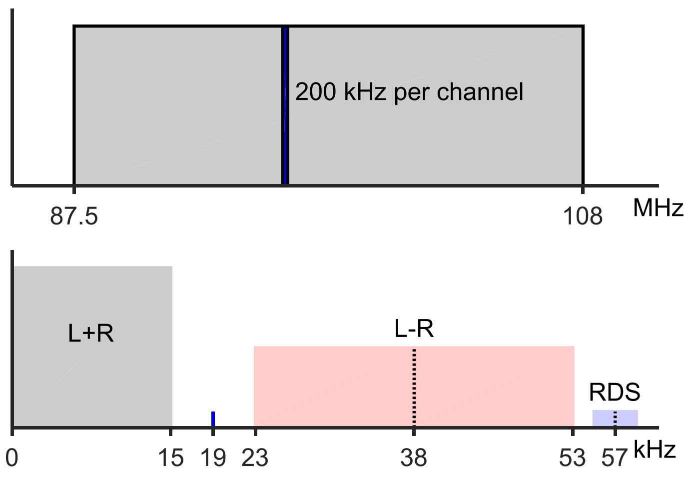
14: FM Radio Receiver

FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\mathrm{MHz}$
Each channel: $\pm 100\,\mathrm{kHz}$

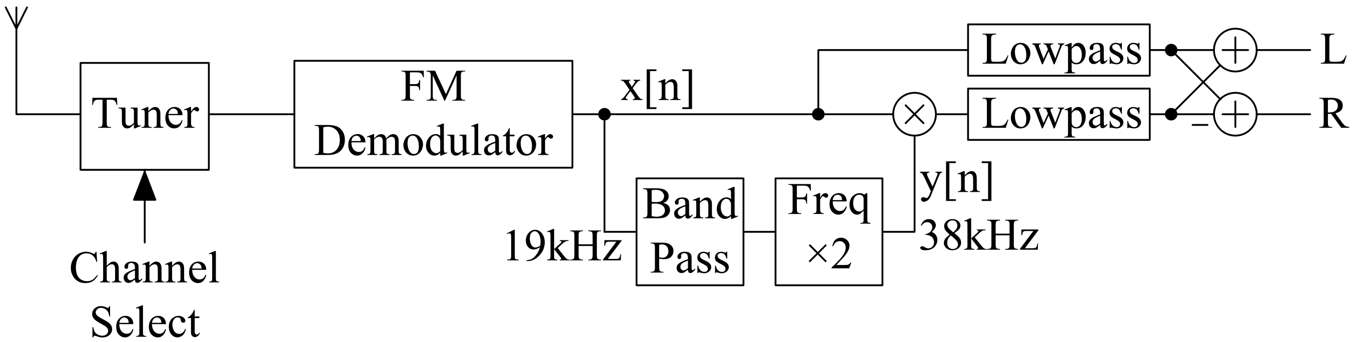


Baseband signal:

Mono (L + R): $\pm 15\,\mathrm{kHz}$
Pilot tone: $19\,\mathrm{kHz}$
Stereo (L – R): $38 \pm 15\,\mathrm{kHz}$
RDS: $57 \pm 2\,\mathrm{kHz}$

FM Modulation:

Freq deviation: $\pm 75\,\mathrm{kHz}$



L–R signal is multiplied by $38\,\mathrm{kHz}$ to shift it to baseband

[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

Aliased ADC

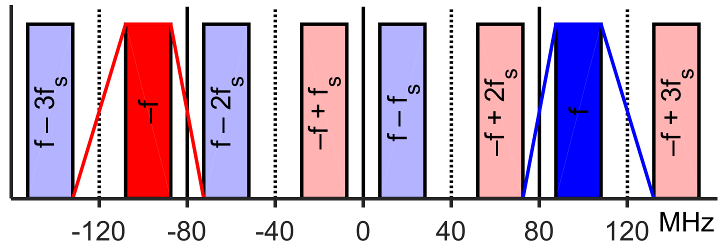
FM band: 87.5 to $108\,\mathrm{MHz}$
Normally sample at $f_s > 2f$

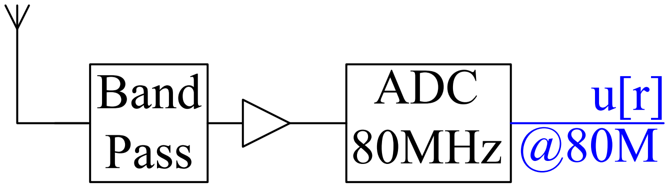
However:

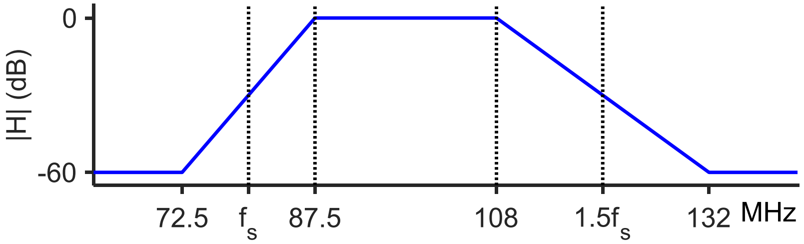
$f_s = 80\,\mathrm{MHz}$ aliases band down to $[7.5,\ 28]\,\mathrm{MHz}$.

–ve frequencies alias to $[-28,\ -7.5]\,\mathrm{MHz}$.

We must suppress other frequencies that alias to the range $\pm[7.5,\ 28]\,\mathrm{MHz}$.







Need an analogue bandpass filter to extract the FM band. Transition band mid-points are at $f_s = 80\,\mathrm{MHz}$ and $1.5f_s = 120\,\mathrm{MHz}$.

You can use an aliased analog-digital converter (ADC) provided that the target band fits entirely between two consecutive multiples of $\frac{1}{2}f_s$.
Lower ADC sample rate ☺. Image = undistorted frequency-shifted copy.

Channel Selection

FM band shifted to 7.5 to $28\,\mathrm{MHz}$ (from 87.5 to $108\,\mathrm{MHz}$)

We need to select a single channel $200\,\mathrm{kHz}$ wide

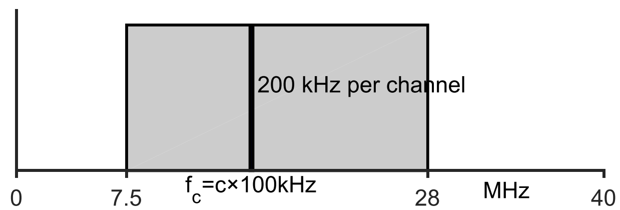
We shift selected channel to DC and then downsample to $f_s = 400\,\mathrm{kHz}$.
Assume channel centre frequency is $f_c = c \times 100\,\mathrm{kHz}$

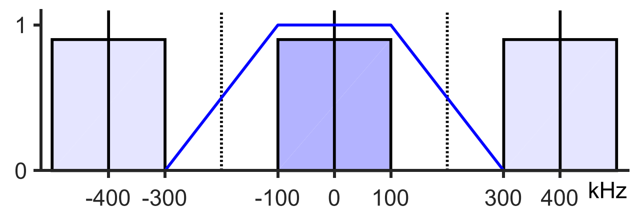
We must apply a filter before downsampling to remove unwanted images

The downsampled signal is complex since positive and negative frequencies contain different information.

We will look at three methods:

1. Freq shift, then polyphase lowpass filter
2. Polyphase bandpass complex filter
3. Polyphase bandpass real filter





Multiply by $e^{-j2\pi r \frac{f_c}{80\,\text{MHz}}}$ to shift channel at f_c to DC.

$f_c = c \times 100\,\text{k} \Rightarrow \frac{f_c}{80\,\text{M}} = \frac{c}{800}$

Result of multiplication is complex (thick lines on diagram)

Next, lowpass filter to $\pm 100\,\text{kHz}$

$\Delta\omega = 2\pi \frac{200\text{ k}}{80\text{ M}} = 0.157$

$\Rightarrow M = \frac{60\text{ dB}}{3.5\Delta\omega} = 1091$

Finally, downsample $200 : 1$

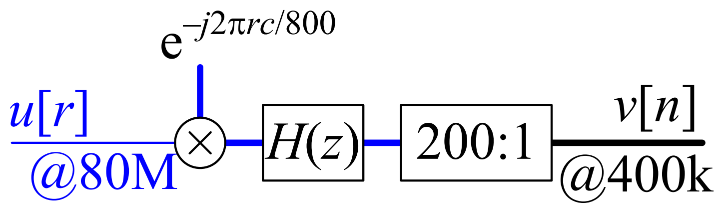
Polyphase:

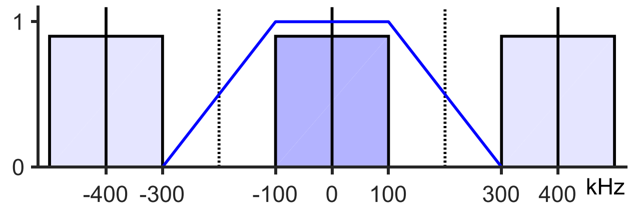
$H_p(z)$ has $\left\lceil \frac{1092}{200} \right\rceil = 6$ taps

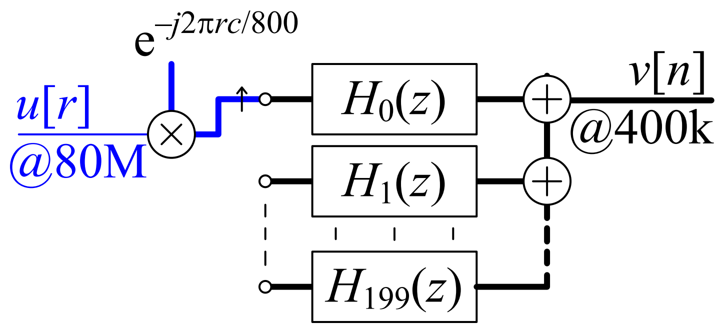
Complex data $\times$ Real Coefficients (needs 2 multiplies per tap)

Multiplication Load:

$2 \times 80\,\text{MHz}$ (freq shift) $+ 12 \times 80\,\text{MHz}$ $(H_p(z))$ $= 14 \times 80\,\text{MHz}$

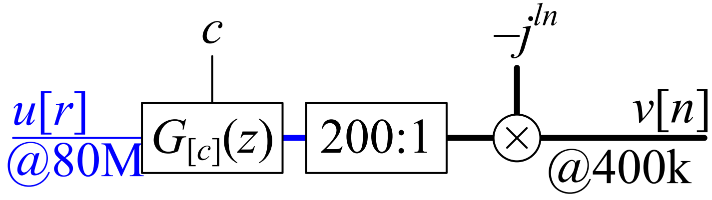


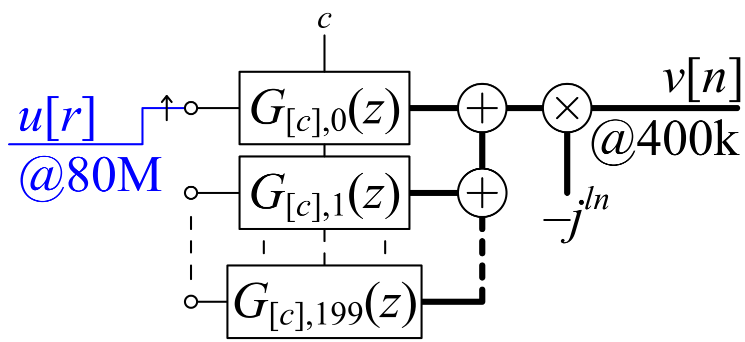




Channel Selection (2)

Channel centre frequency $f_c = c \times 100\,\text{kHz}$ where c is an integer.
Write $c = 4k + l$
 where $k = \left\lfloor \frac{c}{4} \right\rfloor$ and $l = c_{\text{mod } 4}$



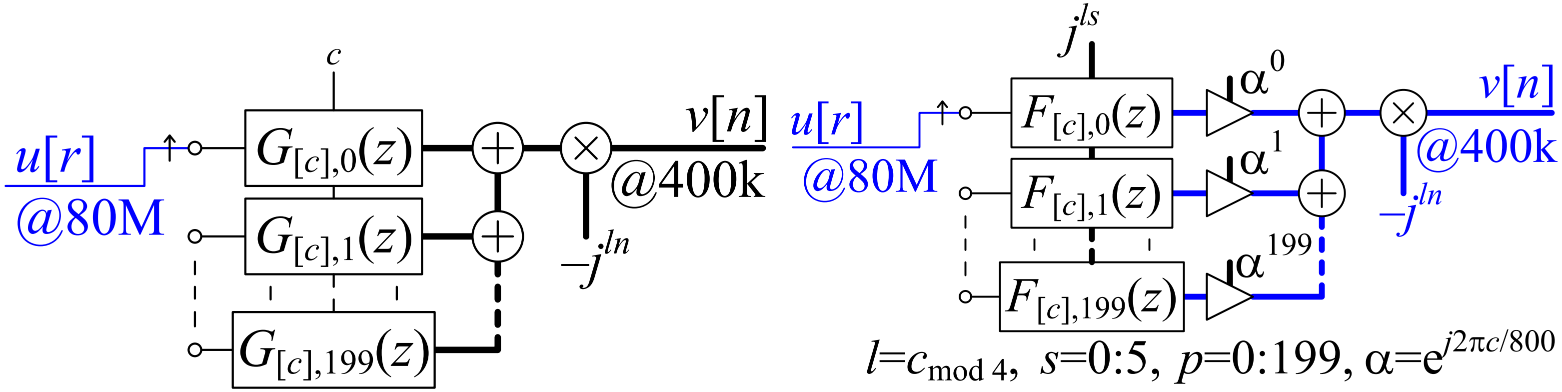


We multiply $u[r]$ by $e^{-j2\pi r \frac{c}{800}}$, convolve with $h[m]$ and then downsample:

$$
\begin{aligned}
v[n] &= \sum_{m=0}^{M} h[m] u[200n - m] e^{-j2\pi(200n-m)\frac{c}{800}} && [r = 200n] \\
&= \sum_{m=0}^{M} h[m] e^{j2\pi\frac{mc}{800}} u[200n - m] e^{-j2\pi 200n \frac{4k+l}{800}} && [c = 4k + 1] \\
&= \sum_{m=0}^{M} g_{[c]}[m] u[200n - m] e^{-j2\pi\frac{ln}{4}} && [g_{[c]}[m] \triangleq h[m] e^{j2\pi\frac{mc}{800}}] \\
&= (-j)^{ln} \sum_{m=0}^{M} g_{[c]}[m] u[200n - m] && [e^{-j2\pi\frac{ln}{4}} \text{ indep of } m]
\end{aligned}
$$

Multiplication Load for polyphase implementation:
 $G_{[c],p}(z)$ has complex coefficients $\times$ real input $\Rightarrow$ 2 mults per tap
 $(-j)^{ln} \in \{+1, -j, -1, +j\}$ so no actual multiplies needed
 Total: $12 \times 80\,\text{MHz}$ (for $G_{[c],p}(z)$) $+ 0$ (for $-j^{ln}$) $= 12 \times 80\,\text{MHz}$

Channel frequency $f_c = c \times 100\,\text{kHz}$ where $c = 4k + l$ is an integer



$$g_{[c]}[m] = h[m]e^{j2\pi\frac{cm}{800}}$$

$$g_{[c],p}[s] = g_c[200s + p] = h[200s + p]e^{j2\pi\frac{c(200s+p)}{800}}$$ [polyphase]

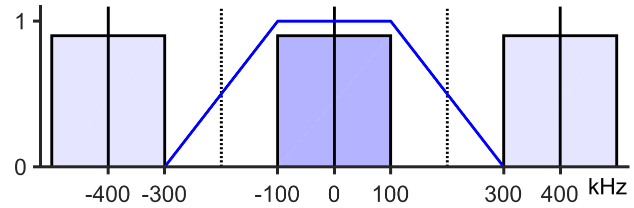
$$= h[200s + p]e^{j2\pi\frac{cs}{4}}e^{j2\pi\frac{cp}{800}} \triangleq h[200s + p]e^{j2\pi\frac{cs}{4}}\alpha^p$$

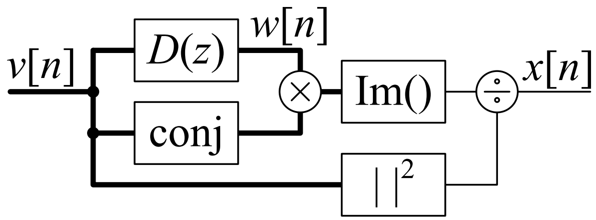
Define $f_{[c],p}[s] = h[200s + p]e^{j2\pi\frac{(4k+l)s}{4}} = j^{ls}h[200s + p]$

Although $f_{[c],p}[s]$ is complex it requires only one multiplication per tap because each tap is either purely real or purely imaginary.

Multiplication Load:

$6 \times 80\,\text{MHz}$ $(F_p(z))$ $+ 4 \times 80\,\text{MHz}$ $(\times e^{j2\pi\frac{cp}{800}})$ $= 10 \times 80\,\text{MHz}$





Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log|v| + j\phi$

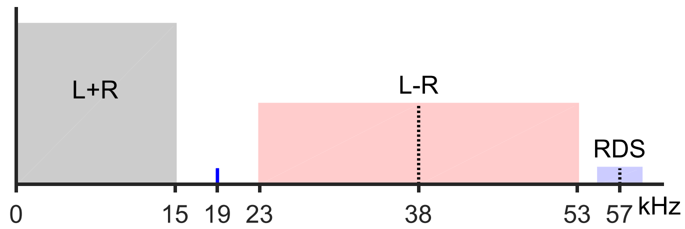
The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

We need to calculate $x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im\left(\frac{1}{v}\frac{dv}{dt}\right) = \frac{1}{|v|^2}\Im\left(v^*\frac{dv}{dt}\right)$

We need:

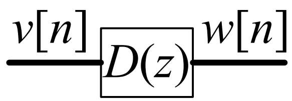
(1) Differentiation filter, $D(z)$
(2) Complex multiply, $w[n] \times v^*[n]$ (only need $\Im$ part)
(3) Real Divide by $|v|^2$

$x[n]$ is baseband signal (real):



Differentiation Filter

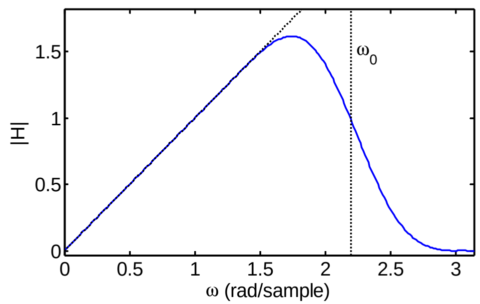
Window design method:
(1) calculate $d[n]$ for the ideal filter
(2) multiply by a window to give finite support

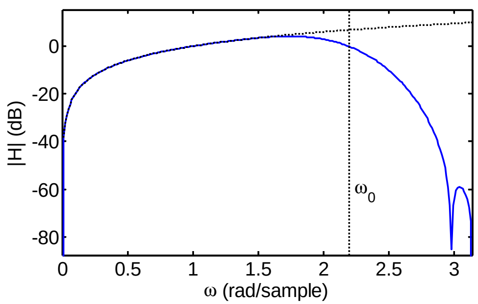


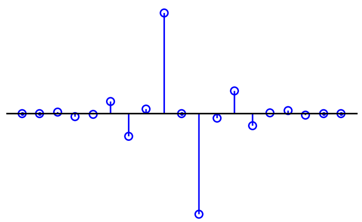
Differentiation: $\frac{d}{dt}e^{j\omega t} = j\omega e^{j\omega t} \quad \Rightarrow \quad D(e^{j\omega}) = \begin{cases} j\omega & |\omega| \le \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases}$

Hence $d[n] = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} j\omega e^{j\omega n} d\omega = \frac{j}{2\pi}\left[\frac{\omega e^{jn\omega}}{jn} - \frac{e^{jn\omega}}{j^2 n^2}\right]_{-\omega_0}^{\omega_0}$ [IDTFT]

$$= \frac{n\omega_0 \cos n\omega_0 - \sin n\omega_0}{\pi n^2}$$

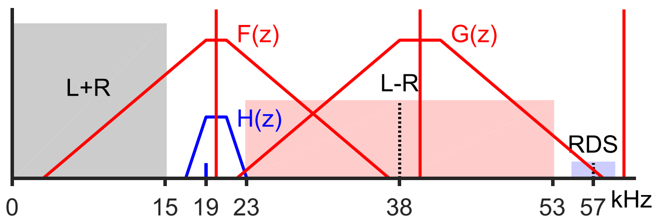


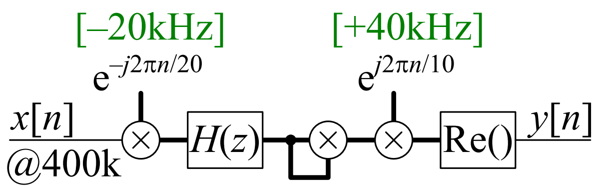




Using $M = 18$, Kaiser window, $\beta = 7$ and $\omega_0 = 2.2 = \frac{2\pi \times 140\,\text{kHz}}{400\,\text{kHz}}$:
Near perfect differentiation for $\omega \le 1.6$ ($\approx 100\,\text{kHz}$ for $f_s = 400\,\text{kHz}$)
Broad transition region allows shorter filter

Pilot tone extraction +

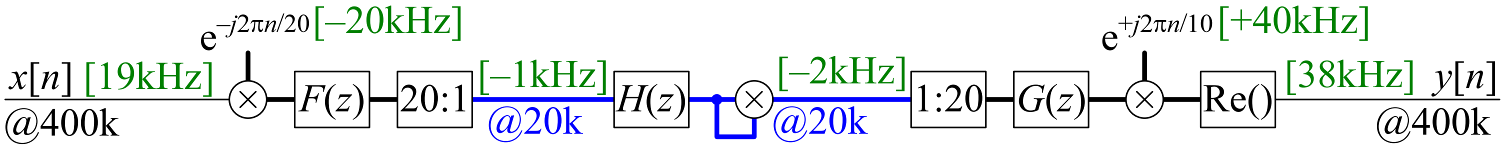




Aim: extract $19\,\text{kHz}$ pilot tone, double freq $\rightarrow$ real $38\,\text{kHz}$ tone.

(1) shift spectrum down by $20\,\text{kHz}$: multiply by $e^{-j2\pi n\frac{20\,\text{kHz}}{400\,\text{kHz}}}$
(2) low pass filter to $\pm 1\,\text{kHz}$ to extract complex pilot at $-1\,\text{kHz}$: $H(z)$
(3) square to double frequency to $-2\,\text{kHz}$ $\left[\left(e^{j\omega t}\right)^2 = e^{j2\omega t}\right]$
(4) shift spectrum up by $40\,\text{kHz}$: multiply by $e^{+j2\pi n\frac{40\,\text{kHz}}{400\,\text{kHz}}}$
(5) take real part

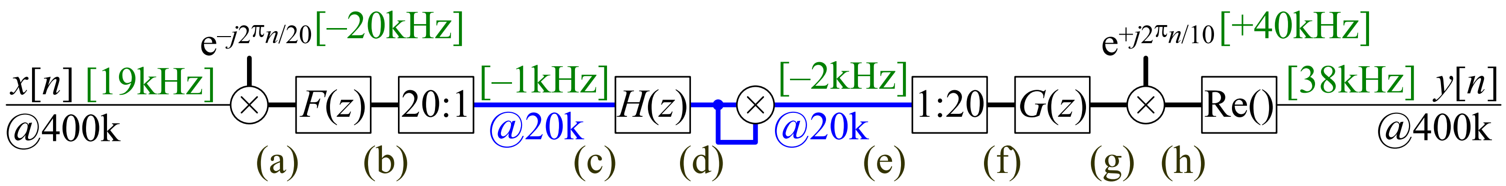
More efficient to do low pass filtering at a low sample rate:

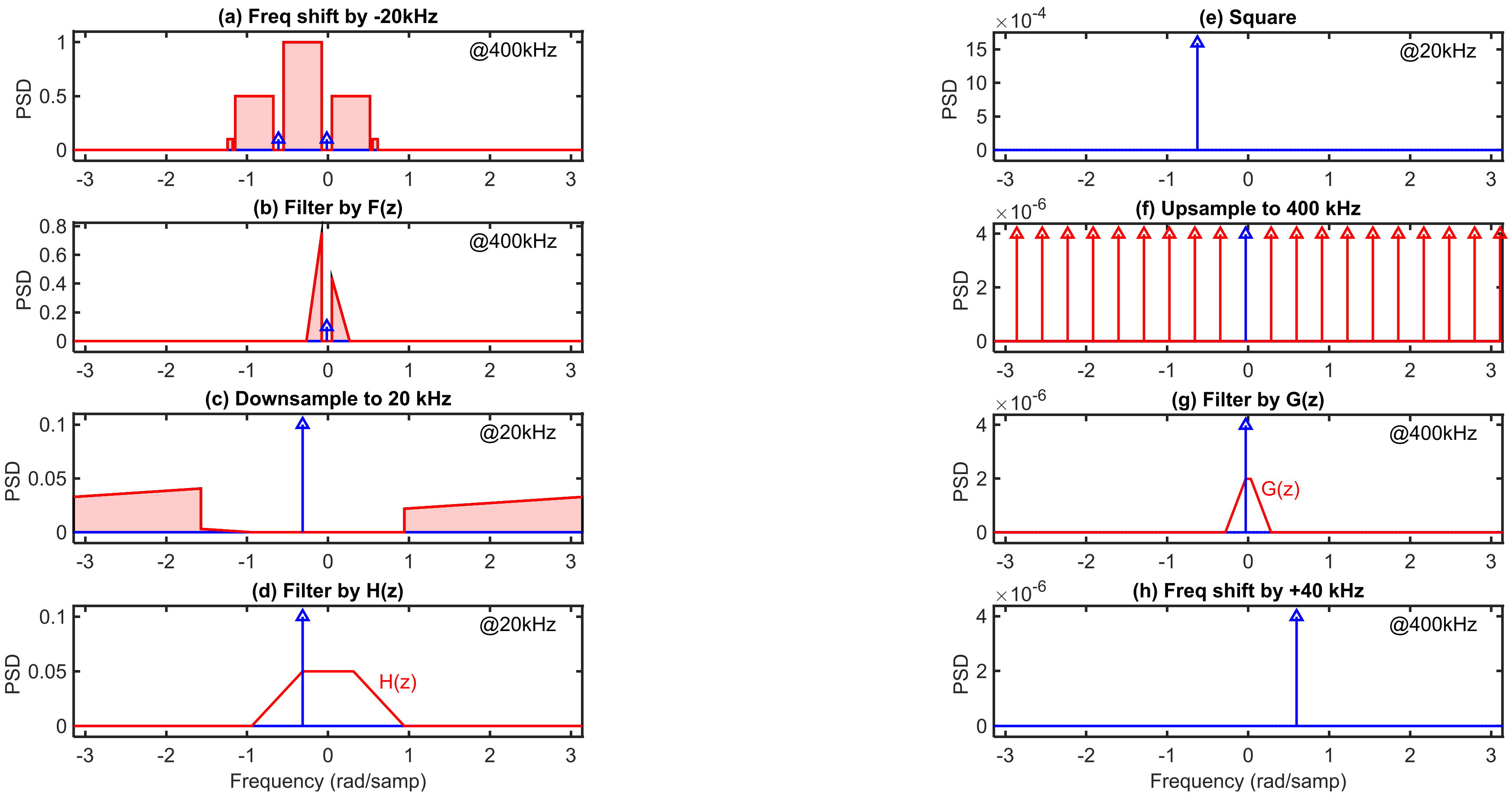


Transition bands:

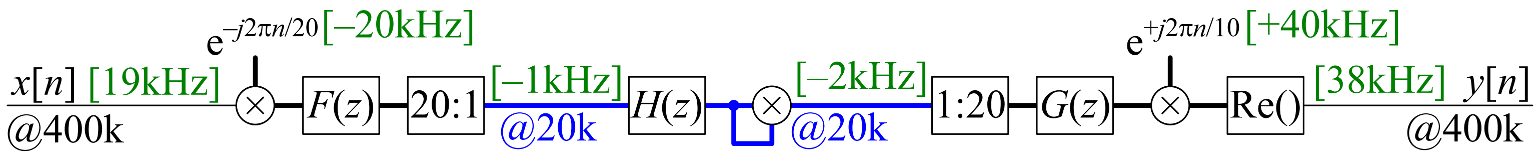
$F(z)$: $1 \rightarrow 17\,\text{kHz}$, $\Delta\omega = 0.25 \Rightarrow M = 68$,
$H(z)$: $1 \rightarrow 3\,\text{kHz}$, $\Delta\omega = 0.63 \Rightarrow 27$,
$G(z)$: $2 \rightarrow 18\,\text{kHz}$ $\Delta\omega = 0.25 \Rightarrow 68$

[Pilot Tone Extraction]





Polyphase Pilot tone



Anti-alias filter: $F(z)$

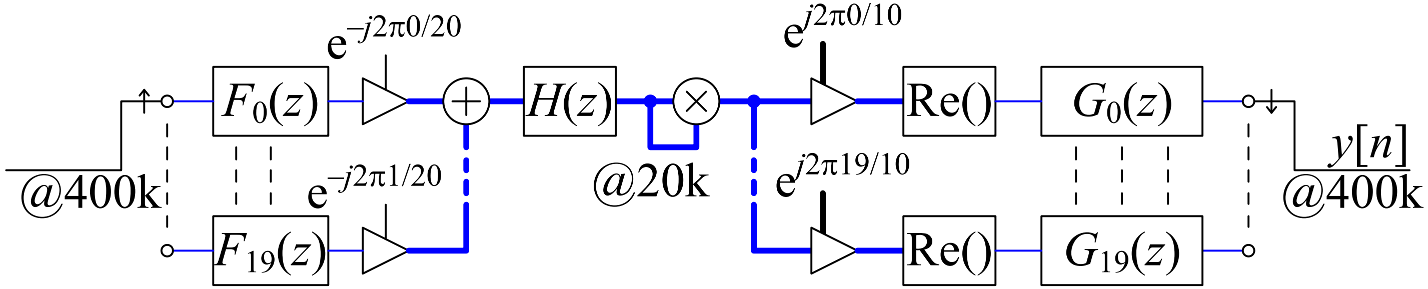
Each branch, $F_p(z)$, gets every 20^{th} sample and an identical $e^{j2\pi\frac{n}{20}}$

So $F_p(z)$ can filter a real signal and then multiply by fixed $e^{j2\pi\frac{p}{20}}$

Anti-image filter: $G(z)$

Each branch, $G_p(z)$, multiplied by identical $e^{j2\pi\frac{n}{10}}$

So $G_p(z)$ can filter a real signal



Multiplies:

F and G each: $(4+2) \times 400\,\text{kHz}$, $H + x^2$: $(2 \times 28 + 4) \times 20\,\text{kHz}$

Total: $15 \times 400\,\text{kHz}$ [Full-rate $H(z)$ needs $273 \times 400\,\text{kHz}$]

Summary

- Aliased ADC allows sampling below the Nyquist frequency
  - Only works because the wanted signal fits entirely within a Nyquist band image
- Polyphase filter can be combined with complex multiplications to select the desired image
  - subsequent multiplication by $-j^{ln}$ shifts by the desired multiple of $\frac{1}{4}$ sample rate
    - No actual multiplications required
- FM demodulation uses a differentiation filter to calculate $\frac{d\phi}{dt}$
- Pilot tone bandpass filter has narrow bandwidth so better done at a low sample rate
  - double the frequency of a complex tone by squaring it

This example is taken from Harris: 13.