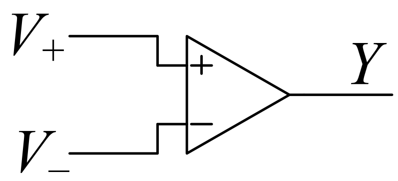
6: Operational Amplifiers

Operational Amplifier

An op amp (operational amplifier) is a circuit with two inputs and one output.

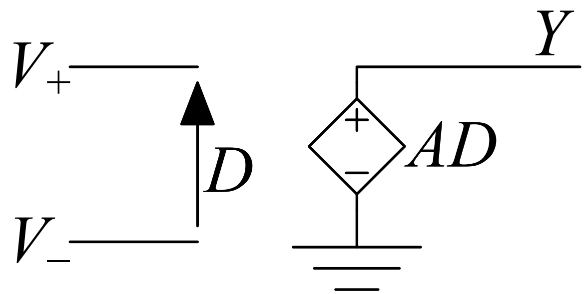


$$Y = A\left(V_+ - V_-\right)$$

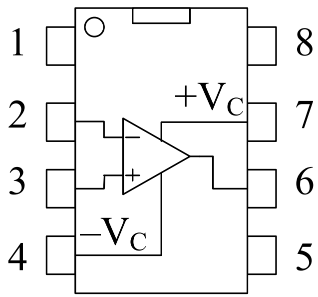
The gain, A, is usually very large: e.g. $A = 10^5$ at low frequencies.

The input currents are very small: e.g. ± 1 nA.

Internally it is a complicated circuit with about 40 components, but we can forget about that and treat it as an almost perfect dependent voltage source.

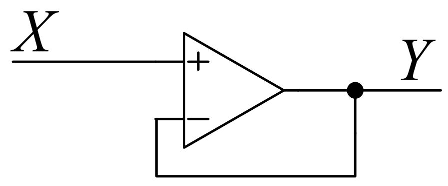


Integrated circuit pins are numbered anti-clockwise from blob or notch (when looking from above).



In a central heating system, if the temperature falls too low the thermostat turns on the heating, when it rises the thermostat turns it off again.
Negative feedback is when the occurence of an event causes something to happen that counteracts the original event.

If op-amp output Y *falls* then V_- will fall by the same amount so $(V_+ - V_-)$ will increase. This causes Y to *rise* since $Y = A(V_+ - V_-)$.



$$Y = A(X - Y)$$

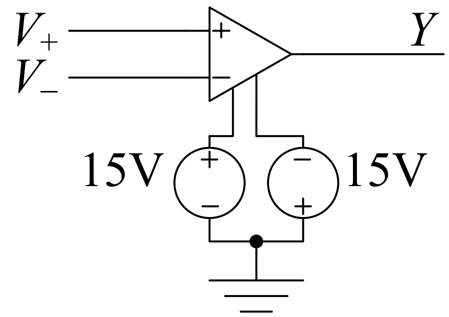
$$Y(1 + A) = AX \quad \Rightarrow \quad Y = \frac{1}{1+{}^{1}/_{A}}X \quad \rightarrow X \text{ for large } A$$

If $Y = A(V_+ - V_-)$ then $V_+ - V_- = \frac{Y}{A}$ which, since $A \simeq 10^5$, is normally *very very* small.

Golden Rule: Negative feedback adjusts the output to make $V_+ \simeq V_-$.

Nodal analysis is simplified by making some assumptions.

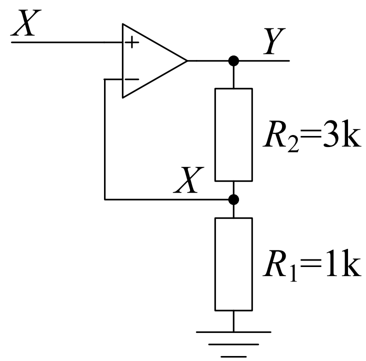
Note: The op-amp needs two power supply connections; usually $+15\,\text{V}$ and $-15\,\text{V}$. These are almost always omitted from the circuit diagram. The currents only sum to zero (KCL) if all five connections are included.



1. Check for negative feedback: to ensure that an increase in Y makes $(V_+ - V_-)$ decrease, Y must be connected (usually via other components) to V_-.
2. Assume $V_+ = V_-$: Since $(V_+ - V_-) = \frac{Y}{A}$, this is the same as assuming that $A = \infty$. Requires negative feedback.
3. Assume zero input current: in most circuits, the current at the op-amp input terminals is much smaller than the other currents in the circuit, so we assume it is zero.
4. Apply KCL at each op-amp input node separately (input currents = 0).
5. Do not apply KCL at output node (output current is unknown).

Non-inverting amplifier

Circuit has input voltage X and output voltage Y. The circuit gain $\triangleq \frac{Y}{X}$.



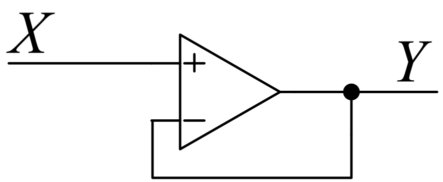
Applying steps 1 to 3:

1. Negative feedback OK.
2. $V_- = V_+ = X$
3. Zero input current at V_- means R_2 and R_1 are in series ($\Rightarrow$ same current) and form a voltage divider. So $X = \frac{R_1}{R_1+R_2}Y$.
   So $Y = \frac{R_1+R_2}{R_1}X = \left(1 + \frac{R_2}{R_1}\right)X = +4X$.

Non-inverting amplifier because the gain $\frac{Y}{X}$ is positive.
Consequence of X connecting to V_+ input.
Can have any gain ≥ 1 by choosing the ratio $\frac{R_2}{R_1}$.

Cause/effect reversal: Potential divider causes $V_- = \frac{1}{4}Y$.
Feedback inverts this so that $Y = 4V_+$.

A special case of the non-inverting amplifier with $R_1 = \infty$ and/or $R_2 = 0$.



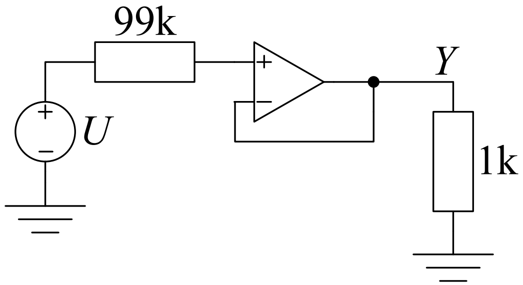
Gain is $1 + \frac{R_2}{R_1} = 1$.

Output Y "follows" input X.

Advantage: Can supply a large current at Y while drawing almost no current from X. Useful if the source supplying X has a high resistance.

Without voltage follower: $Y = 0.01U$.

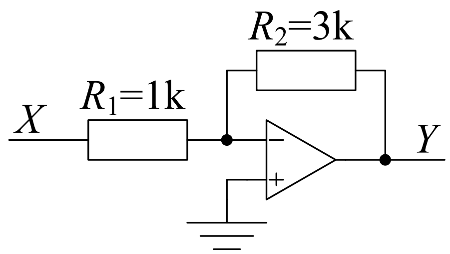
With voltage follower: $Y = U$.



Although the *voltage gain* is only 1, the power gain is much larger.

Negative feedback OK.

Since $V_+ = 0$, we must have $V_- = 0$.



KCL at V_- node: $\frac{0-X}{R_1} + \frac{0-Y}{R_2} = 0 \quad \Rightarrow \quad Y = -\frac{R_2}{R_1}X = -3X$.

Inverting Amplifier because gain $\frac{Y}{X}$ is negative. Consequence of X connecting to the V_- input (via R_1).
Can have any gain ≤ 0 by choosing the ratio $\frac{R_2}{R_1}$.

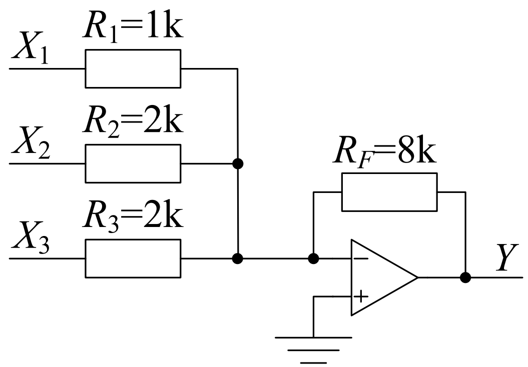
Negative feedback holds V_- very close to V_+.
If $V_+ = 0\,\text{V}$, then V_- is called a *virtual earth* or *virtual ground*.

Nodal Analysis: Do KCL at V_+ and/or V_- to solve circuit. When analysing a circuit, you never do KCL at the output node of an opamp because its output current is unknown. The only exception is if you have already solved the circuit and you want to find out what the op amp output current is (e.g. to check it is not too high).

Inverting Summing Amplifier

We can connect several input signals to the inverting amplifier.

As before, $V_- = 0$ is a virtual earth due to negative feedback and $V_+ = 0$.



KCL at V_- node: $\frac{0-X_1}{R_1} + \frac{0-X_2}{R_2} + \frac{0-X_3}{R_3} + \frac{0-Y}{R_F} = 0$

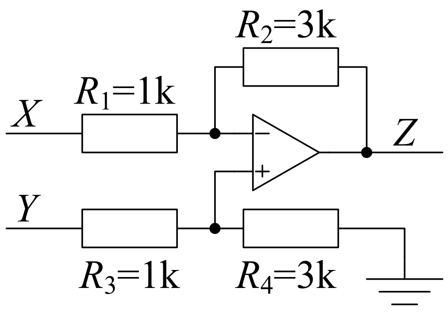
$$\Rightarrow \quad Y = -\left(\frac{R_F}{R_1}X_1 + \frac{R_F}{R_2}X_2 + \frac{R_F}{R_3}X_3\right)$$

$$\Rightarrow \quad Y = -\left(8X_1 + 4X_2 + 4X_3\right).$$

Y is a weighted sum of the input voltages with the weight of X_i equal to $-\frac{R_F}{R_i} = -G_i R_F$.

Input Isolation: The current through R_1 equals $\frac{X_1 - 0}{R_1}$ which is not affected by X_2 or X_3. Because V_- is held at a fixed voltage, the inputs are isolated from each other.

A 2-input circuit combining inverting and non-inverting amplifiers.



Linearity $\Rightarrow$ $Z = aX + bY$.

Use superposition to find a and b.

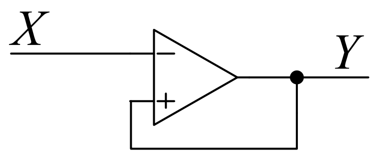
Find a: Set $Y = 0$. KCL at V_+ node $\Rightarrow$ $V_+ = 0$. We now have an inverting amplifier, so $Z = -\frac{R_2}{R_1}X = -3X \Rightarrow a = -3$.

Find b: Set $X = 0$. We can redraw circuit to make it look more familiar: a potential divider followed by a non-inverting amplifier.
R_3 and R_4 are a potential divider (since current into V_+ equals zero), so $V_+ = \frac{R_4}{R_3+R_4}Y = \frac{3}{4}Y$.
The non-inverting amplifier has a gain of $\frac{R_1+R_2}{R_1} = 4$.
The combined gain is $b = \frac{R_4}{R_3+R_4} \times \frac{R_1+R_2}{R_1} = \frac{3}{4} \times 4 = +3$.

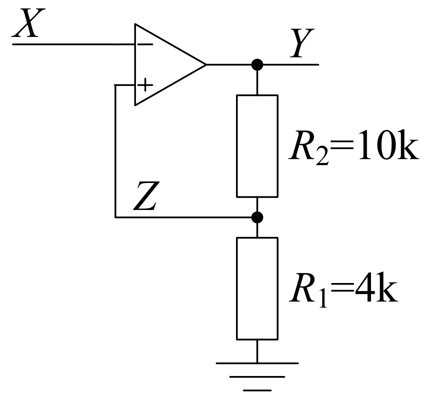
Combining the two gives $Z = 3\,(Y - X)$. The output of a *differential amplifier* is proportional to the diffference between its two inputs.

Schmitt Trigger

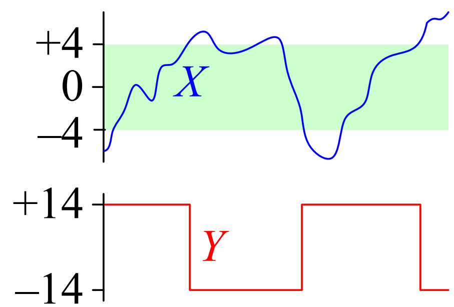
Positive feedback: If op-amp output Y *rises* then $(V_+ - V_-)$ will increase. This causes Y to *rise* even more up to its maximum value (e.g. $+14\,\text{V}$).



If $Y = +14\,\text{V}$, then $Z = 4$. For any $X < 4$, $(V_+ - V_-) > 0$ so the output stays at $+14\,\text{V}$.
If $X > 4$, then $(V_+ - V_-) < 0$, Y will rapidly switch to its minimum value (e.g. $-14\,\text{V}$).
Now $Z = -4$ and Y will only switch back to $+14$ when X falls below -4.



Negative feedback stabilizes the output to make $V_+ \simeq V_-$.
Positive feedback adjusts the output to maximize $|V_+ - V_-|$. Output will switch between its maximum and minimum values, e.g. $\pm 14\,\text{V}$ (slightly less than the $\pm 15\,\text{V}$ power supplies).



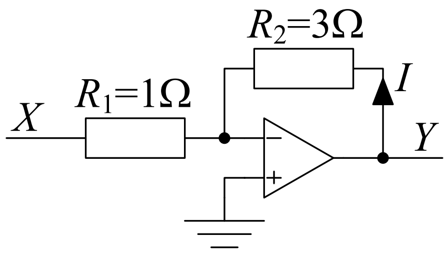
Switching will happen when $V_+ = V_-$.

Choosing Resistor Values

The behaviour of an op-amp circuit depends on the ratio of resistor values: gain $= -R_2/R_1$. How do you choose between $3\,\Omega/1\,\Omega$, $3\,\text{k}\Omega/1\,\text{k}\Omega$, $3\,\text{M}\Omega/1\,\text{M}\Omega$ and $3\,\text{G}\Omega/1\,\text{G}\Omega$?

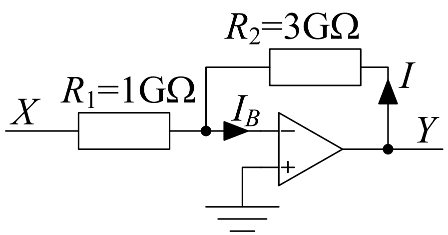
Small resistors cause large currents.

If $X = \pm 1\,\text{V}$, then $Y = \mp 3\,\text{V}$,
and so $I = \frac{Y-0}{R_2} = \mp 1\,\text{A}$.
However typical op-amps can only supply $\pm 5\,\text{mA}$, so the circuit will not work.



Large resistors increase sensitivity to interference and to op-amp input currents.

If the bias current into V_- is $I_B = 1\,\text{nA}$, then KCL at V_- gives



$$\frac{0-Y}{R_2} + \frac{0-X}{R_1} + I_B = 0 \Rightarrow Y = -\frac{R_2}{R_1}X + I_B R_2 = -3X + 3$$

instead of $Y = -3X$.

Within wide limits, the absolute resistor values have little effect. However you should avoid extremes.

Summary

- Ideal properties:
  - Zero input current
  - Infinite gain
  - Do not use KCL at output (except to determine output current).
- Negative Feedback circuits:
  - Assume $V_+ = V_-$ and zero input current
  - Standard amplifier circuits:
    - ▷ Non-inverting gain $= 1 + {}^{R_2}/_{R_1}$
    - ▷ Inverting gain $= {}^{-R_2}/_{R_1}$
    - ▷ Summing amplifier
    - ▷ Differential Amplifier
- Positive feedback circuits:
  - $V_{OUT} = \pm V_{max}$ (no good for an amplifier)
  - Schmitt Trigger: switches when $V_+ = V_-$.
- Choosing resistors: not too low or too high.

For further details see Hayt Ch 6 or Irwin Ch 4.