E1.10 Fourier Series and Transforms

Problem Sheet 1 (Lecture 1)

Key: [A]= easy ... [E]=hard

Questions from RBH textbook: 4.2, 4.8.

1. [B] Using the geometric progression formula, evaluate $\sum_{r=1}^{5} 3^r$.
2. [B] Determine expressions not involving a summation for
   (a) $\sum_{r=1}^{10} 3x^{2r}$, (b) $\sum_{r=0}^{10} \frac{2}{x^r}$, (c) $\sum_{r=0}^{R} x^r y^{r-2}$, (d) $\sum_{r=0}^{R} (-1)^r$.
3. [B] In the expression $\sum_{r=1}^{5} 3^r$, make the substitution $r = m + 1$ and then evaluate the resultant expression.
4. [C] Determine a simplified expression not involving a summation for $\sum_{r=-N}^{N} e^{j\omega r}$ for $N \geq 0$.
5. [C] Determine a simplified expression for $\sum_{r=0}^{R-1} e^{j2\pi r R^{-1}}$ for $R \geq 1$. Ensure your answer is correct even when $R = 1$.
6. [C] Determine the value of $\sum_{n=0}^{N} \sum_{m=1}^{M} 2x^{m-n}$.
7. [C] If $x(t) = \sin t$, determine (a) $\langle x(t) \rangle$, (b) $\langle |x(t)| \rangle$ and (c) $\langle x^2(t) \rangle$ where $\langle \cdots \rangle$ denotes the time-average.
8. [C] The first two normalized Legendre polynomials are $P_0(t) = 1$ and $P_1(t) = \sqrt{3}t$.
   (a) Show that $\langle P_0^2(t) \rangle_{[-1,1]} = \langle P_1^2(t) \rangle_{[-1,1]} = 1$ and $\langle P_0(t) P_1(t) \rangle_{[-1,1]} = 0$ where $\langle \cdots \rangle_{[-1,1]}$ denotes the average over the interval $-1 < t < 1$.
   (b) If $P_2(t) = at^2 + bt + c$, find the coefficients a, b and c such that
   $\langle P_0(t) P_2(t) \rangle_{[-1,1]} = \langle P_1(t) P_2(t) \rangle_{[-1,1]} = 0$ and $\langle P_2^2(t) \rangle_{[-1,1]} = 1$