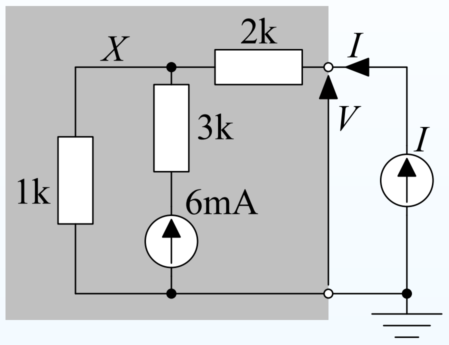
5: Thévenin and Norton Equivalents

Equivalent Networks

From linearity theorem: $V = aI + b$.



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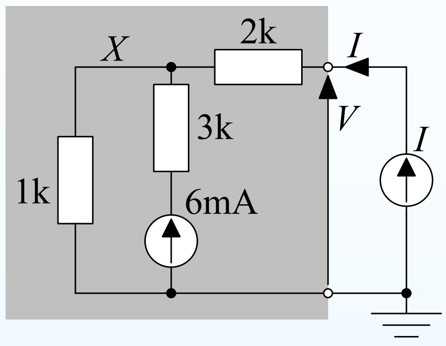

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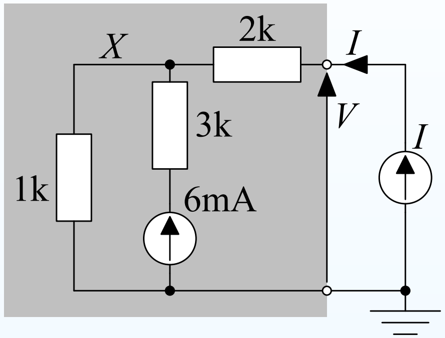

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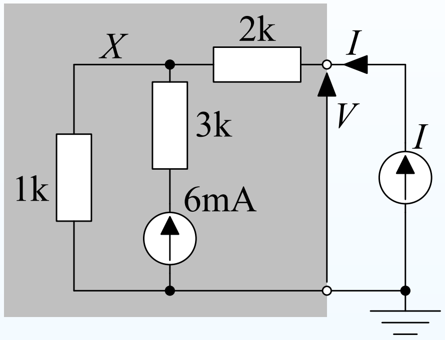

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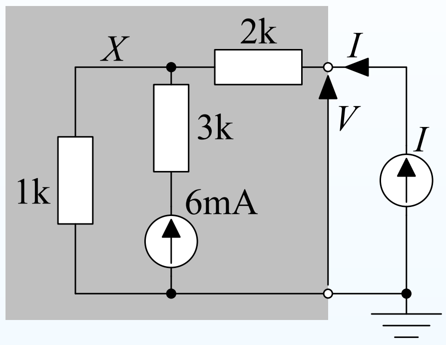

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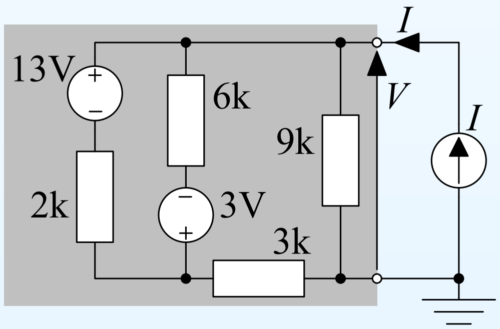

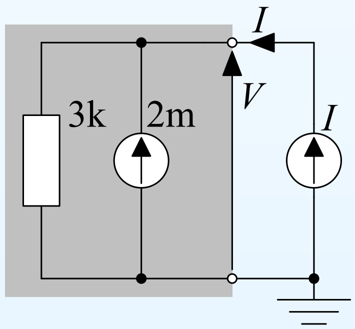

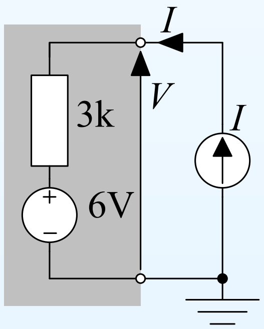

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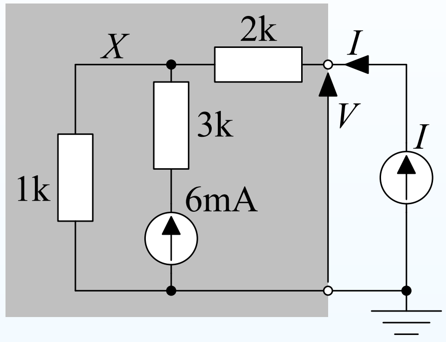

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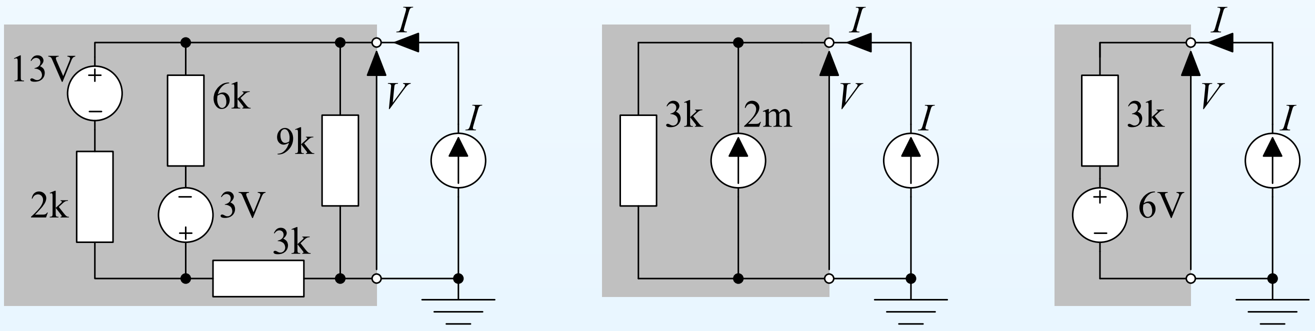

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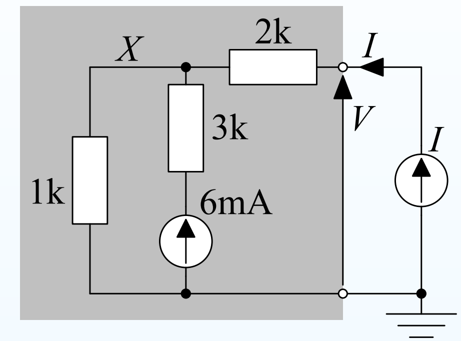

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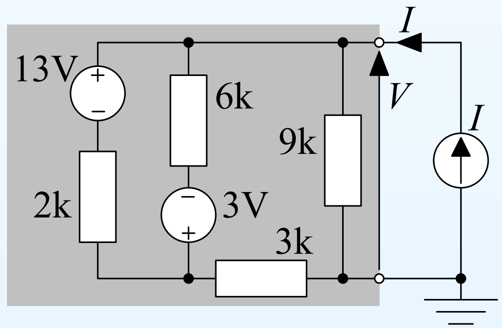

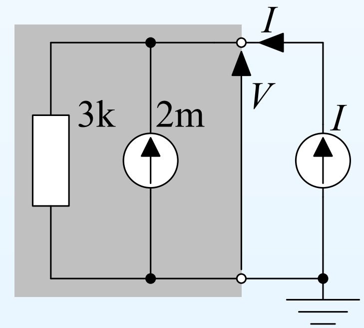

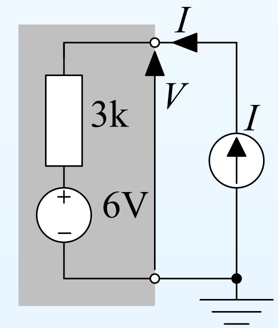

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The last two are particularly simple and are respectively called the *Norton* and *Thévenin* equivalent networks.

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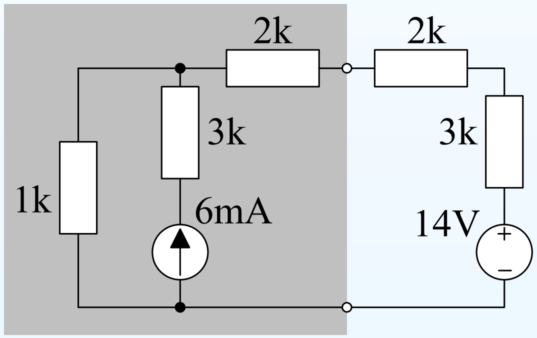
Thévenin Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in series with a fixed voltage source.



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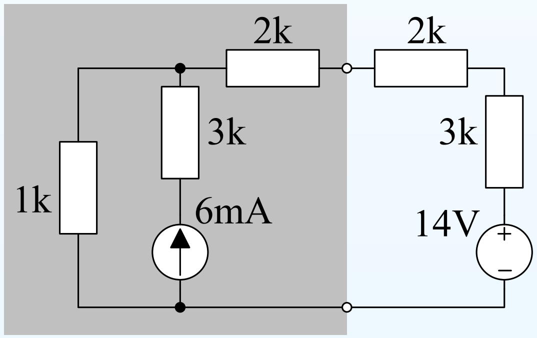

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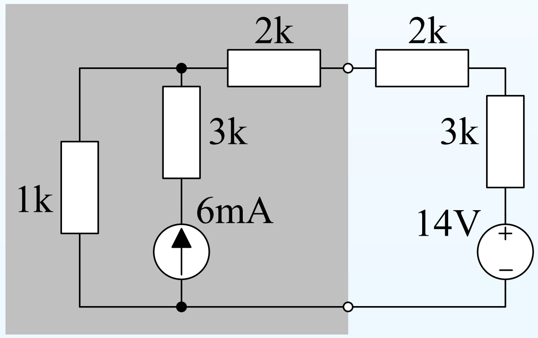

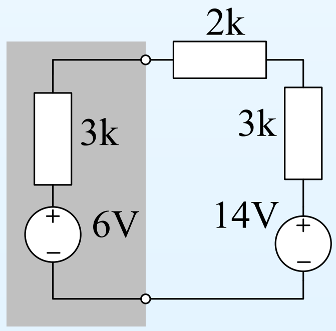

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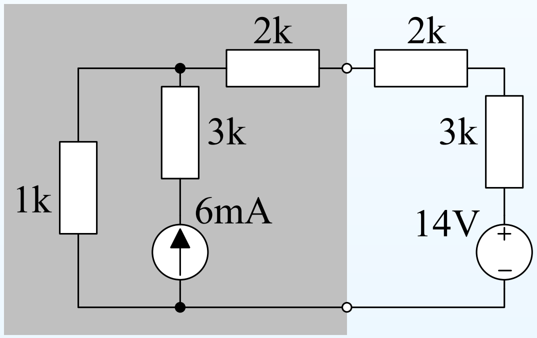

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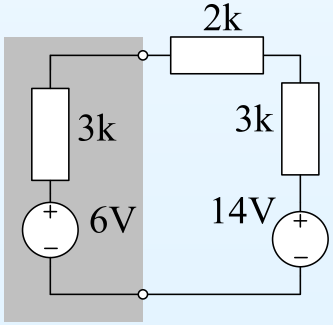

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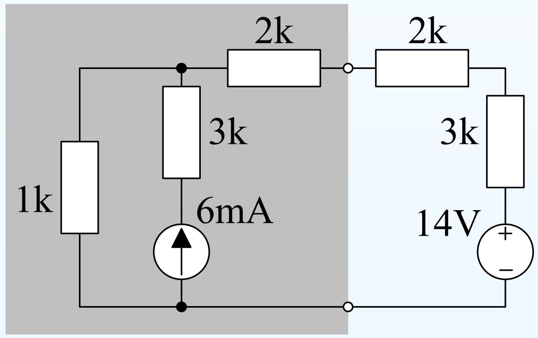

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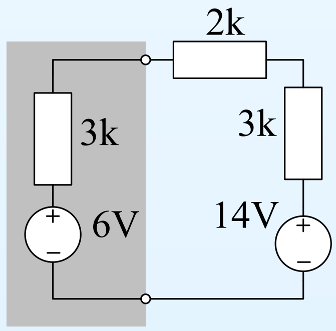

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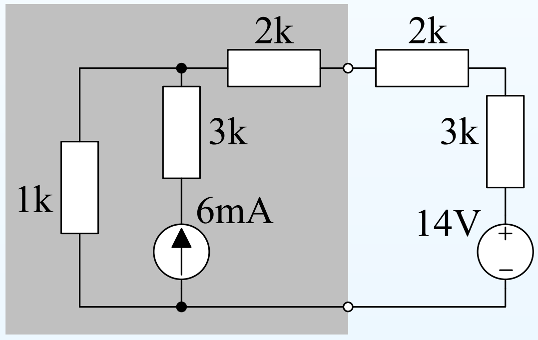

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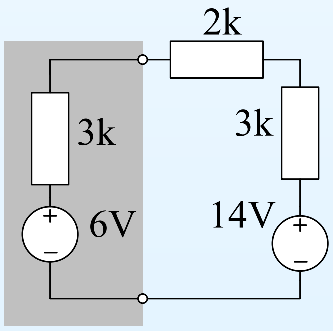
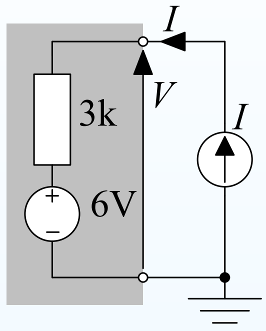


This is often a useful way to simplify a complicated circuit (provided that you do not want to know the voltages and currents in the shaded part).

Thévenin Circuit Properties

A Thévenin equivalent circuit has a straight line characteristic with the equation:

$$V = R_{Th} I + V_{Th}$$



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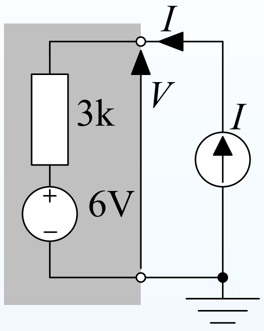

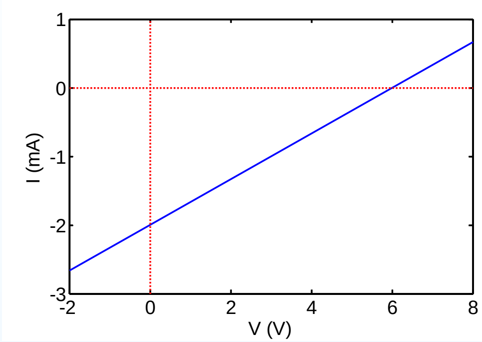

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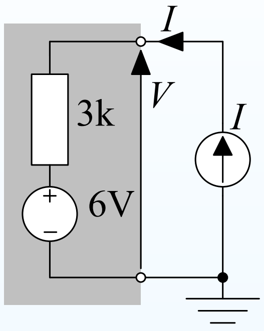

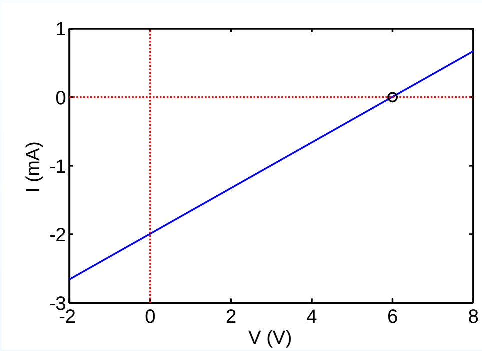

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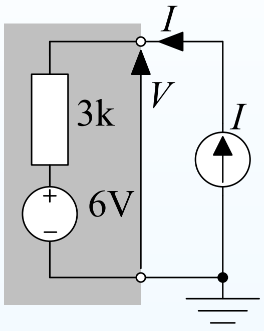

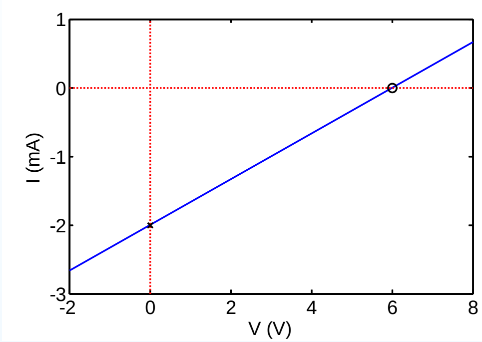

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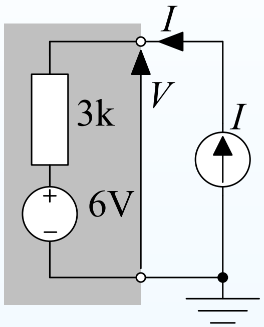

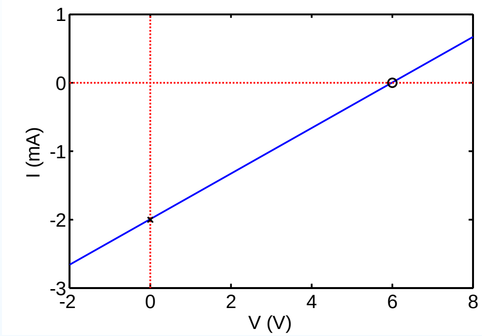

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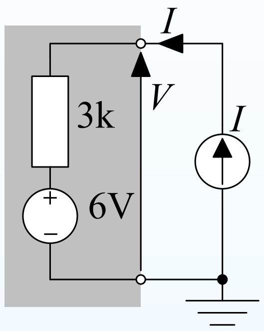

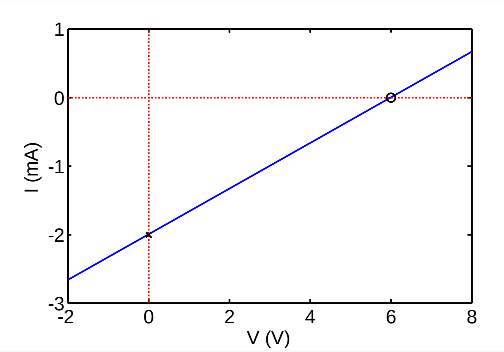

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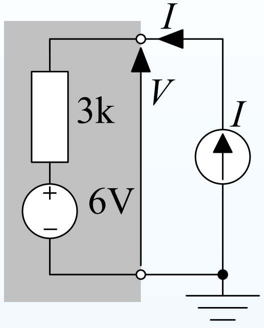

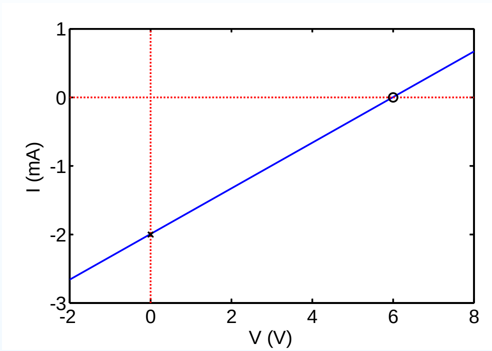

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If we know the value of any two of these three quantities, we can work out V_{Th} and R_{Th}.

In any two-terminal circuit with the same characteristic, the three quantities will have the same values. So if we can determine two of them, we can work out the Thévenin equivalent.

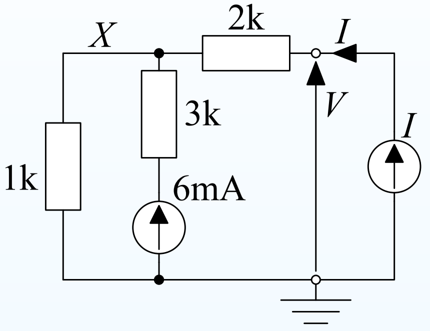
Determining Thévenin Values

We need any two of the following:

Open Circuit Voltage:

Short Circuit Current:

Thévenin Resistance:



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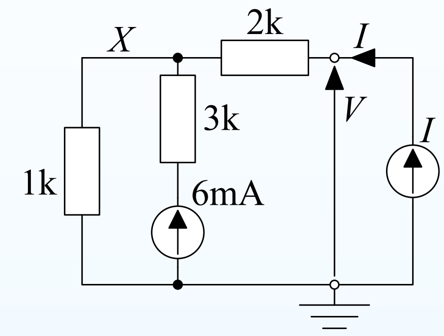

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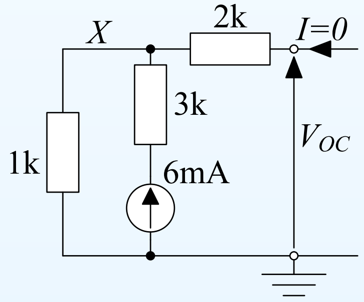

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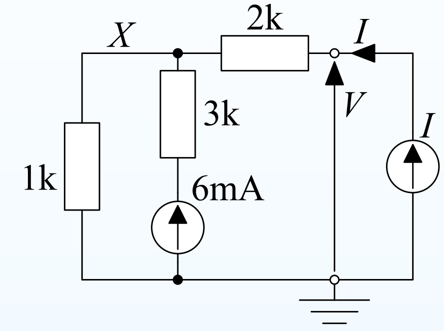

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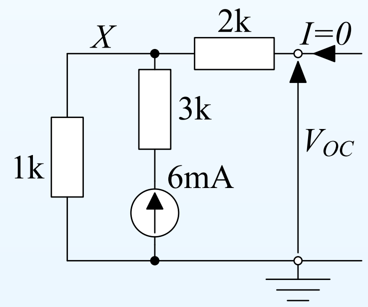

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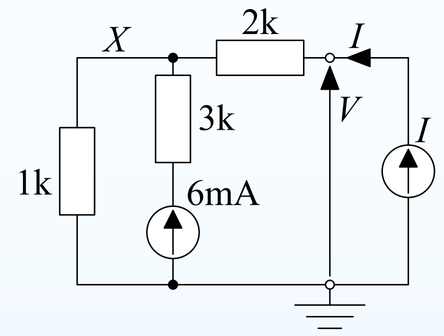

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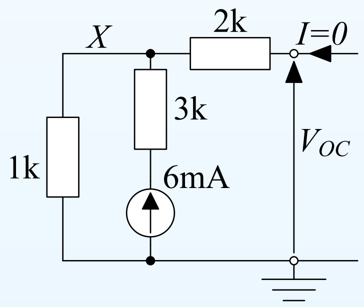

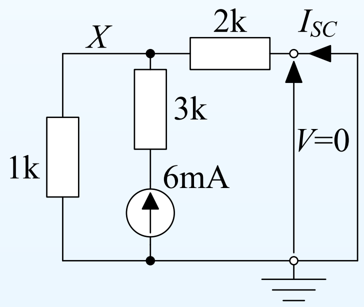

Short Circuit Current:

The $2\,\text{k}$ and $1\,\text{k}$ resistors are in parallel and so form a current divider in which currents are proportional to conductances.

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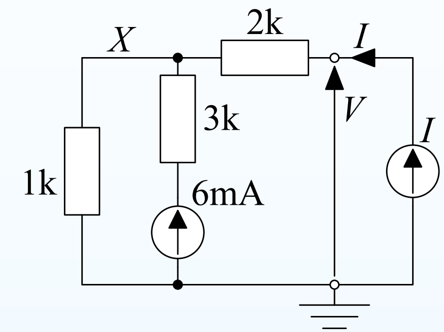

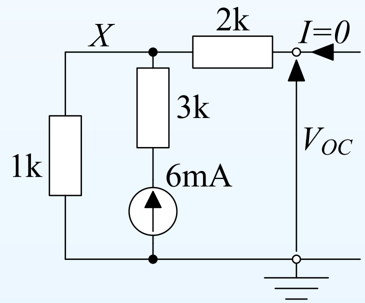

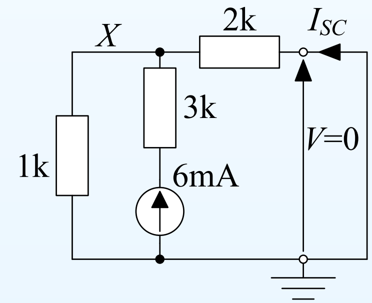

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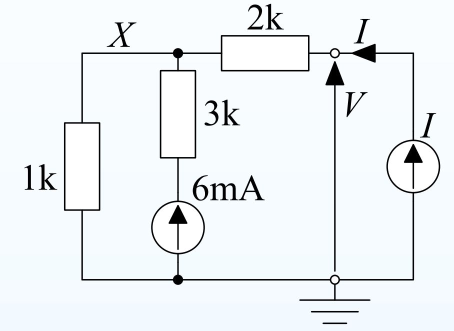

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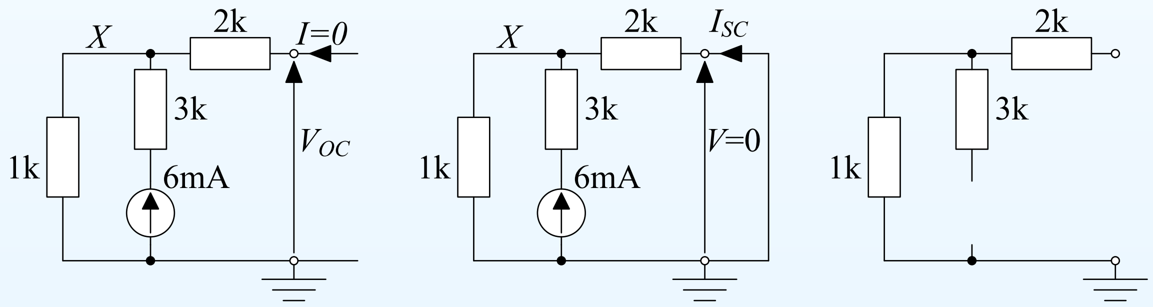

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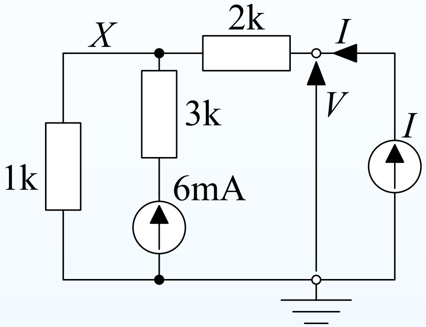
We set all the independent sources to zero (voltage sources $\rightarrow$ short circuit, current sources$\rightarrow$ open circuit). Then we find the equivalent resistance between the two terminals.
The $3\,\mathrm{k}$ resistor has no effect so $R_{Th} = 2\,\mathrm{k} + 1\,\mathrm{k} = 3\,\mathrm{k}$.

Determining Thévenin Values

We need any two of the following:



Open Circuit Voltage: $V_{OC} = V_{Th} = 6\,\text{V}$

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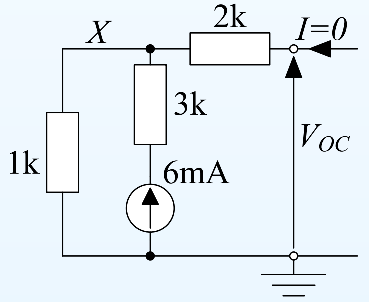

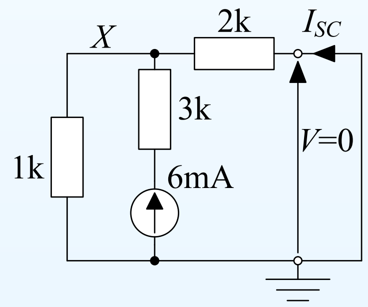

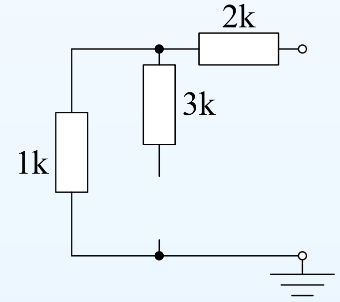

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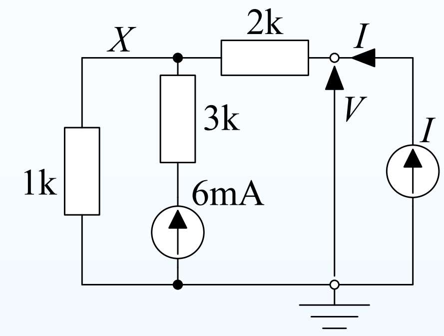

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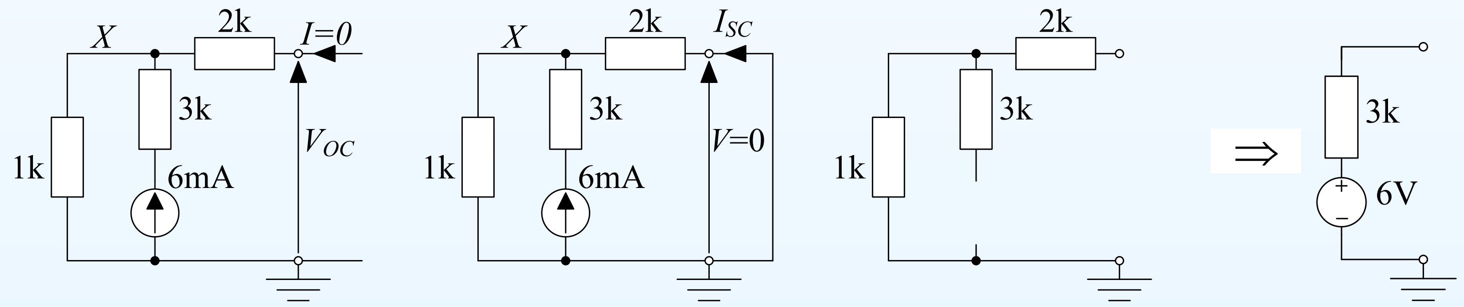

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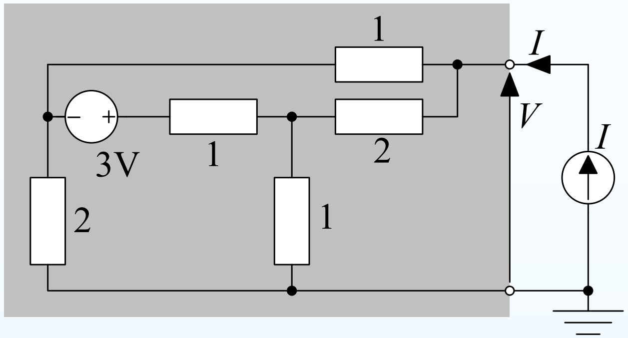
Any measurement gives the same result on an equivalent circuit.

Thévenin of Complicated Circuits

For a complicated circuit, you can use nodal analysis to find the Thévenin equivalent directly in the form:

$$V = V_{Th} + IR_{Th}.$$



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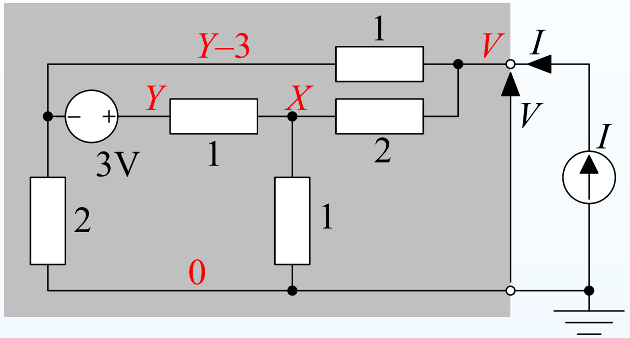

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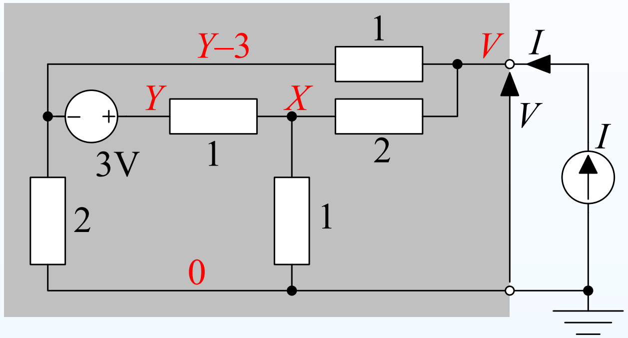

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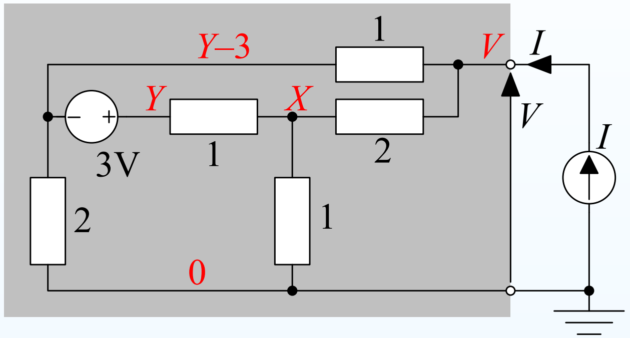

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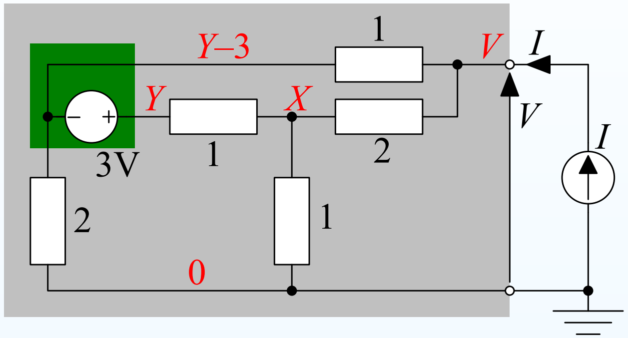

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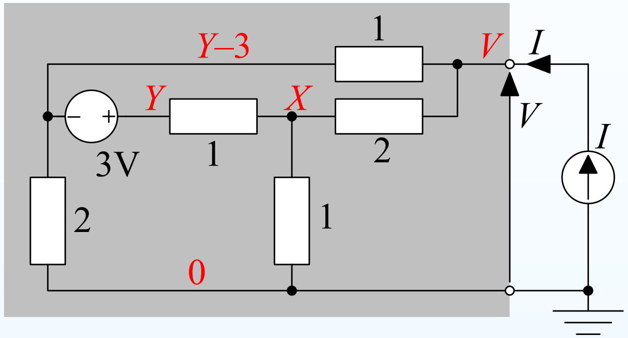

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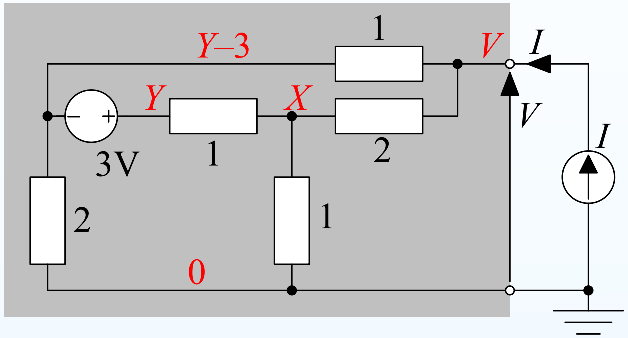

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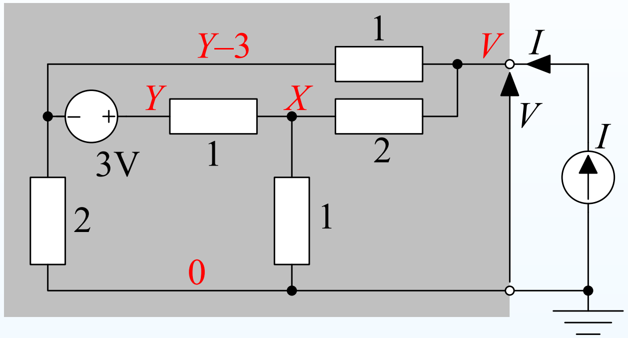

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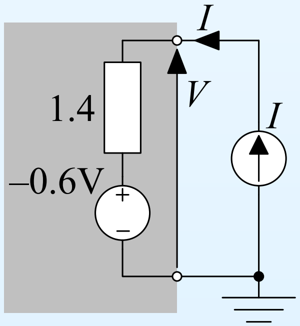

Norton Equivalent

Norton Theorem: Any two-terminal network consisting of resistors, fixed voltage/current sources and linear dependent sources is externally equivalent to a circuit consisting of a resistor in parallel with a fixed current source.

KCL:

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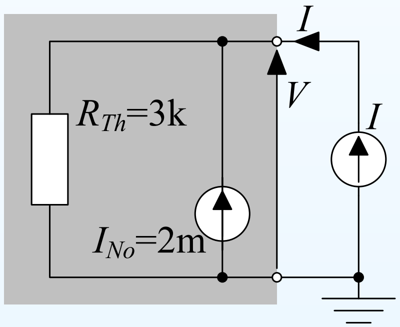

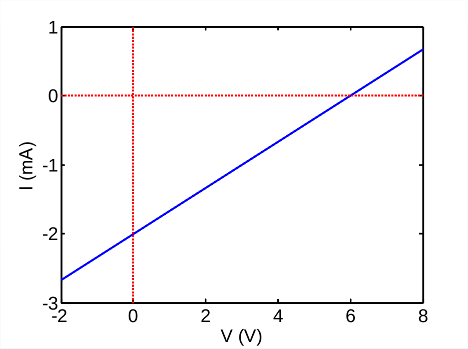

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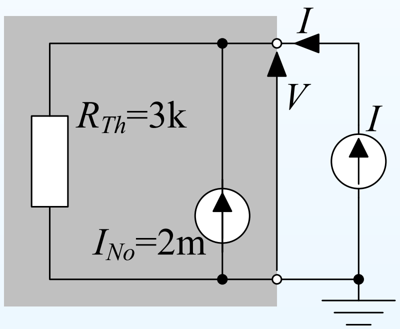

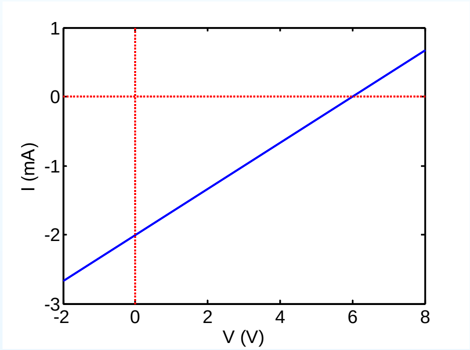

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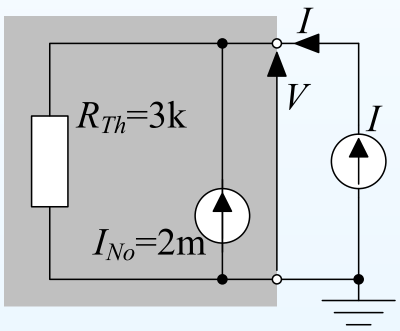

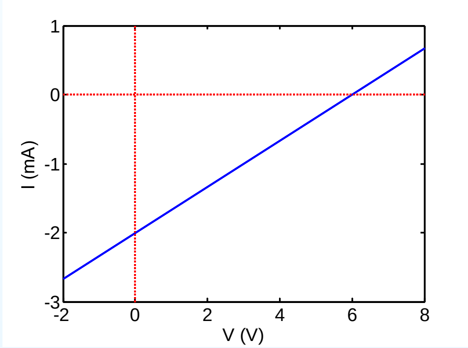

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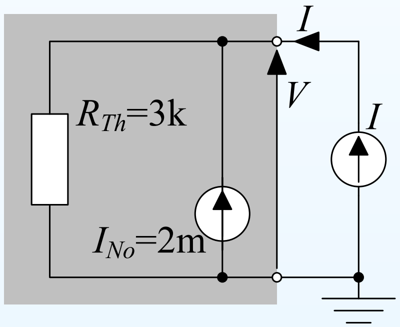

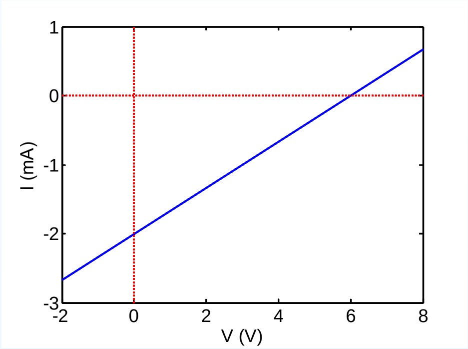

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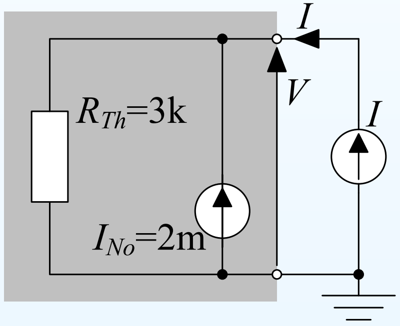

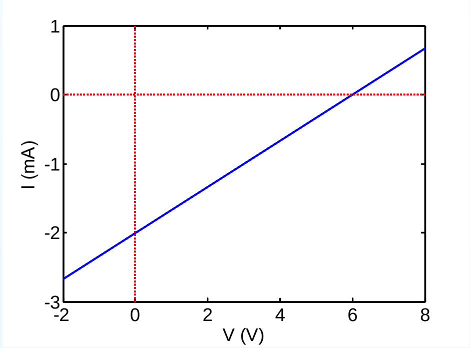

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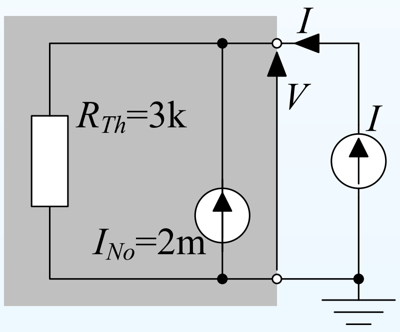

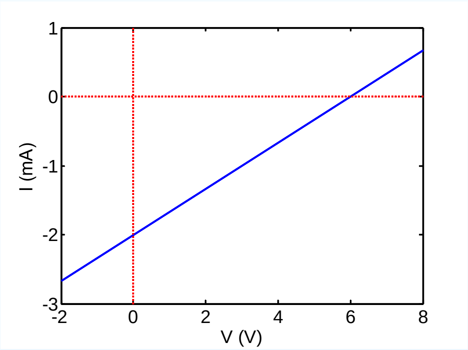

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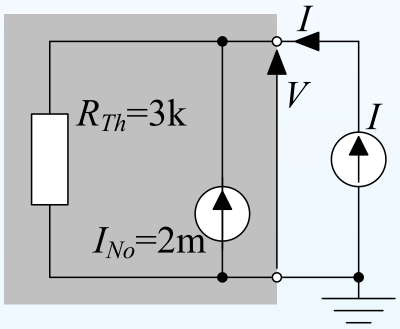

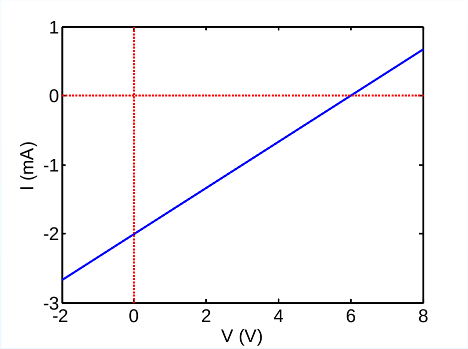

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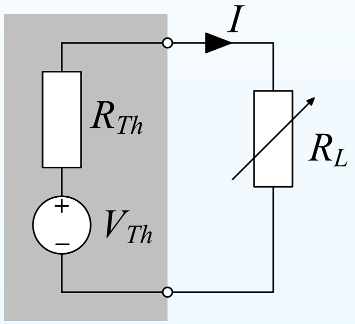
Easy to change between Norton and Thévenin: $V_{Th} = I_{No} R_{Th}$.
Usually best to use Thévenin for small R_{Th} and Norton for large R_{Th} compared to the other impedances in the circuit.

Power Transfer

Suppose we connect a variable resistor, R_L, across a two-terminal network. From Thévenin's theorem, even a complicated network is equivalent to a voltage source and a resistor.



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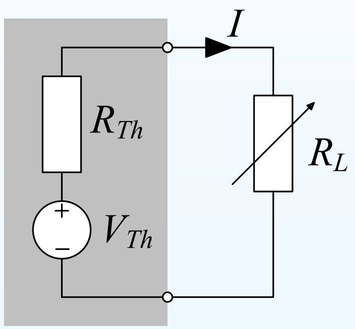

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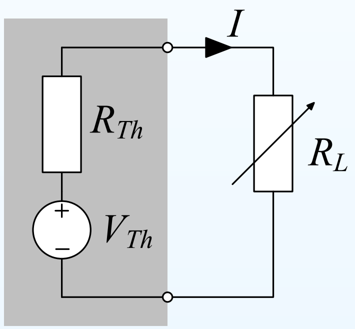

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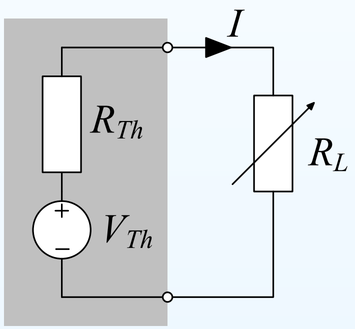

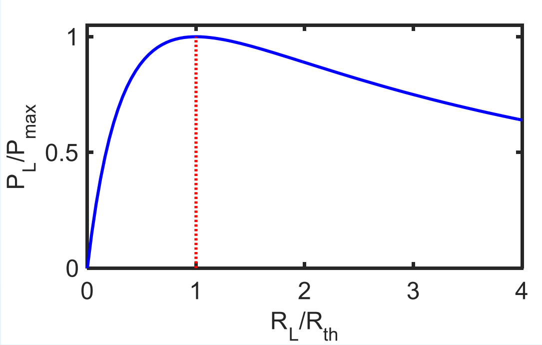

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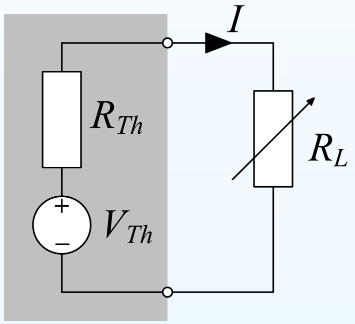

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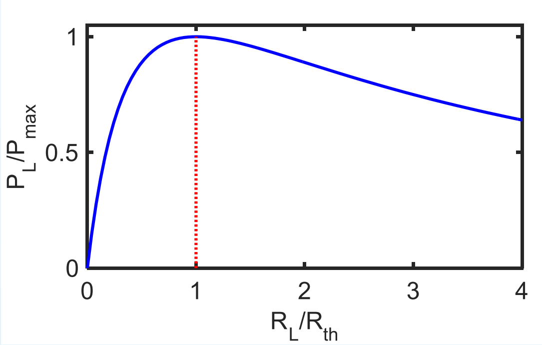

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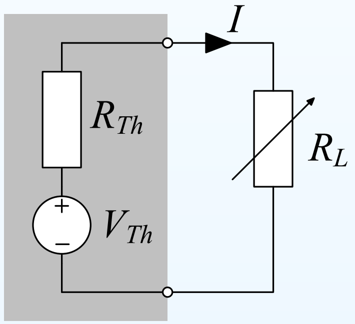

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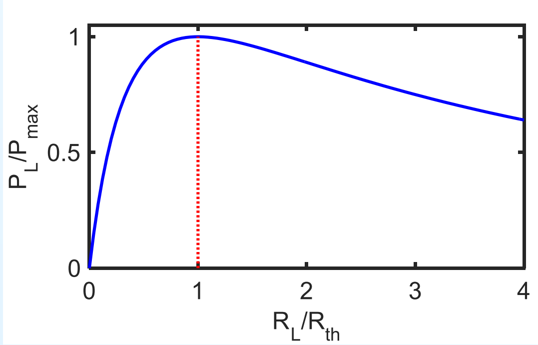

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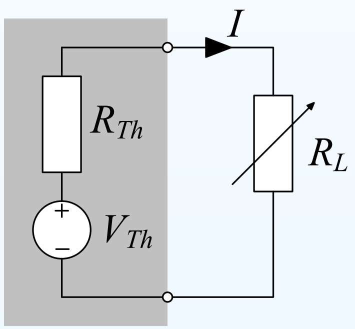

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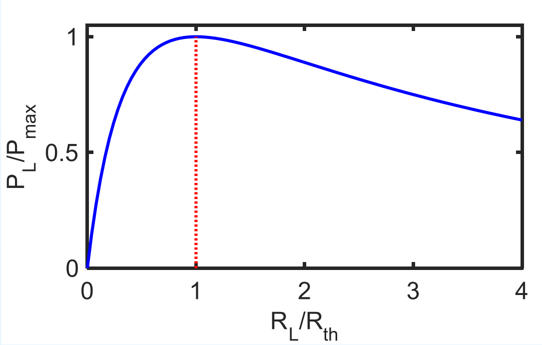

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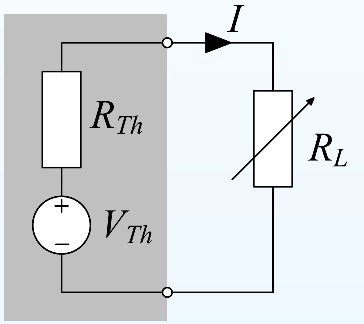

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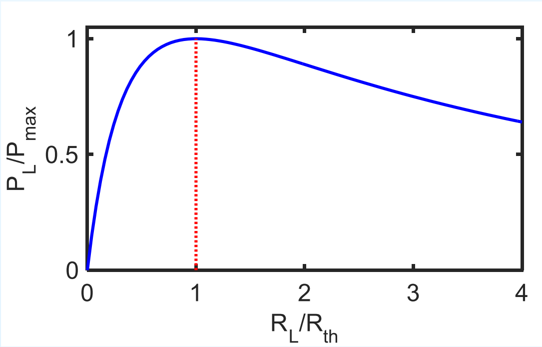

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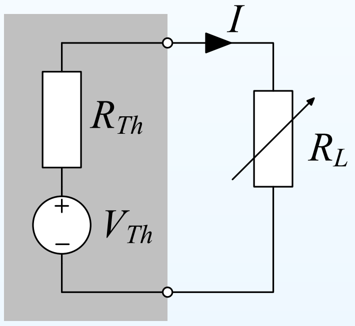

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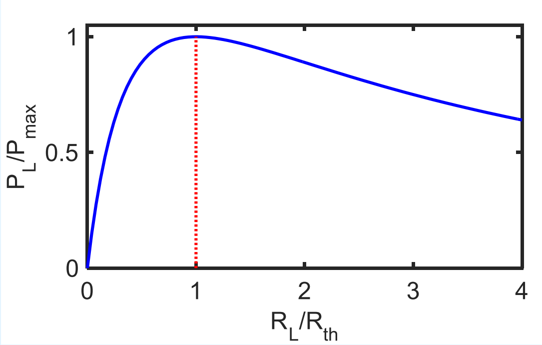

For fixed R_{Th}, the maximum power transfer is when $R_L = R_{Th}$ ("*matched load*").

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Sometimes changing between Thévenin and Norton can simplify a circuit.

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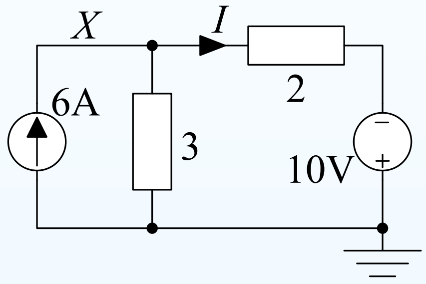

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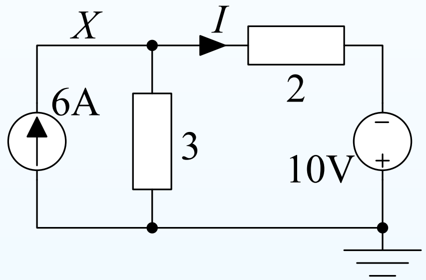

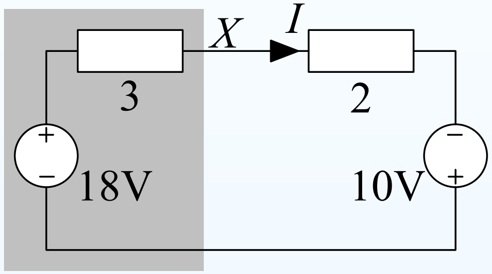

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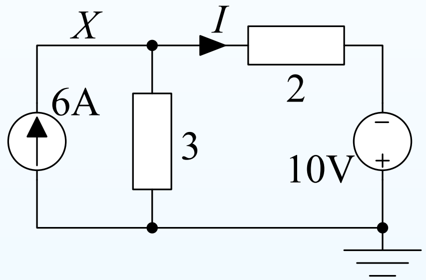

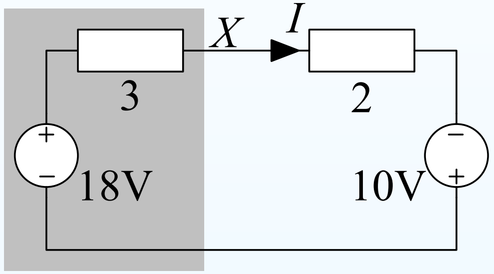

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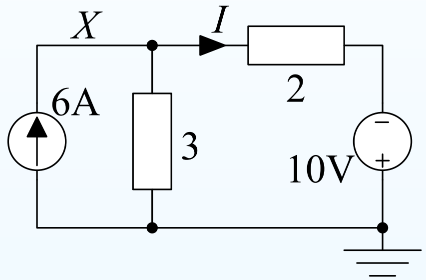

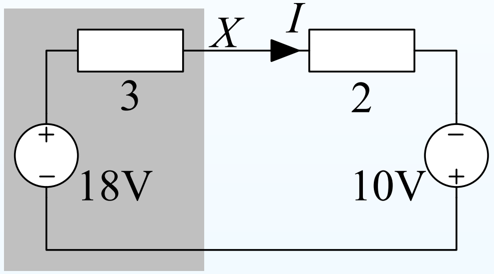

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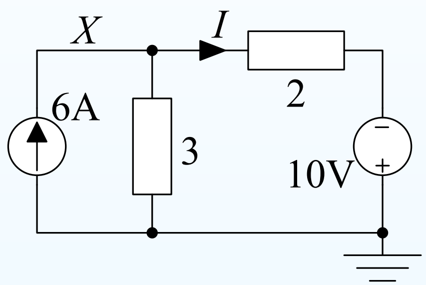

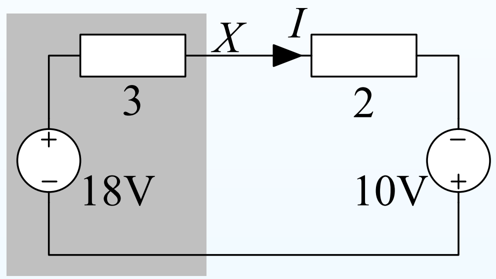

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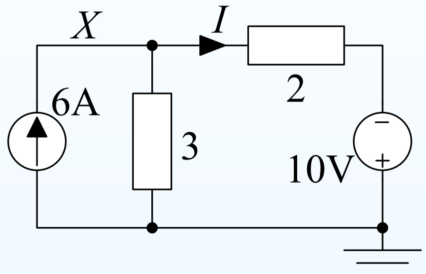

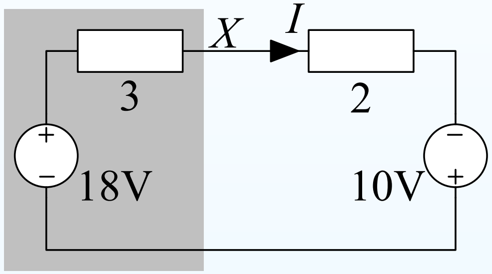

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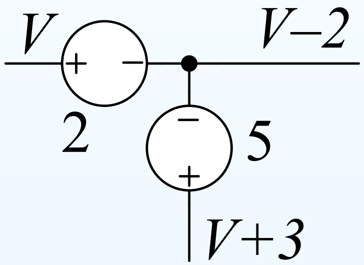

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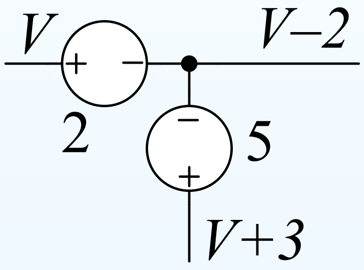

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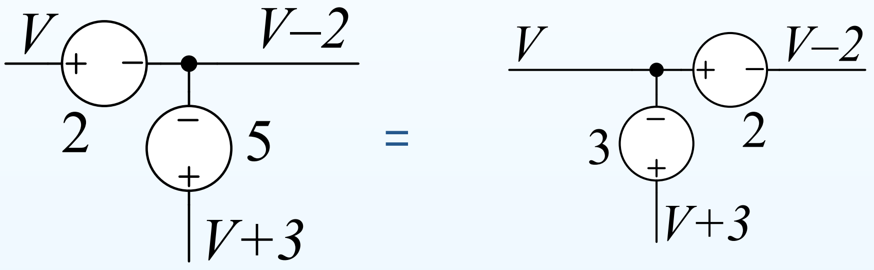

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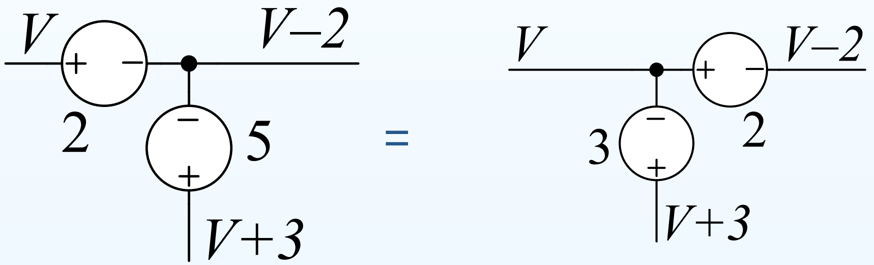

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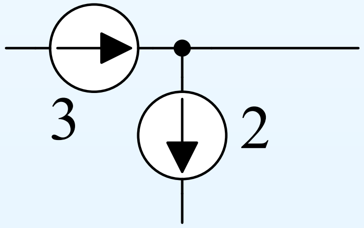

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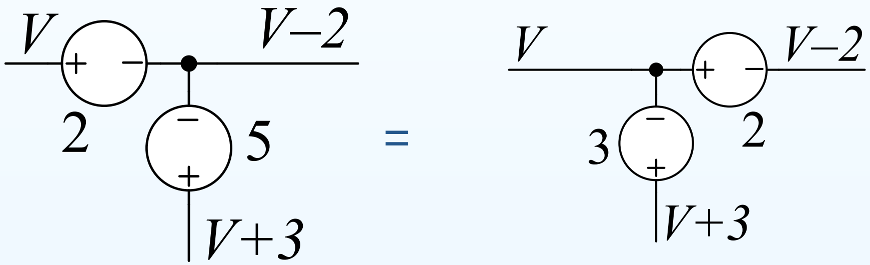

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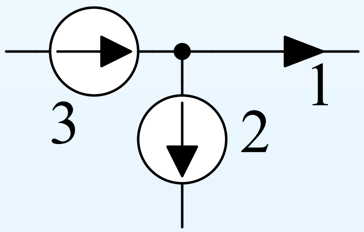

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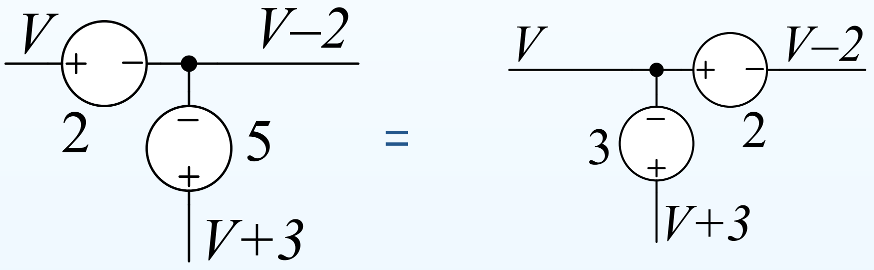

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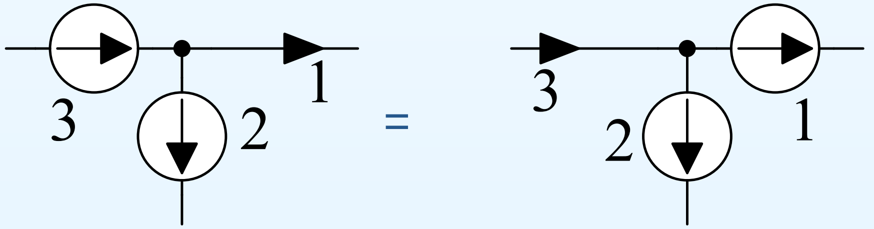

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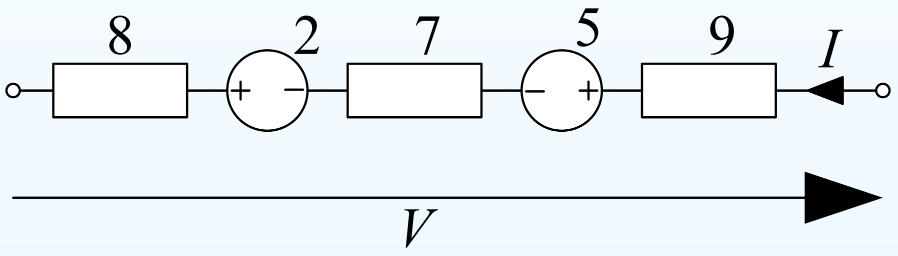

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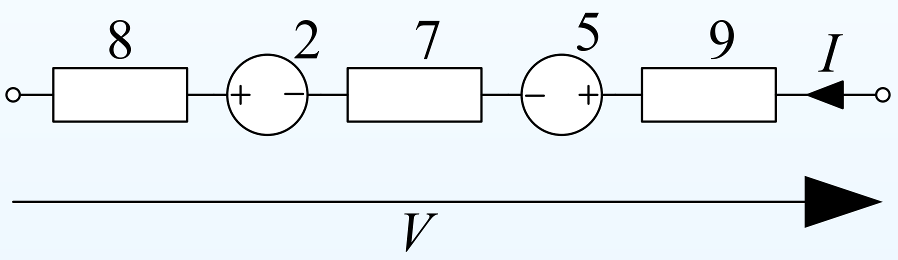

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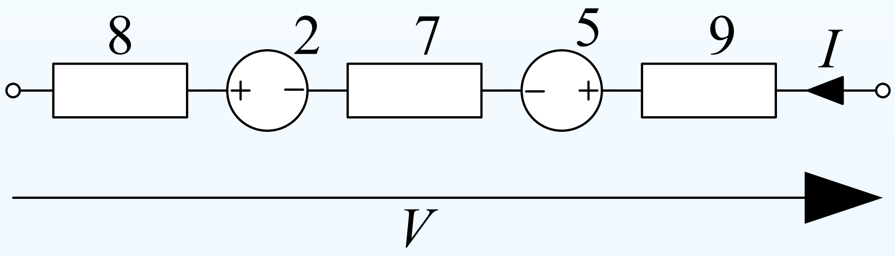

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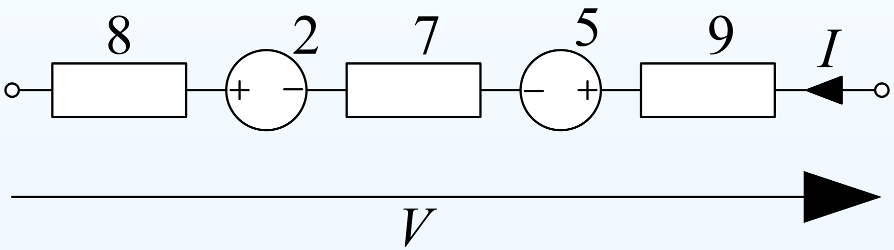

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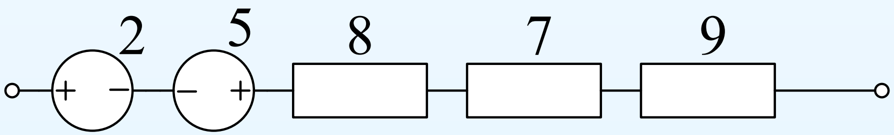

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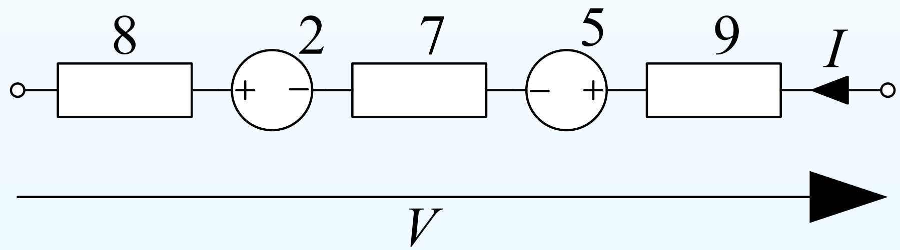

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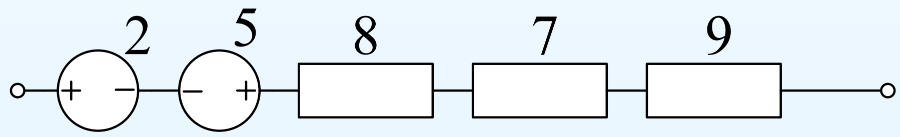

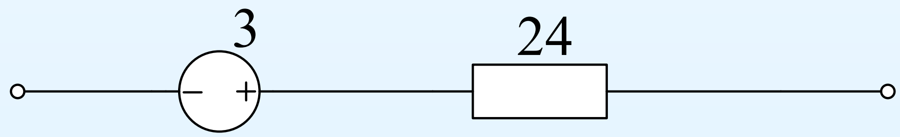

If we move all the voltage sources together and all the resistors together we can merge them and then we get the Thévenin equivalent.

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For further details see Hayt Ch 5 & A3 or Irwin Ch 5.