E1.10 Fourier Series and Transforms

Problem Sheet 2 (Lectures 2, 3)

Key: [A]= easy ... [E]=hard

Questions from RBH textbook: 12.1, 12.2, 12.3, 12.4, 12.5, 12.8, 12.9, 12.10, 12.11, 12.12, 12.13, 12.14, 12.15, 12.17, 12.20, 12.21, 12.22, 12.26.

1. [B] Give the fundamental period of (a) $\cos 1000\pi t$, (b) $\cos 1000\pi t + 0.01\cos 1250\pi t$, (c) $\cos 1000\pi t + \cos 1000t$.

2. [C] A sufficient condition for a periodic function, $u(t)$, to have a Fourier series is that it satisfies the Dirichlet conditions on page 2-5 of the notes. Determine which of the following functions satisfies these conditions. The notation $x \mod n$ means the remainder when x is divided by n.

   (a) $\sin^2 t$, (b) $\frac{1}{\sin t}$, (c) $\sqrt{\frac{1}{|\sin t|}}$, (d) $\frac{1}{1+t^2}$, (e) $t \mod 1$.

3. [B] Determine the fundamental frequency and the Fourier Series coefficients for $u(t) = 1+2\cos(6000\pi t)+ 3\sin(4000\pi t)$.

4. [B] The phasor $2 + 4i$ represents the waveform $2\cos\omega t - 4\sin\omega t$. Give (a) the Fourier coefficients and (b) the complex Fourier coefficients for this waveform.

5. [C] Determine the fundamental frequency and the Fourier Series coefficients for $u(t) = \cos^4(2000\pi t)$.

6. [C] (a) Determine the Fourier coefficients, $\{a_n, b_n\}$ for the waveform, $u(t)$, with period $T = 2$ defined by $u(t) = 3t$ for $-1 \le t < 1$.

   (b) Determine the complex Fourier coefficients, U_n, for the same waveform.

   (c) Determine the complex Fourier coefficients for $v(t) = u(t-1)$.

   (d) Determine the complex Fourier coefficients for $w(t) = 2v(t) + 4$.

7. [B] If $u(t)$ has period $T = \frac{1}{F}$ and Fourier coefficients $a_{0:2} = [5, 2, 3]$ and $b_1 = 1$ with all other coefficients zero. (a) Give an expression for $u(t)$, (b) Determine the complex Fourier coefficients, U_n.

8. [B] Each of the following waveforms has period $T = 2$ and equals the expression given for $-1 \le t < 1$. In each case say whether the complex Fourier coefficients will be (i) real-valued, (ii) purely imaginary or (iii) neither.

   (a) t^2 (b) t^3 (c) $2t+t^2$ (d) t^2+1 (e) t^3+1 (f) $t\sin t$ (g) $t\cos 2t$ (h) $t^2\sin t$.

9. [C] Each of the following waveforms has period $T = 2$ and equals the expression given for $-1 \le t < 1$. In each case say whether or not all the even-numbered Fourier coefficients will equal zero.

   (a) $\sin \pi t$ (b) $\begin{cases} t+1 & t<0 \\ -t & t \ge 0 \end{cases}$ (c) $\begin{cases} t+1 & t<0 \\ 1-t & t \ge 0 \end{cases}$ (d) $t(1-|t|)$ (e) $t^3 - t$.

10. [C] $u(t)$ has period $T = 4$ and is defined by $u(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & 1 \le t < 4 \end{cases}$.

    (a) Find the complex Fourier coefficients, U_n expressing them in polar form: $r \times e^{i\theta}$. Identify which of the coefficients are equal to zero.

    (b) Find the complex Fourier coefficients of $v(t) = u(t + 0.5)$ and explain why they are necessarily real-valued. Explain the relation between the magnitudes $|V_n|$and $|U_n|$.

    (c) Find the complex Fourier coefficients of $w(t) = v(t) + v(t-2)$. Identify which of the coefficients are non-zero and explain how your answer relates to the symmetries of $w(t)$.