From linearity theorem: \( V = aI + b \).
From linearity theorem: \( V = aI + b \).

Use nodal analysis:

- KCL@X: \( \frac{X}{1} - 6 + \frac{X-V}{2} = 0 \)
- KCL@V: \( \frac{V-X}{2} - I = 0 \)
Equivalent Networks

From linearity theorem: \( V = aI + b \).

Use nodal analysis:

\[
\text{KCL@X: } \frac{X}{1} - 6 + \frac{X - V}{2} = 0
\]

\[
\text{KCL@V: } \frac{V - X}{2} - I = 0
\]

Eliminating \( X \) gives: \( V = 3I + 6 \).
Equivalent Networks

From linearity theorem: \( V = aI + b \).

Use nodal analysis:
\[
\text{KCL@X: } \frac{X}{1} - 6 + \frac{X-V}{2} = 0
\]
\[
\text{KCL@V: } \frac{V-X}{2} - I = 0
\]

Eliminating \( X \) gives: \( V = 3I + 6 \).

There are infinitely many networks with the same values of \( a \) and \( b \):
Equivalent Networks

From linearity theorem: \( V = aI + b \).

Use nodal analysis:

\[
\begin{align*}
\text{KCL@X: } & \quad \frac{X}{1} - 6 + \frac{X-V}{2} = 0 \\
\text{KCL@V: } & \quad \frac{V-X}{2} - I = 0
\end{align*}
\]

Eliminating \( X \) gives: \( V = 3I + 6 \).

There are infinitely many networks with the same values of \( a \) and \( b \):
From linearity theorem: \( V = aI + b \).

Use nodal analysis:

\[
\begin{align*}
\text{KCL@X:} & \quad \frac{X}{1} - 6 + \frac{X-V}{2} = 0 \\
\text{KCL@V:} & \quad \frac{V-X}{2} - I = 0
\end{align*}
\]

Eliminating \( X \) gives: \( V = 3I + 6 \).

There are infinitely many networks with the same values of \( a \) and \( b \):

These four shaded networks are **equivalent** because the relationship between \( V \) and \( I \) is **exactly** the same in each case.
Equivalent Networks

From linearity theorem: \( V = aI + b \).

Use nodal analysis:

\[
\text{KCL@X: } \frac{X}{1} - 6 + \frac{X-V}{2} = 0 \\
\text{KCL@V: } \frac{V-X}{2} - I = 0
\]

Eliminating \( X \) gives: \( V = 3I + 6 \).

There are infinitely many networks with the same values of \( a \) and \( b \):

These four shaded networks are equivalent because the relationship between \( V \) and \( I \) is exactly the same in each case.

The last two are particularly simple and are respectively called the Norton and Thévenin equivalent networks.
Thévenin Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.
Thévenin Equivalent

Thévenin Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.

We can replace the shaded part of the circuit with its Thévenin equivalent network.
Thévenin Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.

We can replace the shaded part of the circuit with its Thévenin equivalent network.
**Thévenin Equivalent**

**Thévenin Theorem:** Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.

We can replace the shaded part of the circuit with its Thévenin equivalent network.

The voltages and currents in the unshaded part of the circuit will be identical in both circuits.
**Thévenin Equivalent**

**Thévenin Theorem:** Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.

We can replace the shaded part of the circuit with its Thévenin equivalent network.

The voltages and currents in the unshaded part of the circuit will be identical in both circuits.

The new components are called the *Thévenin equivalent resistance*, $R_{Th}$, and the *Thévenin equivalent voltage*, $V_{Th}$, of the original network.
**Thévenin Equivalent**

**Thévenin Theorem:** Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.

We can replace the shaded part of the circuit with its Thévenin equivalent network.

The voltages and currents in the unshaded part of the circuit will be identical in both circuits.

The new components are called the *Thévenin equivalent resistance*, $R_{Th}$, and the *Thévenin equivalent voltage*, $V_{Th}$, of the original network.

This is often a useful way to simplify a complicated circuit (provided that you do not want to know the voltages and currents in the shaded part).
A Thévenin equivalent circuit has a straight line characteristic with the equation:

\[ V = R_{Th} I + V_{Th} \]
A Thévenin equivalent circuit has a straight line characteristic with the equation:

\[ V = R_{Th}I + V_{Th} \]

\[ \Leftrightarrow I = \frac{1}{R_{Th}} V - \frac{V_{Th}}{R_{Th}} \]
A Thévenin equivalent circuit has a straight line characteristic with the equation:
\[ V = R_{Th} I + V_{Th} \]
\[ \Leftrightarrow I = \frac{1}{R_{Th}} V - \frac{V_{Th}}{R_{Th}} \]

Three important quantities are:

**Open Circuit Voltage:** If \( I = 0 \) then \( V_{OC} = V_{Th} \). (X-intercept: 0)
A Thévenin equivalent circuit has a straight line characteristic with the equation:

\[ V = R_{Th}I + V_{Th} \]

\[ \Leftrightarrow I = \frac{1}{R_{Th}} V - \frac{V_{Th}}{R_{Th}} \]

Three important quantities are:

- **Open Circuit Voltage**: If \( I = 0 \) then \( V_{OC} = V_{Th} \).  
  (X-intercept: \( o \))

- **Short Circuit Current**: If \( V = 0 \) then \( I_{SC} = -\frac{V_{Th}}{R_{Th}} \).  
  (Y-intercept: \( x \))
A Thévenin equivalent circuit has a straight line characteristic with the equation:

\[ V = R_{Th}I + V_{Th} \]

⇔ \[ I = \frac{1}{R_{Th}}V - \frac{V_{Th}}{R_{Th}} \]

Three important quantities are:

- **Open Circuit Voltage:** If \( I = 0 \) then \( V_{OC} = V_{Th} \). (X-intercept: o)

- **Short Circuit Current:** If \( V = 0 \) then \( I_{SC} = -\frac{V_{Th}}{R_{Th}} \). (Y-intercept: x)

- **Thévenin Resistance:** The slope of the characteristic is \( \frac{dI}{dV} = \frac{1}{R_{Th}} \).
A Thévenin equivalent circuit has a straight line characteristic with the equation:

\[ V = R_{Th}I + V_{Th} \]

\[ \iff I = \frac{1}{R_{Th}}V - \frac{V_{Th}}{R_{Th}} \]

Three important quantities are:

**Open Circuit Voltage:** If \( I = 0 \) then \( V_{OC} = V_{Th} \).  \hspace{1cm} (X-intercept: 0)

**Short Circuit Current:** If \( V = 0 \) then \( I_{SC} = \frac{V_{Th}}{R_{Th}} \).  \hspace{1cm} (Y-intercept: x)

**Thévenin Resistance:** The slope of the characteristic is \( \frac{dI}{dV} = \frac{1}{R_{Th}} \).

If we know the value of any two of these three quantities, we can work out \( V_{Th} \) and \( R_{Th} \).
A Thévenin equivalent circuit has a straight line characteristic with the equation:

\[ V = R_{Th} I + V_{Th} \]

\[ \Leftrightarrow I = \frac{1}{R_{Th}} V - \frac{V_{Th}}{R_{Th}} \]

Three important quantities are:

- **Open Circuit Voltage**: If \( I = 0 \) then \( V_{OC} = V_{Th} \).  
  \( \text{(X-intercept: o)} \)

- **Short Circuit Current**: If \( V = 0 \) then \( I_{SC} = -\frac{V_{Th}}{R_{Th}} \).  
  \( \text{(Y-intercept: x)} \)

- **Thévenin Resistance**: The slope of the characteristic is \( \frac{dI}{dV} = \frac{1}{R_{Th}} \).

If we know the value of any two of these three quantities, we can work out \( V_{Th} \) and \( R_{Th} \).

In any two-terminal circuit with the same characteristic, the three quantities will have the same values. So if we can determine two of them, we can work out the Thévenin equivalent.
We need any two of the following:

**Open Circuit Voltage:**

**Short Circuit Current:**

**Thévenin Resistance:**
Determining Thévenin Values

We need any two of the following:

- **Open Circuit Voltage:**
- **Short Circuit Current:**
- **Thévenin Resistance:**

Open Circuit Voltage:
We known that \( I_{1k} = 6 \) because there is nowhere else for the current to go. So \( V_{OC} = 6 \times 1 = 6 \text{ V} \).
Determining Thévenin Values

We need any two of the following:

**Open Circuit Voltage:** \( V_{OC} = V_{Th} = 6 \text{ V} \)

**Short Circuit Current:**

**Thévenin Resistance:**

**Open Circuit Voltage:**

We known that \( I_{1k} = 6 \) because there is nowhere else for the current to go. So \( V_{OC} = 6 \times 1 = 6 \text{ V}. \)
Determining Thévenin Values

We need any two of the following:

Open Circuit Voltage: \( V_{OC} = V_{Th} = 6 \text{ V} \)

Short Circuit Current:

Thévenin Resistance:

Short Circuit Current:

The 2 k and 1 k resistors are in parallel and so form a current divider in which currents are proportional to conductances. So \( I_{SC} = -\frac{1/2}{3/2} \times 6 = -2 \text{ mA} \)
Determining Thévenin Values

We need any two of the following:

**Open Circuit Voltage:** \( V_{OC} = V_{Th} = 6 \text{ V} \)

**Short Circuit Current:** \( I_{SC} = -\frac{V_{Th}}{R_{Th}} = -2 \text{ mA} \)

**Thévenin Resistance:**

Short Circuit Current:
The 2k and 1k resistors are in parallel and so form a current divider in which currents are proportional to conductances.
So \( I_{SC} = -\frac{1/2}{3/2} \times 6 = -2 \text{ mA} \)
Determining Thévenin Values

We need any two of the following:

**Open Circuit Voltage:** $V_{OC} = V_{Th} = 6 \text{ V}$

**Short Circuit Current:** $I_{SC} = -\frac{V_{Th}}{R_{Th}} = -2 \text{ mA}$

**Thévenin Resistance:**

We set all the independent sources to zero (voltage sources $\rightarrow$ short circuit, current sources $\rightarrow$ open circuit). Then we find the equivalent resistance between the two terminals.

The $3 \text{ k}$ resistor has no effect so $R_{Th} = 2 \text{ k} + 1 \text{ k} = 3 \text{ k}$. 

![Thévenin Circuit Diagram](image)
Determining Thévenin Values

We need any two of the following:

**Open Circuit Voltage:** \( V_{OC} = V_{Th} = 6 \text{ V} \)

**Short Circuit Current:** \( I_{SC} = -\frac{V_{Th}}{R_{Th}} = -2 \text{ mA} \)

**Thévenin Resistance:** \( R_{Th} = 2k + 1k = 3k \Omega \)

**Thévenin Resistance:**

We set all the independent sources to zero (voltage sources \( \rightarrow \) short circuit, current sources \( \rightarrow \) open circuit). Then we find the equivalent resistance between the two terminals. The 3 k resistor has no effect so \( R_{Th} = 2k + 1k = 3k \).
Determining Thévenin Values

We need any two of the following:

Open Circuit Voltage: \( V_{OC} = V_{Th} = 6 \text{ V} \)

Short Circuit Current: \( I_{SC} = -\frac{V_{Th}}{R_{Th}} = -2 \text{ mA} \)

Thévenin Resistance: \( R_{Th} = 2 \text{ k} + 1 \text{ k} = 3 \text{ kΩ} \)

Thévenin Resistance:
We set all the independent sources to zero (voltage sources \( \rightarrow \) short circuit, current sources \( \rightarrow \) open circuit). Then we find the equivalent resistance between the two terminals.

The 3 k resistor has no effect so \( R_{Th} = 2 \text{ k} + 1 \text{ k} = 3 \text{ kΩ} \).

Any measurement gives the same result on an equivalent circuit.
For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]
For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

**Step 1:** Label **ground as an output terminal** + label other nodes.
For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

**Step 1:** Label the ground as an output terminal + label other nodes.

**Step 2:** Write down the equations
For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

Step 1: Label **ground as an output terminal** + label other nodes.

Step 2: Write down the equations

\[
\frac{X-V}{2} + \frac{X}{1} + \frac{X-Y}{1} = 0
\]
Thévenin of Complicated Circuits

For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

Step 1: Label **ground as an output terminal** + label other nodes.

Step 2: Write down the equations (Y is a supernode)

\[
\begin{align*}
\frac{X-V}{2} + \frac{X}{1} + \frac{X-Y}{1} &= 0 \\
\frac{Y-3-V}{1} + \frac{Y-X}{1} + \frac{Y-3}{2} &= 0
\end{align*}
\]
For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

**Step 1:** Label ground as an output terminal + label other nodes.

**Step 2:** Write down the equations (\( Y \) is a supernode)

\[
\frac{X-V}{2} + \frac{X}{1} + \frac{X-Y}{1} = 0
\]

\[
\frac{Y-3-V}{1} + \frac{Y-X}{1} + \frac{Y-3}{2} = 0
\]

\[
\frac{V-Y+3}{1} + \frac{V-X}{2} - I = 0
\]
Thévenin of Complicated Circuits

For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

**Step 1:** Label ground as an output terminal + label other nodes.

**Step 2:** Write down the equations (Y is a supernode)

\[
\begin{align*}
\frac{X-V}{2} + \frac{X}{1} + \frac{X-Y}{1} & = 0 \\
\frac{Y-3-V}{1} + \frac{Y-X}{1} + \frac{Y-3}{2} & = 0 \\
\frac{V-Y+3}{1} + \frac{V-X}{2} - I & = 0
\end{align*}
\]

**Step 3:** Eliminate \( X \) and \( Y \) and solve for \( V \) in terms of \( I \):

\[ V = \frac{7}{5}I - \frac{3}{5} = R_{Th}I + V_{Th} \]
Thévenin of Complicated Circuits

For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

\[ V = V_{Th} + IR_{Th}. \]

**Step 1:** Label **ground as an output terminal** + label other nodes.

**Step 2:** Write down the equations (Y is a supernode)

\[
\begin{align*}
\frac{X-V}{2} + \frac{X}{1} + \frac{X-Y}{1} &= 0 \\
\frac{Y-3-V}{1} + \frac{Y-X}{1} + \frac{Y-3}{2} &= 0 \\
\frac{V-Y+3}{1} + \frac{V-X}{2} - I &= 0
\end{align*}
\]

**Step 3:** Eliminate \( X \) and \( Y \) and solve for \( V \) in terms of \( I \):

\[ V = \frac{7}{5} I - \frac{3}{5} = R_{Th} I + V_{Th} \]
Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:
\[-I - I_{NO} + \frac{V}{R_{Th}} = 0\]
Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:
\[-I - I_{No} + \frac{V}{R_{Th}} = 0\]

\(\Leftrightarrow I = \frac{1}{R_{Th}}V - I_{No}\)
Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:
\[-I - I_{No} + \frac{V}{R_{Th}} = 0\]
\[\iff I = \frac{1}{R_{Th}} V - I_{No}\]

c.f. Thévenin (slide 5-4):
Same \( R \) and \( I_{No} = \frac{V_{Th}}{R_{Th}} \)
Norton Equivalent

Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:

\[-I - I_{NO} + \frac{V}{R_{Th}} = 0\]

\[\Leftrightarrow I = \frac{1}{R_{Th}}V - I_{NO}\]

c.f. Thévenin (slide 5-4):
Same \(R\) and \(I_{NO} = \frac{V_{Th}}{R_{Th}}\)

Open Circuit Voltage: If \(I = 0\) then \(V_{OC} = I_{NO}R_{Th}\).
**Norton Theorem:** Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

**KCL:**
\[-I - I_{No} + \frac{V}{R_{Th}} = 0\]
\[\iff I = \frac{1}{R_{Th}}V - I_{No}\]

c.f. Thévenin (slide 5-4): Same $R$ and $I_{No} = \frac{V_{Th}}{R_{Th}}$

**Open Circuit Voltage:** If $I = 0$ then $V_{OC} = I_{No}R_{Th}$.

**Short Circuit Current:** If $V = 0$ then $I_{SC} = -I_{No}$
Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:
\[-I - I_{No} + \frac{V}{R_{Th}} = 0\]
\[\Leftrightarrow I = \frac{1}{R_{Th}} V - I_{No}\]

c.f. Thévenin (slide 5-4):
Same \(R\) and \(I_{No} = \frac{V_{Th}}{R_{Th}}\)

Open Circuit Voltage: If \(I = 0\) then \(V_{OC} = I_{No} R_{Th}\).

Short Circuit Current: If \(V = 0\) then \(I_{SC} = -I_{No}\)

Thévenin Resistance: The slope of the characteristic is \(\frac{1}{R_{Th}}\).
Norton Equivalent

Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:
\[-I - I_{No} + \frac{V}{R_{Th}} = 0\]
\[\Leftrightarrow I = \frac{1}{R_{Th}}V - I_{No}\]

c.f. Thévenin (slide 5-4): Same \(R\) and \(I_{No} = \frac{V_{Th}}{R_{Th}}\)

Open Circuit Voltage: If \(I = 0\) then \(V_{OC} = I_{No}R_{Th}\).

Short Circuit Current: If \(V = 0\) then \(I_{SC} = -I_{No}\)

Thévenin Resistance: The slope of the characteristic is \(\frac{1}{R_{Th}}\).

Easy to change between Norton and Thévenin: \(V_{Th} = I_{No}R_{Th}\).

Usually best to use Thévenin for small \(R_{Th}\) and Norton for large \(R_{Th}\) compared to the other impedances in the circuit.
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th} + R_L}$
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th} + R_L}$

$\Rightarrow$ power in $R_L$ is $P_L = I^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th}+R_L}$

⇒ power in $R_L$ is $P_L = I^2R_L = \frac{V_{Th}^2R_L}{(R_{Th}+R_L)^2}$
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th} + R_L}$

$\Rightarrow$ power in $R_L$ is $P_L = I^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$

To find the $R_L$ that maximizes $P_L$:

$$0 = \frac{dP_L}{dR_L} = \frac{(R_{Th} + R_L)^2 V_{Th}^2 - 2V_{Th}^2 R_L (R_{Th} + R_L)}{(R_{Th} + R_L)^4}$$
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th} + R_L}$

$\Rightarrow$ power in $R_L$ is $P_L = I^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$

To find the $R_L$ that maximizes $P_L$:

$$0 = \frac{dP_L}{dR_L} = \frac{(R_{Th} + R_L)^2 V_{Th}^2 - 2 V_{Th}^2 R_L (R_{Th} + R_L)}{(R_{Th} + R_L)^4}$$

$$= \frac{V_{Th}^2 (R_{Th} + R_L) - 2 V_{Th}^2 R_L}{(R_{Th} + R_L)^3}$$
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th} + R_L}$

$\Rightarrow$ power in $R_L$ is $P_L = I^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$

To find the $R_L$ that maximizes $P_L$:

$$0 = \frac{dP_L}{dR_L} = \frac{(R_{Th} + R_L)^2 V_{Th}^2 - 2V_{Th}^2 R_L (R_{Th} + R_L)}{(R_{Th} + R_L)^4}$$

$$= \frac{V_{Th}^2 (R_{Th} + R_L) - 2V_{Th}^2 R_L}{(R_{Th} + R_L)^3}$$

$\Rightarrow V_{Th}^2 ((R_{Th} + R_L) - 2R_L) = 0$
Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th} + R_L}$

$\Rightarrow$ power in $R_L$ is $P_L = I^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$

To find the $R_L$ that maximizes $P_L$:

$$0 = \frac{dP_L}{dR_L} = \frac{(R_{Th} + R_L)^2 V_{Th}^2 - 2 V_{Th}^2 R_L (R_{Th} + R_L)}{(R_{Th} + R_L)^4}$$

$$= \frac{V_{Th}^2 (R_{Th} + R_L) - 2 V_{Th}^2 R_L}{(R_{Th} + R_L)^3}$$

$\Rightarrow V_{Th}^2 ((R_{Th} + R_L) - 2R_L) = 0$

$\Rightarrow R_L = R_{Th}$  $\Rightarrow P_{(max)} = \frac{V_{Th}^2}{4R_{Th}}$. 

![Diagram of a circuit with a voltage source, a resistor, and a variable resistor connected in parallel.]
Power Transfer

Suppose we connect a variable resistor, $R_L$, across a two-terminal network. From Thévenin’s theorem, even a complicated network is equivalent to a voltage source and a resistor.

We know $I = \frac{V_{Th}}{R_{Th}+R_L}$

$⇒$ power in $R_L$ is $P_L = I^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th}+R_L)^2}$

To find the $R_L$ that maximizes $P_L$:

$0 = \frac{dP_L}{dR_L} = \frac{(R_{Th}+R_L)^2 V_{Th}^2 - 2V_{Th}^2 R_L (R_{Th}+R_L)}{(R_{Th}+R_L)^4}$

$= \frac{V_{Th}^2 (R_{Th}+R_L) - 2V_{Th}^2 R_L}{(R_{Th}+R_L)^3}$

$⇒ V_{Th}^2 ((R_{Th} + R_L) - 2R_L) = 0$

$⇒ R_L = R_{Th} ⇒ P_{(max)} = \frac{V_{Th}^2}{4R_{Th}}$

For fixed $R_{Th}$, the maximum power transfer is when $R_L = R_{Th}$ (“matched load”).
Sometimes changing between Thévenin and Norton can simplify a circuit. Suppose we want to calculate $I$. 

![Circuit Diagram]
Sometimes changing between Thévenin and Norton can simplify a circuit. Suppose we want to calculate $I$.

**Norton → Thévenin** on current source: \[ I = \frac{18 - (-10)}{5} = 5.6 \text{ A} \]
Sometimes changing between Thévenin and Norton can simplify a circuit. Suppose we want to calculate $I$.

\[ I = \frac{18 - (-10)}{5} = 5.6 \text{ A} \]

If you can’t spot any clever tricks, you can always find out everything with nodal analysis.

\[-6 + \frac{X}{3} + \frac{X - (-10)}{2} = 0\]
Sometimes changing between Thévenin and Norton can simplify a circuit. Suppose we want to calculate \( I \).

Norton → Thévenin on current source: \[
I = \frac{18 - (-10)}{5} = 5.6 \text{ A}
\]

If you can’t spot any clever tricks, you can always find out everything with nodal analysis.

\[
-6 + \frac{X}{3} + \frac{X - (-10)}{2} = 0
\]
\[
\Rightarrow \quad 5X = 36 - 30 = 6
\]
Sometimes changing between Thévenin and Norton can simplify a circuit. Suppose we want to calculate $I$.

\[ I = \frac{18 - (-10)}{5} = 5.6 \text{ A} \]

If you can’t spot any clever tricks, you can always find out everything with nodal analysis.

\[-6 + \frac{X}{3} + \frac{X - (-10)}{2} = 0\]

\[\Rightarrow \quad 5X = 36 - 30 = 6\]

\[\Rightarrow \quad X = 1.2\]
Sometimes changing between Thévenin and Norton can simplify a circuit. Suppose we want to calculate $I$.

**Norton → Thévenin** on current source:

$$I = \frac{18 - (-10)}{5} = 5.6 \text{ A}$$

If you can’t spot any clever tricks, you can always find out everything with nodal analysis.

$$-6 + \frac{X}{3} + \frac{X - (-10)}{2} = 0$$

$$\Rightarrow 5X = 36 - 30 = 6$$

$$\Rightarrow X = 1.2$$

$$\Rightarrow I = \frac{X - (-10)}{2} = 5.6$$
Source Rearrangement

If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.
Source Rearrangement

If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.

Voltage Sources:
If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.

**Voltage Sources:**

We can use the left node as the reference.
Source Rearrangement

If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.

Voltage Sources:
We can use the left node as the reference
Source Rearrangement

If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.

**Voltage Sources:**

We can use the left node as the reference.

![Voltage Source Diagram]

**Current Sources:**

![Current Source Diagram]
Source Rearrangement

If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.

Voltage Sources:
We can use the left node as the reference

Current Sources:
KCL gives current into rightmost node
Source Rearrangement

If all but one branches connecting to a node are voltage sources or are current sources, you can choose any of the branches to be the sourceless one.

Voltage Sources:

We can use the left node as the reference

Current Sources:

KCL gives current into rightmost node
Series Rearrangement

If we have any number of voltage sources and resistors in series we can calculate the total voltage across the chain as:

\[ V = 8I - 2 + 7I + 5 + 9I \]
If we have any number of voltage sources and resistors in series we can calculate the total voltage across the chain as:

\[ V = 8I - 2 + 7I + 5 + 9I = (-2 + 5) + (8 + 7 + 9)I \]
If we have any number of voltage sources and resistors in series we can calculate the total voltage across the chain as:

\[ V = 8I - 2 + 7I + 5 + 9I = (-2 + 5) + (8 + 7 + 9)I \]

\[ = 3 + 24I \]
If we have any number of voltage sources and resistors in series we can calculate the total voltage across the chain as:

\[ V = 8I - 2 + 7I + 5 + 9I = (-2 + 5) + (8 + 7 + 9)I \]

\[ = 3 + 24I \]

We can arbitrarily rearrange the order of the components without affecting \( V = 3 + 24I \).
Series Rearrangement

If we have any number of voltage sources and resistors in series we can calculate the total voltage across the chain as:

\[ V = 8I - 2 + 7I + 5 + 9I = (-2 + 5) + (8 + 7 + 9)I \]

\[ = 3 + 24I \]

We can arbitrarily rearrange the order of the components without affecting \( V = 3 + 24I \).

If we move all the voltage sources together and all the resistors together we can merge them and then we get the Thévenin equivalent.
Summary

- Thévenin and Norton Equivalent Circuits
Summary

- Thévenin and Norton Equivalent Circuits
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
Summary

- **Thévenin and Norton Equivalent Circuits**

  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  - How to determine $V_{Th}$, $I_{No}$ and $R_{Th}$
Summary

- **Thévenin and Norton Equivalent Circuits**
  
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  
  - How to determine $V_{Th}$, $I_{No}$ and $R_{Th}$
    - Method 1: Connect current source $\rightarrow$ Nodal analysis
Summary

- **Thévenin and Norton Equivalent Circuits**
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  - How to determine $V_{Th}$, $I_{No}$ and $R_{Th}$
    - Method 1: Connect current source → Nodal analysis
    - Method 2: Find any two of:
      - $V_{OC} = V_{Th}$, the open-circuit voltage
      - $I_{SC} = -I_{No}$, the short-circuit current
      - $R_{Th}$, equivalent resistance with all sources set to zero
Summary

- Thévenin and Norton Equivalent Circuits
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  - How to determine $V_{Th}$, $I_{No}$ and $R_{Th}$
    - Method 1: Connect current source $\rightarrow$ Nodal analysis
    - Method 2: Find any two of:
      - (a) $V_{OC} = V_{Th}$, the open-circuit voltage
      - (b) $I_{SC} = -I_{No}$, the short-circuit current
      - (c) $R_{Th}$, equivalent resistance with all sources set to zero
    - Related by Ohm’s law: $V_{Th} = I_{No}R_{Th}$
### Summary

- **Thévenin and Norton Equivalent Circuits**
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  - How to determine $V_{Th}$, $I_{No}$ and $R_{Th}$
    - Method 1: Connect current source $\rightarrow$ Nodal analysis
    - Method 2: Find any two of:
      1. $V_{OC} = V_{Th}$, the open-circuit voltage
      2. $I_{SC} = -I_{No}$, the short-circuit current
      3. $R_{Th}$, equivalent resistance with all sources set to zero
    - Related by Ohm’s law: $V_{Th} = I_{No}R_{Th}$
  - Load resistor for **maximum power transfer** $= R_{Th}$
5: Thévenin and Norton Equivalents

- Equivalent Networks
- Thévenin Equivalent
- Thévenin Properties
- Determining Thévenin
- Complicated Circuits
- Norton Equivalent
- Power Transfer
- Source Transformation
- Source Rearrangement
- Series Rearrangement
- Summary

Summary

- **Thévenin and Norton Equivalent Circuits**
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  - **How to determine** $V_{Th}$, $I_{No}$ and $R_{Th}$
    - **Method 1**: Connect current source $\rightarrow$ Nodal analysis
    - **Method 2**: Find any two of:
      - (a) $V_{OC} = V_{Th}$, the open-circuit voltage
      - (b) $I_{SC} = -I_{No}$, the short-circuit current
      - (c) $R_{Th}$, equivalent resistance with all sources set to zero
    - Related by Ohm’s law: $V_{Th} = I_{No}R_{Th}$
  - Load resistor for **maximum power transfer** $=$ $R_{Th}$
  - **Source Transformation and Rearrangement**
5: Thévenin and Norton Equivalents
- Equivalent Networks
- Thévenin Equivalent
- Thévenin Properties
- Determining Thévenin
- Complicated Circuits
- Norton Equivalent
- Power Transfer
- Source Transformation
- Source Rearrangement
- Series Rearrangement
- Summary

Summary

- **Thévenin and Norton Equivalent Circuits**
  - A network has Thévenin and Norton equivalents if:
    - only 2 terminals connect it to the outside world
    - it is made of resistors + sources + linear dependent sources
  - How to determine $V_{Th}$, $I_{No}$ and $R_{Th}$
    - Method 1: Connect current source $\rightarrow$ Nodal analysis
    - Method 2: Find any two of:
      - (a) $V_{OC} = V_{Th}$, the open-circuit voltage
      - (b) $I_{SC} = -I_{No}$, the short-circuit current
      - (c) $R_{Th}$, equivalent resistance with all sources set to zero
    - Related by Ohm’s law: $V_{Th} = I_{No}R_{Th}$
  - Load resistor for **maximum power transfer** = $R_{Th}$
  - **Source Transformation and Rearrangement**

For further details see Hayt Ch 5 & A3 or Irwin Ch 5.