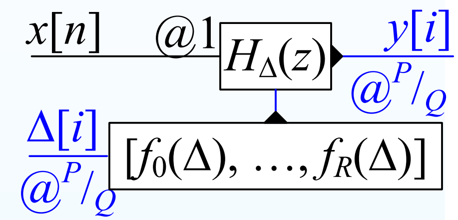

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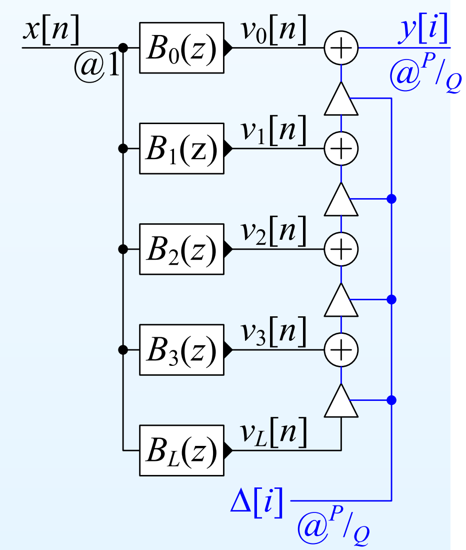

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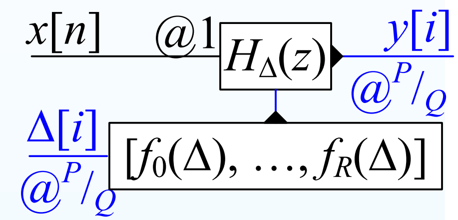

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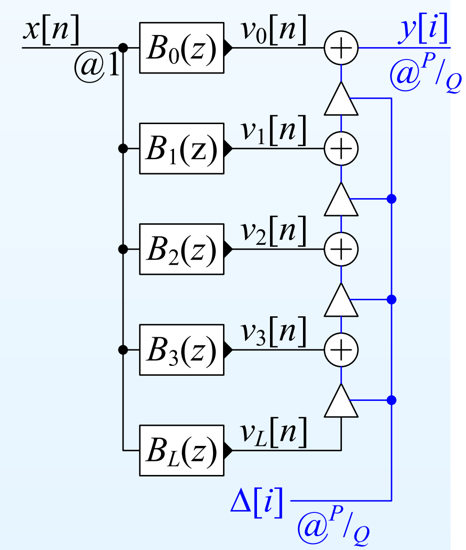

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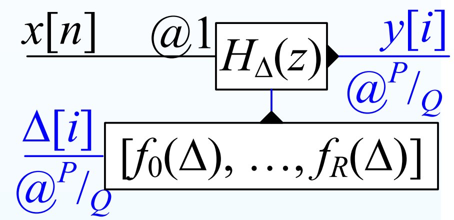

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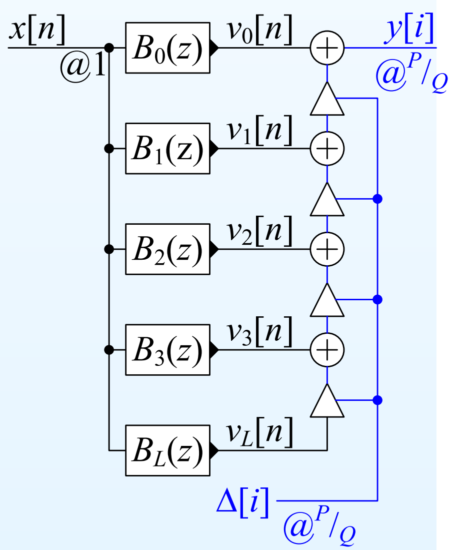
[like a Taylor series expansion]



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For further details see Mitra: 13 and Harris: 7, 8.

MATLAB routines

gcd(p,q)	Find $\alpha p + \beta q = 1$ for coprime p, q
polyfit	Fit a polynomial to data
polyval	Evaluate a polynomial
upfirdn	Perform polyphase filtering
resample	Perform polyphase resampling