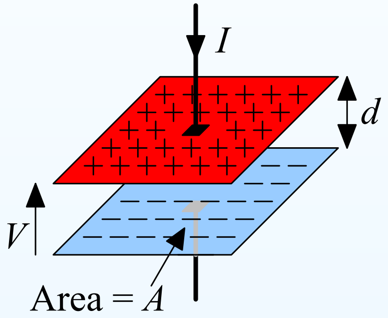
9: Capacitors and Inductors

Capacitors

A capacitor is formed from two conducting plates separated by a thin insulating layer.

If a current i flows, positive change, q, will accumulate on the upper plate. To preserve charge neutrality, a balancing negative charge will be present on the lower plate.



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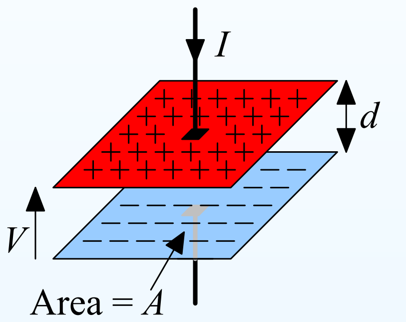

There will be a potential energy difference (or voltage v) between the plates proportional to q.
$v = \frac{d}{A\epsilon}q$ where A is the area of the plates, d is their separation and ϵ is the permittivity of the insulating layer ($\epsilon_0 = 8.85\,{}^{pF}\!/_{\mathrm{m}}$ for a vacuum).

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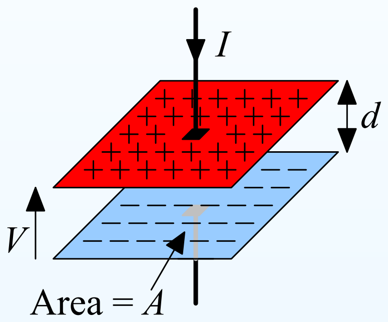

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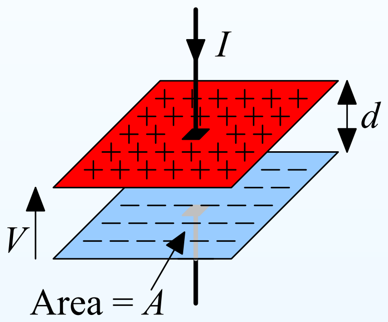

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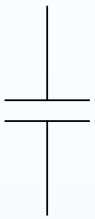
The current, i, is the rate of charge on the plate, hence the

capacitor equation: $i = \frac{dq}{dt} = C\frac{dv}{dt}$.

Types of Capacitor

Capacitor symbol represents the two separated plates. Capacitor types are distinguished by the material used as the insulator.



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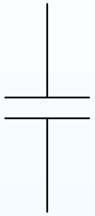

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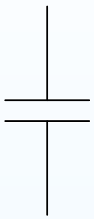

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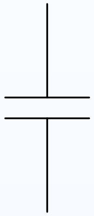

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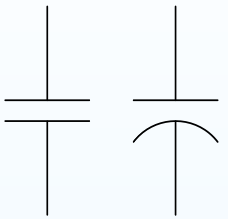

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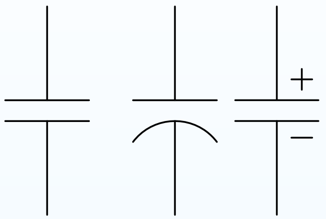

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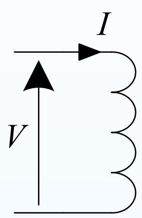
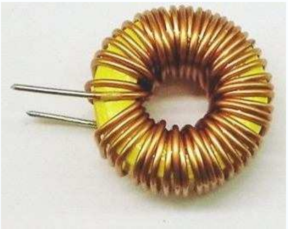
Inductors

Inductors are formed from coils of wire, often around a steel or ferrite core.





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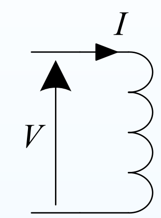

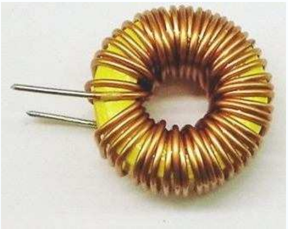

The magnetic flux within the coil is $\Phi = \frac{\mu N A}{l} i$ where N is the number of turns, A is the cross-sectional area of the coil and l is the length of the coil (around the toroid).

μ is a property of the material that the core is made from and is called its *permeability*. For free space (or air): $\mu_0 = 4\pi \times 10^{-7} = 1.26\ {}^{\mu H}/_{\mathrm{m}}$, for steel, $\mu \approx 4000\mu_0 = 5\ {}^{mH}/_{\mathrm{m}}$.

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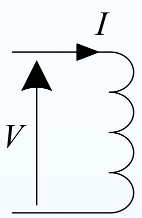

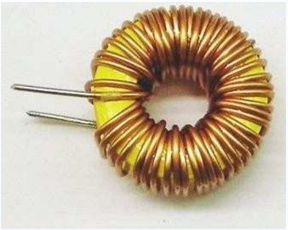

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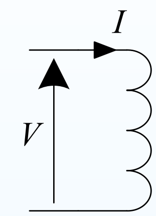

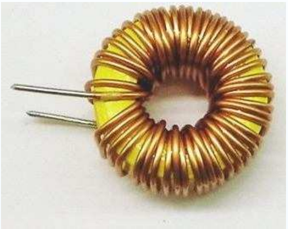

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We measure the *inductance*, $L = \frac{\mu N^2 A}{l}$, in Henrys (H).

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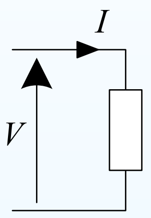

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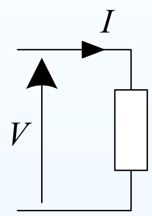
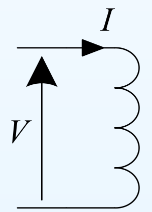


Inductor: $v = L\frac{di}{dt}$



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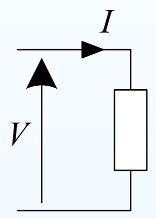

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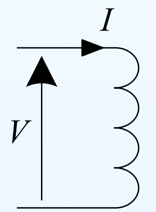
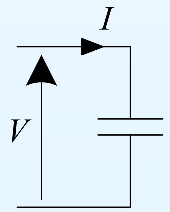


Capacitor: $i = C\frac{dv}{dt}$



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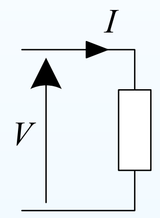

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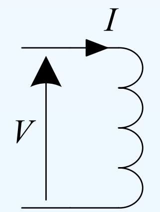

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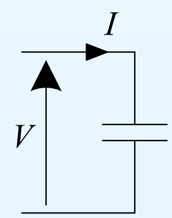
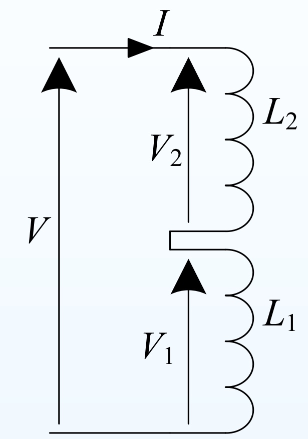


Notes: (1) There are no minus signs anywhere whatever you were taught at school.
(2) We use lower case, v, for time-varying voltages.

Series and Parallel Inductors

$$v = v_1 + v_2$$



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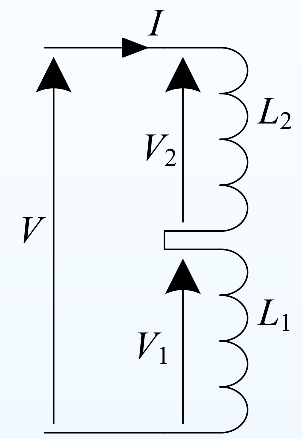

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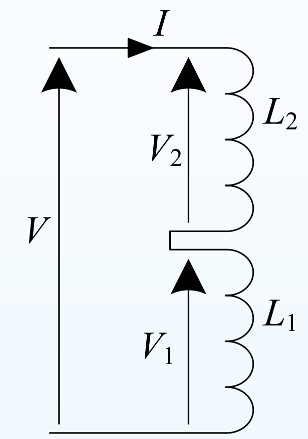

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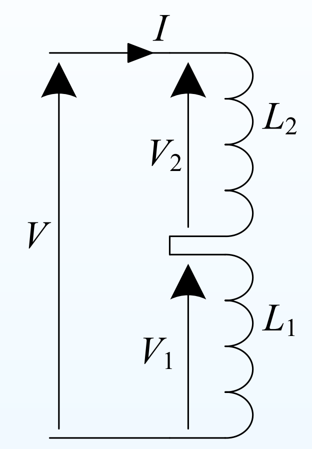
Same equation as a single inductor of value $L_1 + L_2$



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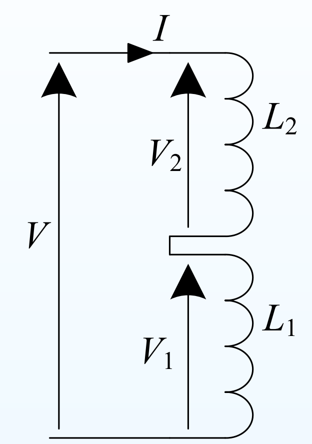
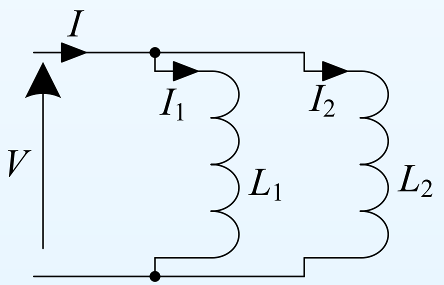


$$\frac{di}{dt} = \frac{d(i_1+i_2)}{dt}$$



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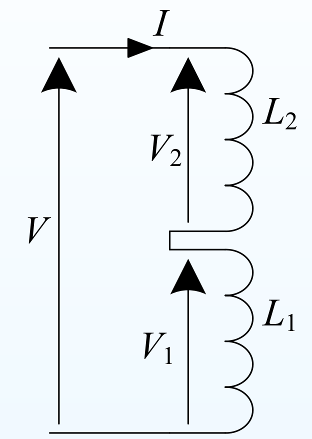

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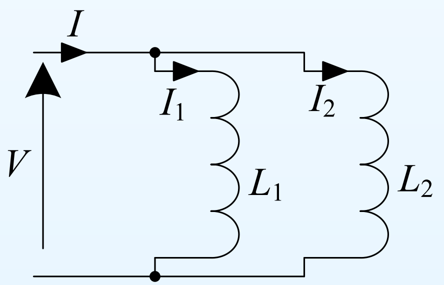

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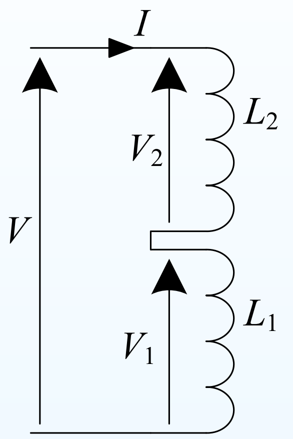

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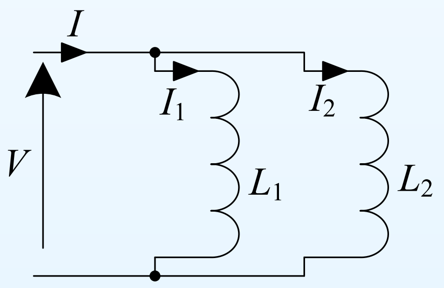

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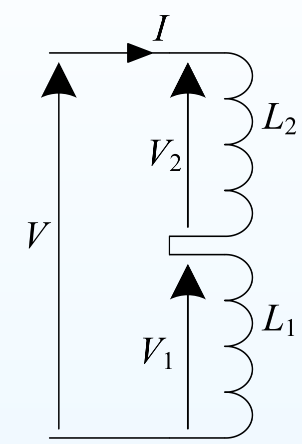

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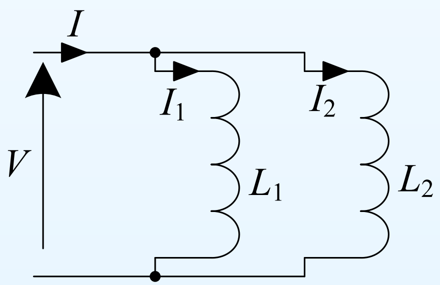

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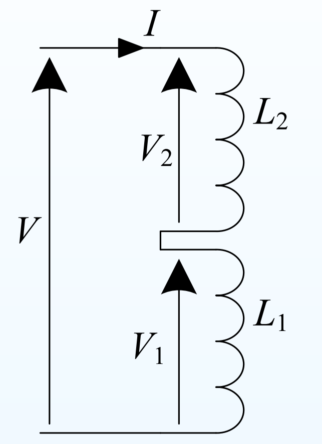

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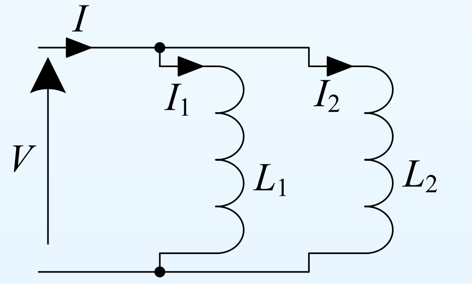

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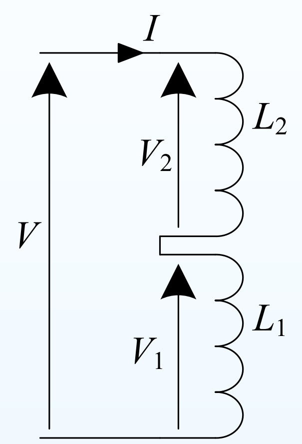

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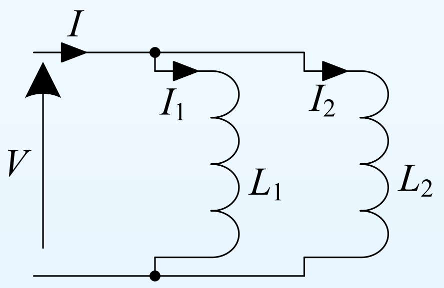
$$v = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} \frac{di}{dt}$$

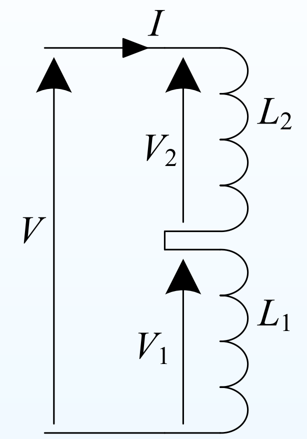


Same as a single inductor of value $\frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$

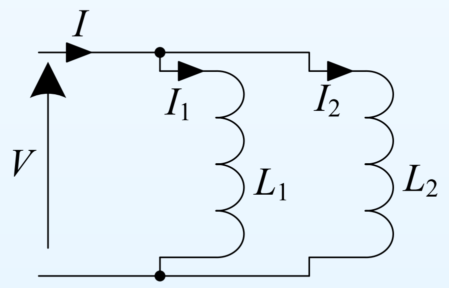
Series and Parallel Inductors

$$v = v_1 + v_2 = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$
$$= (L_1 + L_2) \frac{di}{dt}$$

Same equation as a single inductor of value $L_1 + L_2$



$$\frac{di}{dt} = \frac{d(i_1+i_2)}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$
$$= \frac{v}{L_1} + \frac{v}{L_2} = v\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$
$$v = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}} \frac{di}{dt}$$

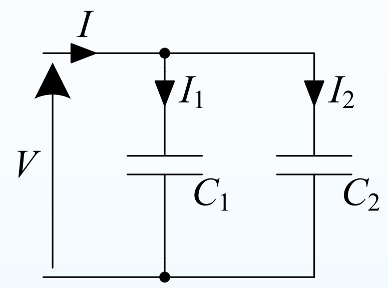


Same as a single inductor of value $\frac{1}{\frac{1}{L_1}+\frac{1}{L_2}} = \frac{L_1 L_2}{L_1+L_2}$

Inductors combine just like resistors.

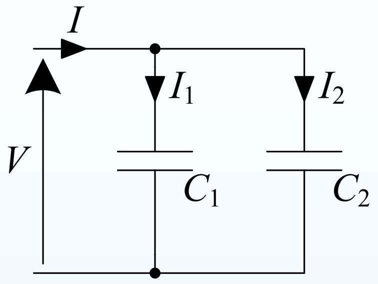
Series and Parallel Capacitors

$$i = i_1 + i_2$$



Series and Parallel Capacitors

$$i = i_1 + i_2 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$



Series and Parallel Capacitors

$$i = i_1 + i_2 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$

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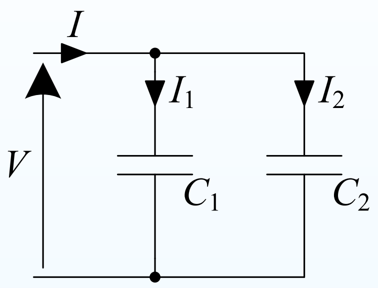

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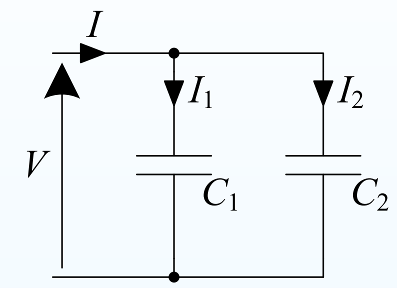

Same equation as a single capacitor of value $C_1 + C_2$

Series and Parallel Capacitors

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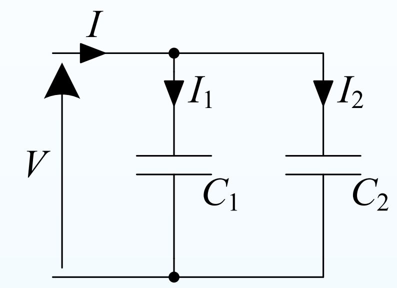
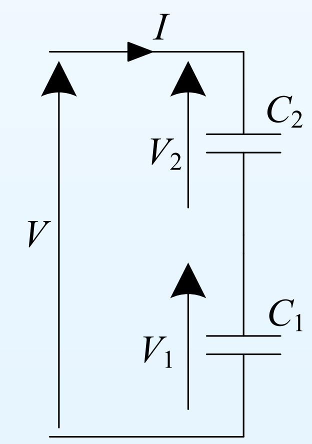


Same equation as a single capacitor of value $C_1 + C_2$

$$\frac{dv}{dt} = \frac{d(v_1 + v_2)}{dt}$$



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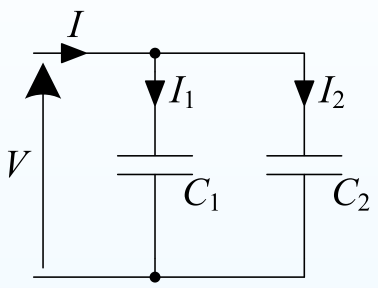

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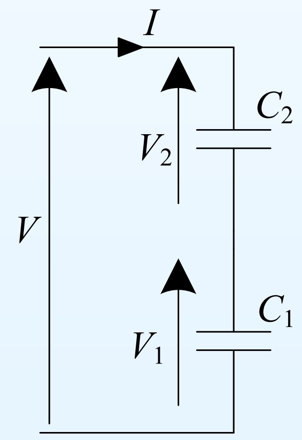

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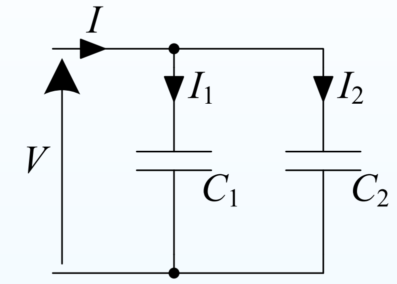

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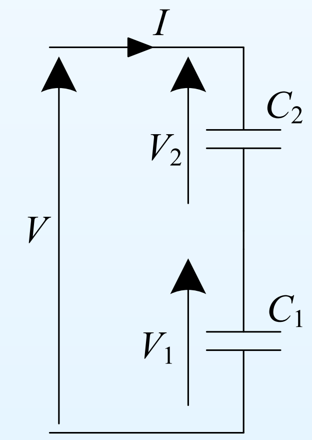

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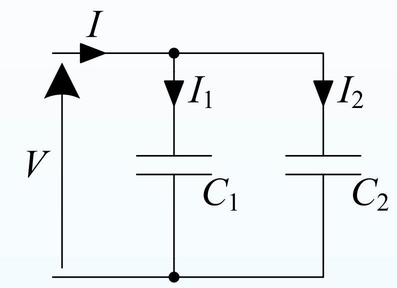

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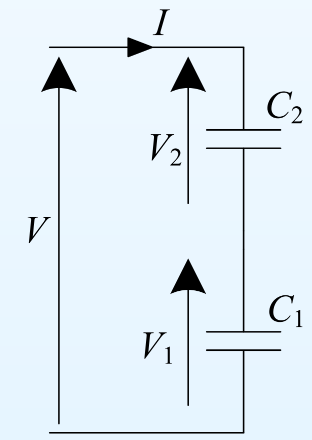

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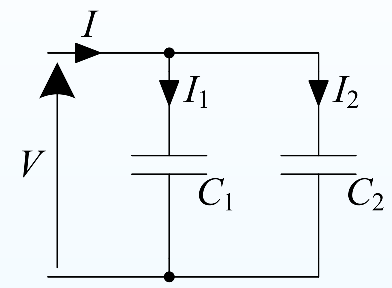

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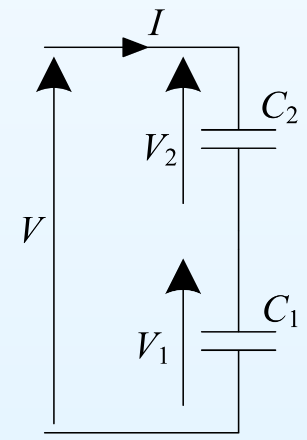

Same as a single capacitor of value $\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$

Series and Parallel Capacitors

$$i = i_1 + i_2 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$
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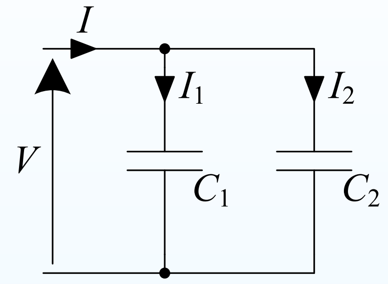

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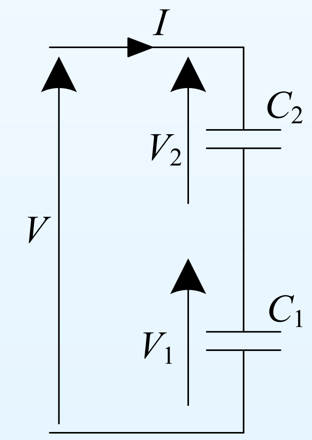

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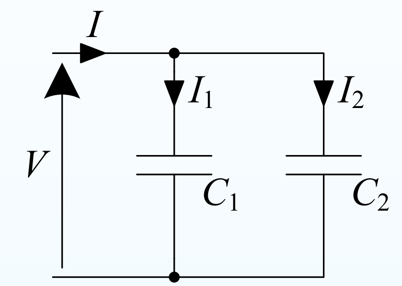

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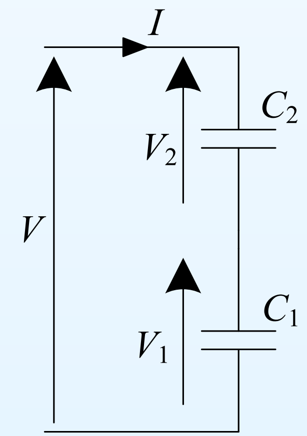
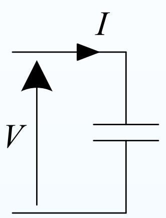


Capacitors combine just like conductances (i.e. parallel capacitors add).

Current/Voltage Continuity

Capacitor: $i = C\frac{dv}{dt}$



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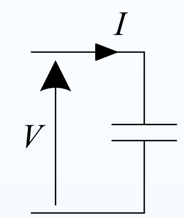

For the voltage to change abruptly

$$\frac{dv}{dt} = \infty \Rightarrow i = \infty.$$

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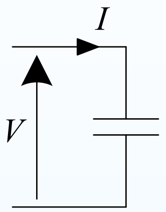

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This never happens so ...

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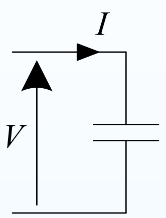

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The voltage across a capacitor never changes instantaneously.

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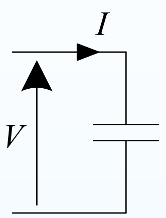
Capacitor: $i = C\frac{dv}{dt}$



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$$\frac{dv}{dt} = \infty \Rightarrow i = \infty.$$

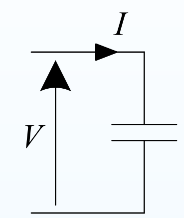
This never happens so ...

The voltage across a capacitor never changes instantaneously.

Informal version: A capacitor "tries" to keep its voltage constant.

Current/Voltage Continuity

Capacitor: $i = C\frac{dv}{dt}$



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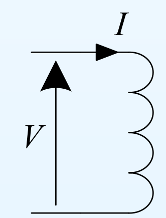
Inductor: $v = L\frac{di}{dt}$



Current/Voltage Continuity

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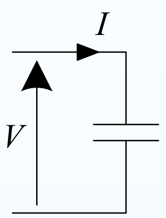

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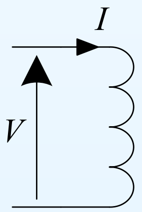
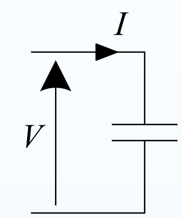


For the current to change abruptly

$$\frac{di}{dt} = \infty \Rightarrow v = \infty.$$

Current/Voltage Continuity

Capacitor: $i = C\frac{dv}{dt}$



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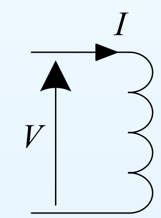

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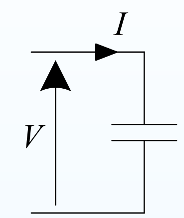

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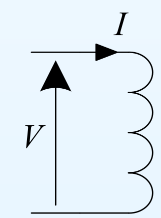

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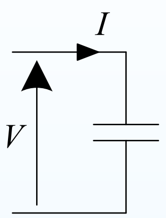
The current through an inductor never changes instantaneously.

Current/Voltage Continuity

Capacitor: $i = C\frac{dv}{dt}$



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Informal version: A capacitor "tries" to keep its voltage constant.

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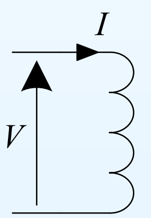


For the current to change abruptly

$$\frac{di}{dt} = \infty \Rightarrow v = \infty.$$

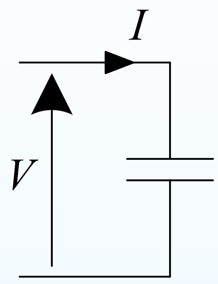
This never happens so ...

The current through an inductor never changes instantaneously.

Informal version: An inductor "tries" to keep its current constant.

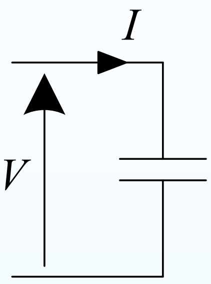
Average Current/Voltage

For a capacitor $i = C\frac{dv}{dt}$.



Average Current/Voltage

For a capacitor $i = C\frac{dv}{dt}$. Take the average of both sides:



$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2} i dt = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} C\frac{dv}{dt} dt$$

Average Current/Voltage

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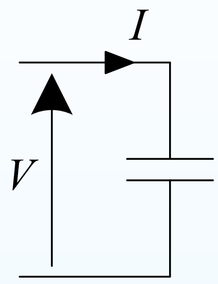

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2} i dt = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} C\frac{dv}{dt}dt = \frac{C}{t_2-t_1}\int_{v(t_1)}^{v(t_2)} dv$$

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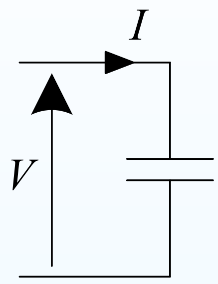
For a capacitor $i = C\frac{dv}{dt}$. Take the average of both sides:



$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2} idt = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} C\frac{dv}{dt}dt = \frac{C}{t_2-t_1}\int_{v(t_1)}^{v(t_2)} dv$$

$$= \frac{C}{t_2-t_1}\left[v\right]_{v(t_1)}^{v(t_2)}$$

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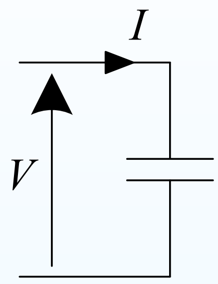

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$$= \frac{C}{t_2-t_1}\left[v\right]_{v(t_1)}^{v(t_2)} = \frac{C}{t_2-t_1}\left(v(t_2) - v(t_1)\right)$$

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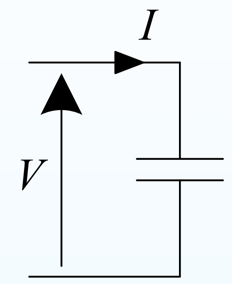

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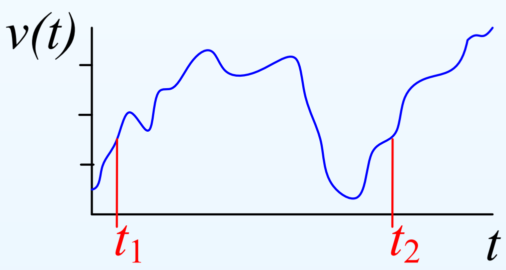
(1) If $v(t_1) = v(t_2)$ then the average current exactly equals zero.



Average Current/Voltage

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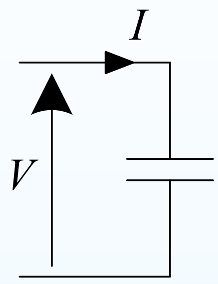

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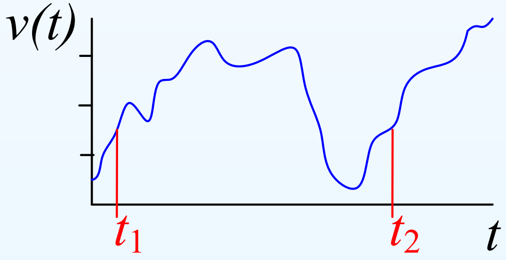

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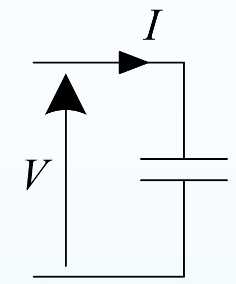

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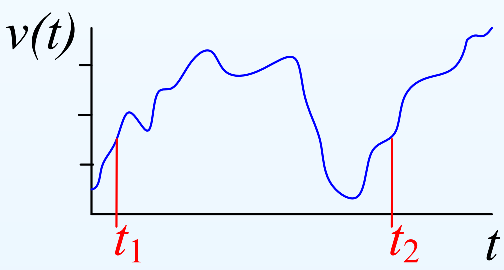

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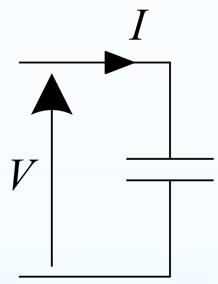

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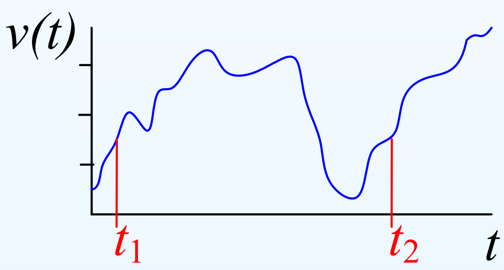

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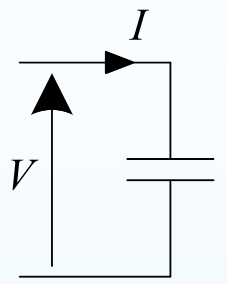

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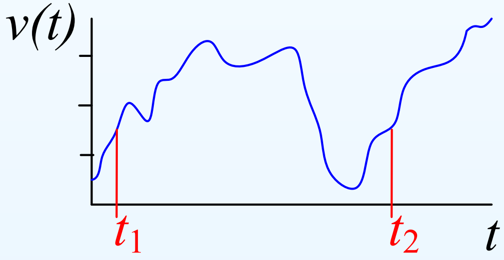

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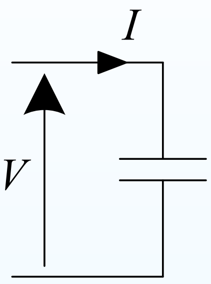

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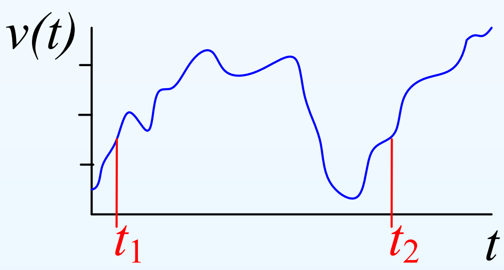

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"Average" can either be over an exact number of periods of a repetitive waveform or else the long-term average (provided v and i remain bounded).

"v is bounded" means $|v|$ always stays less than a predefined maximum value.

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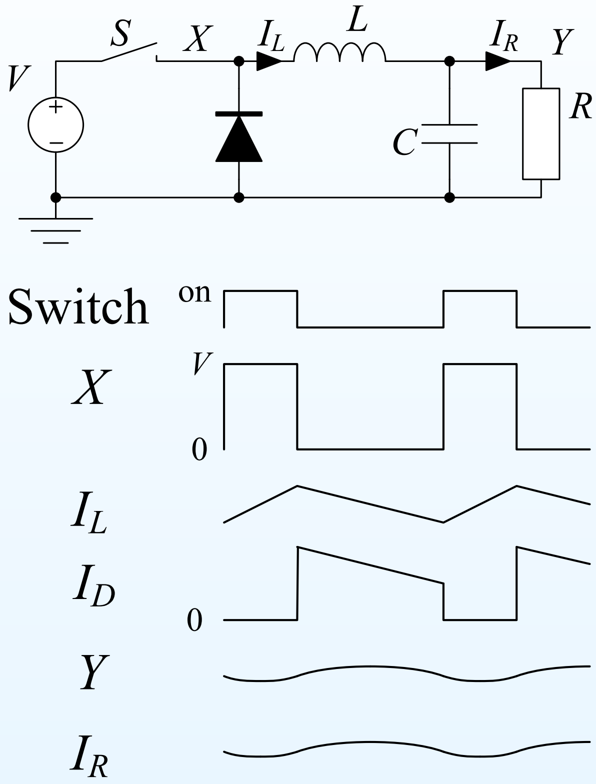
A buck converter converts a high voltage, V, into a lower one, Y.

The switch, S, closes for a fraction a of the time. a is the *duty cycle* and is $\frac{1}{3}$ in this example.



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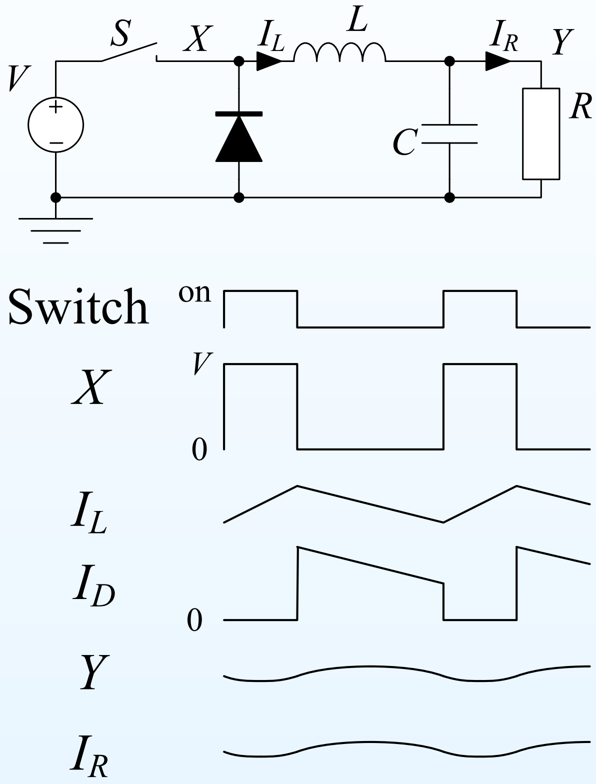

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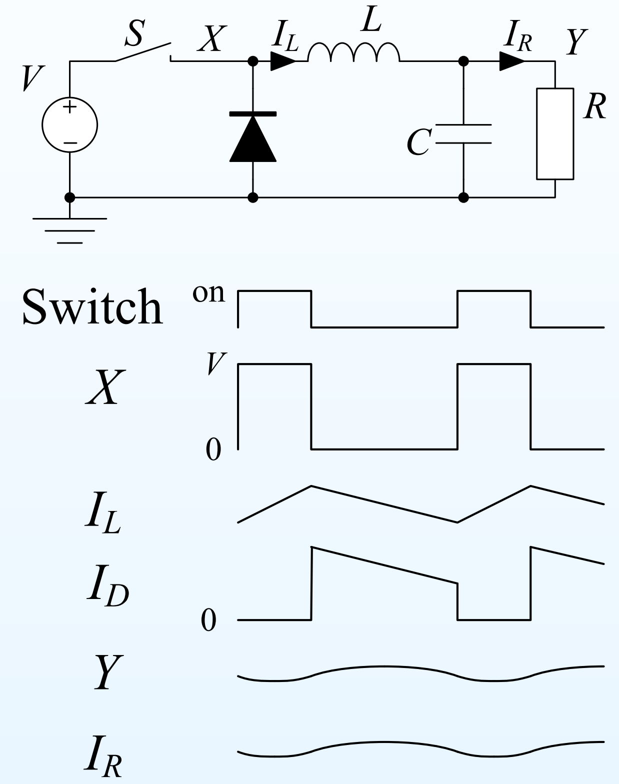

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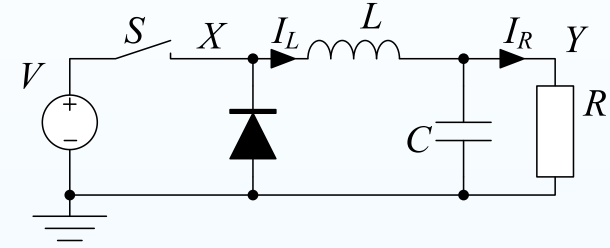

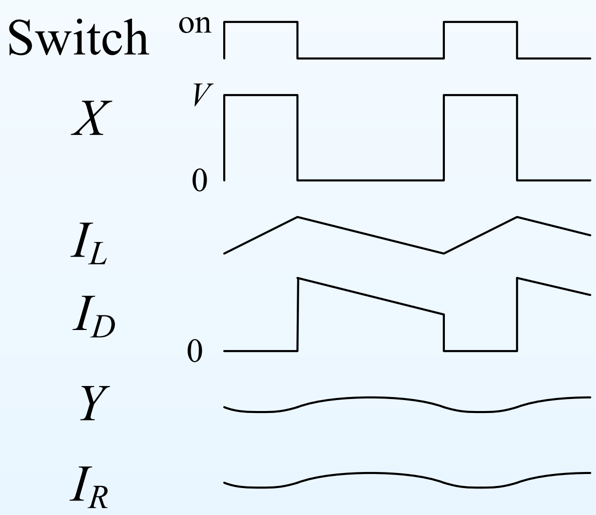

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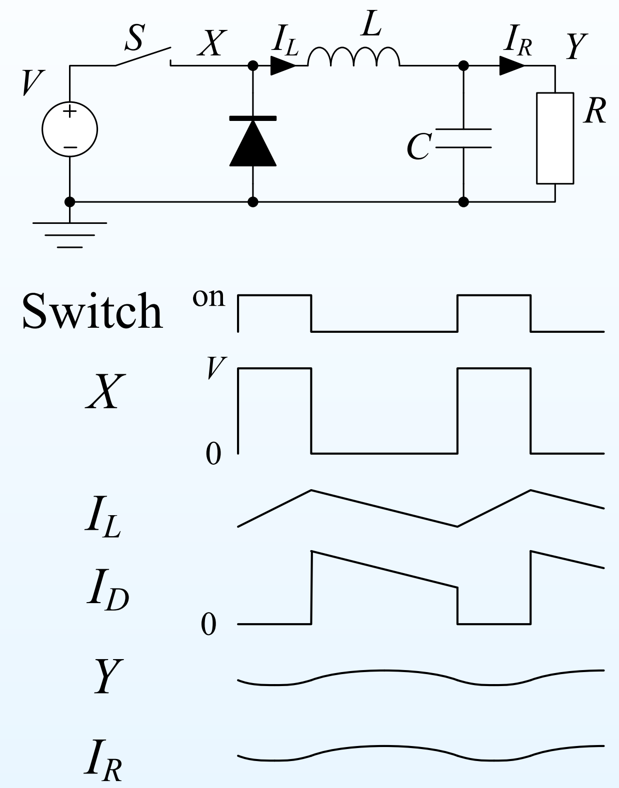

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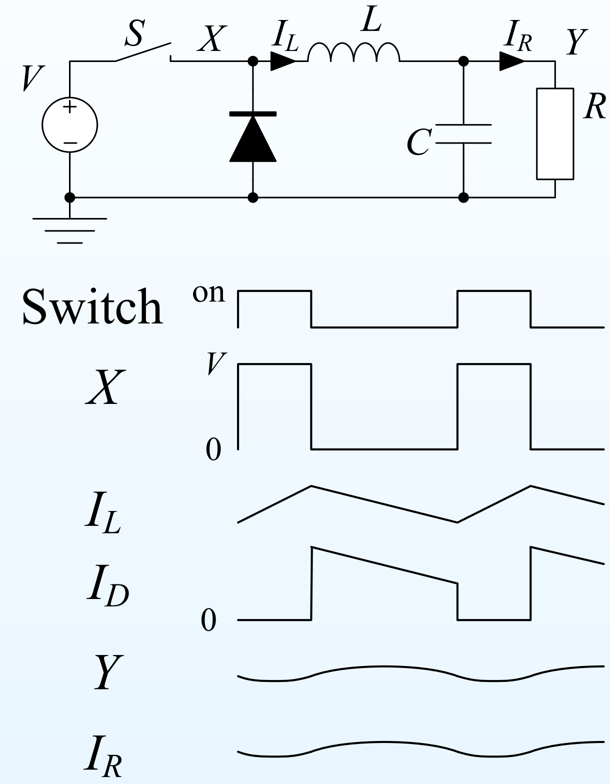

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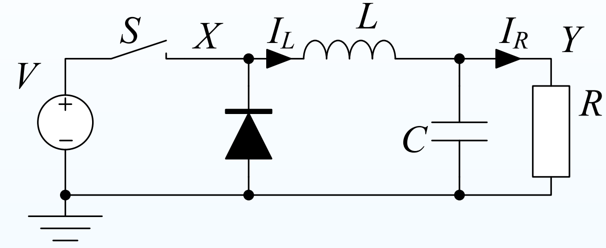
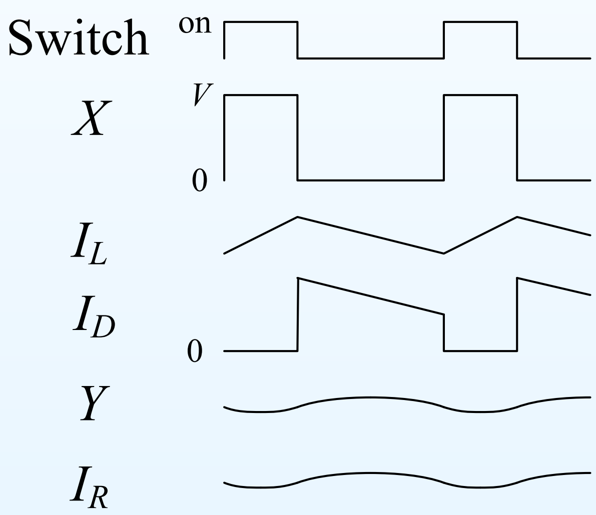
[Do not memorize this circuit]





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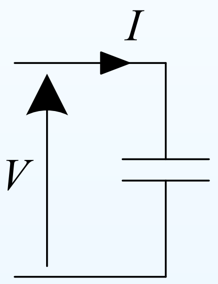

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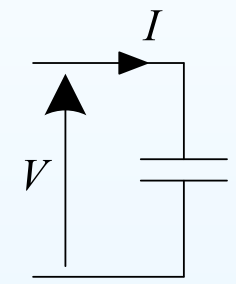

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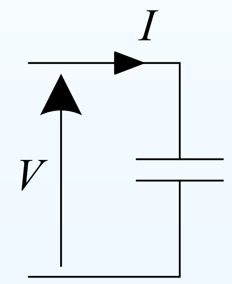

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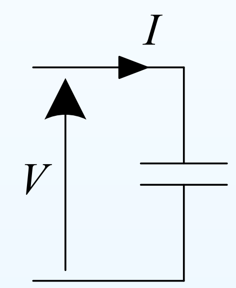

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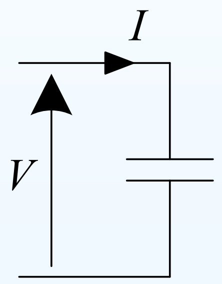

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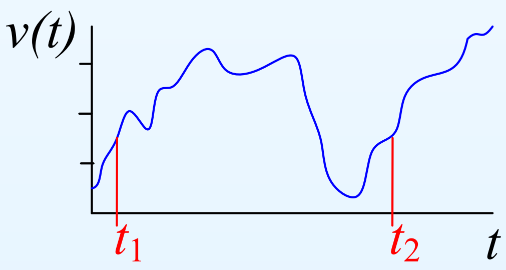

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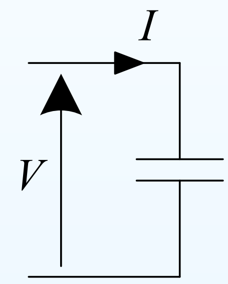

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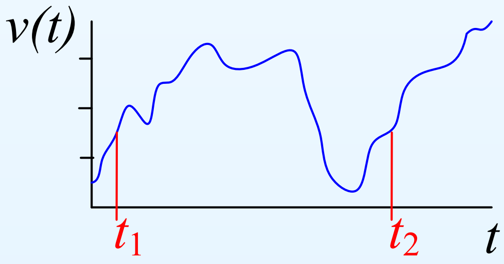

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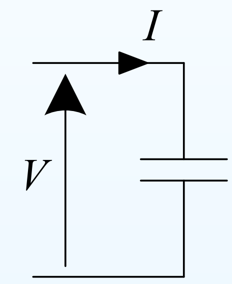

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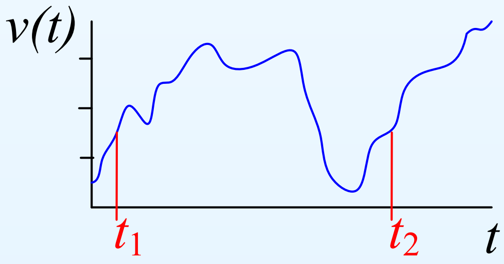

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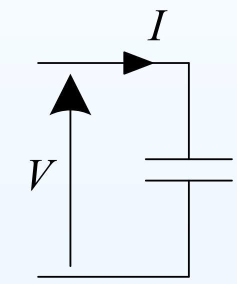

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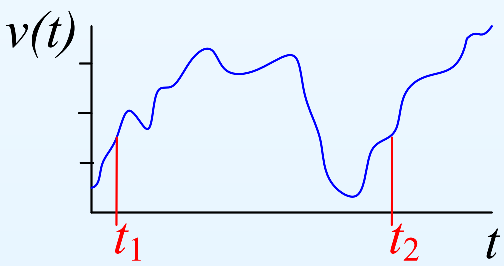

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  - average power absorbed is zero

Summary

- Capacitor:
  - $i = C\frac{dv}{dt}$
  - parallel capacitors add in value
  - average i is zero, v never changes instantaneously.
  - average power absorbed is zero
- Inductor:
  - $v = L\frac{di}{dt}$
  - series inductors add in value (like resistors)
  - average v is zero, i never changes instantaneously.
  - average power absorbed is zero

For further details see Hayt Ch 7 or Irwin Ch 6.