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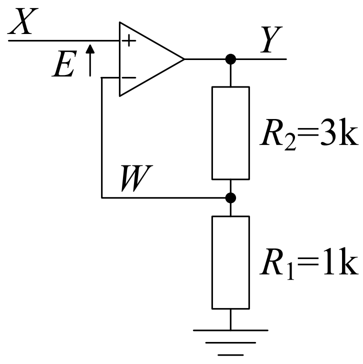
Block Diagram

In the non-inverting op amp circuit we take a fraction of the output signal, Y, and subtract it from the input signal, X.

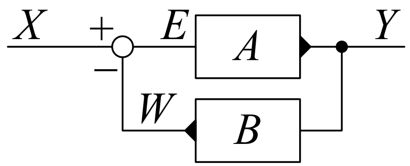


We can represent this using a block diagram:

$A = \frac{Y}{E}$: the gain of the op amp

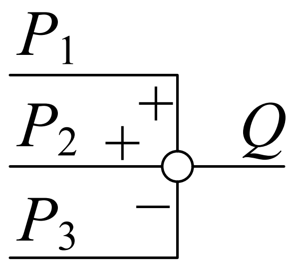
$B = \frac{W}{Y} = \frac{1}{4}$: gain of the feedback path

The "+" and "–" signs indicate that the feedback is subtracted from X to give an "error" signal, E.



A *gain block* has one input and one output (indicated here by an arrow): $V = A \times U$

An *adder block* many inputs and one output. The signs indicate whether each input is added or subtracted: $Q = P_1 + P_2 - P_3$



Normally, inputs are on the left and outputs are on the right.

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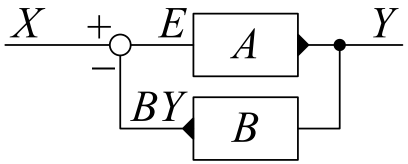
- Label inputs, output and adder outputs



- Write down equations for the output and all adder outputs

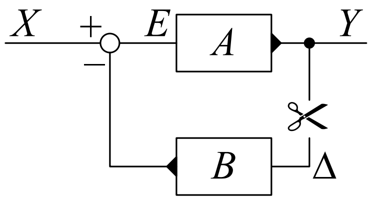
$$Y = AE$$
$$E = X - BY$$

Never use Kichoff's current law in block diagrams.

- Solve the equations by eliminating unwanted variables

$$Y = AE = A(X - BY) = AX - ABY$$
$$\Rightarrow Y(1 + AB) = AX \quad \Rightarrow \quad \frac{Y}{X} = \frac{A}{1+AB}$$

AB is called the *loop gain* of the circuit. If you break the loop at any point and inject a signal Δ after the break, this will cause the other side of the break to change by $-\Delta \times AB$.



Inverting Amplifier

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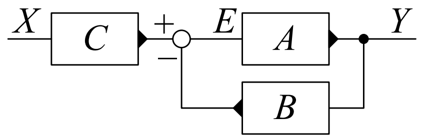
Sometimes we have an additional block at the input shown here as C.

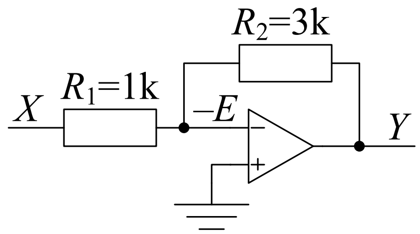


We see that $E = CX - BY$ and, as before,
$Y = AE$

Eliminating E : $\frac{Y}{X} = \frac{CA}{1+AB} = \frac{C}{A^{-1}+B} \approx \frac{C}{B}$ provided $A^{-1} \ll B$.

$\frac{Y}{X}$ equals the forward gain, CA, divided by the loop gain plus one.

Inverting Amplifier



Error signal is $E \triangleq V_+ - V_-$

Hence $V_+ = 0 \quad \Rightarrow \quad V_- = -E$

Op-amp output is $Y = AE$ where $A \approx 10^5$ is the op-amp gain.

Use superposition, nodal analysis or weighted average formula to find an expression for $-E$ in terms of X and Y:

$$-E = \frac{\frac{1}{1}X + \frac{1}{3}Y}{\frac{1}{1}+\frac{1}{3}} = \frac{3}{4}X + \frac{1}{4}Y = -\left(CX - BY\right)$$

Hence $C = -\frac{3}{4}$ and $B = +\frac{1}{4}$ and $\frac{Y}{X} \approx \frac{C}{B} = -3$

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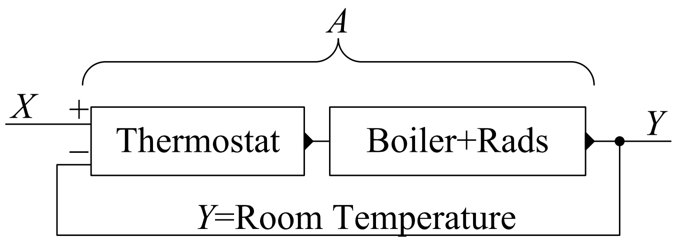
Central Heating:

X: Desired temperature
Y: Actual room temperature
A: Rather complicated system of boiler and radiators



Steam Engine Governor:

X: Desired Speed
Y: Actual Speed
A: Rotational speed causes weights to fly apart (centrifugal force) which adjusts the steam supply via a throttle valve.

Many Other Examples:

Economics: Demand↑ ⇒Price↑ ⇒Supply↑ ⇒Supply=Demand
Biology: More rabbits ⇒Not enough food ⇒Less rabbits ⇒Enough food

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Benefits of Negative Feedback

1) Gain Stabilization

The gain of a feedback system is almost entirely determined by the feedback path and not by the gain of the amplification path. This means that you can get predictable gains even when the gain of the amplification path is unknown or time-varying.

2) Distortion Reduction

High power amplifiers are often non-linear, e.g. their gain decreases at high signal amplitudes. Since the gain of a feedback system does not depend much on the gain of the amplification path, the non-linearity has little effect.

3) Interference Rejection

External disturbances have little effect on the output of a feedback system because the feedback adjusts to compensate for them.

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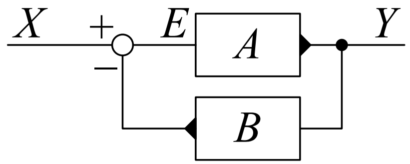
Gain is $\frac{Y}{X} = \frac{A}{1+AB} = \frac{1}{A^{-1}+B}$

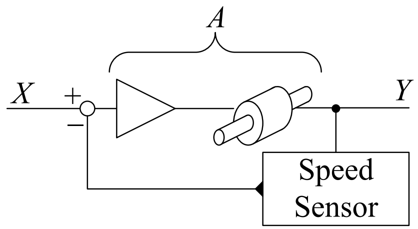


If A is very large then $\frac{Y}{X} \approx \frac{1}{B}$ and the precise value of A makes no difference.

"very large" means $A^{-1} \ll B \Leftrightarrow A \gg \frac{1}{B}$. So as long as A is much larger than the desired gain, its actual value does not matter.
For an op amp $A \approx 10^5$ at low frequencies but less at high frequencies.

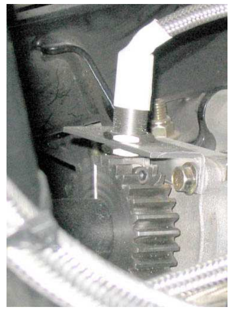
Motor Speed Control:

A is the "gain" of the amplifier and motor
(units = rotation speed per volt = rad.$s^{-1}V^{-1}$).
A cannot be precisely known: it depends on mechanical load and friction.



However this is OK so long as it is large enough.

We can sense the motor speed using gear-teeth and a magnetic (Hall effect) sensor together with a circuit that converts frequency to voltage.



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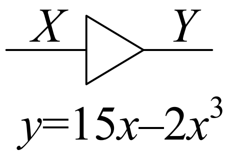
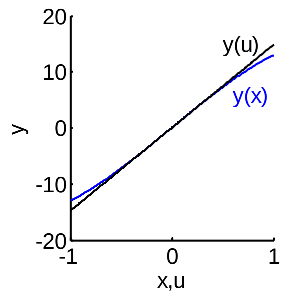
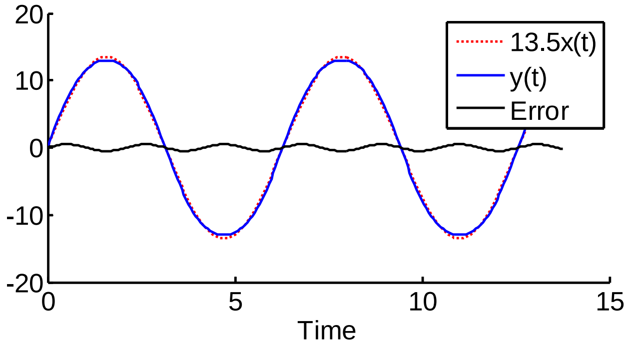
If A includes a high-power amplifier and/or a mechanical system (e.g. a motor) it is almost always non-linear.
$y = 15x - 2x^3$: gain decreases at high $|x|$





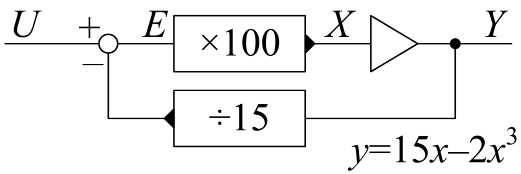
$x = \sin t \;\Rightarrow\; y = 15\sin t - 2\sin^3 t$
$\Rightarrow y = 13.5\sin t + 0.5\sin 3t$

The gain is only 13.5 instead of 15 and *harmonic distortion* is added at a multiple of the original frequency.
The total harmonic distortion (THD) is equal to $\frac{0.5^2}{13.5^2} = 0.14\%$.



Use feedback to reduce distortion

Put in feedback loop with $\times 100$ gain,
$A = \frac{Y}{E} = 100\frac{Y}{X}$ and $B = \frac{1}{15}$



Even though A depends on the signal amplitude, the gain is $\frac{Y}{U} \approx \frac{1}{B} = 15$.

[Trigonometrical Identities]

The easiest way to derive trigonometrical identities is to use De Moivre's theorem

$$\cos 3t + i \sin 3t = (\cos t + i \sin t)^3 = \cos^3 t + 3i \sin t \cos^2 t - 3 \sin^2 t \cos t - i \sin^3 t.$$

Taking the imaginary part of both sides gives

$$\sin 3t = 3 \sin t \cos^2 t - \sin^3 t = 3 \sin t \left(1 - \sin^2 t\right) - \sin^3 t = 3 \sin t - 4 \sin^3 t$$

and hence

$$\sin^3 t = \tfrac{3}{4} \sin t - \tfrac{1}{4} \sin 3t.$$

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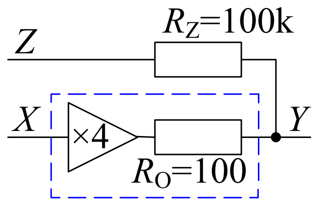
Interference Rejection

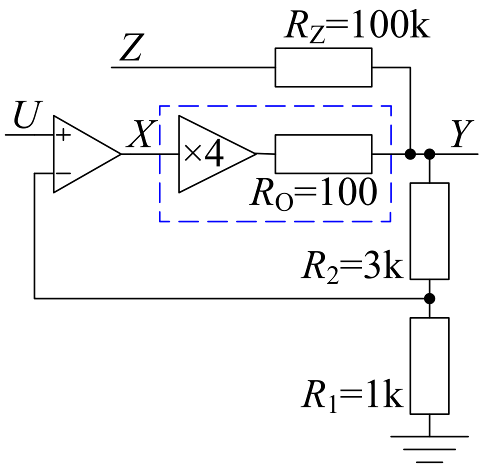
The amplifier output, Y, is affected by interference, Z.
Y = average of $4X$ and Z weighted by conductances:

$$Y = \frac{\frac{1}{R_O}4X + \frac{1}{R_Z}Z}{\frac{1}{R_O}+\frac{1}{R_Z}} = 3.996X + \frac{1}{1001}Z$$

Z is often much bigger than X (e.g. mains @ 230V).
R_O is amplifier *output resistance*.



Use feedback to reject interference



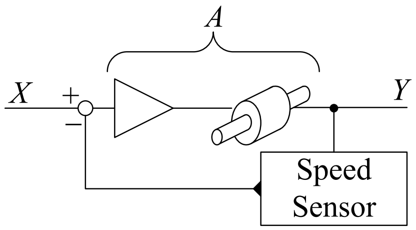
Opamp gain $= A \approx 10^5 \Rightarrow X = A\left(U - \frac{Y}{4}\right)$

$$Y = \frac{\frac{1}{R_O}4X + \frac{1}{R_Z}Z + \frac{1}{4k}0}{\frac{1}{R_O}+\frac{1}{R_Z}+\frac{1}{4k}} = 3.899X + \frac{1}{1026}Z$$

Eliminate X: $Y = 4U + \frac{1}{100001026}Z$

Interference reduced by the loop gain $\approx 10^5$.

"Interference" includes any external influence that may affect the output.
E.g. the mechanical load changing on a motor or an opened window in a heating system.



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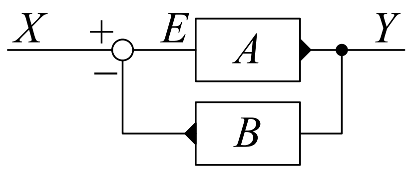
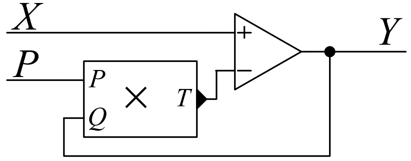


If multiplying by B is easier than dividing by B, use feedback to multiply by $\frac{1}{B}$.

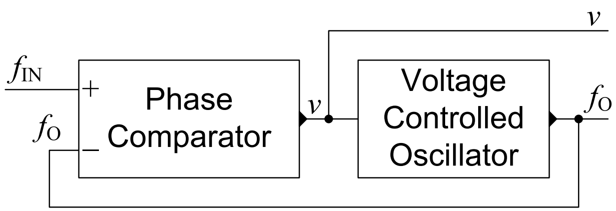
Division Circuit



Multiplier circuit is quite easy to make: $T = P \times Q$

Use in feedback loop to give $Y = \frac{X}{P}$
P must be +ve to ensure negative feedback.

Phase Lock Loop



Easy to make a voltage controlled oscillator with $f_O = k \times v$

Phase comparator output is $v \propto \int (f_{IN} - f_O)\, dt$ so v increases whenever $f_O < f_{IN}$ and decreases when $f_O > f_{IN}$. When v reaches equilibrium, we must have $f_O = f_{IN}$ so $v = \frac{1}{k} \times f_{IN}$.
We have generated a voltage proportional to the input frequency.
Used in FM radios and in many other circuits.

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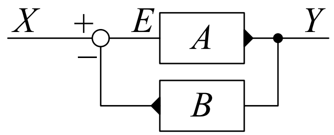
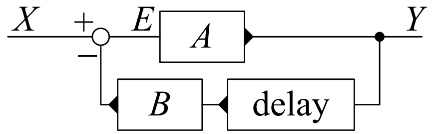
The biggest problem of feedback systems is the possibility of instability.



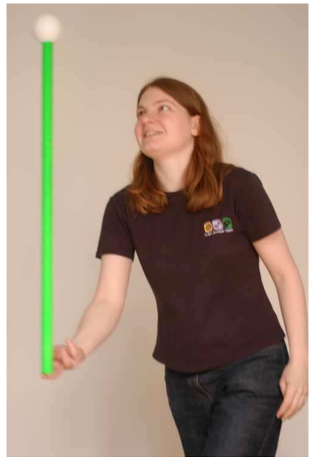
Gain is $\frac{Y}{X} = \frac{A}{1+AB}$. We have four cases:



$$\begin{array}{rl} AB > 0 & \text{Normal: } \frac{Y}{X} \approx \frac{1}{B} < A \\ -1 < AB < 0 & \text{Increased Gain: } \frac{Y}{X} > A \\ AB = -1 & \frac{Y}{X} = \infty \\ AB < -1 & \text{Usually saturates or oscillates if } AB > 0 \text{ at DC} \end{array}$$

Delays are Death

For a sine wave, a delay anywhere within the loop of half a period (e.g. 0.5 ms for 1 kHz) is the same as multiplying by -1. At this frequency the loop gain, AB, is large and negative so the system becomes unstable and oscillates.



© Science made simple

Quite a common problem: steering a boat, walking when drunk, balancing a stick.

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Why negative feedback is wonderful:

- The precise value of A does not matter as long as it is big enough because the gain is determined by the feedback, B.
- It makes no difference if A varies with time or with signal amplitude (i.e. A is non-linear).
- The effect of external interference at the output is reduced by the loop gain, AB.
- If making a gain B is easy, you can use feedback to make B^{-1}.

The one thing that can go wrong:

- Phase lags or delays can make a feedback system unstable (oscillate).
- Must make sure that as frequency increases, the loop gain falls below 1 before the phase shift reaches -180°.