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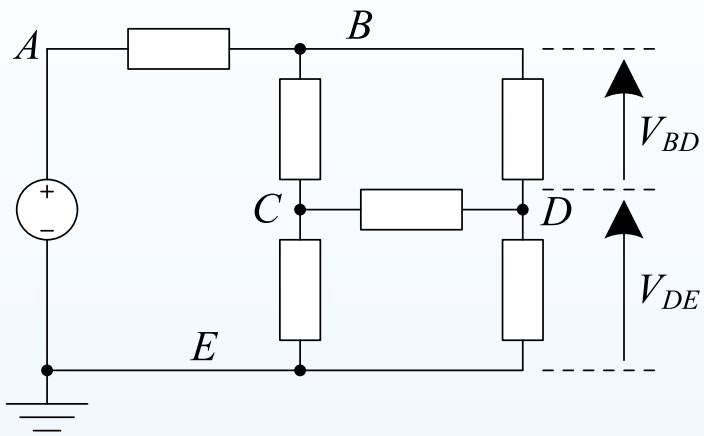
Each component that links a pair of nodes is called a *branch* of the network.



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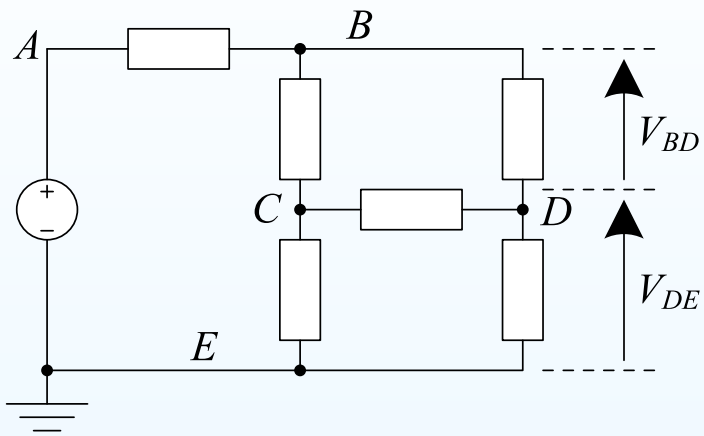

Kirchoff's Voltage Law (KVL) is a consequence of the fact that the work done in moving a charge from one node to another does not depend on the route you take; in particular the work done in going from one node back to the same node by any route is zero.

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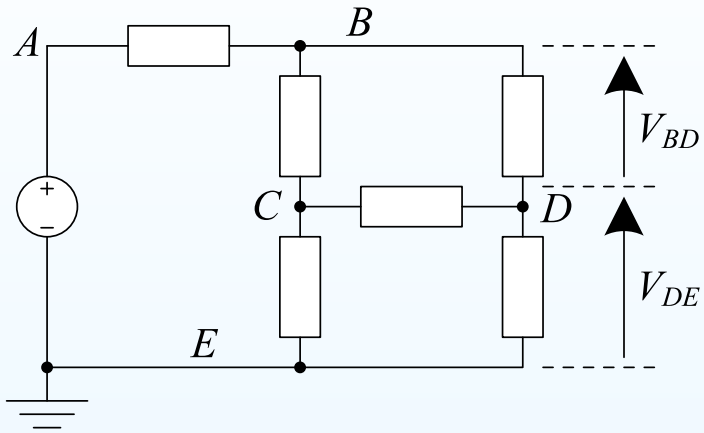

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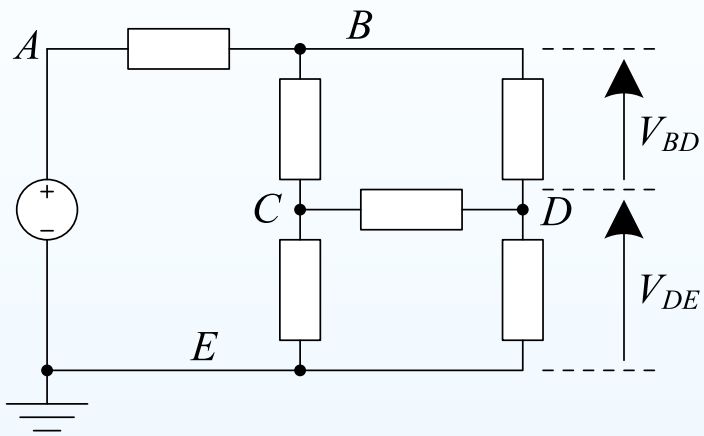

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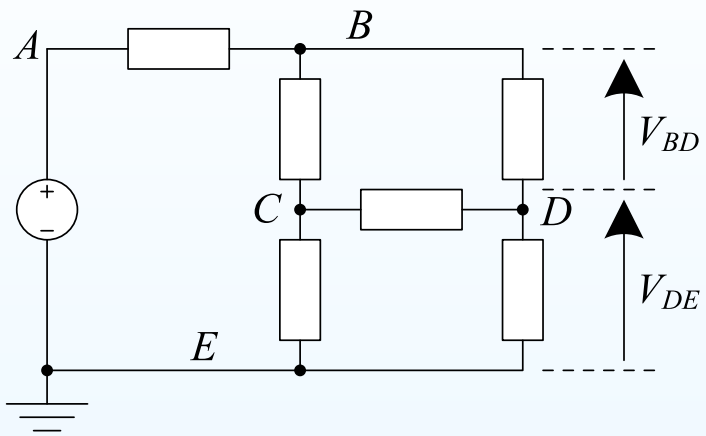

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Equivalent formulation:

$V_{XY} = V_{XE} - V_{YE} = V_X - V_Y$ for any nodes X and Y.

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Wherever charges are free to move around, they will move to ensure charge neutrality everywhere at all times.

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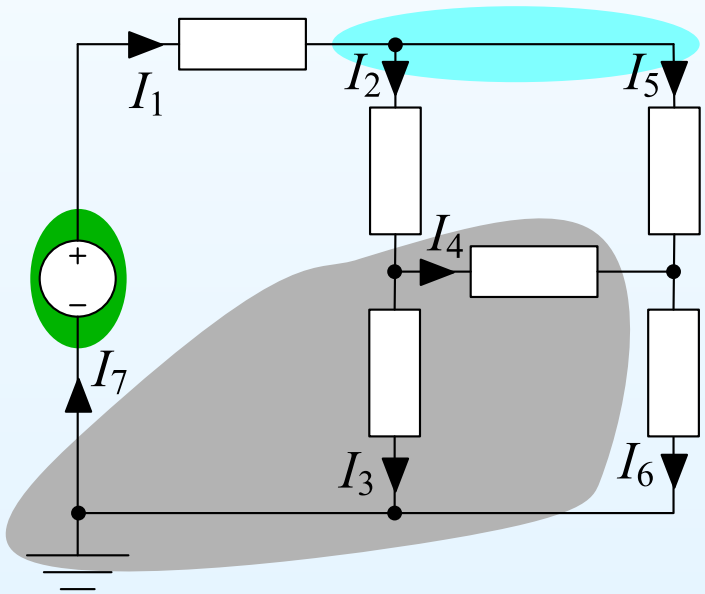

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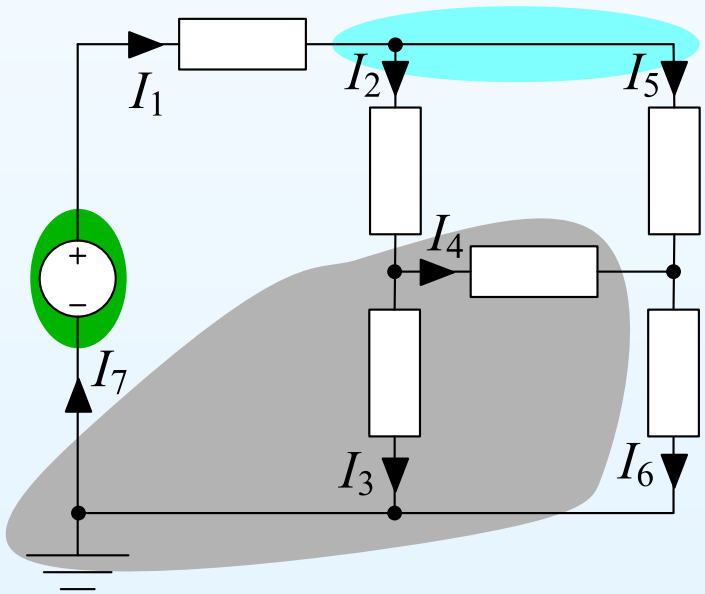

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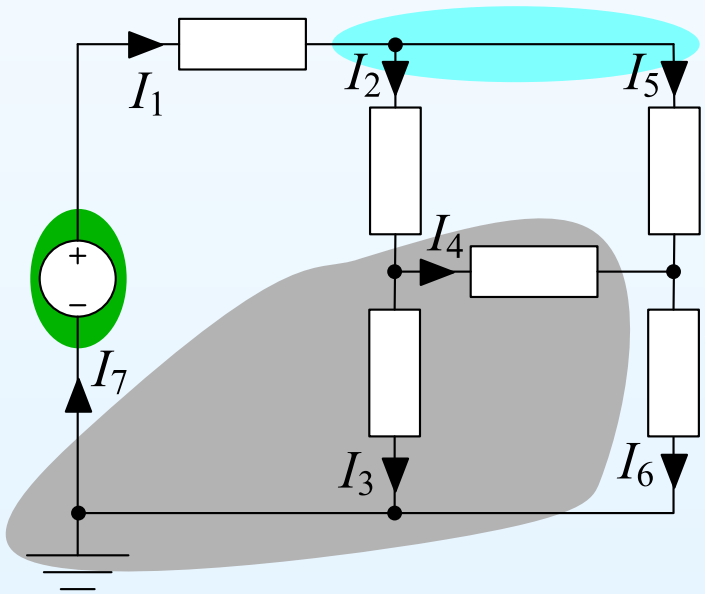

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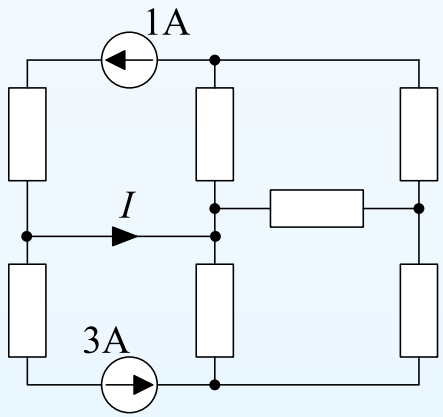

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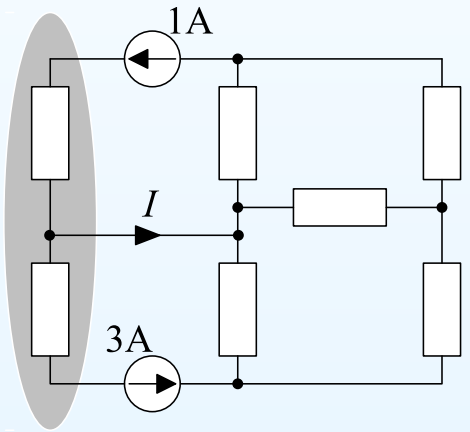

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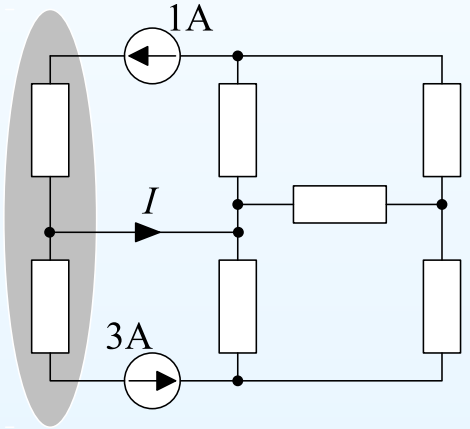

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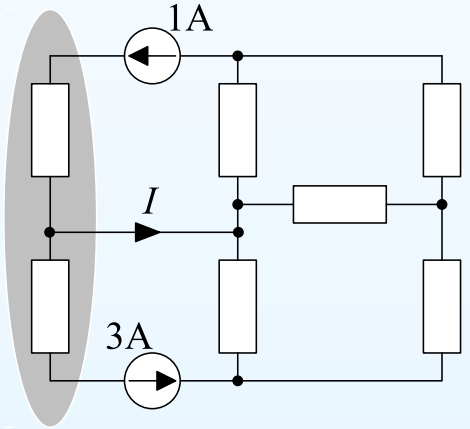

Note that here I ends up negative which means we chose the wrong arrow direction to label the circuit. This does not matter. You can choose the directions arbitrarily and let the algebra take care of reality.

Series and Parallel

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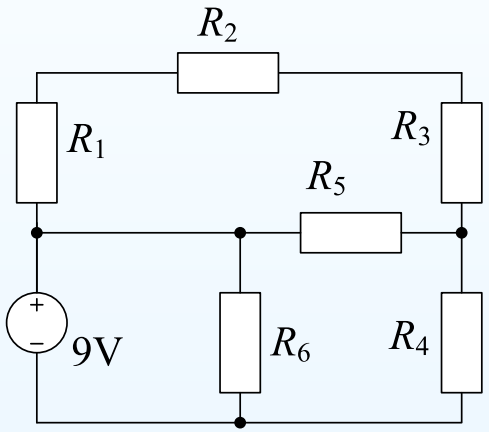

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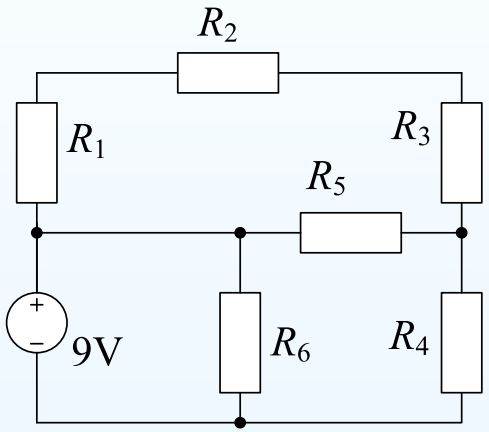

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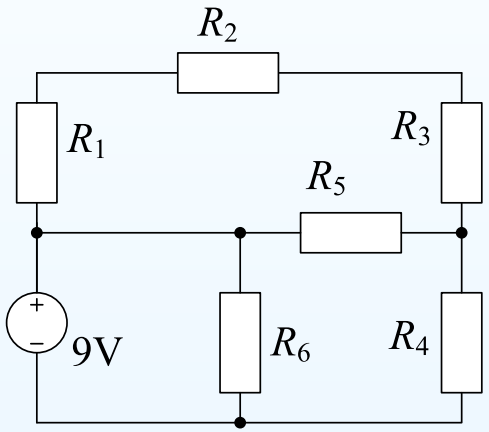

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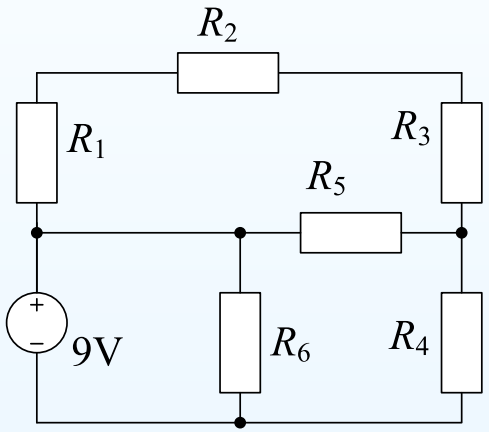

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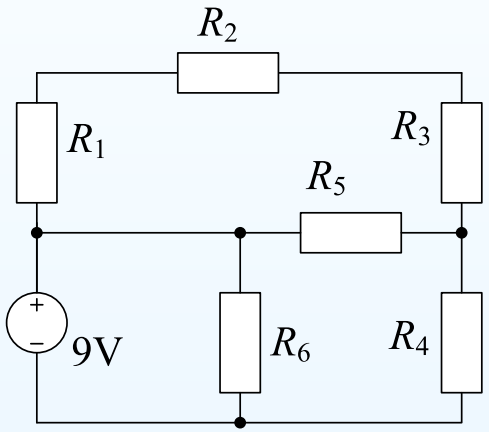

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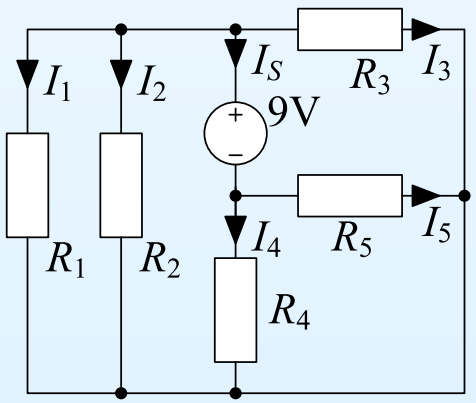

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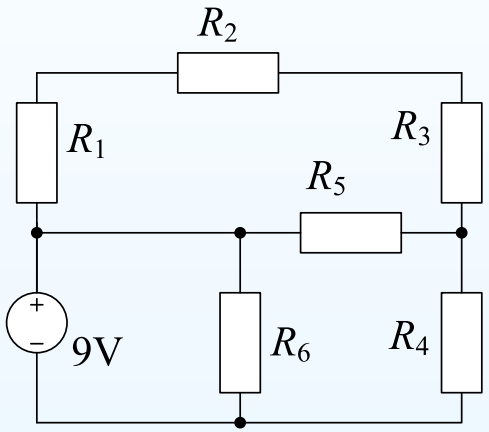

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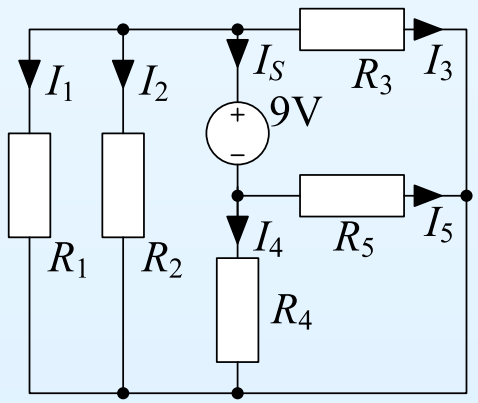

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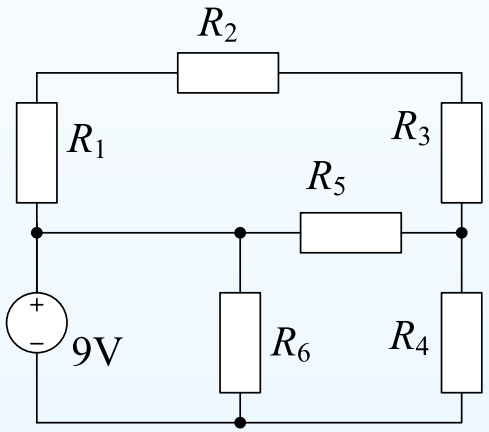

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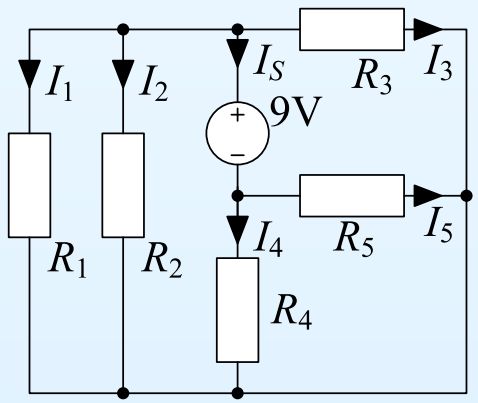

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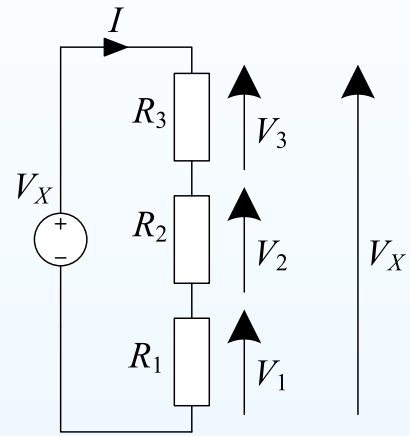
$$V_X = V_1 + V_2 + V_3$$



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$$V_X = V_1 + V_2 + V_3$$
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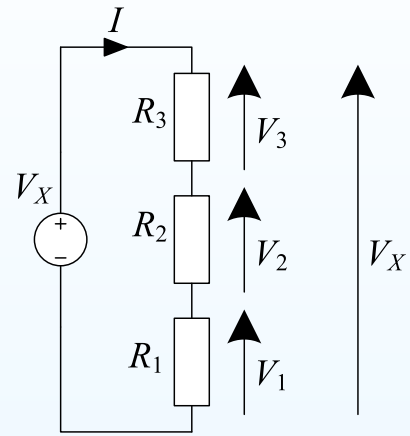

Series Resistors: Voltage Divider

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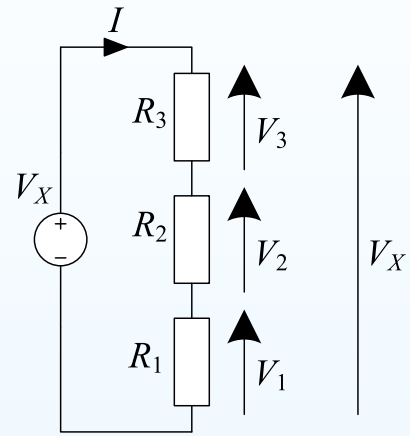

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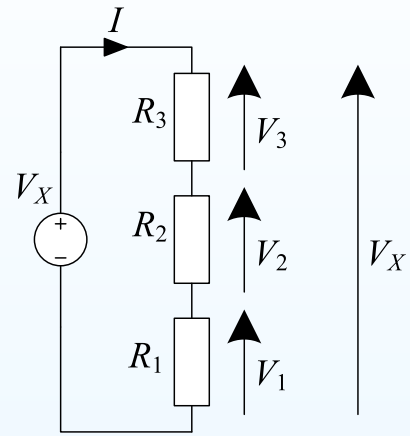

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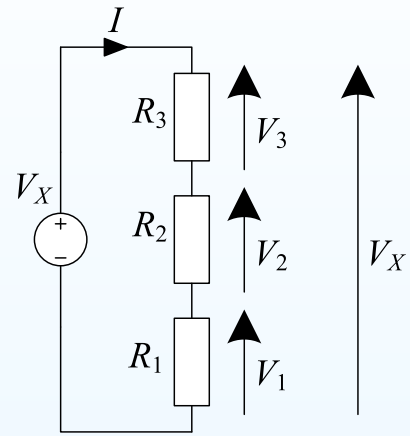

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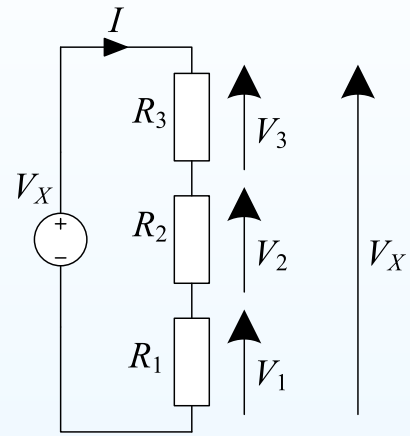
where $R_T = R_1 + R_2 + R_3$ is the total resistance of the chain.



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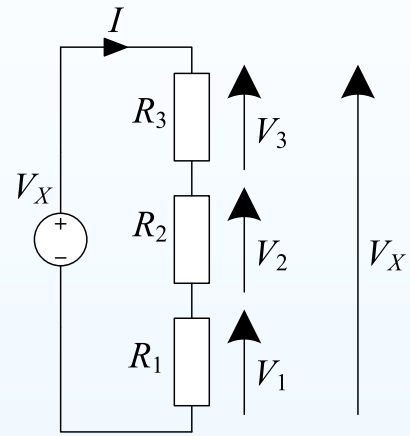

V_X is divided into $V_1 : V_2 : V_3$ in the proportions $R_1 : R_2 : R_3$.

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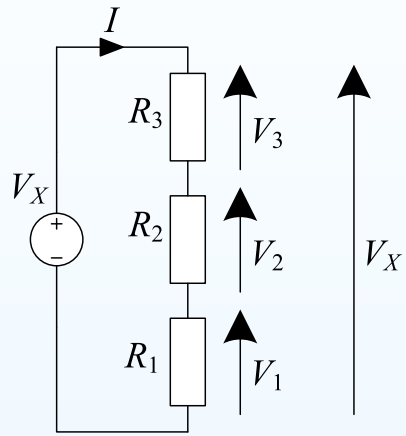
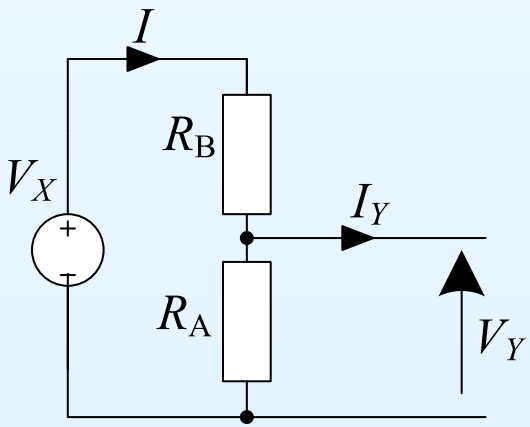


V_X is divided into $V_1 : V_2 : V_3$ in the proportions $R_1 : R_2 : R_3$.

Approximate Voltage Divider:

If $I_Y = 0$, then $V_Y = \frac{R_A}{R_A+R_B} V_X$.



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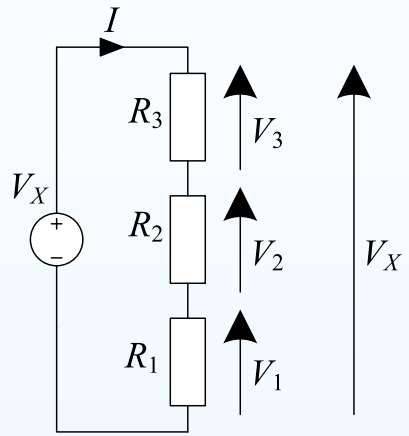

V_X is divided into $V_1 : V_2 : V_3$ in the proportions $R_1 : R_2 : R_3$.

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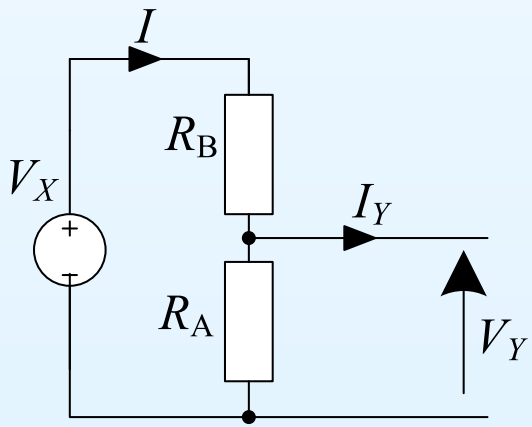
If $I_Y \ll I$, then $V_Y \approx \frac{R_A}{R_A+R_B} V_X$.



Parallel Resistors: Current Divider

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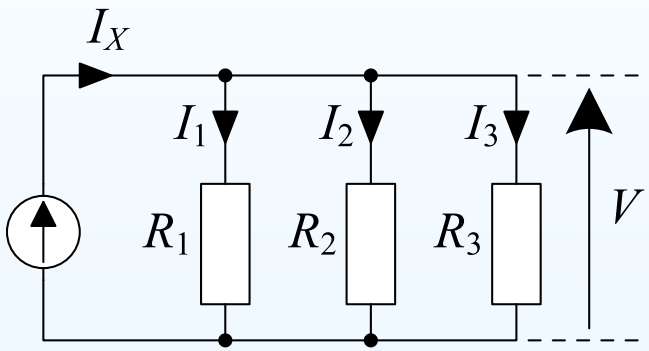
Parallel resistors all share the same V.



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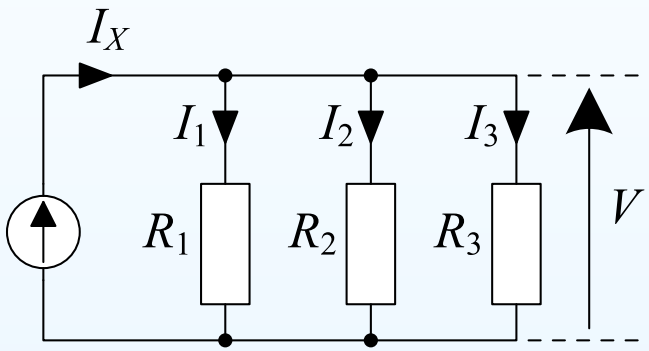

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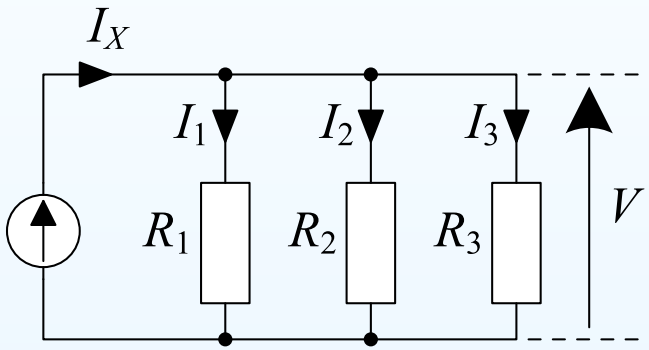

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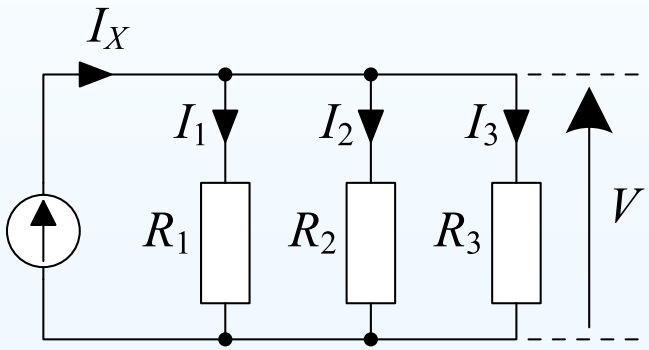

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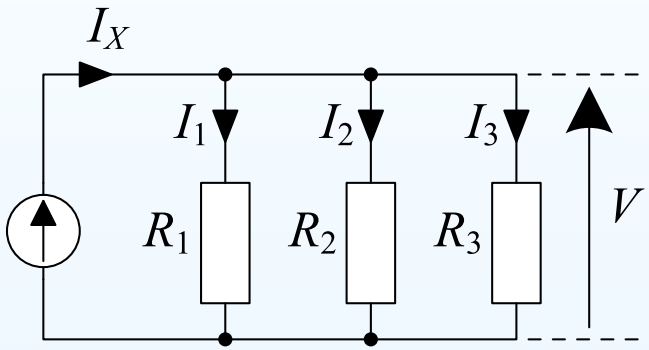

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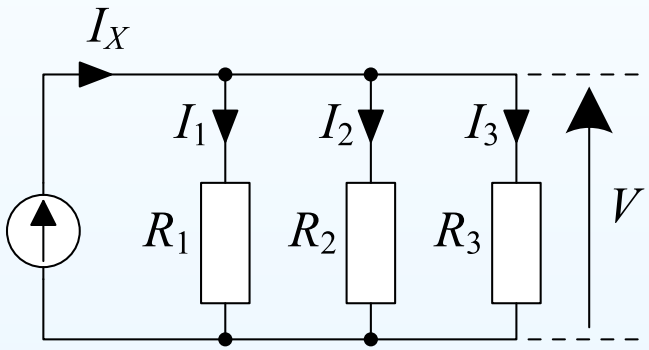

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Special case for only two resistors:

$$I_1 : I_2 = G_1 : G_2 = R_2 : R_1 \;\Rightarrow\; I_1 = \frac{R_2}{R_1+R_2} I_X.$$

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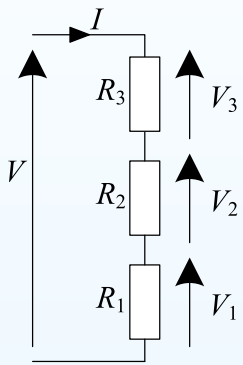
So we can replace the three resistors by a single *equivalent resistor* of value R_T without affecting the relationship between V and I.



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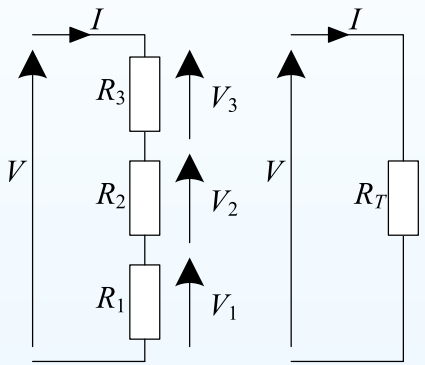

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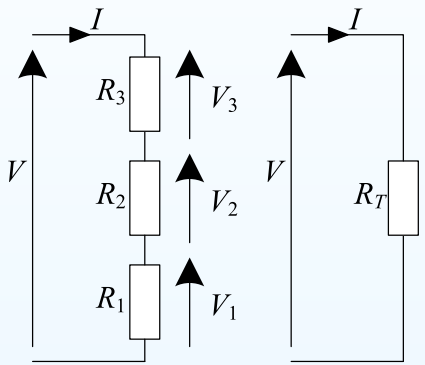
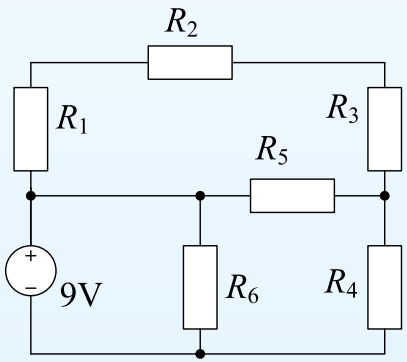


Replacing series resistors by their equivalent resistor will not affect any of the voltages or currents in the rest of the circuit.



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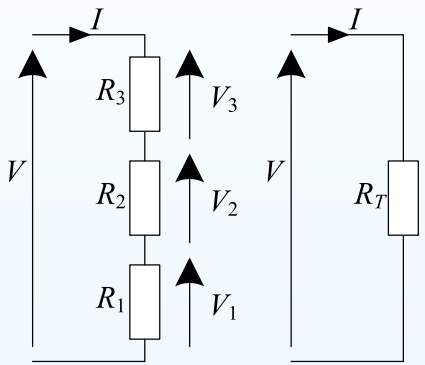
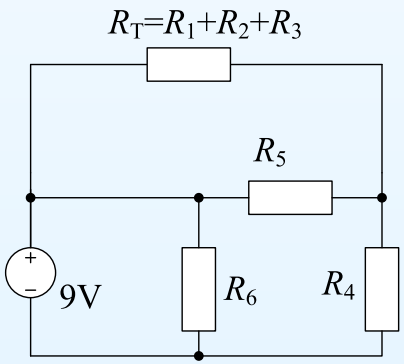


Replacing series resistors by their equivalent resistor will not affect any of the voltages or currents in the rest of the circuit.

However the individual voltages V_1, V_2 and V_3 are no longer accessible.



Equivalent Resistance: Parallel

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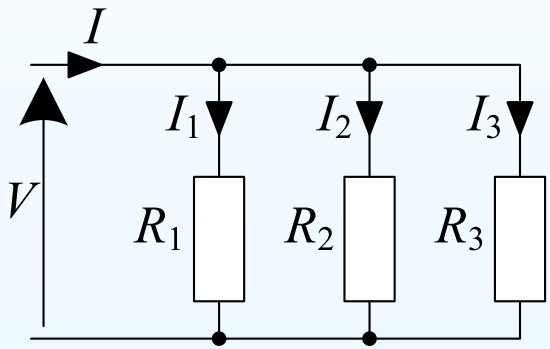
Similarly we known that $I = I_1 + I_2 + I_3 = V(G_1 + G_2 + G_3) = VG_P$.



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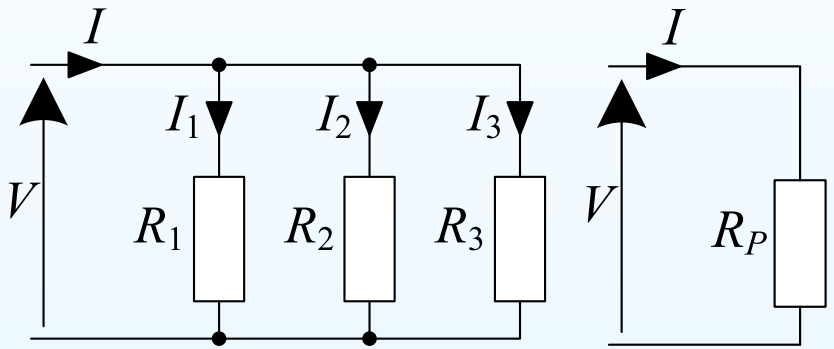

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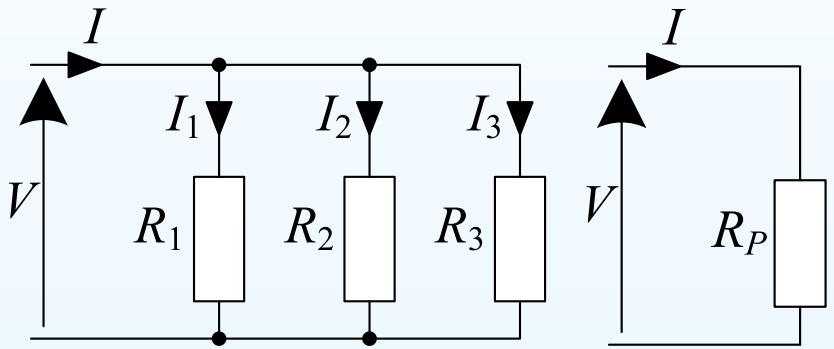

Equivalent Resistance: Parallel

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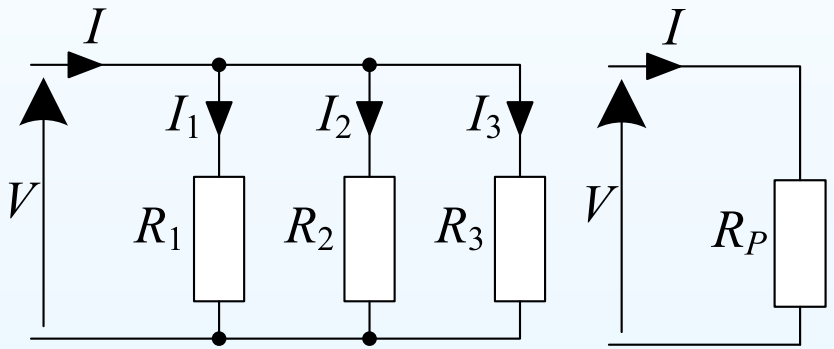

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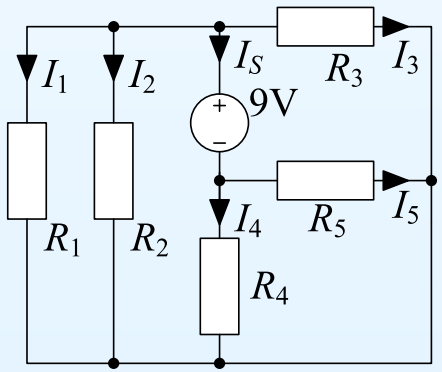

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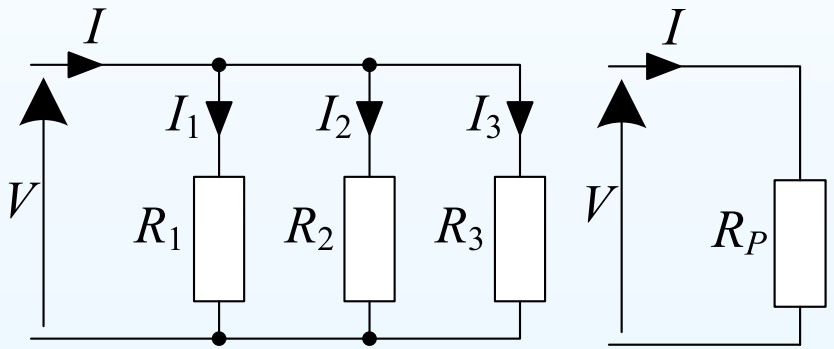

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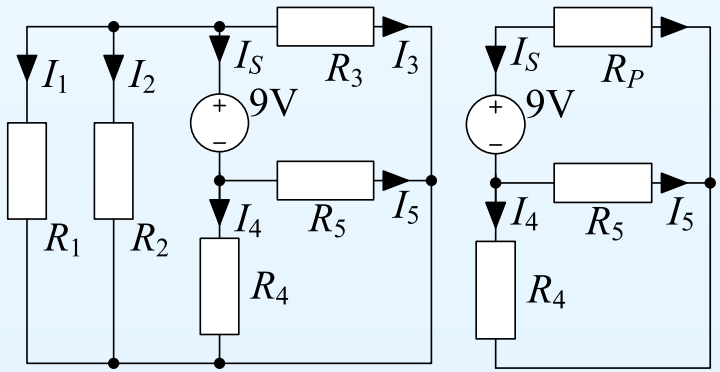

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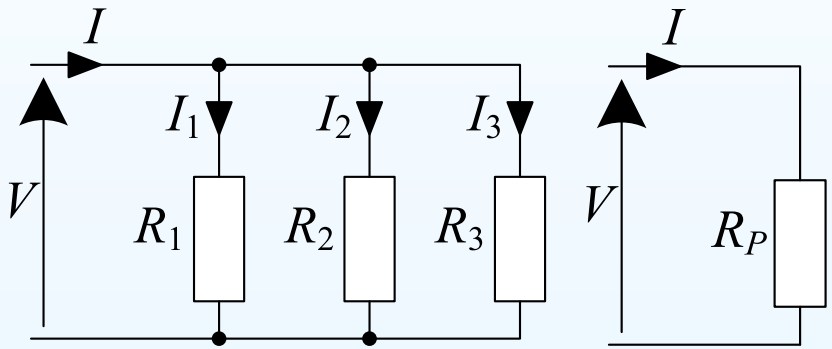

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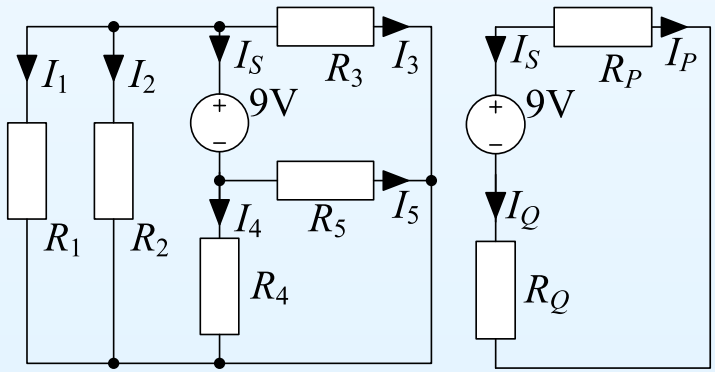
$R_4$and R_5 are also in parallel.



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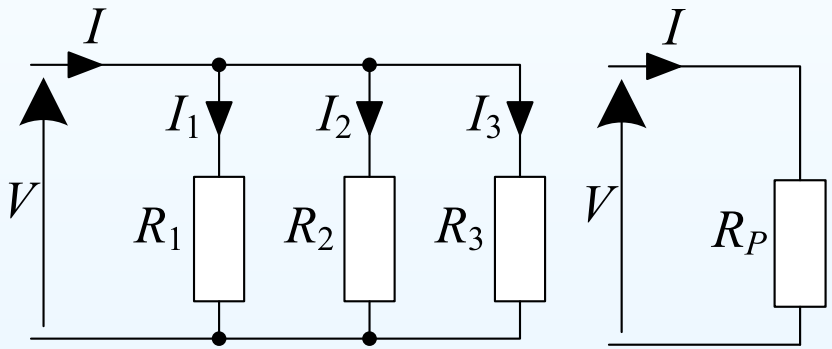

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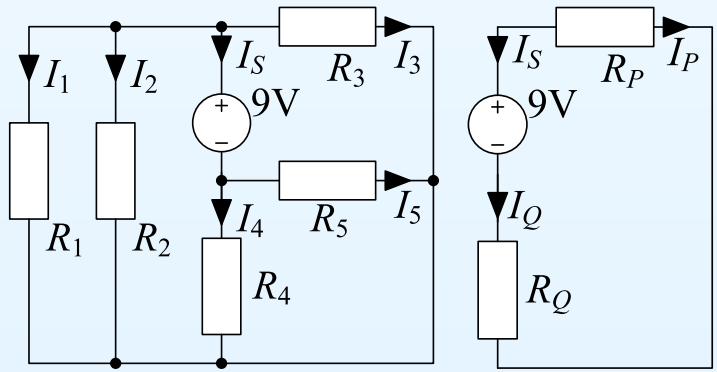

Much simpler - although none of the original currents $I_1, \cdots, I_5$ are now accessible. Current I_S and the three node voltages are identical.

Equivalent Resistance: Parallel Formulae

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For parallel resistors $G_P = G_1 + G_2 + G_3$

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These formulae work for any number of resistors.

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Example: $1\,\mathrm{k\Omega} \,||\, 99\,\mathrm{k\Omega} = \frac{99}{100}\,\mathrm{k\Omega} = \left(1 - \frac{1}{100}\right)\,\mathrm{k\Omega}$

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Important: The equivalent resistance of parallel resistors is always less than any of them.

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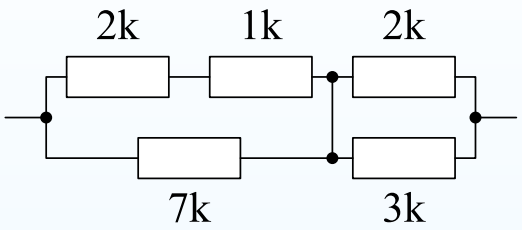

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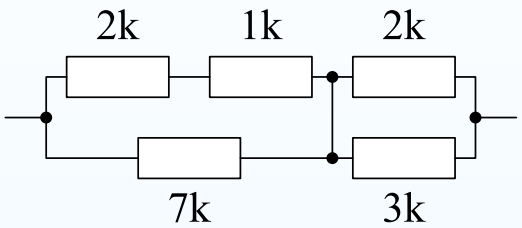
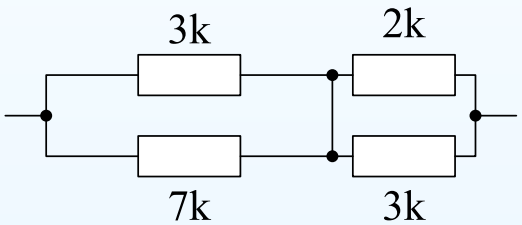
Series: $2\,\mathrm{k} + 1\,\mathrm{k} = 3\,\mathrm{k}$





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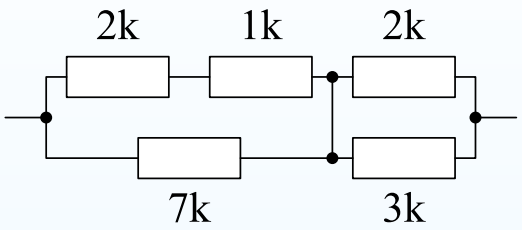

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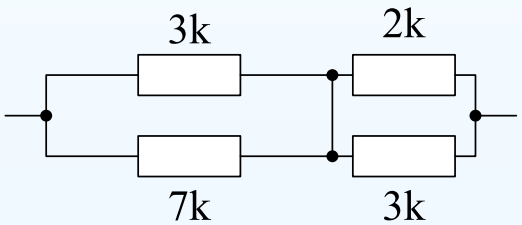

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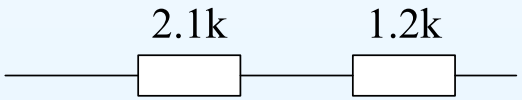

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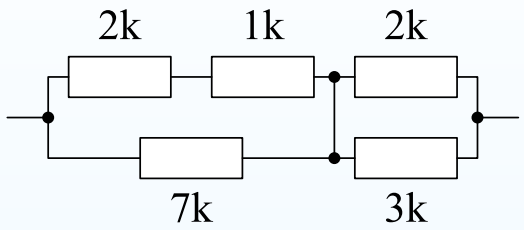

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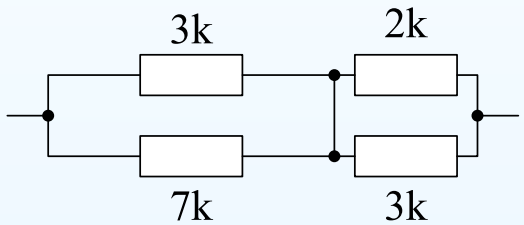

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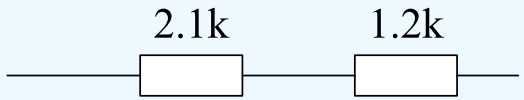

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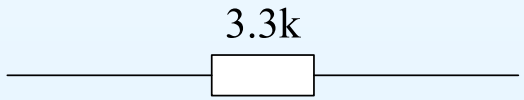

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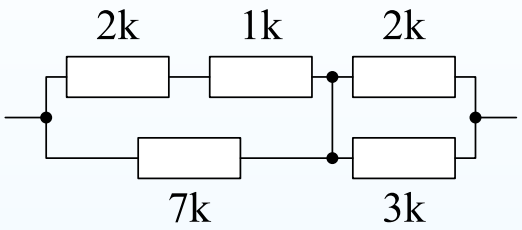

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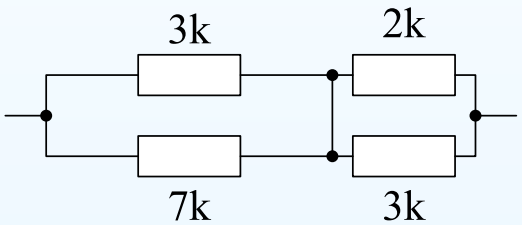

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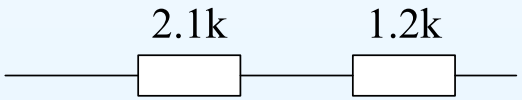

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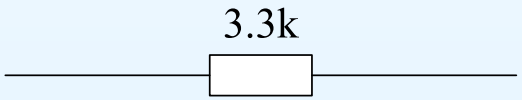
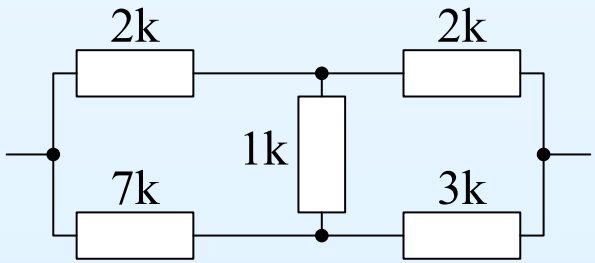


Sadly this method does not always work: there are no series or parallel resistors here.



Non-ideal Voltage Source

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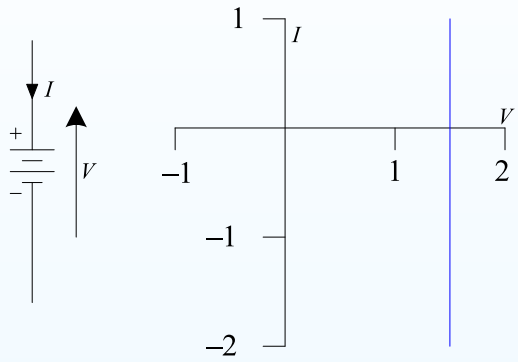
Normally a battery is supplying energy so V and I have opposite signs, so $I \leq 0$.



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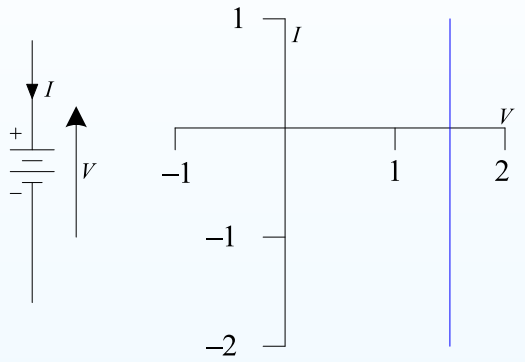
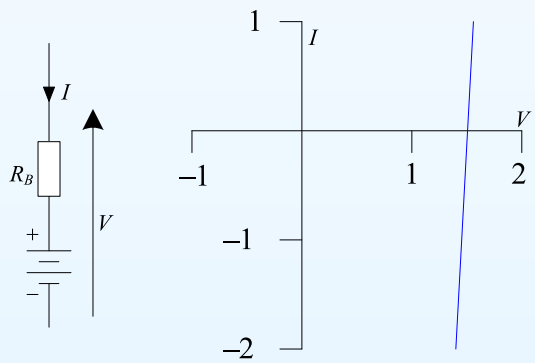


An real battery has a characteristic that has a slight positive slope: battery voltage decreases as the (negative) current increases.

Model this by including a small resistor in series. $V = V_B + IR_B$.



Non-ideal Voltage Source

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- Kirchoff's Voltage Law
- Kirchoff's Current Law
- KCL Example
- Series and Parallel
- Dividers
- Equivalent Resistance: Series
- Equivalent Resistance: Parallel
- Equivalent Resistance: Parallel Formulae
- Simplifying Resistor Networks
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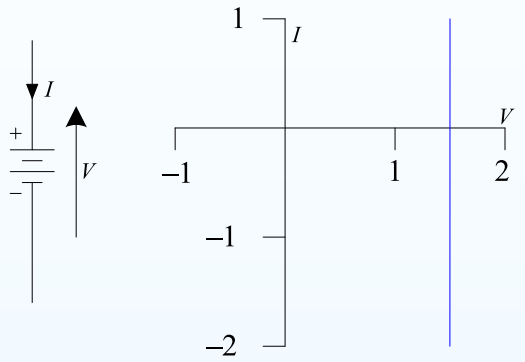
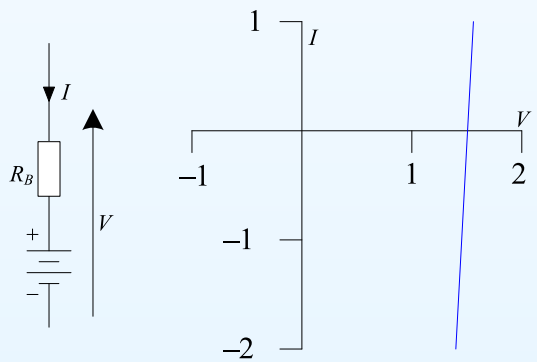
- Summary

An ideal battery has a characteristic that is vertical: battery voltage does not vary with current.
Normally a battery is supplying energy so V and I have opposite signs, so $I \leq 0$.

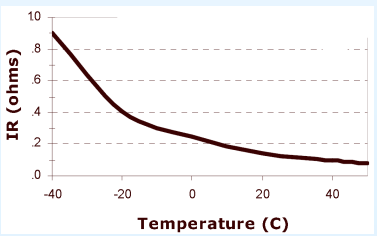


An real battery has a characteristic that has a slight positive slope: battery voltage decreases as the (negative) current increases.

Model this by including a small resistor in series. $V = V_B + IR_B$.



The equivalent resistance for a battery increases at low temperatures.



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- Kichoff's Voltage and Current Laws
- Series and Parallel components
- Voltage and Current Dividers
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- Battery Internal Resistance

For further details see Hayt Ch 3 or Irwin Ch 2.