6: Operational Amplifiers

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- Summary
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Integrated circuit pins are numbered anti-clockwise from blob or notch (when looking from above).
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**Golden Rule:** Negative feedback adjusts the output to make $V_+ \simeq V_-$. 
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3. Zero input current at $V_-$ means $R_2$ and $R_1$ are in series ($\Rightarrow$ same current) and form a voltage divider. So $X = \frac{R_1}{R_1+R_2}Y$. 

[Diagram of the non-inverting amplifier circuit with $R_1=1k$ and $R_2=3k$.]
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So $Y = \frac{R_1 + R_2}{R_1} X = \left(1 + \frac{R_2}{R_1}\right) X = +4X$. 
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**Non-inverting amplifier** because the gain $\frac{Y}{X}$ is positive.

Consequence of $X$ connecting to $V_+$ input.
Can have any gain $\geq 1$ by choosing the ratio $\frac{R_2}{R_1}$. 
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Circuit has input voltage \( X \) and output voltage \( Y \). The circuit gain \( \triangleq \frac{Y}{X} \).

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*Cause/effect reversal:* Potential divider causes \( V_- = \frac{1}{4} Y \).

Feedback inverts this so that \( Y = 4V_+ \).
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Output $Y$ “follows” input $X$. 
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**Advantage:** Can supply a large current at \( Y \) while drawing almost no current from \( X \). Useful if the source supplying \( X \) has a high resistance.
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Although the *voltage gain* is only 1, the *power gain* is much larger.
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![Inverting Amplifier Circuit Diagram]
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KCL at $V_-$ node: \(|\frac{0-X}{R_1} + \frac{0-Y}{R_2}| = 0 \Rightarrow Y = -\frac{R_2}{R_1}X = -3X. \)
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If \( V_+ = 0 \text{ V} \), then \( V_- \) is called a *virtual earth* or *virtual ground*.
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Nodal Analysis: Do KCL at \( V_+ \) and/or \( V_- \) to solve circuit. When analysing a circuit, you never do KCL at the output node of an opamp because its output current is unknown. The only exception is if you have already solved the circuit and you want to find out what the op amp output current is (e.g. to check it is not too high).
Inverting Summing Amplifier

We can connect several input signals to the inverting amplifier.
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As before, $V_- = 0$ is a virtual earth due to negative feedback and $V_+ = 0$. 

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Diagram:

- $X_1$, $X_2$, $X_3$ are input sources
- $R_1 = 1k\Omega$, $R_2 = 2k\Omega$, $R_3 = 2k\Omega$, $R_F = 8k\Omega$
- Output $Y$
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$Y$ is a weighted sum of the input voltages with the weight of $X_i$ equal to

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**Input Isolation:** The current through $R_1$ equals $\frac{X_1-0}{R_1}$ which is not affected by $X_2$ or $X_3$. Because $V_-$ is held at a fixed voltage, the inputs are isolated from each other.
A 2-input circuit combining inverting and non-inverting amplifiers.
Differential Amplifier

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Linearity $\Rightarrow Z = aX + bY$.

Use superposition to find $a$ and $b$. 

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The non-inverting amplifier has a gain of \( \frac{R_1 + R_2}{R_1} = 4 \).
Differential Amplifier

A 2-input circuit combining inverting and non-inverting amplifiers.

Linearity ⇒ $Z = aX + bY$.

Use superposition to find $a$ and $b$.

Find $a$: Set $Y = 0$. KCL at $V_+$ node ⇒ $V_+ = 0$. We now have an inverting amplifier, so $Z = -\frac{R_2}{R_1}X = -3X$ ⇒ $a = -3$.

Find $b$: Set $X = 0$. We can redraw circuit to make it look more familiar: a potential divider followed by a non-inverting amplifier.

$R_3$ and $R_4$ are a potential divider (since current into $V_+$ equals zero), so $V_+ = \frac{R_4}{R_3+R_4} Y = \frac{3}{4} Y$.

The non-inverting amplifier has a gain of $\frac{R_1+R_2}{R_1} = 4$.

The combined gain is $b = \frac{R_4}{R_3+R_4} \times \frac{R_1+R_2}{R_1} = \frac{3}{4} \times 4 = +3$. 
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Combining the two gives \( Z = 3 \left(Y - X\right) \). The output of a *differential amplifier* is proportional to the difference between its two inputs.
Positive feedback: If op-amp output $Y$ rises then $(V_+ - V_-)$ will increase. This causes $Y$ to rise even more up to its maximum value (e.g. $+14 \text{ V}$).
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If $Y = +14 \, \text{V}$, then $Z = 4$. 

![Schmitt Trigger Diagram](image)
**Schmitt Trigger**

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If $Y = +14$ V, then $Z = 4$. For any $X < 4$, $(V_+ - V_-) > 0$ so the output stays at $+14$ V.
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**Positive feedback** adjusts the output to maximize $|V_+ - V_-|$. Output will switch between its maximum and minimum values, e.g. $\pm 14\,\text{V}$ (slightly less than the $\pm 15\,\text{V}$ power supplies).

Switching will happen when $V_+ = V_-$. 
The behaviour of an op-amp circuit depends on the ratio of resistor values: \( \text{gain} = -\frac{R_2}{R_1} \). How do you choose between 3 Ω/1 Ω, 3 kΩ/1 kΩ, 3 MΩ/1 MΩ and 3 GΩ/1 GΩ?
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If $X = \pm 1\ V$, then $Y = \mp 3\ V$, and so $I = \frac{Y-0}{R_2} = \mp 1\ A$.

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Within wide limits, the absolute resistor values have little effect.
However you should avoid extremes.
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For further details see Hayt Ch 6 or Irwin Ch 4.