6: Operational Amplifiers

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- Summary
An op amp (operational amplifier) is a circuit with two inputs and one output.

\[ Y = A (V_+ - V_-) \]

The gain, \( A \), is usually very large: e.g. \( A = 10^5 \) at low frequencies.

The input currents are very small: e.g. \( \pm 1 \text{nA} \).

Internally it is a complicated circuit with about 40 components, but we can forget about that and treat it as an almost perfect dependent voltage source.

Integrated circuit pins are numbered anti-clockwise from blob or notch (when looking from above).
Negative Feedback

In a central heating system, if the temperature falls too low the thermostat turns on the heating, when it rises the thermostat turns it off again. **Negative feedback** is when the occurrence of an event causes something to happen that counteracts the original event.

If op-amp output $Y$ falls then $V_-$ will fall by the same amount so $(V_+ - V_-)$ will increase. This causes $Y$ to rise since $Y = A (V_+ - V_-)$.

$$Y = A (X - Y)$$

$$Y (1 + A) = AX \quad \Rightarrow \quad Y = \frac{1}{1+1/A} X \quad \rightarrow X \text{ for large } A$$

If $Y = A(V_+ - V_-)$ then $V_+ - V_- = \frac{Y}{A}$ which, since $A \approx 10^5$, is normally very very small.

**Golden Rule:** Negative feedback adjusts the output to make $V_+ \approx V_-$. 
Analysing op-amp circuits

Nodal analysis is simplified by making some assumptions.

**Note:** The op-amp needs two power supply connections; usually +15 V and −15 V. These are almost always omitted from the circuit diagram. The currents only sum to zero (KCL) if all five connections are included.

1. **Check for negative feedback:** to ensure that an increase in \( Y \) makes \( (V_+ - V_-) \) decrease, \( Y \) must be connected (usually via other components) to \( V_- \).
2. **Assume** \( V_+ = V_- \): Since \( (V_+ - V_-) = \frac{Y}{A} \), this is the same as assuming that \( A = \infty \). Requires negative feedback.
3. **Assume zero input current:** in most circuits, the current at the op-amp input terminals is much smaller than the other currents in the circuit, so we assume it is zero.
4. **Apply KCL at each op-amp input node separately** (input currents = 0).
5. **Do not apply KCL at output node** (output current is unknown).
Non-inverting amplifier

Circuit has input voltage $X$ and output voltage $Y$. The circuit gain $\triangleq \frac{Y}{X}$.

Applying steps 1 to 3:
1. Negative feedback OK.
2. $V_– = V_+ = X$
3. Zero input current at $V_–$ means $R_2$ and $R_1$ are in series ($\Rightarrow$ same current) and form a voltage divider. So $X = \frac{R_1}{R_1 + R_2} Y$.
   
   So $Y = \frac{R_1 + R_2}{R_1} X = \left(1 + \frac{R_2}{R_1}\right) X = +4X$.

*Non-inverting amplifier* because the gain $\frac{Y}{X}$ is positive.

Consequence of $X$ connecting to $V_+$ input.
Can have any gain $\geq 1$ by choosing the ratio $\frac{R_2}{R_1}$.

*Cause/effect reversal*: Potential divider causes $V_– = \frac{1}{4} Y$.
Feedback inverts this so that $Y = 4V_+$. 
Voltage Follower

A special case of the non-inverting amplifier with \( R_1 = \infty \) and/or \( R_2 = 0 \).

Gain is \( 1 + \frac{R_2}{R_1} = 1 \).

Output \( Y \) “follows” input \( X \).

**Advantage:** Can supply a large current at \( Y \) while drawing almost no current from \( X \). Useful if the source supplying \( X \) has a high resistance.

**Without voltage follower:** \( Y = 0.01U \).

**With voltage follower:** \( Y = U \).

Although the *voltage gain* is only 1, the *power gain* is much larger.
Inverting Amplifier

Negative feedback OK.

Since \( V_+ = 0 \), we must have \( V_- = 0 \).

KCL at \( V_- \) node: \( \frac{0-X}{R_1} + \frac{0-Y}{R_2} = 0 \) \( \Rightarrow \) \( Y = -\frac{R_2}{R_1} X = -3X \).

*Inverting Amplifier* because gain \( \frac{Y}{X} \) is negative. Consequence of \( X \) connecting to the \( V_- \) input (via \( R_1 \)).

Can have any gain \( \leq 0 \) by choosing the ratio \( \frac{R_2}{R_1} \).

Negative feedback holds \( V_- \) very close to \( V_+ \).

If \( V_+ = 0 \) V, then \( V_- \) is called a *virtual earth* or *virtual ground*.

**Nodal Analysis**: Do KCL at \( V_+ \) and/or \( V_- \) to solve circuit. When analysing a circuit, you never do KCL at the output node of an opamp because its output current is unknown. The only exception is if you have already solved the circuit and you want to find out what the op amp output current is (e.g. to check it is not too high).
We can connect several input signals to the inverting amplifier.

As before, \( V_- = 0 \) is a virtual earth due to negative feedback and \( V_+ = 0 \).

KCL at \( V_- \) node: \( \frac{0-X_1}{R_1} + \frac{0-X_2}{R_2} + \frac{0-X_3}{R_3} + \frac{0-Y}{R_F} = 0 \)

\[ \Rightarrow \quad Y = -\left( \frac{R_F}{R_1} X_1 + \frac{R_F}{R_2} X_2 + \frac{R_F}{R_3} X_3 \right) \]

\[ \Rightarrow \quad Y = -(8X_1 + 4X_2 + 4X_3). \]

\( Y \) is a weighted sum of the input voltages with the weight of \( X_i \) equal to \( -\frac{R_F}{R_i} = -G_i R_F \).

**Input Isolation:** The current through \( R_1 \) equals \( \frac{X_1-0}{R_1} \) which is not affected by \( X_2 \) or \( X_3 \). Because \( V_- \) is held at a fixed voltage, the inputs are isolated from each other.
Differential Amplifier

A 2-input circuit combining inverting and non-inverting amplifiers.

Linearity $\Rightarrow Z = aX + bY$.

Use superposition to find $a$ and $b$.

Find $a$: Set $Y = 0$. KCL at $V_+$ node $\Rightarrow V_+ = 0$. We now have an inverting amplifier, so $Z = -\frac{R_2}{R_1} X = -3X \Rightarrow a = -3$.

Find $b$: Set $X = 0$. We can redraw circuit to make it look more familiar: a potential divider followed by a non-inverting amplifier.

$R_3$ and $R_4$ are a potential divider (since current into $V_+$ equals zero), so $V_+ = \frac{R_4}{R_3+R_4} Y = \frac{3}{4} Y$.

The non-inverting amplifier has a gain of $\frac{R_1+R_2}{R_1} = 4$.

The combined gain is $b = \frac{R_4}{R_3+R_4} \times \frac{R_1+R_2}{R_1} = \frac{3}{4} \times 4 = +3$.

Combining the two gives $Z = 3 (Y - X)$. The output of a differential amplifier is proportional to the difference between its two inputs.
Positive feedback: If op-amp output $Y$ *rises* then $(V_+ - V_-)$ will increase. This causes $Y$ to *rise* even more up to its maximum value (e.g. $+14 \text{ V}$).

If $Y = +14 \text{ V}$, then $Z = 4$. For any $X < 4$, $(V_+ - V_-) > 0$ so the output stays at $+14 \text{ V}$.

If $X > 4$, then $(V_+ - V_-) < 0$, $Y$ will rapidly switch to its minimum value (e.g. $-14 \text{ V}$).

Now $Z = -4$ and $Y$ will only switch back to $+14$ when $X$ falls below $-4$.

Negative feedback stabilizes the output to make $V_+ \simeq V_-$. Positive feedback adjusts the output to maximize $|V_+ - V_-|$. Output will switch between its maximum and minimum values, e.g. $\pm 14 \text{ V}$ (slightly less than the $\pm 15 \text{ V}$ power supplies).

Switching will happen when $V_+ = V_-$. 
Choosing Resistor Values

The behaviour of an op-amp circuit depends on the ratio of resistor values: gain $= -\frac{R_2}{R_1}$. How do you choose between $\frac{3\Omega}{1\Omega}$, $\frac{3k\Omega}{1k\Omega}$, $\frac{3M\Omega}{1M\Omega}$ and $\frac{3G\Omega}{1G\Omega}$?

**Small resistors** cause large currents.
- If $X = \pm 1\,\text{V}$, then $Y = \mp 3\,\text{V}$,
  and so $I = \frac{Y - 0}{R_2} = \mp 1\,\text{A}$.
  However typical op-amps can only supply $\pm 5\,\text{mA}$, so the circuit **will not work**.

**Large resistors** increase sensitivity to interference and to op-amp input currents.
- If the bias current into $V_-$ is $I_B = 1\,\text{nA}$,
  then KCL at $V_-$ gives
  \[
  \frac{0 - Y}{R_2} + \frac{0 - X}{R_1} + I_B = 0 \implies Y = -\frac{R_2}{R_1}X + I_B R_2 = -3X + 3
  \]
  instead of $Y = -3X$.

Within wide limits, the absolute resistor values have little effect. However you should avoid extremes.
Summary

- **Ideal properties:**
  - Zero input current
  - Infinite gain
  - Do not use KCL at output (except to determine output current).

- **Negative Feedback circuits:**
  - Assume $V_+ = V_-$ and zero input current
  - Standard amplifier circuits:
    - Non-inverting gain $= 1 + \frac{R_2}{R_1}$
    - Inverting gain $= -\frac{R_2}{R_1}$
    - Summing amplifier
    - Differential Amplifier

- **Positive feedback circuits:**
  - $V_{OUT} = \pm V_{max}$ (no good for an amplifier)
  - Schmitt Trigger: switches when $V_+ = V_-$.  

- **Choosing resistors:** not too low or too high.

For further details see Hayt Ch 6 or Irwin Ch 4.