14: FM Radio Receiver

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- Pilot tone extraction
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- Summary
FM Radio Block Diagram

FM spectrum: 87.5 to 108 MHz

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200 kHz per channel

[This example is taken from Ch 13 of Harris: Multirate Signal Processing]
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Mono (L + R): ±15 kHz

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Stereo (L – R): 38 ± 15 kHz

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**FM Modulation:**

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FM Modulation:
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L–R signal is multiplied by 38 kHz to shift it to baseband

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You can use an aliased analog-digital converter (ADC) provided that the target band fits entirely between two consecutive multiples of $\frac{1}{2}f_s$. 
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Lower ADC sample rate 😊. Image = undistorted frequency-shifted copy.
Channel Selection

FM band shifted to 7.5 to 28 MHz (from 87.5 to 108 MHz)
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We shift selected channel to DC and then downsample to $f_s = 400$ kHz. Assume channel centre frequency is $f_c = c \times 100$ kHz
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We will look at three methods:

1. Freq shift, then polyphase lowpass filter
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Assume channel centre frequency is \( f_c = c \times 100 \text{ kHz} \)

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We will look at three methods:

1. Freq shift, then polyphase lowpass filter
2. Polyphase bandpass complex filter
3. Polyphase bandpass real filter
Multiply by $e^{-j2\pi \frac{f_c}{80 \text{ MHz}}}$ to shift channel at $f_c$ to DC.

$f_c = c \times 100 \text{ k} \Rightarrow \frac{f_c}{80 \text{ M}} = \frac{c}{800}$
Channel Selection (1)

Multiply by $e^{-j2\pi \frac{f_c}{80\text{ MHz}}}$ to shift channel at $f_c$ to DC. 

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Result of multiplication is complex (thick lines on diagram)
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Next, lowpass filter to $\pm 100 \text{ kHz}$

$$\Delta \omega = 2\pi \frac{200 \text{ k}}{80 \text{ M}} = 0.157$$
**Channel Selection (1)**

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Next, **lowpass filter** to $\pm 100$ kHz

\[ \Delta \omega = 2\pi \frac{200\text{ k}}{80\text{M}} = 0.157 \]

\[ \Rightarrow M = \frac{60 \text{ dB}}{3.5\Delta \omega} = 1091 \]

![Diagram](image-url)
Channel Selection (1)

Multiply by \( e^{-j2\pi f_c/80 \text{MHz}} \) to shift channel at \( f_c \) to DC.

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f_c = c \times 100 \text{ k} \Rightarrow \frac{f_c}{80 \text{ MHz}} = \frac{c}{800}
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Finally, downsample 200 : 1
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Polyphase:

$H_p(z)$ has $\left\lceil\frac{1092}{200}\right\rceil = 6$ taps
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Next, **lowpass filter** to \( \pm 100 \text{kHz} \)

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**Polyphase:**

\[
H_p(z) \text{ has } \left\lceil \frac{1092}{200} \right\rceil = 6 \text{ taps}
\]

Complex data \( \times \) Real Coefficients (needs 2 multiplies per tap)

**Multiplication Load:**

\( 2 \times 80 \text{MHz} \text{ (freq shift)} + 12 \times 80 \text{MHz} (H_p(z)) = 14 \times 80 \text{MHz} \)
Channel Selection (2)

Channel centre frequency $f_c = c \times 100\,\text{kHz}$ where $c$ is an integer.
Channel Selection (2)

Channel centre frequency \( f_c = c \times 100 \text{ kHz} \) where \( c \) is an integer. Write \( c = 4k + l \)
where \( k = \left\lfloor \frac{c}{4} \right\rfloor \) and \( l = c \mod 4 \)
Channel Selection (2)

Channel centre frequency $f_c = c \times 100 \text{ kHz}$ where $c$ is an integer.

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$e^{-j2\pi rc/800}$

$u[r]$  \hspace{1cm} H(z) \hspace{1cm} 200:1 \hspace{1cm} v[n]$
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\[
e^{-j2\pi rc/800}
\]

\[
u[r] @80M \times H(z) \xrightarrow{200:1} v[n] @400k
\]

We multiply \( u[r] \) by \( e^{-j2\pi r \frac{c}{800}} \), convolve with \( h[m] \) and then downsample:

\[
v[n] = \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi (200n-m) \frac{c}{800}} \quad [r = 200n]
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\[
e^{-j2\pi rc/800}
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\[
\begin{align*}
\frac{u[r]}{@80M} \times H(z) \rightarrow 200:1 \rightarrow \frac{v[n]}{@400k}
\end{align*}
\]

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v[n] = \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi (200n - m) \frac{c}{800}}
\]

\[
= \sum_{m=0}^{M} h[m] e^{j2\pi \frac{mc}{800}} u[200n - m] e^{-j2\pi 200n \frac{4k+l}{800}} \quad [r = 200n] \]

\[
[c = 4k + 1]
\]
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$$e^{-j2\pi rc/800}$$

$$\begin{align*}
\frac{u[r]}{800M} \times H(z) & \downarrow 200:1
\end{align*}$$

$$\frac{v[n]}{400k}$$

We multiply $u[r]$ by $e^{-j2\pi r \frac{c}{800}}$, convolve with $h[m]$ and then downsample:

$$v[n] = \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi (200n-m) \frac{c}{800}} \quad [r = 200n]$$

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$$= \sum_{m=0}^{M} g[c][m] u[200n - m] e^{-j2\pi \frac{ln}{4}} \quad [g[c][m] \triangleq h[m] e^{j2\pi \frac{mc}{800}}]$$
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\[ e^{-j2\pi c/800} \]

\[ u[r] \]

\[ H(z) \]

\[ 200:1 \]

\[ v[n] \]

\[ @80M \]

\[ @400k \]

We multiply \( u[r] \) by \( e^{-j2\pi r \frac{c}{800}} \), convolve with \( h[m] \) and then downsample:

\[
\begin{align*}
v[n] &= \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi (200n - m) \frac{c}{800}} & [r = 200n] \\
&= \sum_{m=0}^{M} h[m] e^{j2\pi \frac{mc}{800}} u[200n - m] e^{-j2\pi 200n \frac{4k+l}{800}} & [c = 4k + 1] \\
&= \sum_{m=0}^{M} g[c][m] u[200n - m] e^{-j2\pi \frac{ln}{4}} & [g[c][m] \triangleq h[m] e^{j2\pi \frac{mc}{800}}] \\
&= (-j)^{ln} \sum_{m=0}^{M} g[c][m] u[200n - m] & [e^{-j2\pi \frac{ln}{4}} \text{ indep of } m]
\end{align*}
\]
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Channel centre frequency $f_c = c \times 100 \text{ kHz}$ where $c$ is an integer.

Write $c = 4k + l$

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We multiply $u[r]$ by $e^{-j 2\pi r \frac{c}{800}}$, convolve with $h[m]$ and then downsample:

$$v[n] = \sum_{m=0}^{M} h[m] u[200n - m] e^{-j 2\pi (200n - m) \frac{c}{800}} \quad [r = 200n]$$

$$= \sum_{m=0}^{M} h[m] e^{j 2\pi \frac{mc}{800}} u[200n - m] e^{-j 2\pi 200n \frac{4k+l}{800}} \quad [c = 4k + 1]$$

$$= \sum_{m=0}^{M} g[c][m] u[200n - m] e^{-j 2\pi \frac{ln}{4}} \quad [g[c][m] \triangleq h[m] e^{j 2\pi \frac{mc}{800}}]$$

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\[
u[r] \rightarrow 80\text{M} \quad G_c(z) \quad 200:1 \quad \times \quad v[n] \rightarrow 400\text{k}
\]

We multiply \( u[r] \) by \( e^{-j2\pi \frac{r}{800}} \), convolve with \( h[m] \) and then downsample:

\[
v[n] = \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi \frac{200n-m}{800}}
\]

\[
= \sum_{m=0}^{M} h[m] e^{j2\pi \frac{mc}{800}} u[200n - m] e^{-j2\pi 200n \frac{4k+l}{800}} \quad [r = 200n]
\]

\[
= \sum_{m=0}^{M} g[c][m] u[200n - m] e^{-j2\pi \frac{ln}{4}} \quad [g[c][m] \triangleq h[m] e^{j2\pi \frac{mc}{800}}]
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= (-j)^{ln} \sum_{m=0}^{M} g[c][m] u[200n - m] \quad [e^{-j2\pi \frac{ln}{4}} \text{ indep of } m]
\]

Multiplication Load for polyphase implementation:

\( G_c,p(z) \) has complex coefficients \( \times \) real input \( \Rightarrow \) 2 mults per tap
Channel centre frequency \( f_c = c \times 100 \text{ kHz} \) where \( c \) is an integer.

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v[n] = \sum_{m=0}^{M} h[m]u[200n - m]e^{-j2\pi(200n-m)\frac{c}{800}} \\
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Multiplication Load for polyphase implementation:

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\((-j)^{ln} \in \{+1, -j, -1, +j\} \) so no actual multiplies needed
Channel Selection (2)

Channel centre frequency \( f_c = c \times 100 \text{ kHz} \) where \( c \) is an integer.

Write \( c = 4k + l \)

where \( k = \left\lfloor \frac{c}{4} \right\rfloor \) and \( l = c_{\text{mod} \ 4} \)

\[
\begin{align*}
\text{we multiply } u[r] & \text{ by } e^{-j2\pi \frac{r}{800}} , \text{ convolve with } h[m] \text{ and then downsample:} \\
v[n] & = \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi\frac{200n-m}{800}} c \\
& = \sum_{m=0}^{M} h[m] e^{j2\pi\frac{mc}{800}} u[200n - m] e^{-j2\pi200n\frac{4k+l}{800}} [c = 4k + 1] \\
& = \sum_{m=0}^{M} g[c][m] u[200n - m] e^{-j2\pi\frac{ln}{4}} [g[c][m] \triangleq h[m] e^{j2\pi\frac{mc}{800}} ] \\
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\end{align*}
\]

Multiplication Load for polyphase implementation:

\( G[c,p](z) \) has complex coefficients \( \times \) real input \( \Rightarrow \) 2 mults per tap

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Total: \( 12 \times 80 \text{ MHz} \) (for \( G[c,p](z) \)) + 0 (for \(-j^{ln}) = 12 \times 80 \text{ MHz} \)
Channel Selection (3)

Channel frequency \( f_c = c \times 100 \text{ kHz} \) where \( c = 4k + l \) is an integer

\[
f_c = c \times 100 \text{ kHz}
\]

\[
f_c = 4k + l
\]

where \( c \) is an integer.

\[
u[r] \quad \text{@80M}
\]

\[
G_{[c],0}(z)
\]

\[
G_{[c],1}(z)
\]

\[
G_{[c],199}(z)
\]

\[
v[n] \quad \text{@400k}
\]

\[
@400k
\]

\[
@400k
\]

\[
@400k
\]
Channel Selection (3)

Channel frequency \( f_c = c \times 100 \text{kHz} \) where \( c = 4k + l \) is an integer

\[
g_c[m] = h[m]e^{j2\pi \frac{cm}{800}}
\]
Channel frequency $f_c = c \times 100 \text{ kHz}$ where $c = 4k + l$ is an integer

\[ g[c][m] = h[m]e^{j2\pi \frac{cm}{800}} \]

\[ g[c],p[s] = g_c[200s + p] = h[200s + p]e^{j2\pi \frac{c(200s+p)}{800}} \]
Channel Selection (3)

Channel frequency \( f_c = c \times 100 \text{ kHz} \) where \( c = 4k + l \) is an integer

\[
g[c][m] = h[m]e^{j2\pi \frac{cm}{800}}
\]

\[
g[c,p][s] = g_c[200s + p] = h[200s + p]e^{j2\pi \frac{c(200s+p)}{800}} \quad \text{[polyphase]}
\]

\[
= h[200s + p]e^{j2\pi \frac{c_s}{4}} e^{j2\pi \frac{cp}{800}}
\]
Channel Selection (3)

Channel frequency \( f_c = c \times 100 \text{ kHz} \) where \( c = 4k + l \) is an integer

\[
g[c][m] = h[m]e^{j2\pi \frac{cm}{800}}
\]

\[
g[c,p][s] = g_c[200s + p] = h[200s + p]e^{j2\pi \frac{c(200s+p)}{800}} \tag{polyphase}
\]

\[
= h[200s + p]e^{j2\pi \frac{c\xi}{4}}e^{j2\pi \frac{cp}{800}} \triangleq h[200s + p]e^{j2\pi \frac{c\xi}{4} \alpha^p}
\]
**Channel Selection (3)**

Channel frequency $f_c = c \times 100 \text{ kHz}$ where $c = 4k + l$ is an integer

\[ g[c][m] = h[m]e^{j2\pi \frac{cm}{800}} \]

\[ g[c],[s] = g_c[200s + p] = h[200s + p]e^{j2\pi \frac{c(200s+p)}{800}} \quad \text{[polyphase]} \]

\[ = h[200s + p]e^{j2\pi \frac{c}{4}} e^{j2\pi \frac{c}{800} p} \triangleq h[200s + p]e^{j2\pi \frac{c}{4} \alpha^p} \]

Define $f[c],[s] = h[200s + p]e^{j2\pi \frac{(4k+l)s}{4}} = jls h[200s + p]$
Channel Selection (3)

**Channel frequency** $f_c = c \times 100 \text{ kHz}$ where $c = 4k + l$ is an integer

\[ g[c][m] = h[m]e^{j2\pi \frac{cm}{800}} \]

\[ g[c],p[s] = g_c[200s + p] = h[200s + p]e^{j2\pi \frac{c(200s + p)}{800}} \quad [\text{polyphase}] \]

\[ = h[200s + p]e^{j2\pi \frac{cs}{4}} e^{j2\pi \frac{cp}{800}} \triangleq h[200s + p]e^{j2\pi \frac{cs}{4}} \alpha^p \]

**Define** $f[c],p[s] = h[200s + p]e^{j2\pi \frac{(4k+l)s}{4}} = jls h[200s + p]$

Although $f[c],p[s]$ is complex it requires only one multiplication per tap because each tap is either purely real or purely imaginary.
Channel frequency \( f_c = c \times 100 \text{ kHz} \) where \( c = 4k + l \) is an integer

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g[c, p][s] = g_c[200s + p] = h[200s + p]e^{j2\pi \frac{c(200s+p)}{800}} \quad \text{[polyphase]}
\]

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define f[c, p][s] = h[200s + p]e^{j2\pi \frac{(4k+l)s}{4}} = jls h[200s + p]
\]

Although \( f[c, p][s] \) is complex it requires only one multiplication per tap because each tap is either purely real or purely imaginary.

Multiplication Load:

\[
6 \times 80 \text{ MHz} \ (F_p(z)) + 4 \times 80 \text{ MHz} \ (\times e^{j2\pi \frac{c_p}{800}}) = 10 \times 80 \text{ MHz}
\]
Complex FM signal centred at DC: $v(t) = |v(t)| e^{j\phi(t)}$
Complex FM signal centred at DC: \( v(t) = |v(t)|e^{j\phi(t)} \)

We know that \( \log v = \log |v| + j\phi \)
Complex FM signal centred at DC: \( v(t) = |v(t)|e^{j\phi(t)} \)

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The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).
Complex FM signal centred at DC: \( v(t) = |v(t)| e^{j\phi(t)} \)

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The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).

We need to calculate \( x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} \)
Complex FM signal centred at DC: \( v(t) = |v(t)| e^{j\phi(t)} \)

We know that \( \log v = \log |v| + j\phi \)

The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).

We need to calculate \( x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im\left( \frac{1}{v} \frac{dv}{dt} \right) \)
**FM Demodulator**

Complex FM signal centred at DC: \( v(t) = |v(t)| e^{j\phi(t)} \)

We know that \( \log v = \log |v| + j\phi \)

The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).

We need to calculate \( x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im \left( \frac{1}{|v|} \frac{dv}{dt} \right) = \frac{1}{|v|^2} \Im \left( v^* \frac{dv}{dt} \right) \)
FM Demodulator

Complex FM signal centred at DC: \( v(t) = |v(t)| e^{j\phi(t)} \)

We know that \( \log v = \log |v| + j\phi \)

The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).

We need to calculate \( x(t) = \frac{d\phi}{dt} = \frac{d\Re(\log v)}{dt} = \Re \left( \frac{1}{|v|} \frac{dv}{dt} \right) = \frac{1}{|v|^2} \Im (v^* \frac{dv}{dt}) \)
Complex FM signal centred at DC: \( v(t) = |v(t)| e^{j\phi(t)} \)
We know that \( \log v = \log |v| + j\phi \)

The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).

We need to calculate \( x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im(\frac{1}{v} \frac{dv}{dt}) = \frac{1}{|v|^2} \Im(v^* \frac{dv}{dt}) \)

We need:
(1) Differentiation filter, \( D(z) \)
Complex FM signal centred at DC: $v(t) = |v(t)| e^{j\phi(t)}$

We know that $\log v = \log |v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

We need to calculate $x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im\left(\frac{1}{v} \frac{dv}{dt}\right) = \frac{1}{|v|^2} \Im\left(v^* \frac{dv}{dt}\right)$

We need:

1. Differentiation filter, $D(z)$
2. Complex multiply, $w[n] \times v^*[n]$ (only need $\Im$ part)
Complex FM signal centred at DC: \( v(t) = |v(t)|e^{j\phi(t)} \)

We know that \( \log v = \log |v| + j\phi \)

The instantaneous frequency of \( v(t) \) is \( \frac{d\phi}{dt} \).

We need to calculate \( x(t) = \frac{d\phi}{dt} = \frac{d \Im(\log v)}{dt} = \Im \left( \frac{1}{|v|} \frac{dv}{dt} \right) = \frac{1}{|v|^2} \Im \left( v^* \frac{dv}{dt} \right) \)

We need:

1. Differentiation filter, \( D(z) \)
2. Complex multiply, \( w[n] \times v^*[n] \) (only need \( \Im \) part)
3. Real Divide by \( |v|^2 \)
Complex FM signal centred at DC: $v(t) = |v(t)| e^{j\phi(t)}$

We know that $\log v = \log |v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

We need to calculate $x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im\left(\frac{1}{v} \frac{dv}{dt}\right) = \frac{1}{|v|^2} \Im\left(v^* \frac{dv}{dt}\right)$

**We need:**

1. Differentiation filter, $D(z)$
2. Complex multiply, $w[n] \times v^*[n]$ (only need $\Im$ part)
3. Real Divide by $|v|^2$

$x[n]$ is baseband signal (real):
Differentiation Filter

\[ \frac{v[n]}{D(z)} w[n] \]

14: FM Radio Receiver
- FM Radio Block Diagram
- Aliased ADC
- Channel Selection
- Channel Selection (1)
- Channel Selection (2)
- Channel Selection (3)
- FM Demodulator
  - Differentiation Filter
- Pilot tone extraction
- Polyphase Pilot tone
- Summary
Differentiation Filter

Window design method:
(1) calculate $d[n]$ for the ideal filter
(2) multiply by a window to give finite support

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Differentiation Filter

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1. calculate $d[n]$ for the ideal filter
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Differentiation: $\frac{d}{dt}e^{j\omega t} = j\omega e^{j\omega t}$
Differentiation Filter

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Differentiation: $\frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t}$  \Rightarrow  $D(e^{j\omega}) = \begin{cases} j\omega & |\omega| \leq \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases}$
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Hence $d[n] = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} j\omega e^{j\omega n} d\omega = \frac{j}{2\pi} \left[ \frac{e^{j\omega n}}{jn} - \frac{e^{j\omega n}}{j^2n^2} \right]_{-\omega_0}^{\omega_0} [\text{IDTFT}]$

\[ = \frac{n\omega_0 \cos n\omega_0 - \sin n\omega_0}{\pi n^2} \]
Differentiation Filter

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Differentiation:
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\end{cases}
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\[
d[n] = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} j\omega e^{jn\omega} d\omega = \frac{j}{2\pi} \left[ \frac{e^{-jn\omega}}{jn} - \frac{e^{jn\omega}}{j^2 n^2} \right]_{-\omega_0}^{\omega_0} = n\omega_0 \cos n\omega_0 - \frac{\sin n\omega_0}{\pi n^2}
\]

Using \( M = 18 \), Kaiser window, \( \beta = 7 \) and \( \omega_0 = 2.2 = \frac{2\pi \times 140 \text{ kHz}}{400 \text{ kHz}} \):
Differentiation Filter

Window design method:
(1) calculate $d[n]$ for the ideal filter
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$= \frac{n\omega_0 \cos n\omega_0 - \sin n\omega_0}{\pi n^2}$

Using $M = 18$, Kaiser window, $\beta = 7$ and $\omega_0 = 2.2 = \frac{2\pi \times 140\text{ kHz}}{400\text{ kHz}}$:
Near perfect differentiation for $\omega \leq 1.6$ ($\approx 100\text{ kHz}$ for $f_s = 400\text{ kHz}$)
Differentiation Filter

Window design method:
(1) calculate \( d[n] \) for the ideal filter
(2) multiply by a window to give finite support

Differentiation: \( \frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t} \)  \( \Rightarrow \) \( D(e^{j\omega}) = \begin{cases} j\omega & |\omega| \leq \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases} \)

Hence \( d[n] = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} j\omega e^{j\omega n} d\omega = \frac{j}{2\pi} \left[ \frac{\omega e^{j\omega n}}{jn} - \frac{e^{j\omega n}}{j^2 n^2} \right]_{-\omega_0}^{\omega_0} \)

\[ = \frac{n\omega_0 \cos n\omega_0 - \sin n\omega_0}{\pi n^2} \]  [IDTFT]

Using \( M = 18 \), Kaiser window, \( \beta = 7 \) and \( \omega_0 = 2.2 = \frac{2\pi \times 140 \text{ kHz}}{400 \text{ kHz}} \):
Near perfect differentiation for \( \omega \leq 1.6 \) (\( \approx 100 \text{ kHz} \) for \( f_s = 400 \text{ kHz} \))
Broad transition region allows shorter filter
Aim: extract 19 kHz pilot tone, double freq → real 38 kHz tone.
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**Aim**: extract 19 kHz pilot tone, double freq → real 38 kHz tone.

1. Shift spectrum down by 20 kHz: multiply by \( e^{-j \frac{2 \pi n}{20}} \).
Pilot tone extraction

Aim: Extract 19 kHz pilot tone, double freq → real 38 kHz tone.

(1) Shift spectrum down by 20 kHz: multiply by $e^{-j \frac{2\pi}{20} n}$
(2) Low pass filter to $\pm 1$ kHz to extract complex pilot at $-1$ kHz: $H(z)$
Aim: extract 19 kHz pilot tone, double freq → real 38 kHz tone.

(1) shift spectrum down by 20 kHz: multiply by \( e^{-\frac{j2\pi n}{400\,\text{kHz}}} \)
(2) low pass filter to ±1 kHz to extract complex pilot at −1 kHz: \( H(z) \)
(3) square to double frequency to −2 kHz

\[
\left( e^{j\omega t} \right)^2 = e^{j2\omega t}
\]
Pilot tone extraction

Aim: extract 19 kHz pilot tone, double freq → real 38 kHz tone.

(1) shift spectrum down by 20 kHz: multiply by \( e^{-j2\pi n \frac{20 \text{ kHz}}{400 \text{ kHz}}} \)
(2) low pass filter to \( \pm 1 \text{ kHz} \) to extract complex pilot at \(-1 \text{ kHz}\): \( H(z) \)
(3) square to double frequency to \(-2 \text{ kHz}\)
(4) shift spectrum up by 40 kHz: multiply by \( e^{+j2\pi n \frac{40 \text{ kHz}}{400 \text{ kHz}}} \)
Aim: extract 19 kHz pilot tone, double freq → real 38 kHz tone.

1. Shift spectrum down by 20 kHz: multiply by $e^{-j\frac{2\pi n}{20\text{kHz}}}$
2. Low pass filter to ±1 kHz to extract complex pilot at $-1$ kHz: $H(z)$
3. Square to double frequency to $-2$ kHz
4. Shift spectrum up by 40 kHz: multiply by $e^{+j2\frac{\pi n}{40\text{kHz}}}$
5. Take real part
Pilot tone extraction

Aim: extract 19 kHz pilot tone, double freq $\rightarrow$ real 38 kHz tone.

1. Shift spectrum down by 20 kHz: multiply by $e^{-j\frac{2\pi}{20k}n \frac{20}{400k} Hz}$
2. Low pass filter to $\pm 1$ kHz to extract complex pilot at $-1$ kHz: $H(z)$
3. Square to double frequency to $-2$ kHz
4. Shift spectrum up by 40 kHz: multiply by $e^{+j\frac{2\pi}{40k}n \frac{40}{400k} Hz}$
5. Take real part

More efficient to do low pass filtering at a low sample rate:
Aim: extract 19 kHz pilot tone, double freq → real 38 kHz tone.

1. shift spectrum down by 20 kHz: multiply by $e^{-j\frac{2\pi}{20}n}$
2. low pass filter to ±1 kHz to extract complex pilot at $-1$ kHz: $H(z)$
3. square to double frequency to $-2$ kHz
4. shift spectrum up by 40 kHz: multiply by $e^{+j\frac{2\pi}{40}n}$
5. take real part

More efficient to do low pass filtering at a low sample rate:

Transition bands:
$F(z): 1 \rightarrow 17$ kHz, $H(z): 1 \rightarrow 3$ kHz
Pilot tone extraction

Aim: extract 19 kHz pilot tone, double freq → real 38 kHz tone.

1. Shift spectrum down by 20 kHz: multiply by $e^{-j2\pi n \frac{20}{400 \text{kHz}}}$
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3. Square to double frequency to $-2 \text{ kHz}$
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More efficient to do low pass filtering at a low sample rate:

Transition bands:
- $F(z): 1 \rightarrow 17 \text{ kHz}$
- $H(z): 1 \rightarrow 3 \text{ kHz}$
- $G(z): 2 \rightarrow 18 \text{ kHz}$
**Pilot tone extraction**

**Aim:** extract 19 kHz pilot tone, double freq → real 38 kHz tone.

1. shift spectrum down by 20 kHz: multiply by \( e^{-j\frac{2\pi n}{20 \text{ kHz}}} \)
2. low pass filter to ±1 kHz to extract complex pilot at \(-1 \text{ kHz}\): \( H(z) \)
3. square to double frequency to \(-2 \text{ kHz}\)
4. shift spectrum up by 40 kHz: multiply by \( e^{+j\frac{2\pi n}{40 \text{ kHz}}} \)
5. take real part

More efficient to do low pass filtering at a low sample rate:

Transition bands:

\( F(z) : 1 \rightarrow 17 \text{ kHz}, \quad H(z) : 1 \rightarrow 3 \text{ kHz}, \quad G(z) : 2 \rightarrow 18 \text{ kHz} \)

\( \Delta \omega = 0.25 \Rightarrow M = 68, \quad \Delta \omega = 0.63 \Rightarrow 27, \quad \Delta \omega = 0.25 \Rightarrow 68 \)
Polyphase Pilot tone

\[ x[n] \left[ 19\text{kHz} \right] \downarrow e^{-j2\pi n/20}[20\text{kHz}] \times F(z) \downarrow 20:1 \left[ -1\text{kHz} \right] \downarrow H(z) \downarrow 20:1 \left[ -2\text{kHz} \right] \downarrow G(z) \downarrow \text{Re} \left[ 38\text{kHz} \right] y[n] \uparrow \left[ +40\text{kHz} \right] \downarrow 1:20 \downarrow 20k \left[ 1\text{kHz} \right] \downarrow 20k \left[ 20\text{kHz} \right] \downarrow F(z) \downarrow e^{-j2\pi n/20}[20\text{kHz}] \left[ 19\text{kHz} \right] \downarrow x[n] \]
Polyphase Pilot tone

\[
x[n] \quad [19\text{kHz}] \quad x[n] e^{-j\pi/20}[\text{\(-20\text{kHz}\)}] \quad x[n] e^{+j\pi/10}[\text{\(+40\text{kHz}\)}]
\]

\[
\frac{x[n]}{400k} \times F(z) \quad 20:1 \quad \frac{H(z)}{20k} \quad 1:20 \quad \frac{G(z)}{20k} \quad \frac{y[n]}{400k}
\]

\[
x[n] e^{-j\pi/20} \quad \frac{F_0(z)}{400k} \quad F_1(z)\quad e^{-j\pi/20} \quad H(z) \quad e^{j\pi/10} \quad \frac{G_0(z)}{400k}
\]

\[
\frac{F_0(z)}{400k} + \frac{F_1(z)}{20k} \quad \frac{H(z)}{20k} \quad \frac{G_0(z)}{400k}
\]
Polyphase Pilot tone

Anti-alias filter: $F(z)$

Each branch, $F_p(z)$, gets every $20^{th}$ sample and an identical $e^{j2\pi \frac{n}{20}}$.
**Polyphase Pilot tone**

Anti-alias filter: $F(z)$

Each branch, $F_p(z)$, gets every $20^{th}$ sample and an identical $e^{j \frac{2\pi}{20} \frac{n}{20}}$.

So $F_p(z)$ can filter a real signal and then multiply by fixed $e^{j \frac{2\pi}{20} \frac{p}{20}}$. 

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DSP and Digital Filters (2017-10178)
Polyphase Pilot tone

Anti-alias filter: $F(z)$
- Each branch, $F_p(z)$, gets every $20^{th}$ sample and an identical $e^{j2\pi \frac{n}{20}}$
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Anti-image filter: $G(z)$
- Each branch, $G_p(z)$, multiplied by identical $e^{j2\pi \frac{n}{10}}$
Polyphase Pilot tone

Anti-alias filter: $F(z)$

Each branch, $F_p(z)$, gets every $20^{th}$ sample and an identical $e^{j2\pi \frac{n}{20}}$

So $F_p(z)$ can filter a real signal and then multiply by fixed $e^{j2\pi \frac{P}{20}}$

Anti-image filter: $G(z)$

Each branch, $G_p(z)$, multiplied by identical $e^{j2\pi \frac{n}{10}}$

So $G_p(z)$ can filter a real signal
Polyphase Pilot tone

Anti-alias filter: $F(z)$
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Anti-image filter: $G(z)$
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Multiplies:
$F$ and $G$ each: $(4 + 2) \times 400 \text{ kHz}$
Polyphase Pilot tone

Anti-alias filter: $F(z)$
Each branch, $F_p(z)$, gets every $20^{th}$ sample and an identical $e^{j2\pi n/20}$
So $F_p(z)$ can filter a real signal and then multiply by fixed $e^{j2\pi p/20}$

Anti-image filter: $G(z)$
Each branch, $G_p(z)$, multiplied by identical $e^{j2\pi n/10}$
So $G_p(z)$ can filter a real signal

Multiplies:
$F$ and $G$ each: $(4 + 2) \times 400$ kHz, $H + x^2$: $(2 \times 28 + 4) \times 20$ kHz
Polyphase Pilot tone

Anti-alias filter: \( F(z) \)
- Each branch, \( F_p(z) \), gets every 20\textsuperscript{th} sample and an identical \( e^{j2\pi \frac{n}{20}} \)
- So \( F_p(z) \) can filter a real signal and then multiply by fixed \( e^{j2\pi \frac{p}{20}} \)

Anti-image filter: \( G(z) \)
- Each branch, \( G_p(z) \), multiplied by identical \( e^{j2\pi \frac{n}{10}} \)
- So \( G_p(z) \) can filter a real signal

Multiplies:
- \( F \) and \( G \) each: \((4 + 2) \times 400 \text{ kHz}, H + x^2: (2 \times 28 + 4) \times 20 \text{ kHz}\)
- Total: \( 15 \times 400 \text{ kHz} \)
**Polyphase Pilot tone**

Anti-alias filter: $F(z)$

Each branch, $F_p(z)$, gets every $20^{th}$ sample and an identical $e^{j2\pi n/20}$

So $F_p(z)$ can filter a real signal and then multiply by fixed $e^{j2\pi p/20}$

Anti-image filter: $G(z)$

Each branch, $G_p(z)$, multiplied by identical $e^{j2\pi n/10}$

So $G_p(z)$ can filter a real signal

Multiplies:

$F$ and $G$ each: $(4 + 2) \times 400 \text{ kHz}$, $H + x^2$: $(2 \times 28 + 4) \times 20 \text{ kHz}$

Total: $15 \times 400 \text{ kHz}$

[Full-rate $H(z)$ needs $273 \times 400 \text{ kHz}$]
Summary

- **Aliased ADC** allows sampling below the Nyquist frequency
  - Only works because the wanted signal fits entirely within a Nyquist band image
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  - Only works because the wanted signal fits entirely within a Nyquist band image

- **Polyphase filter can be combined with complex multiplications** to select the desired image
  - Subsequent multiplication by $-j^{\ln}$ shifts by the desired multiple of $\frac{1}{4}$ sample rate
    - No actual multiplications required
Summary

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- **Pilot tone bandpass filter** has narrow bandwidth so better done at a low sample rate
  - double the frequency of a complex tone by squaring it
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*This example is taken from Harris: 13.*