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$\sin 2\pi ft$ makes $f$ complete repetitions every time $t$ increases by $1$; this gives a frequency of $f$ cycles per second, or $f$ Hz.

We often use the angular frequency, $\omega = 2\pi f$ instead.

$\omega$ is measured in radians per second. E.g. $50$ Hz $\simeq 314$ rad.$s^{-1}$. 
Rotating Rod

A useful way to think of a cosine wave is as the projection of a rotating rod onto the horizontal axis.

For a unit-length rod, the projection has length $\cos \theta$. 
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v = \cos 2\pi ft \quad \text{and} \quad v = \sin 2\pi ft = \cos \left(2\pi ft - \frac{\pi}{2}\right)
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\( \sin 2\pi ft \) lags \( \cos 2\pi ft \) by \( 90^\circ \) (or \( \frac{\pi}{2} \) radians) because its peaks occurs \( \frac{1}{4} \) of a cycle later (equivalently \( \cos \) leads \( \sin \)).
If the rod has length $A$ and starts at an angle $\phi$ then the projection onto the horizontal axis is

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The argument of the phasor, $\phi = \arctan \frac{Y}{X}$, gives the phase shift relative to $\cos 2\pi ft$.

If $\phi > 0$, it is leading and if $\phi < 0$, it is lagging relative to $\cos 2\pi ft$. 
Phasor Examples

$V = 1, f = 50 \text{ Hz}$
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Casio: \( \text{Pol}(X, Y) \rightarrow A, \phi, \text{Rec}(A, \phi) \rightarrow X, Y \). Saved \( \rightarrow X \ & \ Y \) mems.
Phasor arithmetic

Phasors

\[ V = P + jQ \]

Waveforms

\[ v(t) = P \cos \omega t - Q \sin \omega t \]

where \( \omega = 2\pi f \).
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Phasor arithmetic

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Waveforms

$$v(t) = P \cos \omega t - Q \sin \omega t$$

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Phasor arithmetic

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Wave equation: 

$v(t) = P \cos \omega t - Q \sin \omega t$
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Adding or scaling is the same for waveforms and phasors.
Phasor arithmetic

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\dot{V} = (-\omega Q) + j (\omega P) \\
\frac{dv}{dt} = -\omega P \sin \omega t - \omega Q \cos \omega t \\
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\[
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<td>$v(t) = P \cos \omega t - Q \sin \omega t$</td>
</tr>
</tbody>
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where $\omega = 2\pi f$.

$aV$

$a \times v(t) = aP \cos \omega t - aQ \sin \omega t$

$V_1 + V_2$

$v_1(t) + v_2(t)$

Adding or scaling is the same for waveforms and phasors.

$\dot{V} = (-\omega Q) + j (\omega P)$

$= j\omega (P + jQ)$

$= j\omega V$

$\frac{dv}{dt} = -\omega P \sin \omega t - \omega Q \cos \omega t$

$= (-\omega Q) \cos \omega t - (\omega P) \sin \omega t$

Differentiating waveforms corresponds to multiplying phasors by $j\omega$. 
Phasor arithmetic

<table>
<thead>
<tr>
<th>Phasors</th>
<th>Waveforms</th>
</tr>
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Adding or scaling is the same for waveforms and phasors.

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\dot{V} = (-\omega Q) + j(\omega P) \\
= j\omega (P + jQ) \\
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Phasor arithmetic

Phasors

\[ V = P + jQ \]

Waveforms

\[ v(t) = P \cos \omega t - Q \sin \omega t \]

where \( \omega = 2\pi f \).

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\[ a \times v(t) = aP \cos \omega t - aQ \sin \omega t \]

\[ V_1 + V_2 \]

\[ v_1(t) + v_2(t) \]

Adding or scaling is the same for waveforms and phasors.

\[ \dot{V} = -(\omega Q) + j (\omega P) \]

\[ = j\omega (P + jQ) \]

\[ = j\omega V \]

\[ \frac{dv}{dt} = -\omega P \sin \omega t - \omega Q \cos \omega t \]

\[ = (-\omega Q) \cos \omega t - (\omega P) \sin \omega t \]

Differentiating waveforms corresponds to multiplying phasors by \( j\omega \).

Rotate anti-clockwise \( 90^\circ \) and scale by \( \omega = 2\pi f \).
Complex Impedances

Resistor:

\[ v(t) = Ri(t) \]
Complex Impedances

Resistor:

\[ v(t) = Ri(t) \Rightarrow V = RI \]
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\[ i(t) = C \frac{dv(t)}{dt} \]
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For all three components, phasors obey Ohm’s law if we use the complex impedances \( j\omega L \) and \( \frac{1}{j\omega C} \) as the “resistance” of an inductor or capacitor.
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For all three components, phasors obey Ohm’s law if we use the complex impedances \( j\omega L \) and \( \frac{1}{j\omega C} \) as the “resistance” of an inductor or capacitor.

If all sources in a circuit are sine waves having the same frequency, we can do circuit analysis exactly as before by using complex impedances.
Given \( v = 10 \sin \omega t \) where \( \omega = 2\pi \times 1000 \), find \( v_C(t) \).
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1) Find capacitor complex impedance

$$Z = \frac{1}{j\omega C} = \frac{1}{6.28 \times 10^{-4}} = -1592j$$
Given \( v = 10 \sin \omega t \) where \( \omega = 2\pi \times 1000 \), find \( v_C(t) \).

(1) Find capacitor complex impedance
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Z = \frac{1}{j\omega C} = \frac{1}{6.28 \times 10^{-4}} = -1592j
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(2) Solve circuit with phasors
\[
V_C = V \times \frac{Z}{R + Z}
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2. Solve circuit with phasors
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   V_C = V \times \frac{Z}{R+Z} = -10j \times \frac{-1592j}{1000-1592j} = -4.5 - 7.2j = 8.47 \angle -122^\circ
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(3) Draw a phasor diagram showing KVL:

- \( V = -10j \)
- \( V_C = -4.5 - 7.2j \)
- \( V_R = V - V_C = 4.5 - 2.8j = 5.3\angle -32^\circ \)
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\begin{align*}
V_C &= V \times \frac{Z}{R+Z} \\
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\]

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3. Phasors add like vectors
Capacitors: \( i = C \frac{dv}{dt} \) \( \Rightarrow I \) leads \( V \)

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Casio fx-991 (available in all exams except Maths) will do complex arithmetic (\( +, -, \times, ÷, x^2, \frac{1}{x}, |x|, x^* \)) in CMPLX mode.

**Learn how to use this: it will save lots of time and errors.**
Impedance and Admittance

For any network (resistors+capacitors+inductors):
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(1) **Impedance** = Resistance + $j \times$ Reactance

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Note:
\[ Y = G + jB = \frac{1}{Z} \]
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Note:
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Y = G + jB = \frac{1}{Z} = \frac{1}{R+jX}
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**Note:**

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So

$$G = \frac{R}{R^2+X^2} = \frac{R}{|Z|^2}$$

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**Beware:** $G \neq \frac{1}{R}$ unless $X = 0$. 
Sine waves are the only bounded signals whose shape is unchanged by differentiation.
Summary

- Sine waves are the only bounded signals whose shape is unchanged by differentiation.

- Think of a sine wave as the projection of a rotating rod onto the horizontal (or real) axis.
  - A **phasor** is a complex number representing the length and position of the rod at time $t = 0$. 
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See Hayt Ch 10 or Irwin Ch 8