4: Linearity and Superposition

- Linearity Theorem
- Zero-value sources
- Superposition
- Superposition and dependent sources
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- Superposition and Power
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- Summary

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**Linearity Theorem:** For any circuit containing resistors and independent voltage and current sources, every node voltage and branch current is a linear function of the source values and has the form $\sum a_i U_i$ where the $U_i$ are the source values and the $a_i$ are suitably dimensioned constants.
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Also true for a circuit containing *dependent* sources providing their values are sums of multiples of other voltages and/or currents in the circuit.
Zero-value sources

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![Diagram of zero-value source](image)
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Superposition

From the linearity theorem, we know that $X = a_1 U_1 + a_2 U_2$ so all we need to do is find the values of $a_1$ and $a_2$. 

![Circuit Diagram](image)
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If we set $U_2 = 0$ then $X = a_1 U_1$. For $U_2 = 0$ the current source becomes an *open circuit* and now the $3 \, \text{k}$ resistor plays no part in the circuit.
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2 k and 1 k form a potential divider and so

$$a_1 = \frac{1 \text{k}}{2 \text{k} + 1 \text{k}} = \frac{1}{3}.$$
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If we set $U_1 = 0$ then $X = a_2 U_2$. For $U_1 = 0$ the voltage source becomes a short circuit and the $2 \, \text{k}$ and $1 \, \text{k}$ are in parallel.

$$2 \, \text{k} \parallel 1 \, \text{k} = \frac{2 \, \text{k} \times 1 \, \text{k}}{2 \, \text{k} + 1 \, \text{k}} = \frac{2}{3} \, \text{k}.$$
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Now $X = \frac{2}{3} U_2$ and so $a_2 = \frac{2}{3}$. 

\[\text{Diagram of the circuit with the resistance values and terminals labeled.}\]
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2 \text{k} \parallel 1 \text{k} = \frac{2 \text{k} \times 1 \text{k}}{2 \text{k} + 1 \text{k}} = \frac{2}{3} \text{k}.
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Now \( X = \frac{2}{3} U_2 \) and so \( a_2 = \frac{2}{3} \).

Combining these two gives \( X = a_1 U_1 + a_2 U_2 = \frac{1}{3} U_1 + \frac{2}{3} U_2 \).
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Now \( X = \frac{2}{3} U_2 \) and so \( a_2 = \frac{2}{3} \).

Combining these two gives \( X = a_1 U_1 + a_2 U_2 = \frac{1}{3} U_1 + \frac{2}{3} U_2 \).

**Superposition:** Any voltage or current in a circuit may be found by adding up the values due to each of the independent sources in the circuit while setting all the other independent sources to zero.
A dependent source is one that is determined by the voltage and/or current elsewhere in the circuit. Here $V \triangleq Y - X$. 
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$$Y = 2U_1$$
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**Step 1:** Pretend all sources are independent and use superposition to find expressions for the node voltages:

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X = \frac{10}{3}U_1 + 2U_2 \\
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$$X = \frac{10}{3} U_1 + 2U_2 + \frac{1}{6} (Y - X) \Rightarrow \frac{7}{6} X - \frac{1}{6} Y = \frac{10}{3} U_1 + 2U_2$$
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**Note:** This is an alternative to nodal analysis: you get the same answer.
Any current or voltage can be written \( X = a_1 U_1 + a_2 U_2 + a_3 U_3 + \ldots \).
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Suppose we know $U_2 = 6$ mA.

Then $X = \frac{1}{3}U_1 + \frac{2}{3}U_2 = \frac{1}{3}U_1 + 4$. 
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If all the independent sources except for \( U_1 \) have known fixed values, then

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This has a straight line graph.
The power absorbed (or \textit{dissipated}) by a component always equals $VI$ where the measurement directions of $V$ and $I$ follow the passive sign convention.

For a resistor $VI = \frac{V^2}{R} = I^2 R$. 
Superposition and Power

The power absorbed (or *dissipated*) by a component always equals $V I$ where the measurement directions of $V$ and $I$ follow the passive sign convention.

For a resistor $V I = \frac{V^2}{R} = I^2 R$.

Power in resistor is $P = \frac{(U_1+U_2)^2}{10} = 6.4 \text{ W}$.

![Resistor Circuit Diagram]
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$P \neq P_1 + P_2 \implies$ Power does not obey superposition.

You must use superposition to calculate the total $V$ and/or the total $I$ and then calculate the power.
From the linearity theorem, all voltages and currents have the form \( \sum a_i U_i \) where the \( U_i \) are the values of the independent sources.

If you multiply \textit{all} the independent sources by the same factor, \( k \), then all voltages and currents in the circuit will be multiplied by \( k \).

The power dissipated in any component will be multiplied by \( k^2 \).
**Proportionality**

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**Special Case:**
If there is only one independent source, \( U \), then all voltages and currents are proportional to \( U \) and all power dissipations are proportional to \( U^2 \).
Summary

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- **Superposition:** sometimes simpler than nodal analysis, often more insight.
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  - Dependent sources - treat as independent and add dependency as an extra equation
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- If all sources are fixed except for \( U_1 \) then all voltages and currents in the circuit have the form \( aU_1 + b \).
- Power **does not obey** superposition.
- **Proportionality:** multiplying all sources by \( k \) multiplies all voltages and currents by \( k \) and all powers by \( k^2 \).
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  - Zero-value voltage and current sources
  - Dependent sources - treat as independent and add dependency as an extra equation

- If all sources are fixed except for \( U_1 \) then all voltages and currents in the circuit have the form \( aU_1 + b \).

- Power **does not obey** superposition.

- **Proportionality**: multiplying all sources by \( k \) multiplies all voltages and currents by \( k \) and all powers by \( k^2 \).

For further details see Hayt et al. Chapter 5.