4: Linearity and Superposition

Linearity Theorem

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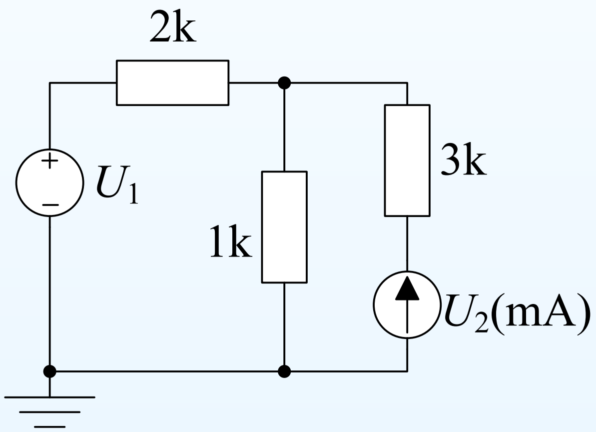

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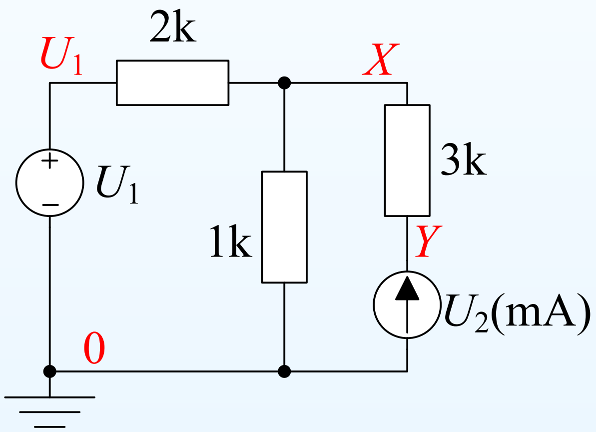

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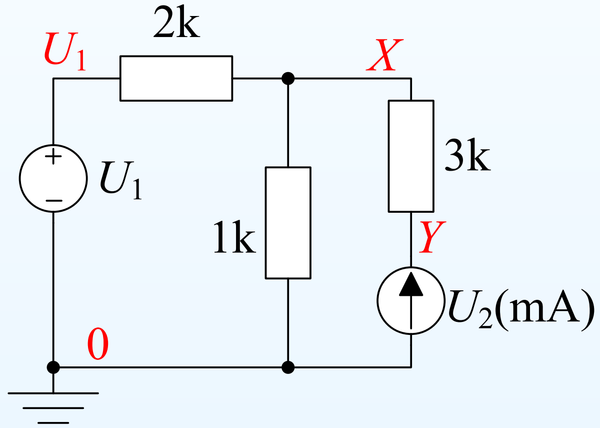

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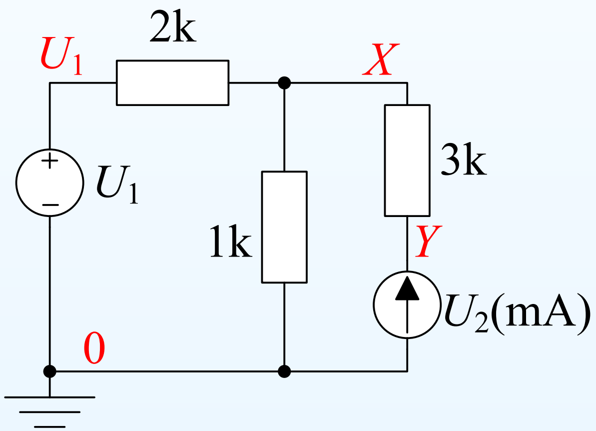

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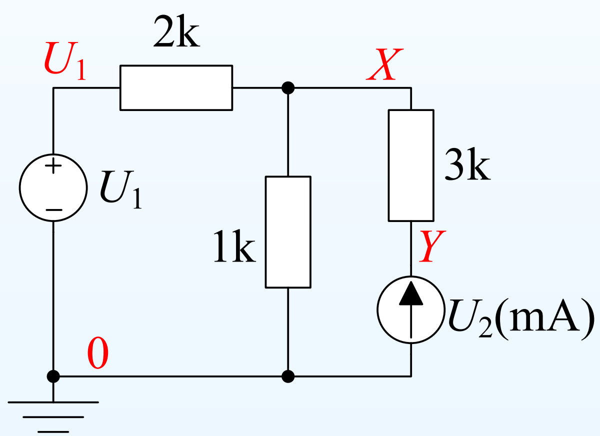

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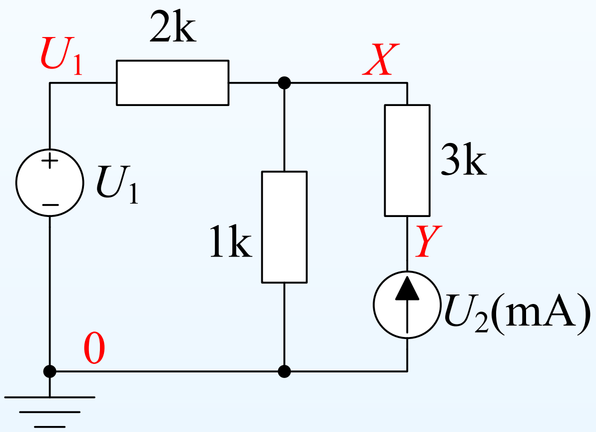

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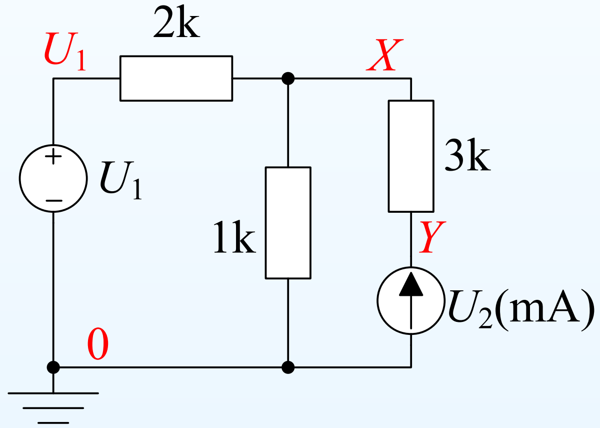

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Also true for a circuit containing *dependent* sources whose values are proportional to voltages or currents elsewhere in the circuit.

Zero-value sources

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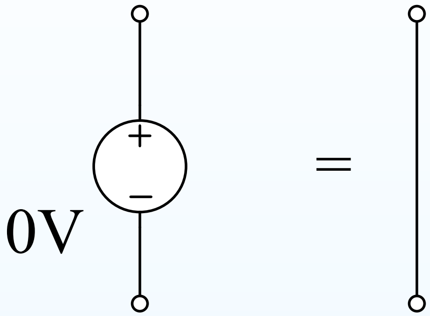

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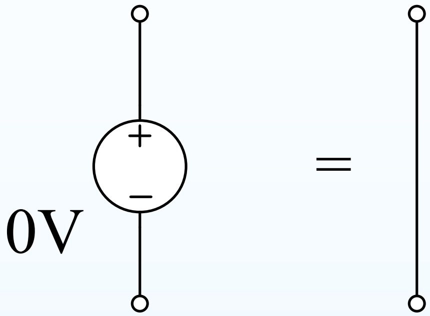

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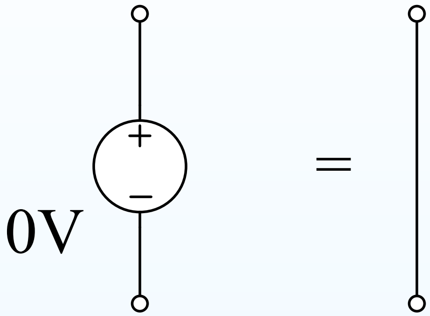

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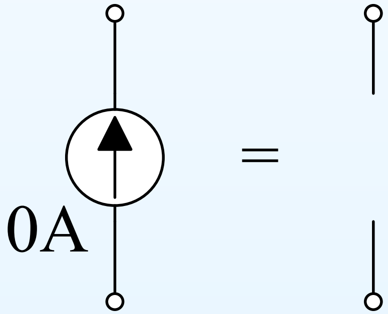

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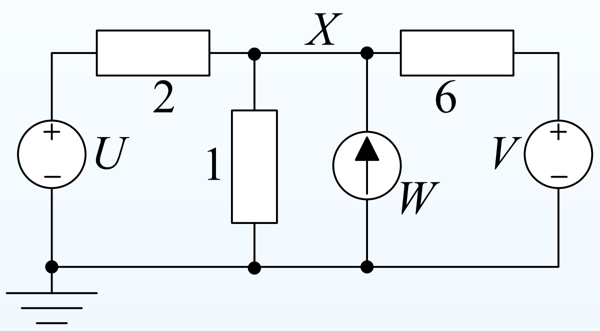

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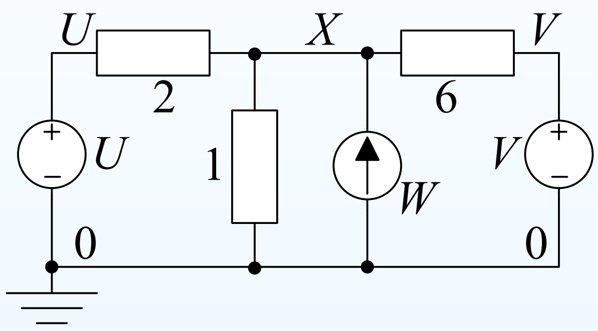

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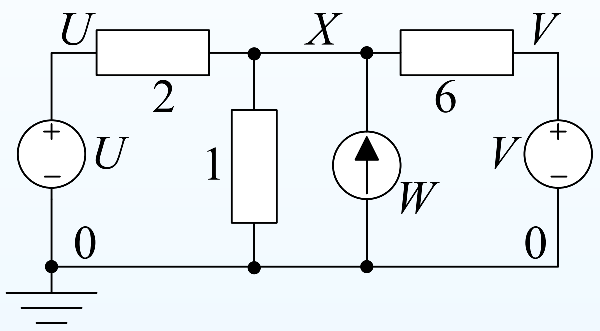

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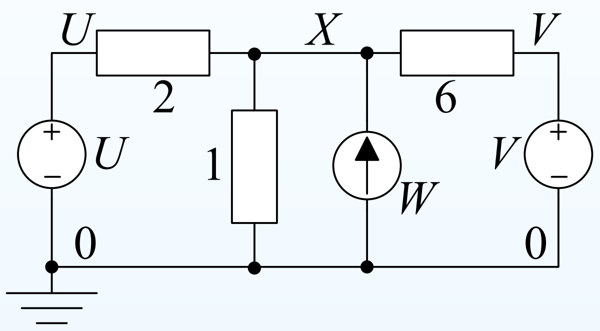

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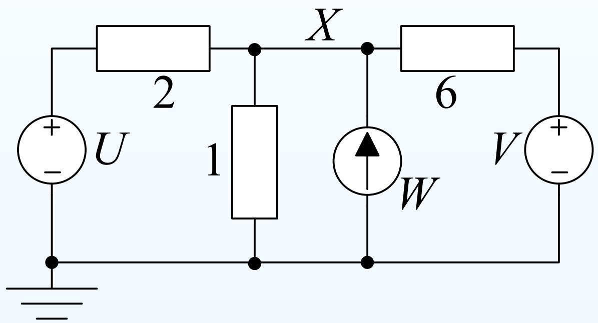

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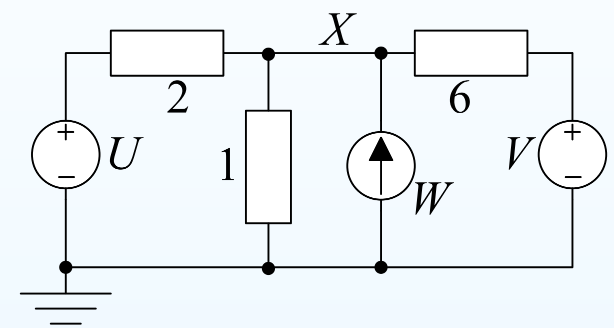

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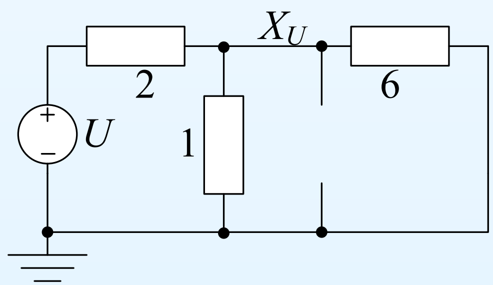

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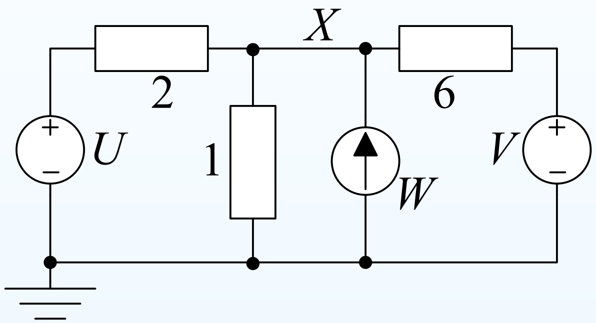

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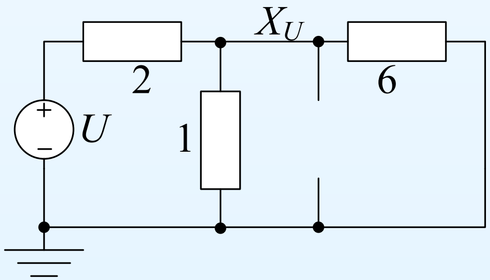

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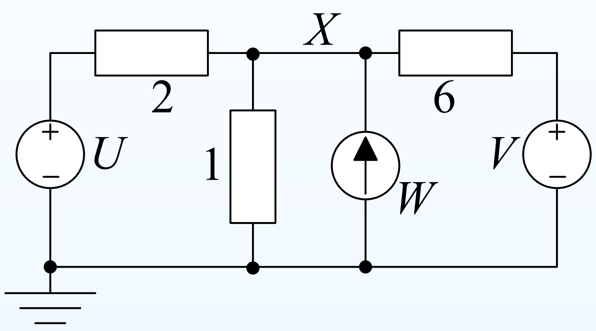

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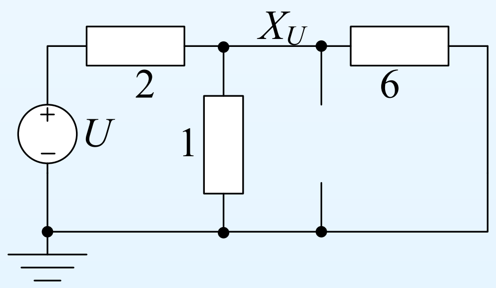

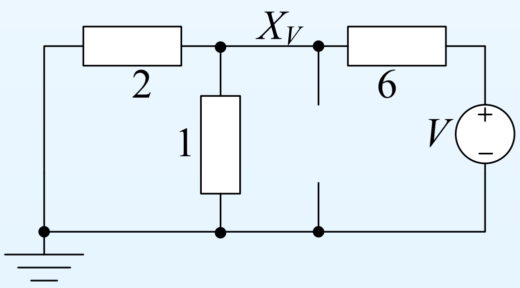

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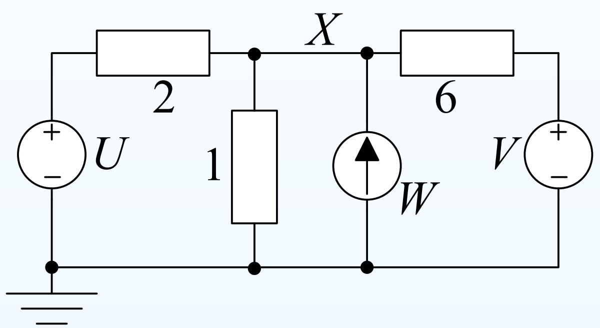

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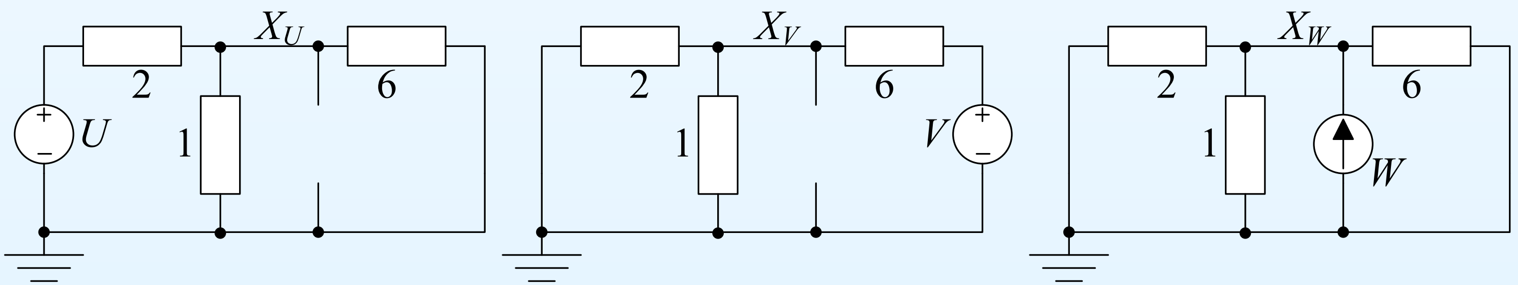

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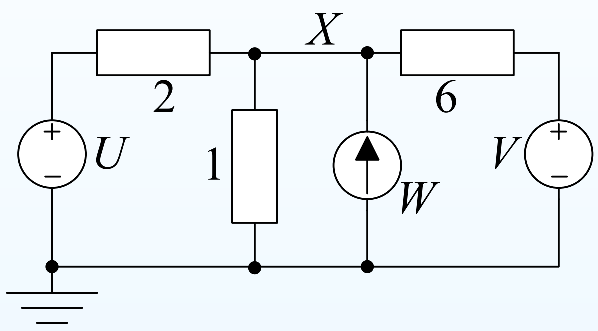

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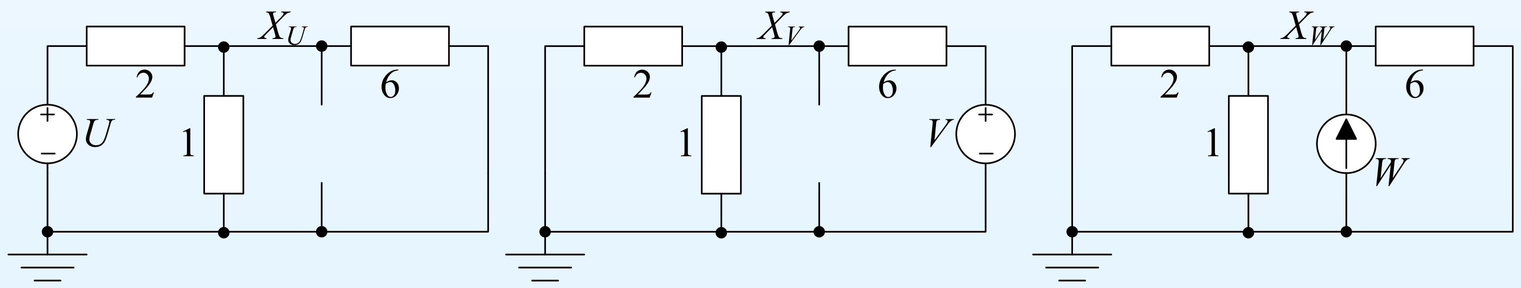

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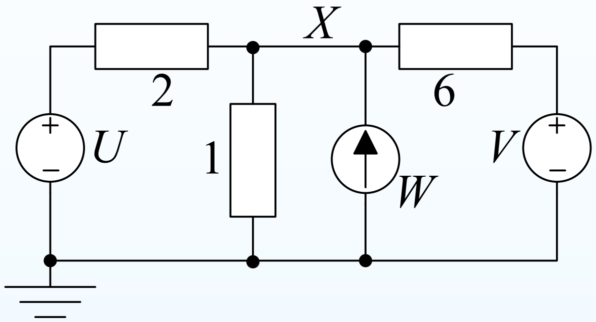
Similarly, $X_V = bV$ and $X_W = cW \quad \Rightarrow \quad X = X_U + X_V + X_W$.

Superposition Calculation

Superposition:

Find the effect of each source on its own by setting all other sources to zero. Then add up the results.



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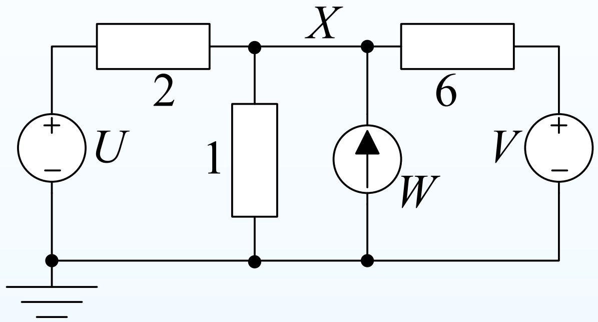

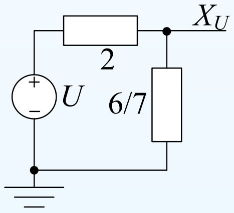

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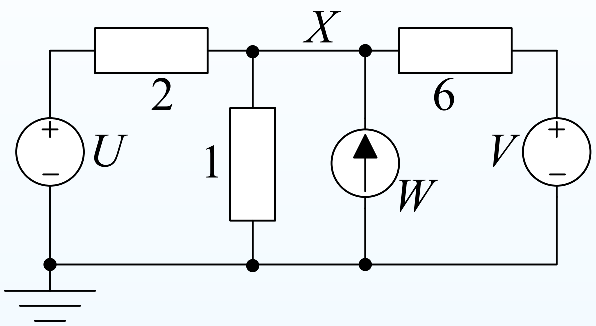

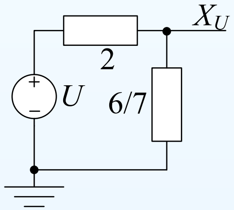

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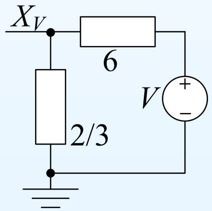

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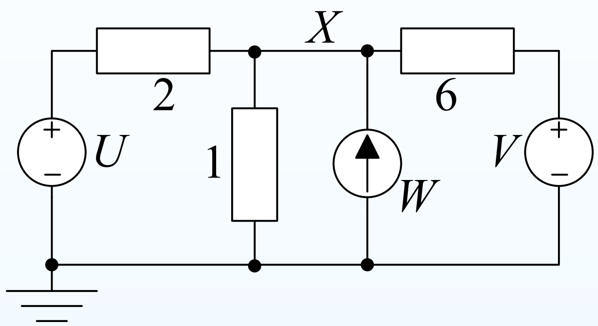

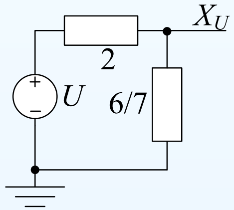

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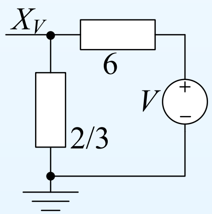

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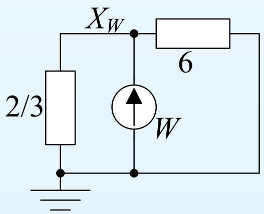

$$X_W = \frac{6}{6+\frac{2}{3}}W \times \frac{2}{3} = \frac{12}{20}W = 0.6W$$

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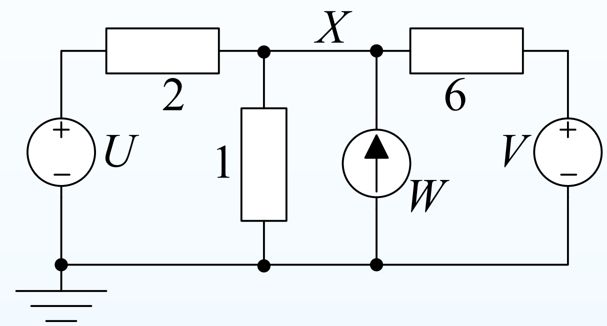

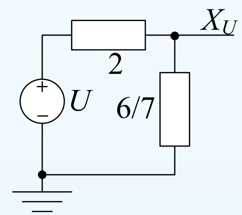

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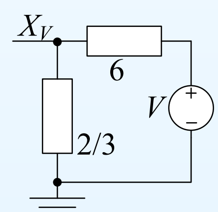

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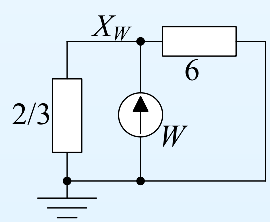

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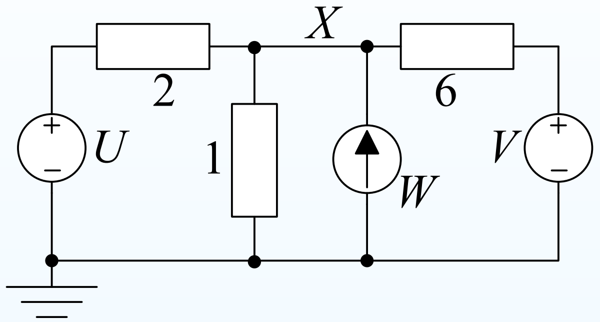

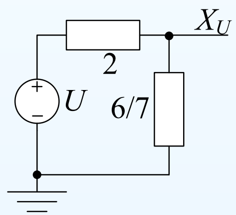

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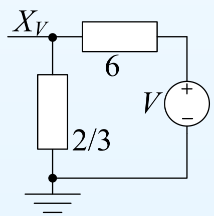

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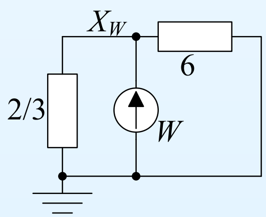

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Adding them up: $X = X_U + X_V + X_W = 0.3U + 0.1V + 0.6W$

Superposition and dependent sources

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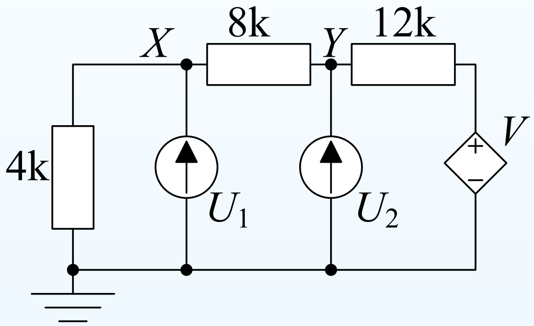

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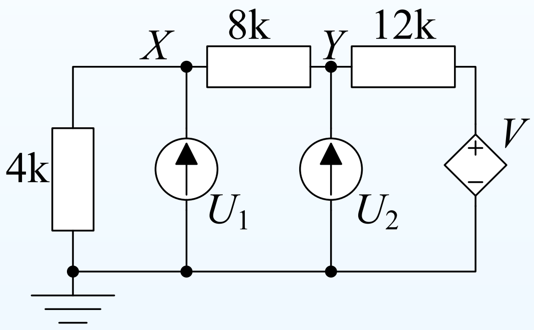

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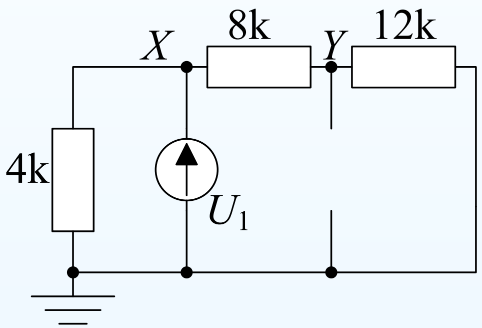

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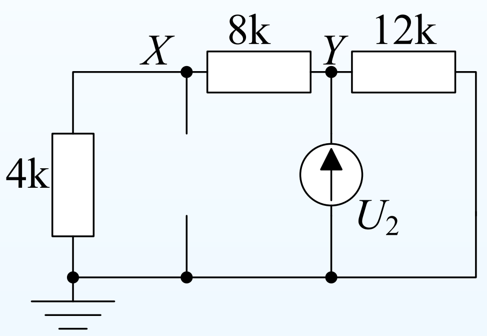

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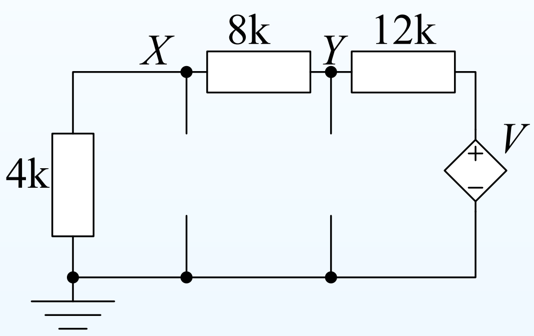

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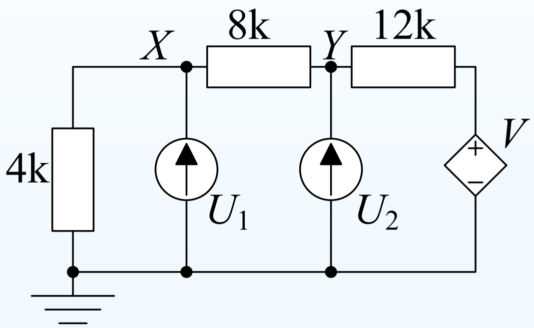

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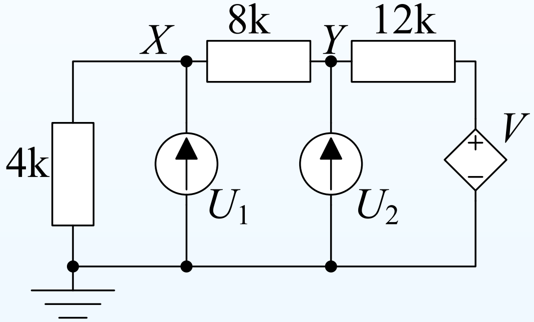

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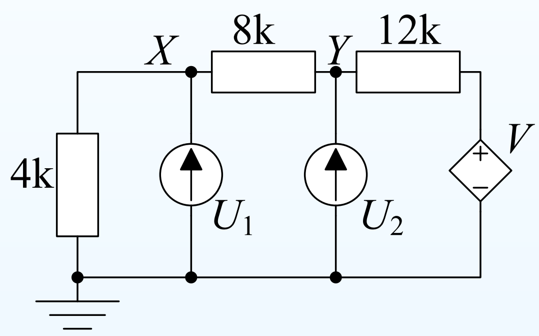

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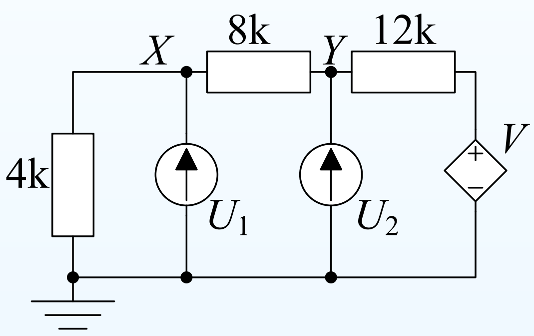

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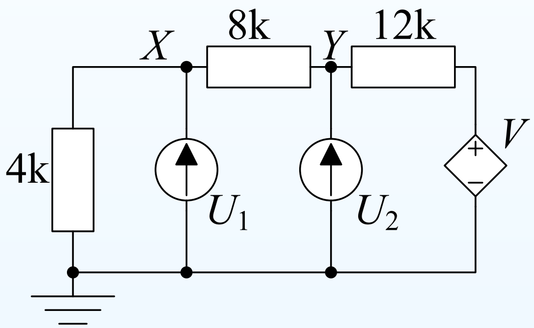

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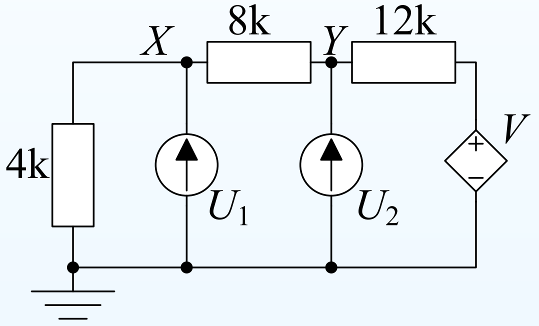

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Note: This is an alternative to nodal anlysis: you get the same answer.

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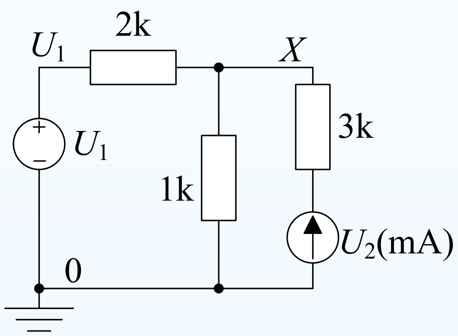
Any current or voltage can be written $X = a_1U_1 + a_2U_2 + a_3U_3 + \ldots$.



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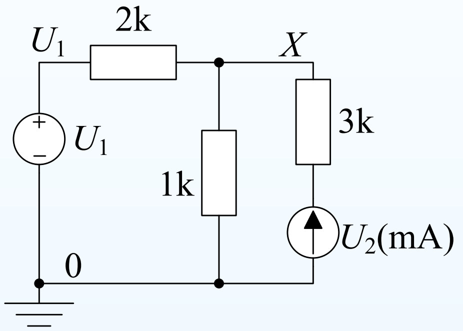

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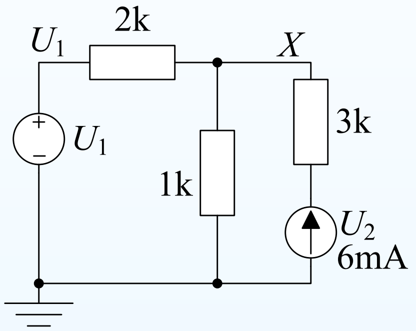

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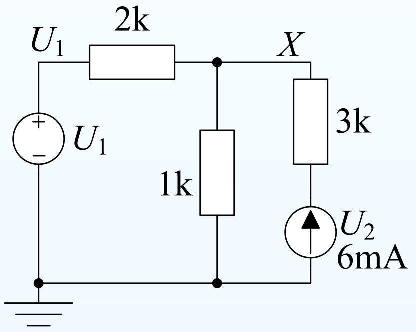

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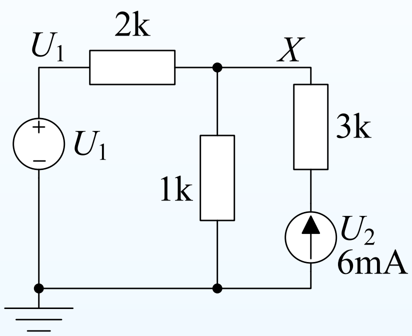

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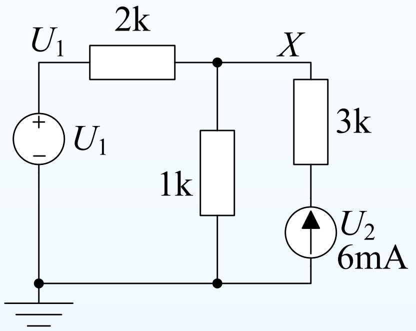

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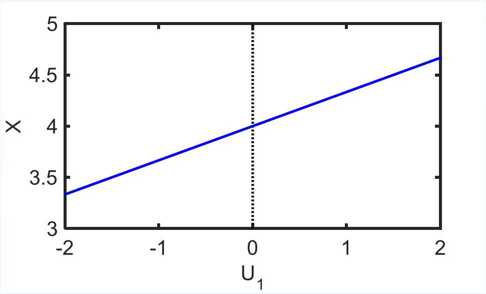

This has a straight line graph.

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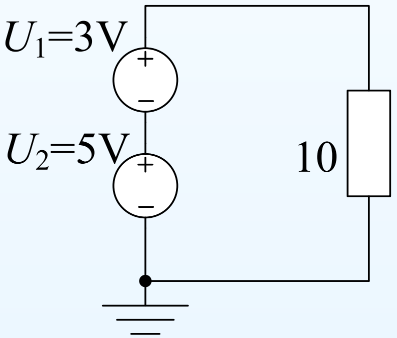

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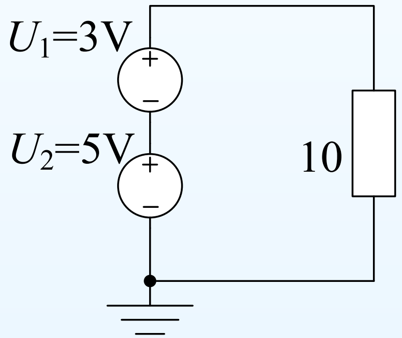

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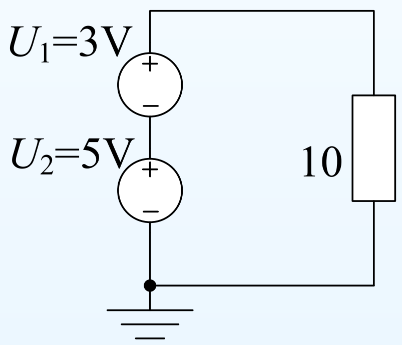

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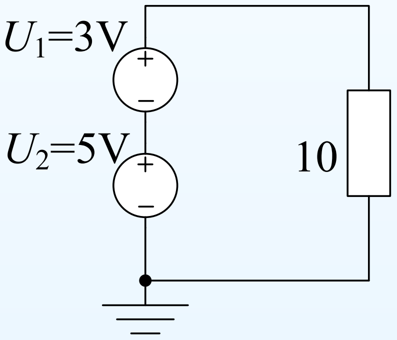

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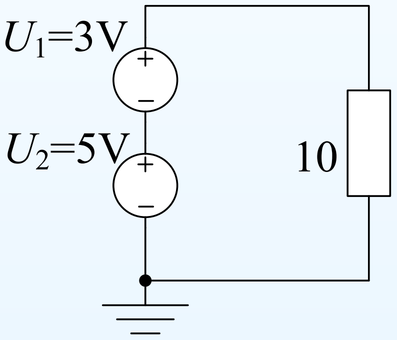

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You must use superposition to calculate the total V and/or the total I and then calculate the power.

Proportionality

From the linearity theorem, all voltages and currents have the form $\sum a_i U_i$ where the U_i are the values of the independent sources.

If you multiply *all* the independent sources by the same factor, k, then all voltages and currents in the circuit will be multiplied by k.

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Special Case:
If there is only one independent source, U, then all voltages and currents are proportional to U and all power dissipations are proportional to U^2.

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For further details see Hayt Ch 5 or Irwin Ch 5.