E1.1 Analysis of Circuits

Mike Brookes

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What are circuits?
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- 18 lectures: feel free to ask questions
- Buy the textbook: Hayt, Kemmerly & Durbin "Engineering Circuit Analysis" ISBN: 0071217066 (£44) or Irwin, Nelms & Patnaik "Engineering Circuit Analysis" ISBN: 1118960637 (£37)
- Weekly study group: Problem sheets - KEEP UP TO DATE
- Fortnightly tutorial: tutorial problems
- Lecture slides (including animations) and problem sheets + answers available via Blackboard or from my website: http://www.ee.ic.ac.uk/hp/staff/dmb/courses/ccts1/ccts1.htm
  - Quite dense: you should understand every word
- Email me with any errors or confusions in slides or problems/answers
- Christmas Test in January
- Exam in June (sample papers + solutions available via Blackboard)

What are circuits?

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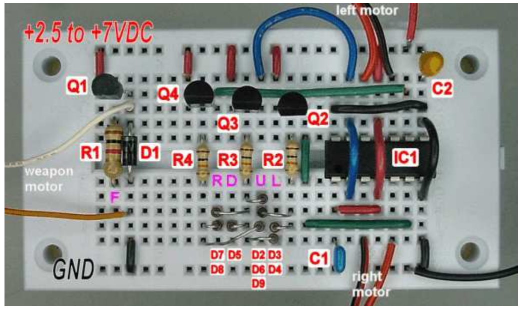
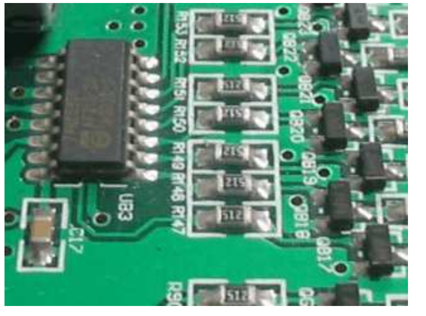
- A *circuit* consists of electrical or electronic components interconnected with metal wires
- Every electrical or electronic device is a circuit



Breadboard



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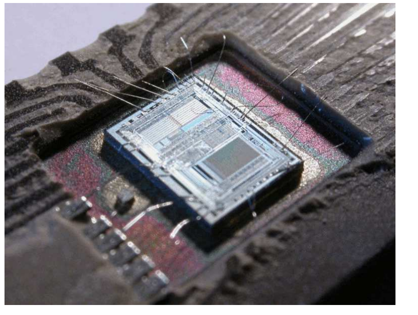

Integrated

- The function of the circuit is determined by which components are used and how they are interconnected: the physical positioning of the components usually has hardly any effect.

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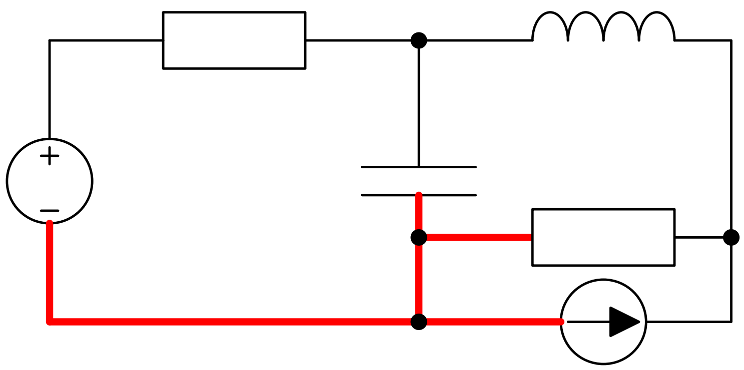
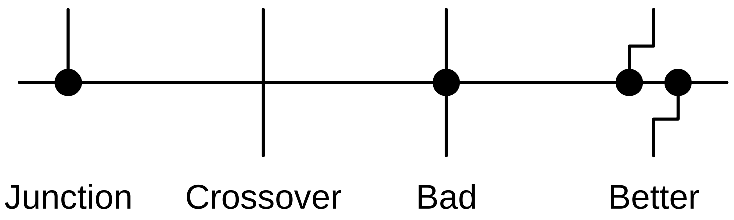
A *circuit diagram* shows the way in which the components are connected

- Each component has a special symbol
- The interconnecting wires are shown as lines



A *node* in a circuit is all the points that are connected together via the interconnecting wires. One of the four nodes in the diagram is coloured red.
Assumption: Interconnecting wires have zero resistance so everywhere along a node has the same voltage.



Indicate three meeting wires with a • and crossovers without one.

Avoid having four meeting wires in case the • disappears; stagger the wires instead.

Charge

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Charge is an electrical property possessed by some atomic particles
Charge is measured in Colombs (abbreviated C)
An electron has a charge $-1.6 \times 10^{-19}\mathrm{C}$, a proton $+1.6 \times 10^{-19}\mathrm{C}$
Unlike charges attract, like charges repel: the force is fantastically huge

Two people $384,000\,\mathrm{km}$ apart
Each with 1% extra electrons

Force $= 2 \times 10^8\mathrm{N}$
$= 20,000\,\mathrm{tonne-force}$
$= 360,000\times$ their weight

Consequence: Charge never accumulates in a conductor: everywhere in a conducting path stays electrically neutral at all times.

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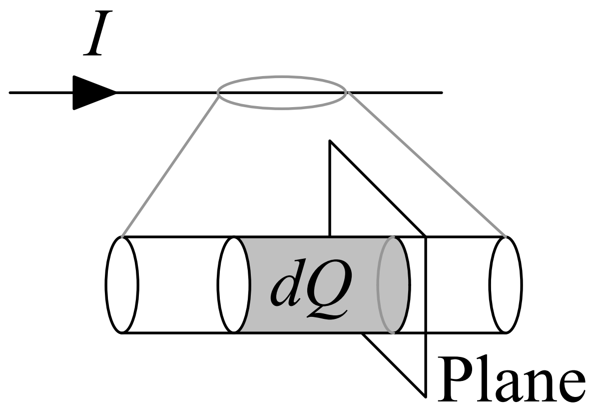
Current is the flow of charged particles past a measurement boundary
Using an ammeter, we measure current in Ampères (usually abbreviated to Amps or A): $1\text{ A} = 1\text{ C/s}$
Analogy: the flow of water in a pipe or river is measured in litres per second

The arrow in a circuit diagram indicates the direction we choose to measure the current.
$I = +1\text{ A} \Rightarrow 1\text{ C}$ of +ve charge passes each point every second in the direction of the arrow (or else 1 C of –ve charge in the opposite direction)



$I = -1\text{ A} \Rightarrow 1\text{ C}$ of +ve charge in the direction opposite to the arrow

- Average electron velocity is surprisingly slow (e.g. 1 mm/s) but (like a water pipe) the signal travels much faster.
- In metals the charge carriers (electrons) are actually –ve: in this course you should ignore this always.

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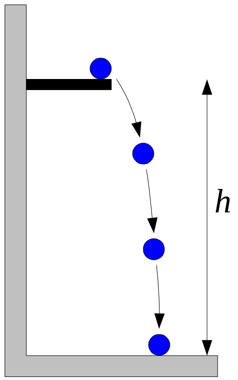
When a ball falls from a shelf, it loses potential energy of mgh or, equivalently, gh per kg.



The potential energy per kg of any point on a mountain range is equal to gh where h is measured relative to an equipotential reference surface (e.g. the surface of a lake).

The potential energy difference between any two points is the energy needed to move 1 kg from one point to the other.
The potential energy difference does not depend on the route taken between the points.
The potential enegy difference does not depend on your choice of reference surface (e.g. lake surface or sea level).

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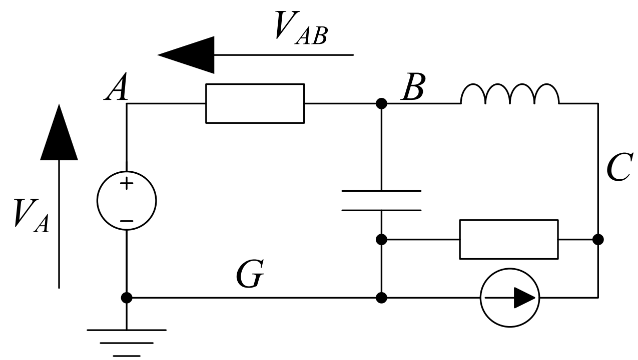
The *electrical potential difference* (or *voltage difference*) between any two nodes in a circuit is the energy per coulomb needed to move a small +ve charge from one node to the the other.
We usually pick one of the nodes as a reference and define the *voltage at a node* to be the voltage difference between that node and the reference.

The four nodes are labelled A, B, C, G.

We have chosen G as the reference node; indicated by the "ground" symbol.



The potential difference between A and the ground reference, G, is written V_A and is also called "the voltage at A".
The potential difference between A and B is written as V_{AB} and shown as an arrow pointing towards A. This is the energy per coulomb in going from B to A and satisfies $V_{AB} = V_A - V_B$. (Different from vectors)

Easy algebra shows that $V_{AB} = -V_{BA}$ and that $V_{AC} = V_{AB} + V_{BC}$.

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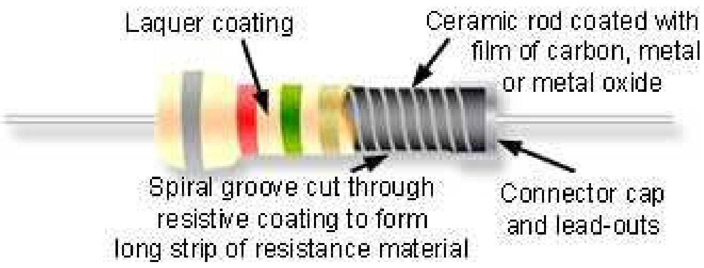
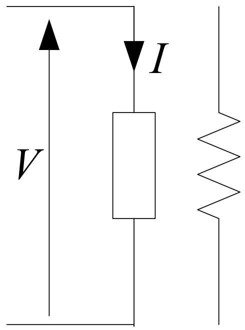
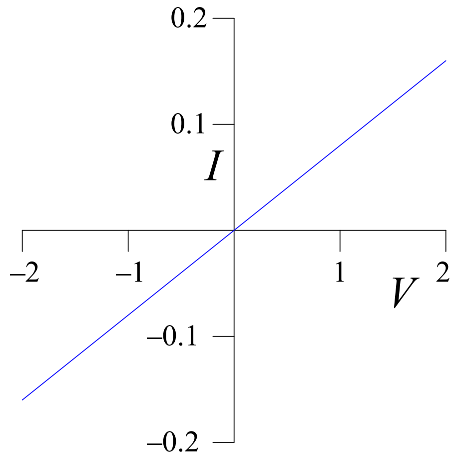
A *resistor* is made from a thin strip of metal film deposited onto an insulating ceramic base.



The *characteristic* of a component is a graph showing how the voltage and current are related. We always choose the current and voltage arrows in opposite directions: this is *the passive sign convention*.



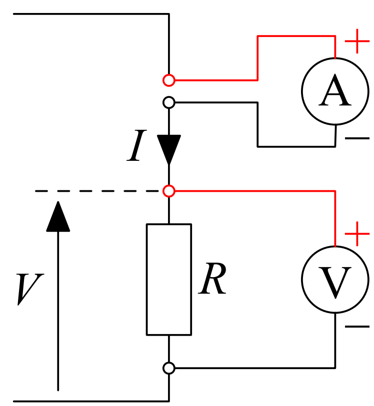


For a resistor, $I \propto V$ and $\frac{V}{I} = R$, its *resistance* which is measured in Ohms (Ω). This is Ohm's Law. Sometimes it is more convenient to work in terms of the *conductance*, $G = \frac{1}{R} = \frac{I}{V}$ measured in Siemens (S).

The graph shows the characteristic of a $12.5\,\Omega$ resistor. The gradient of the graph equals the conductance $G = 80\,\text{mS}$. Alternative zigzag symbol.

[Passive Sign Convention]

To measure the voltage in a physical circuit, you use a voltmeter (V in the figure) which has two test leads connected to it usually coloured red (marked +) and black (marked −) respectively. The reading on the voltmeter shows the voltage at the red lead relative to that at the black lead (or equivalently the red voltage minus the black voltage). To measure the voltage V in the figure, you would connect the red lead to the top end of the arrow (pointed end) and the black lead to the bottom (blunt end).



To measure current you use an ammeter (A in the figure) which also has two test leads coloured red and black respectively. The reading shows the current flowing through the ammeter into the red lead and out of the black lead. To measure the current I on the previous slide, you would need to break the wire carrying the current and insert the ammeter as shown in the figure.

With the connections shown in the figure, the readings on V and A will always have the same sign: either both positive or both negative and will satisfy Ohm's law: $V = IR$. However, if the connections are reversed on either V or A, then the two readings will have opposite signs and $V = -IR$ which does not satisfy Ohm's law.

So, if you want Ohm's law to be true you must be sure to connect the measuring devices the right way round according to the passive sign convention.

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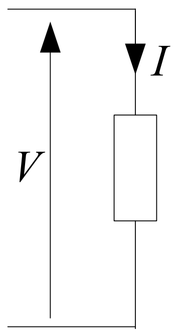
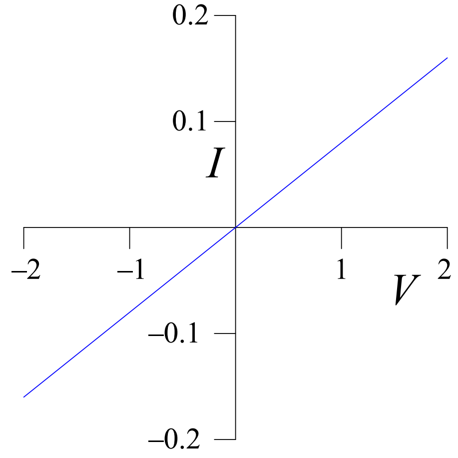
Ohm's law relates the voltage drop across a resistor to the current flowing in it.





If the voltage, V, is fixed elsewhere in the circuit, it is convenient to think that V *causes* the current I to flow.

If the current, I, is fixed elsewhere in the circuit, it is more convenient to think that V is *caused by* the current I flowing through the resistor.

Neither statement is "more true" than the other. It is perhaps truer to say that I and V are *constrained to satisfy* $V = I \times R$.

Resistor Power Dissipation

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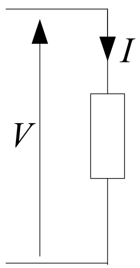
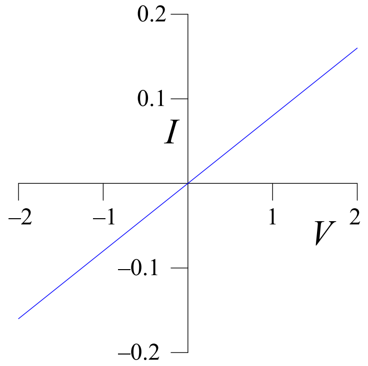
Gravitational potential energy, mgh, lost by a falling object is transformed into kinetic energy or heat.

Current in a resistor always flows from a high voltage (more positive) to a low voltage (more negative).





When current flows through a resistor, the electrical potential energy that is lost is transformed into heat.

The power dissipated as heat in a resistor is equal to VI Watts (W). 1 Watt equals one Joule of energy per second. Since V and I always have the same sign (see graph) the power dissipation is always positive.

Any component: $P = VI$ gives the power absorbed by any component.

For a resistor only: $\frac{V}{I} = R \quad \Rightarrow \quad P = VI = \frac{V^2}{R} = I^2 R.$

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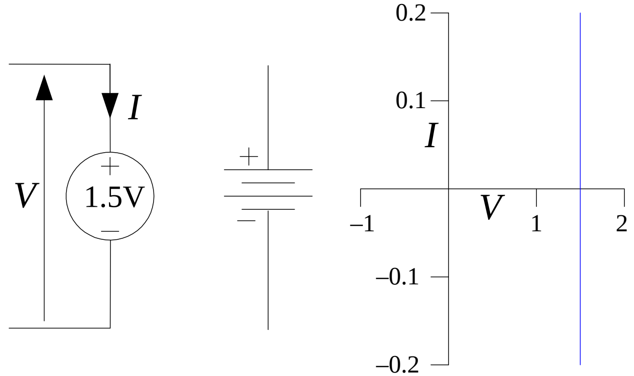
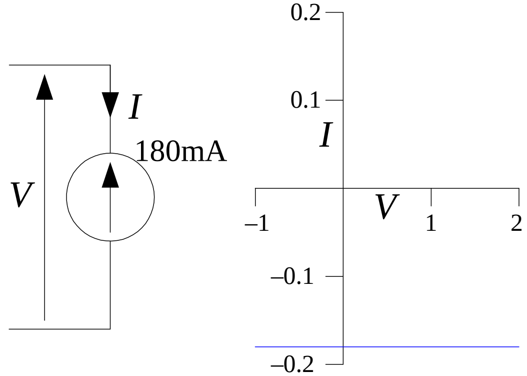
Energy in an electrical circuit is supplied by voltage and current sources

An *ideal voltage source* maintains the same value of V for all currents. Its characteristic is a vertical line with infinite gradient. There are two common symbols.



An *ideal current source* maintains the same value of I for all voltages. Its characteristic is a horizontal line with zero gradient. Notice that I is negative.



If the source is supplying electrical energy to a circuit, then $VI < 0$.
However, when a recharcheable battery is charging, $VI > 0$.

Power Conservation

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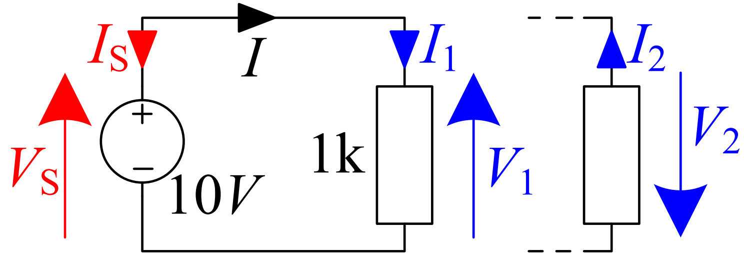
In any circuit some circuit elements will be supplying energy and others absorbing it. At all times, the power absorbed by all the elements will sum to zero.

The circuit has two nodes whose potential difference is 10 V.



Ohm's Law:
$I = \frac{V}{R} = 0.01\text{ A}$

Power absorbed by resistor:

$$P_R = V_1 \times I_1 = (+10) \times (+0.01) = +0.1\text{ W}$$

For Ohm's law or power dissipation, V and I can be measured either way round but must be in opposite directions (passive sign convention).

$$P_R = V_2 \times I_2 = (-10) \times (-0.01) = +0.1\text{ W}$$

Power absorbed by voltage source:

$$P_S = V_S \times I_S = (+10) \times (-0.01) = -0.1\text{ W}$$

Total power absorbed by circuit elements: $P_S + P_R = 0$

Units and Multipliers

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Quantity	Letter	Unit	Symbol
Charge	Q	Coulomb	C
Conductance	G	Siemens	S
Current	I	Amp	A
Energy	W	Joule	J
Potential	V	Volt	V
Power	P	Watt	W
Resistance	R	Ohm	Ω

Value	Prefix	Symbol
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Value	Prefix	Symbol
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

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- ☐ Charge, Current and Voltage
- ☐ Resistors, Voltage Source and Current Sources
- ☐ Power Dissipation and Power Conservation

For further details see Hayt Ch 2 or Irwin Ch 1.