10: Sine waves and phasors

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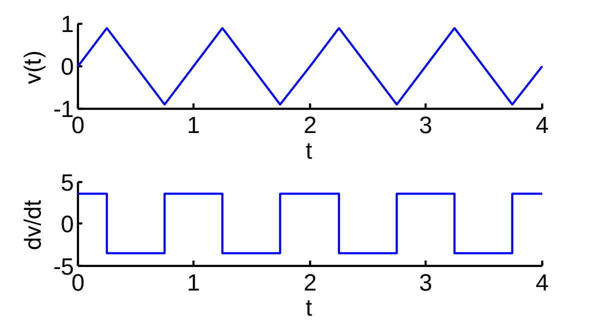

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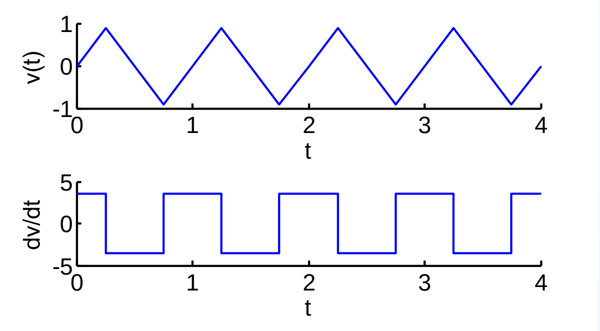

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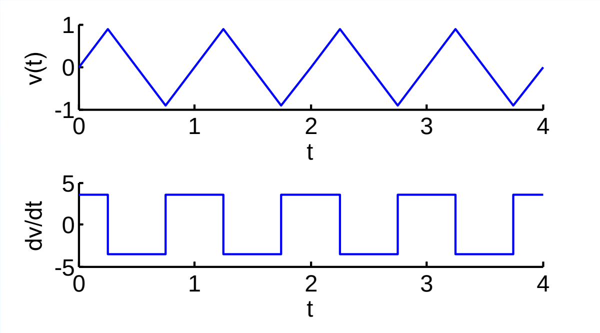

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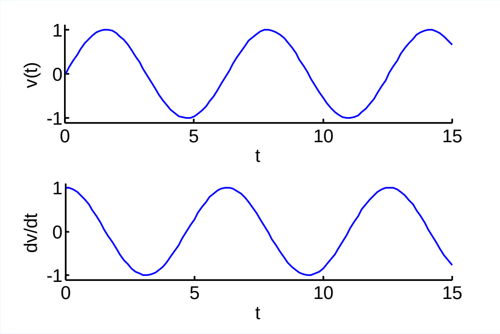

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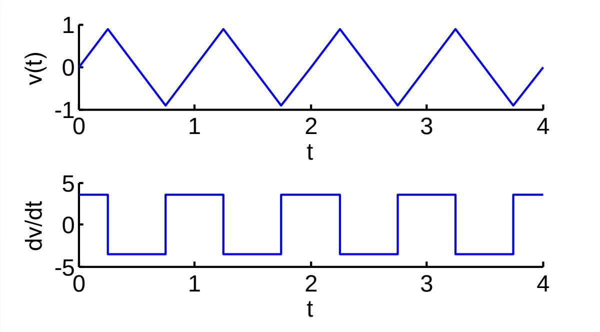

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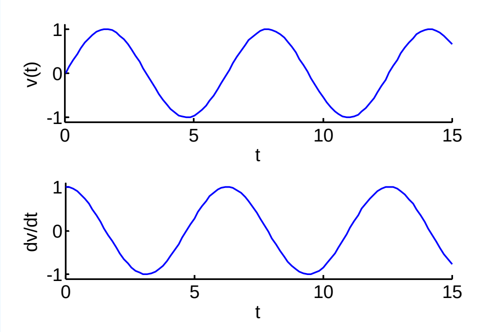

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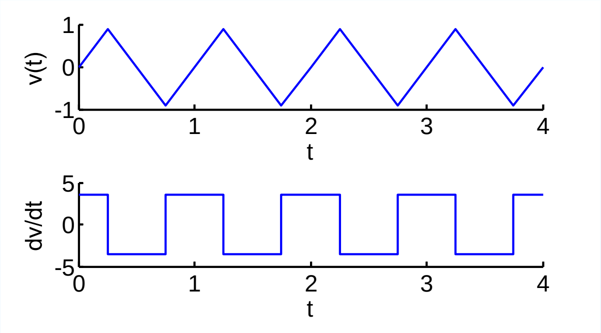

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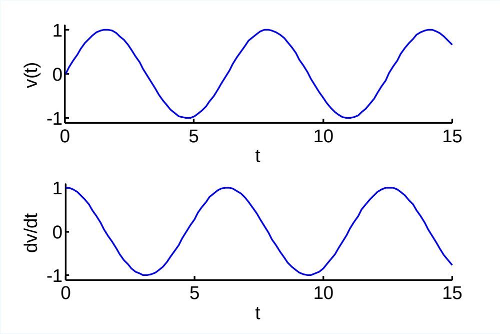

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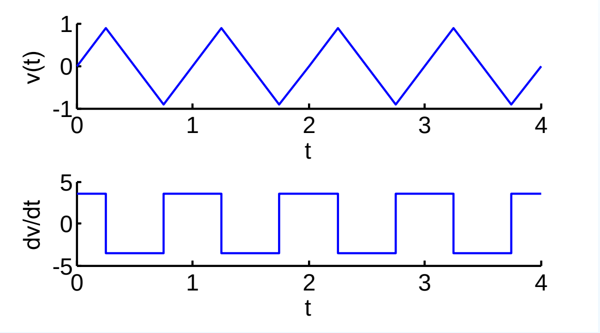

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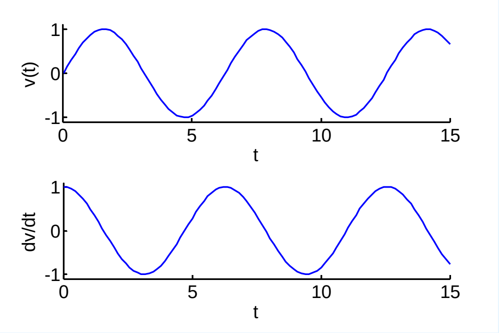

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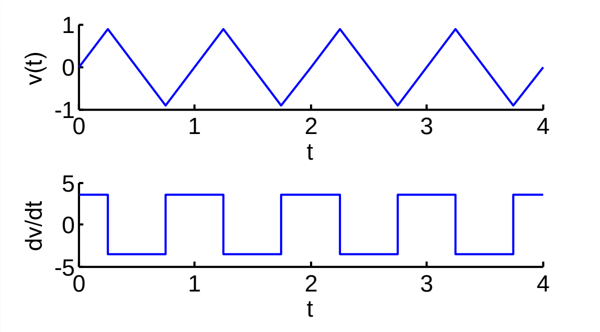

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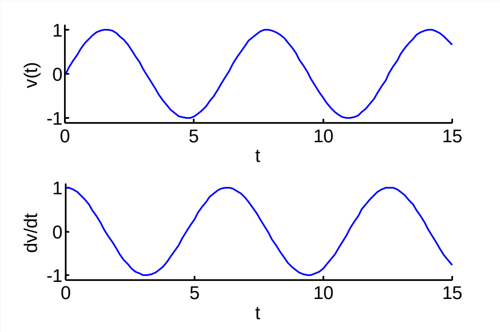
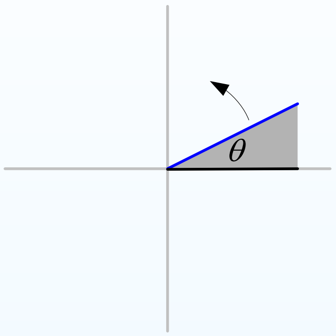


$\sin 2\pi f t$ makes f complete repetitions every time t increases by 1; this gives a *frequency* of f cycles per second, or $f\,\mathrm{Hz}$.
We often use the *angular frequency*, $\omega = 2\pi f$ instead.
ω is measured in radians per second. E.g. $50\,\mathrm{Hz} \simeq 314\,\mathrm{rad.s}^{-1}$.

Rotating Rod

A useful way to think of a cosine wave is as the projection of a rotating rod onto the horizontal axis.

For a unit-length rod, the projection has length $\cos\theta$.



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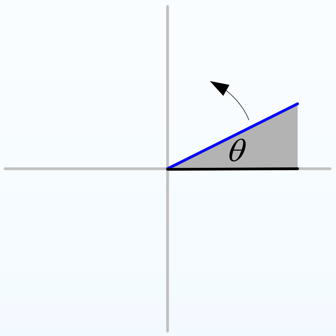

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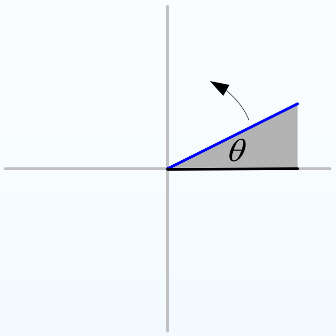

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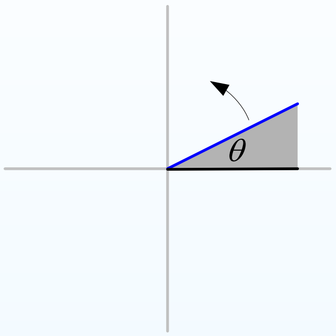

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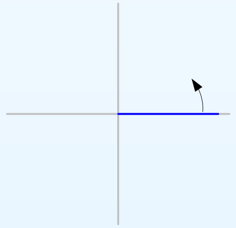

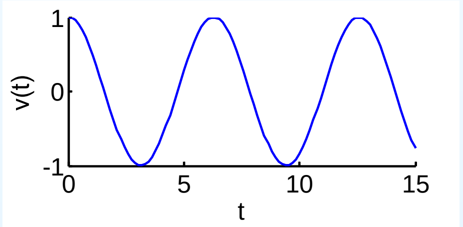

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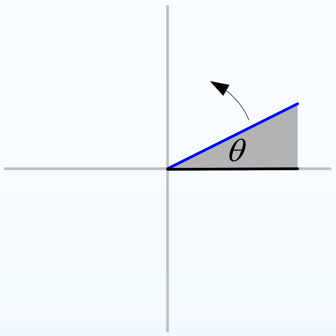

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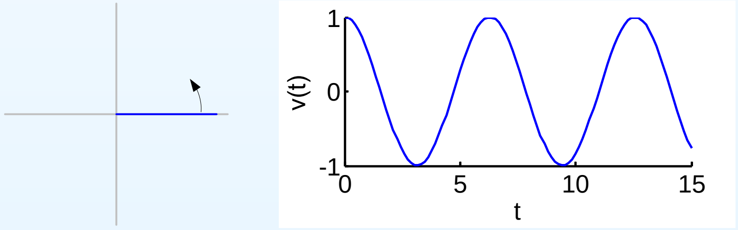

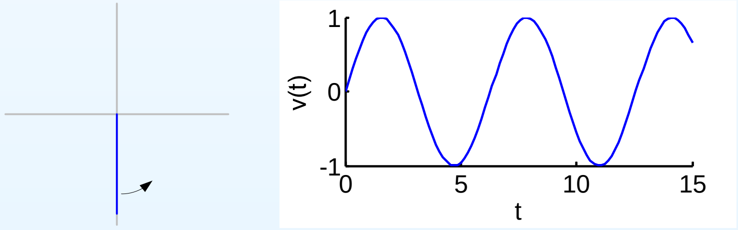

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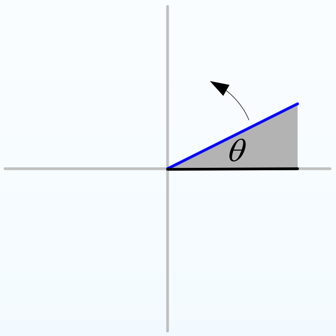

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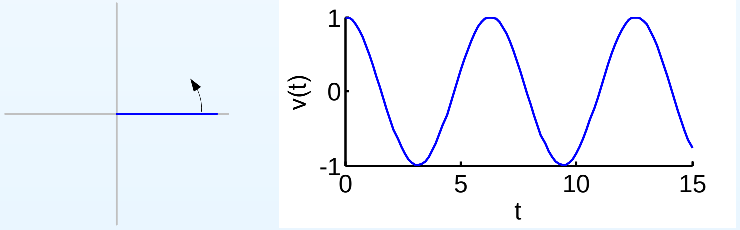

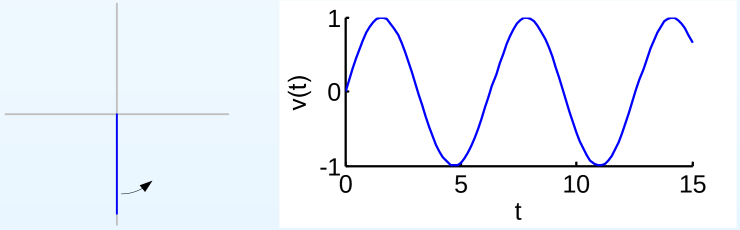

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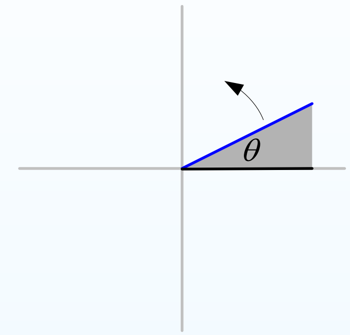

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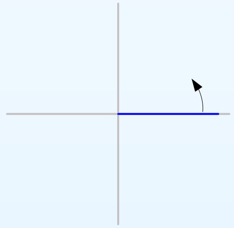

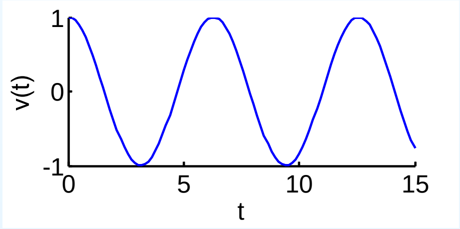

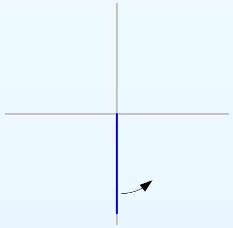

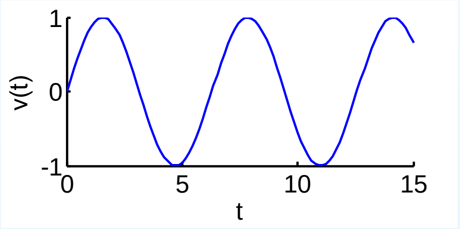

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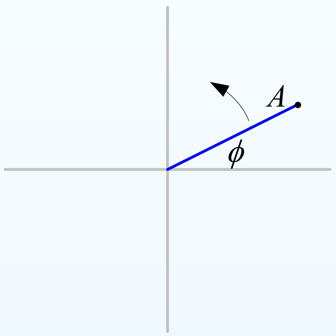
$\sin 2\pi f t$ *lags* $\cos 2\pi f t$ by 90° (or $\frac{\pi}{2}$ radians) because its peaks occurs $\frac{1}{4}$ of a cycle later (equivalently $\cos$ *leads* $\sin$) .

Phasors

If the rod has length A and starts at an angle ϕ then the projection onto the horizontal axis is

$$A\cos(2\pi ft + \phi)$$



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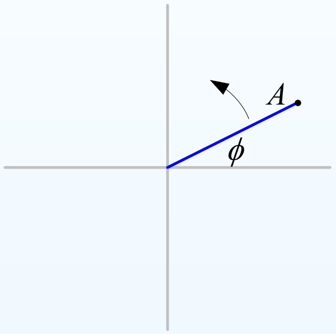

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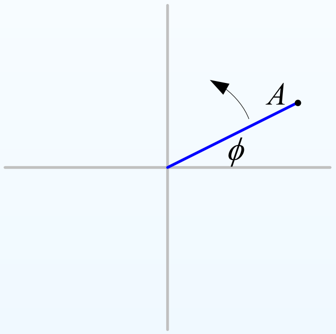

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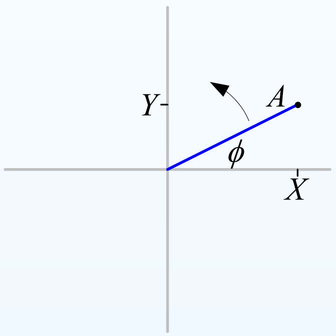

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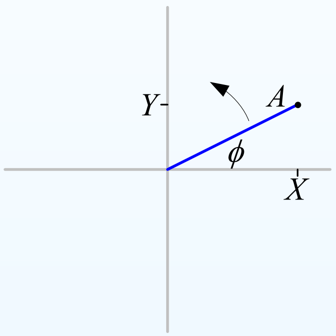

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If we think of the plane as an Argand Diagram (or complex plane), then the complex number $X + jY$ corresponding to the tip of the rod at $t = 0$ is called a *phasor*.

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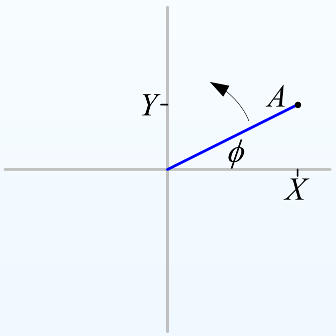

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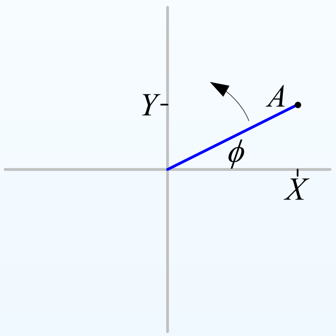

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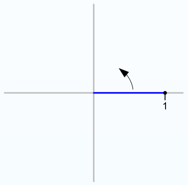
The *argument* of the phasor, $\phi = \arctan\frac{Y}{X}$, gives the phase shift relative to $\cos 2\pi ft$.
If $\phi > 0$, it is *leading* and if $\phi < 0$, it is *lagging* relative to $\cos 2\pi ft$.

Phasor Examples

$$V = 1,\ f = 50\,\mathrm{Hz}$$



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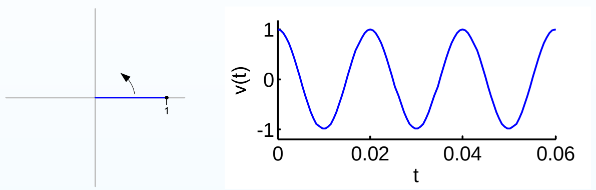

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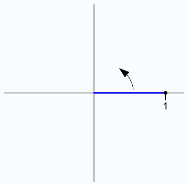
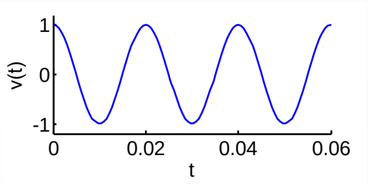
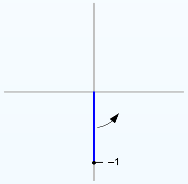




$$V = -j$$



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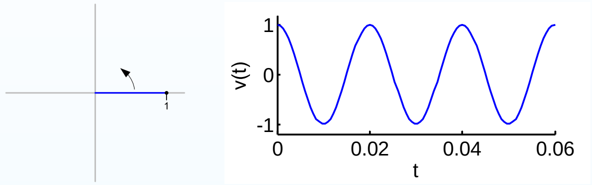

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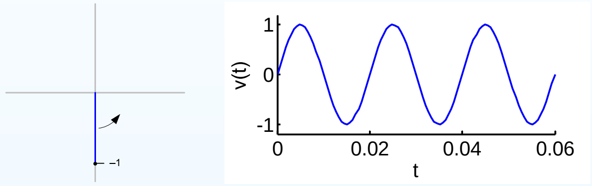

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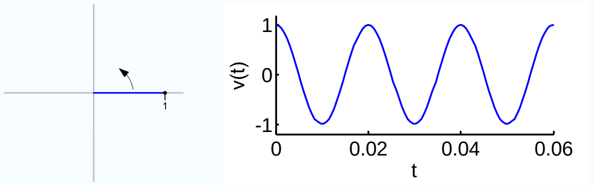

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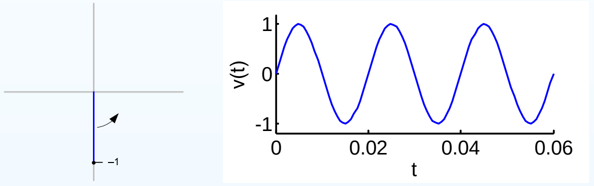
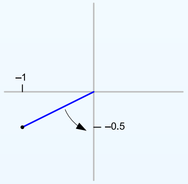


$$V = -1 - 0.5j$$



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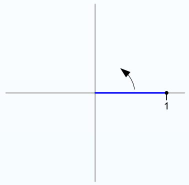

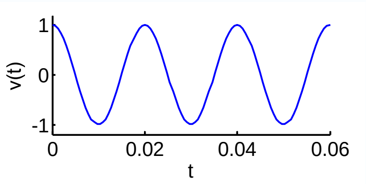

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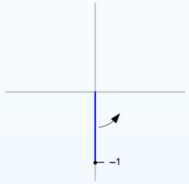

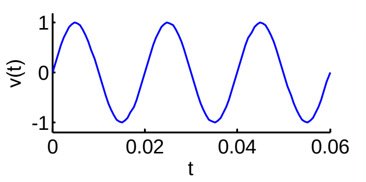

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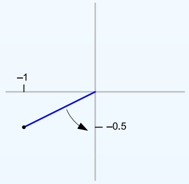

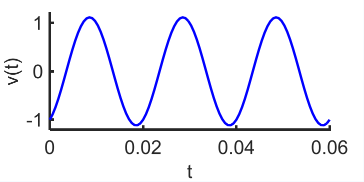

Phasor Examples

+

$$V = 1,\ f = 50\,\mathrm{Hz}$$
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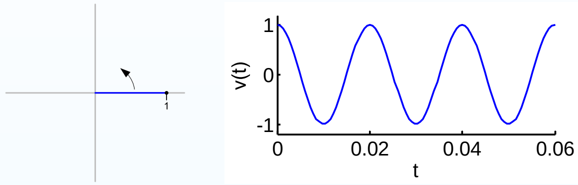

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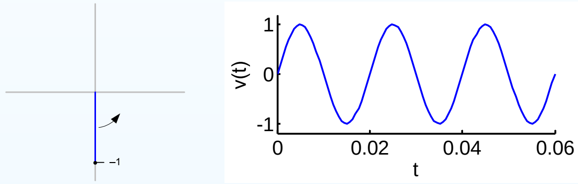

$$V = -1 - 0.5j = 1.12\angle -153^\circ$$
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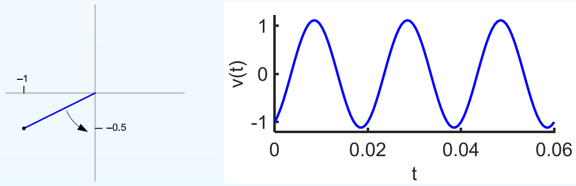

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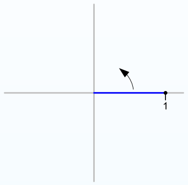

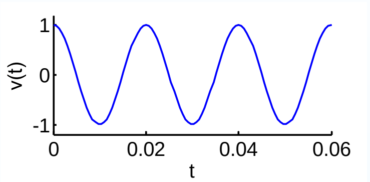

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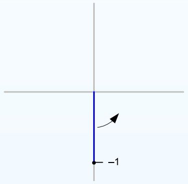

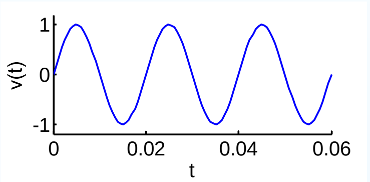

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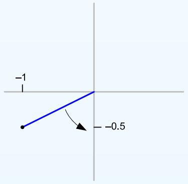

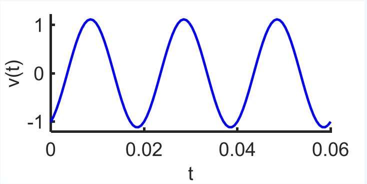

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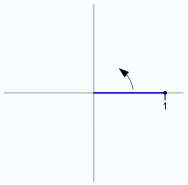

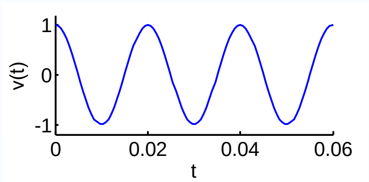

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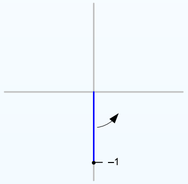

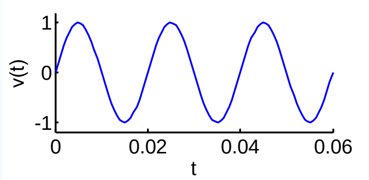

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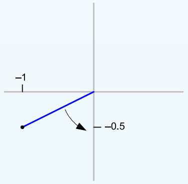

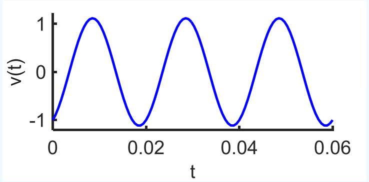

$$V = X + jY$$

Phasor Examples

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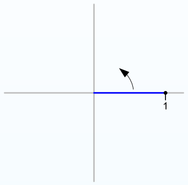
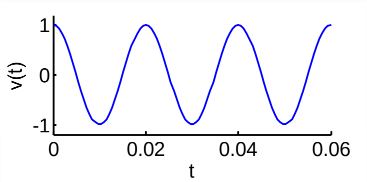

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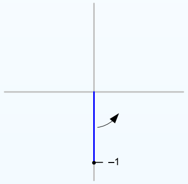
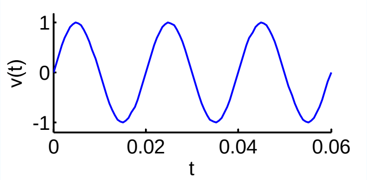

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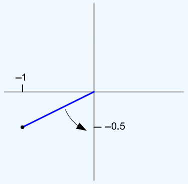
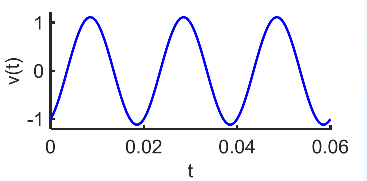

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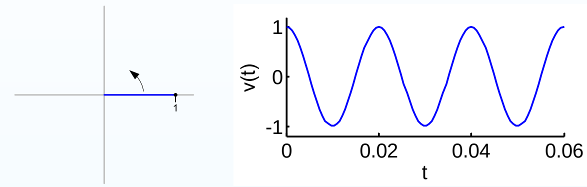

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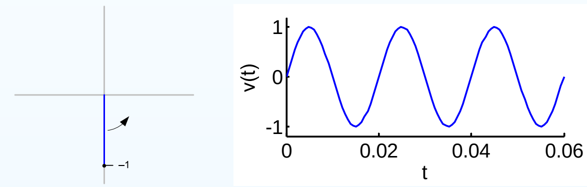

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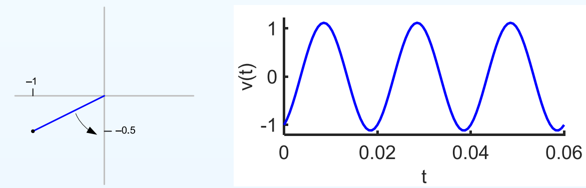

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Beware minus sign.

Phasor Examples

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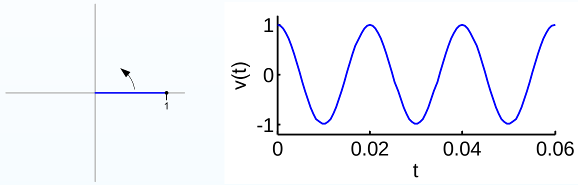

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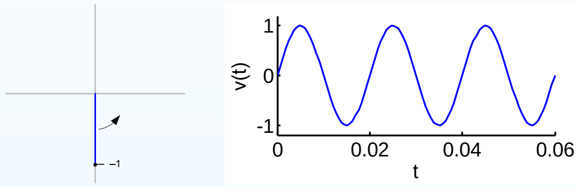

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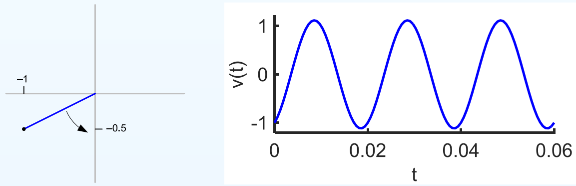

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Phasor Examples

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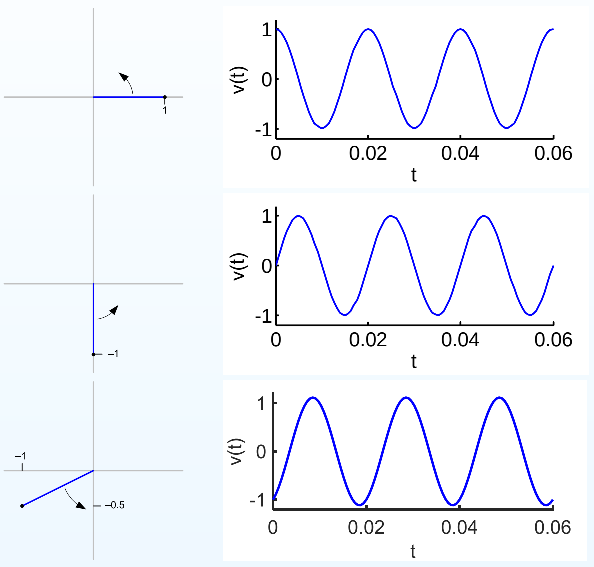

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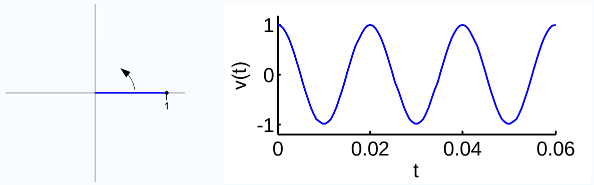

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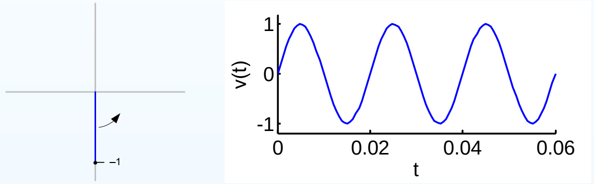

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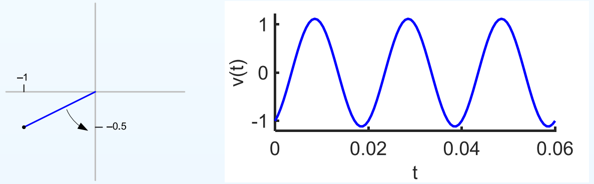

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Phasor Examples

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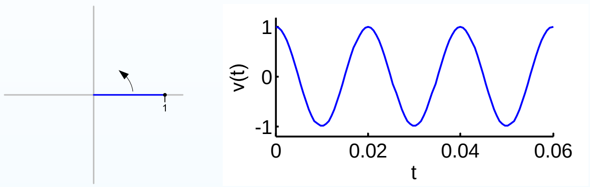

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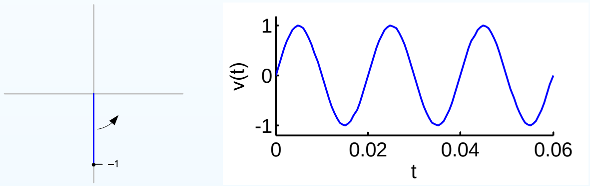

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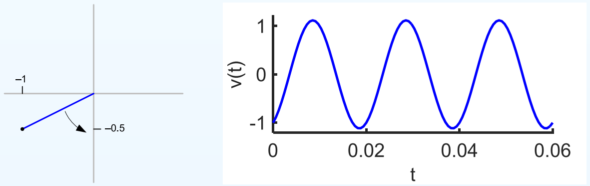

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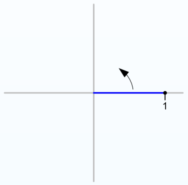

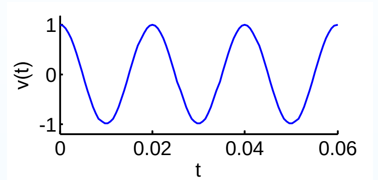

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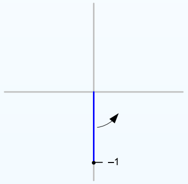

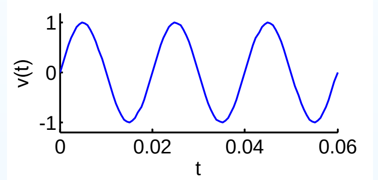

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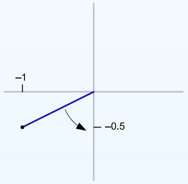

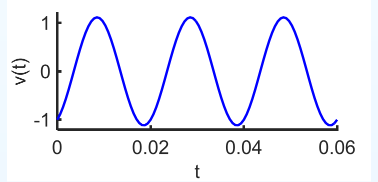

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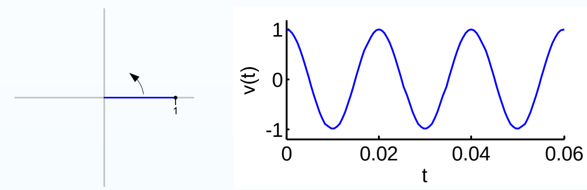

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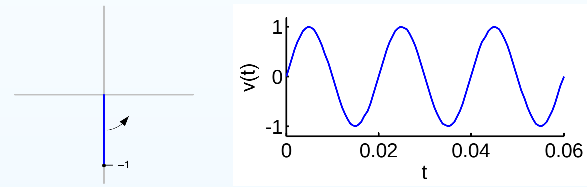

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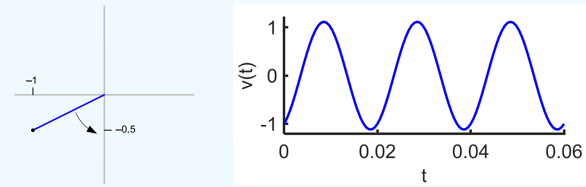

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Casio: $\mathrm{Pol}(X, Y) \to A, \phi$, $\mathrm{Rec}(A, \phi) \to X, Y$. Saved $\to$ X & Y mems.

Phasor arithmetic

Phasors

$V = P + jQ$

Waveforms

$v(t) = P\cos\omega t - Q\sin\omega t$
where $\omega = 2\pi f$.

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$v_1(t) + v_2(t)$

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Adding or scaling is the same for waveforms and phasors.

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$$= (-\omega Q)\cos\omega t - (\omega P)\sin\omega t$$

Phasor arithmetic

Phasors	Waveforms
$V = P + jQ$	$v(t) = P\cos\omega t - Q\sin\omega t$ where $\omega = 2\pi f$.
aV	$a \times v(t) \; = aP\cos\omega t - aQ\sin\omega t$
$V_1 + V_2$	$v_1(t) + v_2(t)$

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Differentiating waveforms corresponds to multiplying phasors by $j\omega$.

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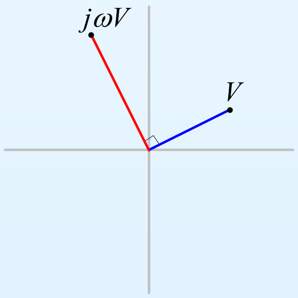

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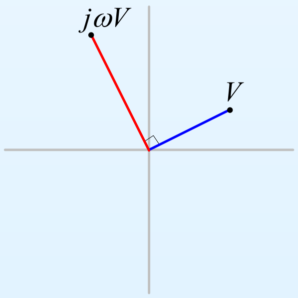
Differentiating waveforms corresponds to multiplying phasors by $j\omega$.

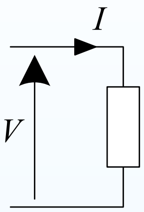
Rotate anti-clockwise 90° and scale by $\omega = 2\pi f$.



Complex Impedances

Resistor:

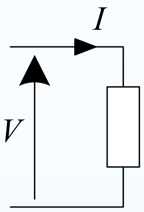
$$v(t) = Ri(t)$$



Complex Impedances

Resistor:

$$v(t) = Ri(t) \Rightarrow V = RI$$



Complex Impedances

Resistor:

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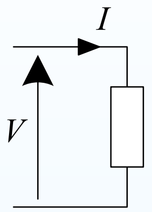

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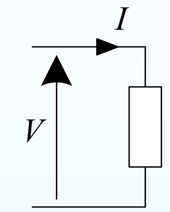
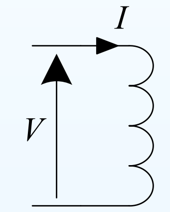


Inductor:

$$v(t) = L\frac{di}{dt}$$



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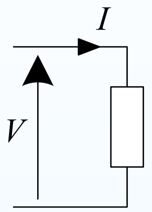
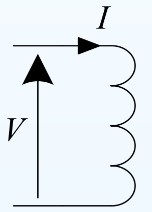


Inductor:

$$v(t) = L\frac{di}{dt} \Rightarrow V = j\omega LI$$



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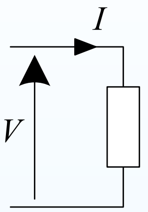

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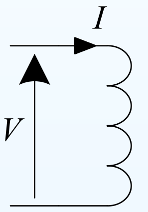

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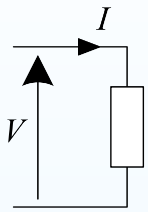

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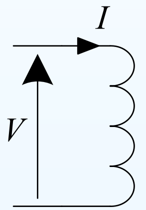
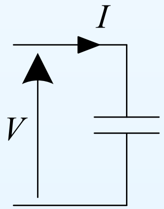


Capacitor:

$$i(t) = C\frac{dv}{dt}$$



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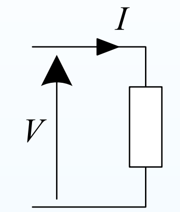

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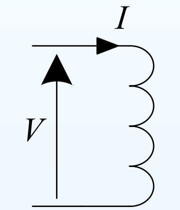

Capacitor:

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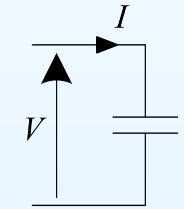

Complex Impedances

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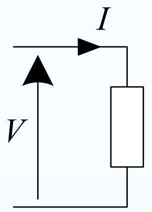

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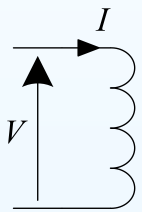

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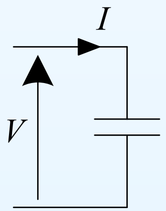

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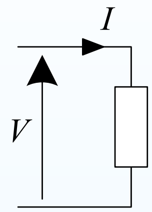

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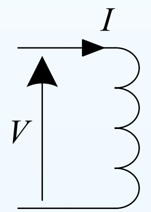

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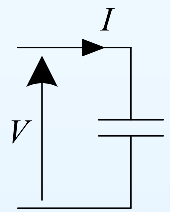

For all three components, phasors obey Ohm's law if we use the *complex impedances* $j\omega L$ and $\frac{1}{j\omega C}$ as the "resistance" of an inductor or capacitor.

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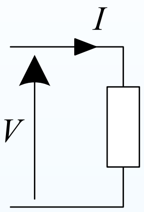

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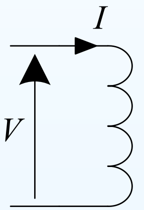

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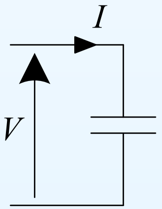
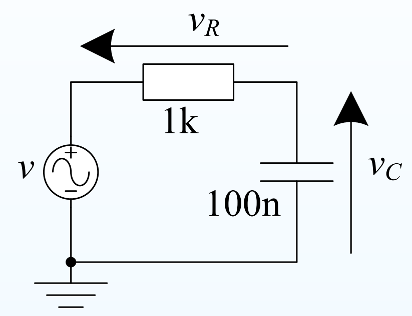


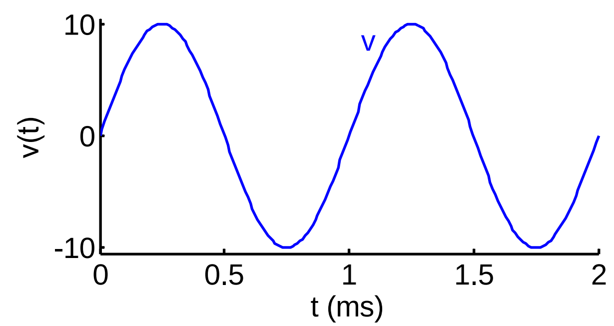
For all three components, phasors obey Ohm's law if we use the *complex impedances* $j\omega L$ and $\frac{1}{j\omega C}$ as the "resistance" of an inductor or capacitor.

If all sources in a circuit are sine waves having the same frequency, we can do circuit analysis exactly as before by using complex impedances.

Phasor Analysis

Given $v = 10\sin\omega t$ where $\omega = 2\pi \times 1000$, find $v_C(t)$.





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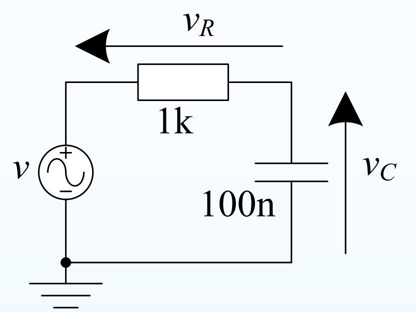

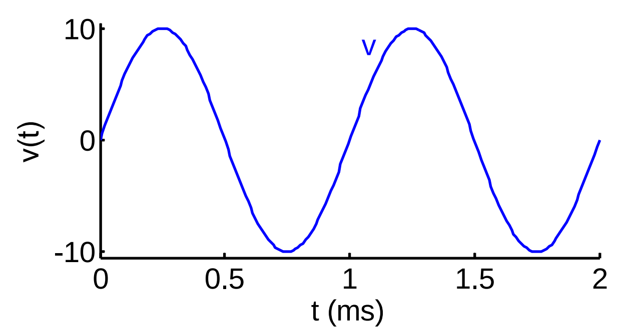

(1) Find capacitor complex impedance

$$Z = \frac{1}{j\omega C} = \frac{1}{6.28j \times 10^{-4}} = -1592j$$

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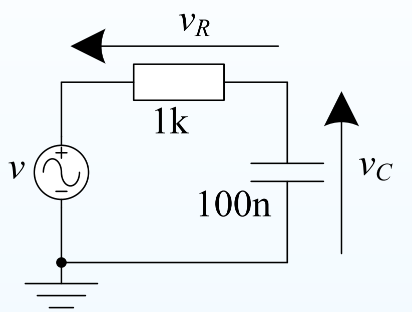

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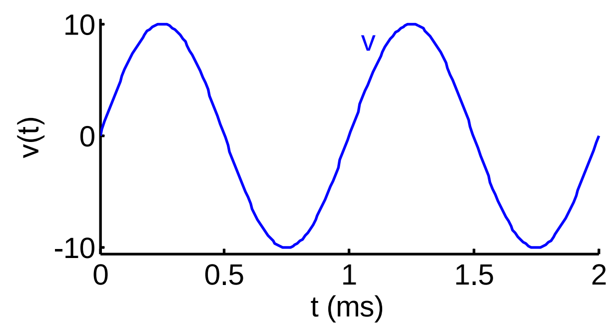
(2) Solve circuit with phasors

$$V_C = V \times \frac{Z}{R+Z}$$



Phasor Analysis

+

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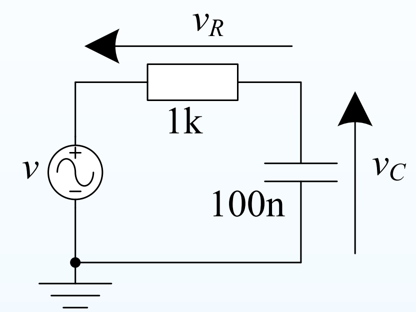

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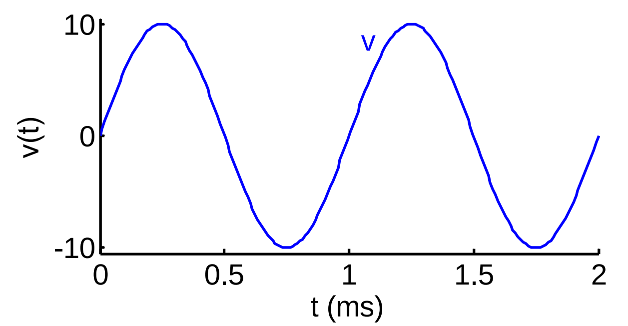

(2) Solve circuit with phasors

$$V_C = V \times \frac{Z}{R+Z}$$

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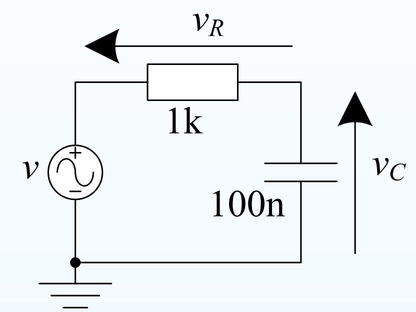

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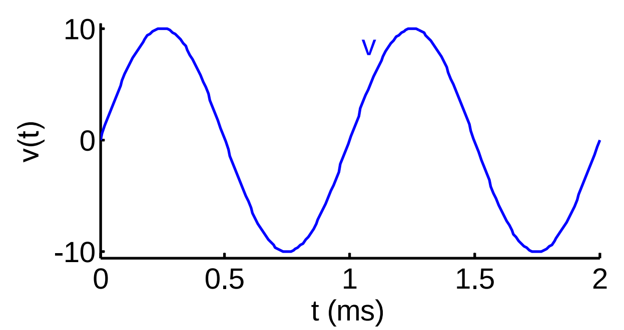
$$= -4.5 - 7.2j = 8.47\angle - 122°$$



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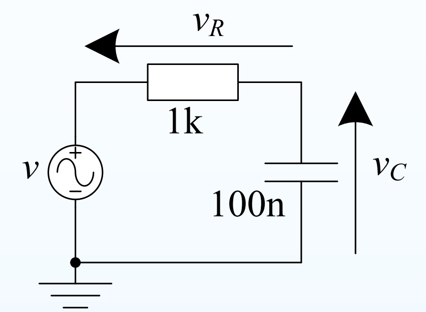

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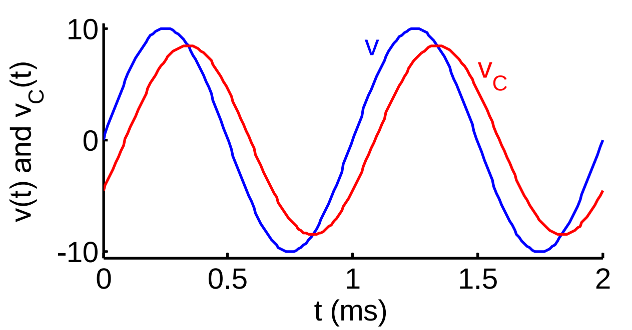
$$v_C = 8.47 \cos\left(\omega t - 122°\right)$$



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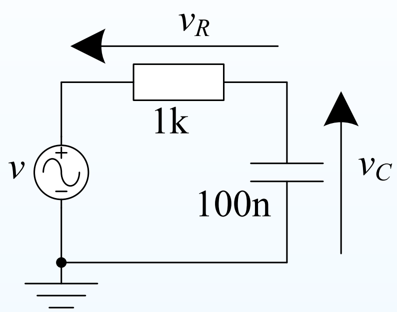

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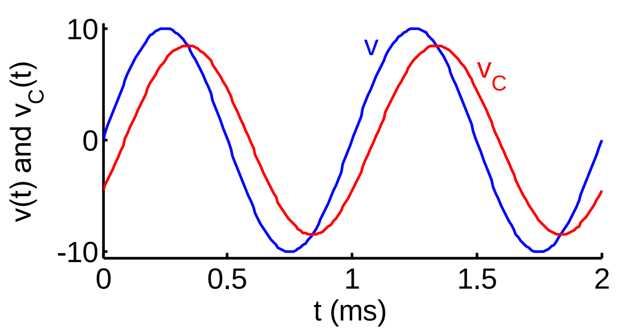

(3) Draw a *phasor diagram* showing KVL:

$$V = -10j$$
$$V_C = -4.5 - 7.2j$$
$$V_R = V - V_C = 4.5 - 2.8j = 5.3\angle -32°$$

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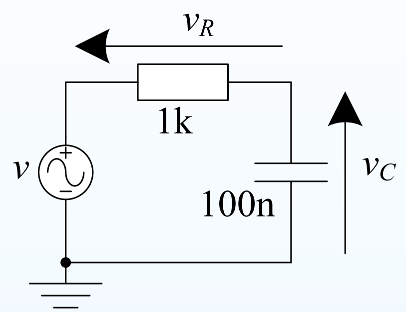

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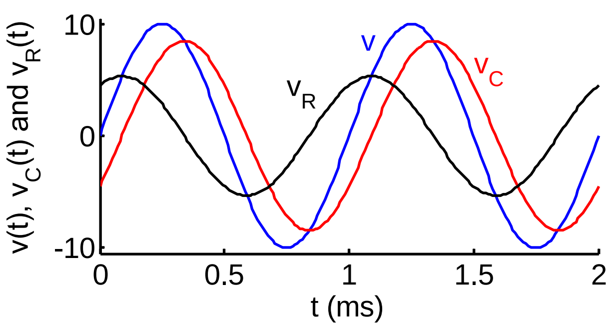

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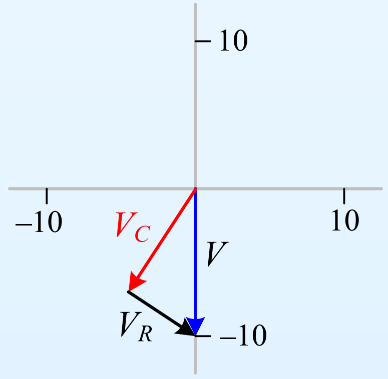
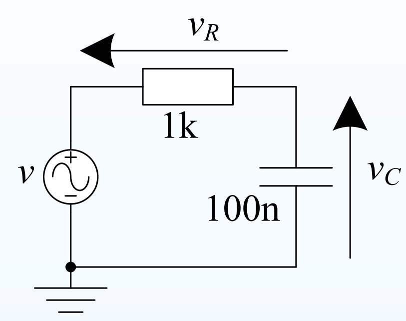


Phasor Analysis

+

Given $v = 10 \sin \omega t$ where $\omega = 2\pi \times 1000$, find $v_C(t)$.

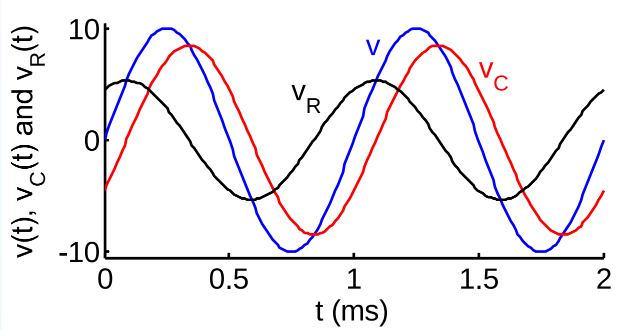


(1) Find capacitor complex impedance

$$Z = \frac{1}{j\omega C} = \frac{1}{6.28j \times 10^{-4}} = -1592j$$

(2) Solve circuit with phasors

$$V_C = V \times \frac{Z}{R+Z}$$
$$= -10j \times \frac{-1592j}{1000-1592j}$$
$$= -4.5 - 7.2j = 8.47\angle -122°$$
$$v_C = 8.47 \cos\left(\omega t - 122°\right)$$



(3) Draw a *phasor diagram* showing KVL:

$$V = -10j$$
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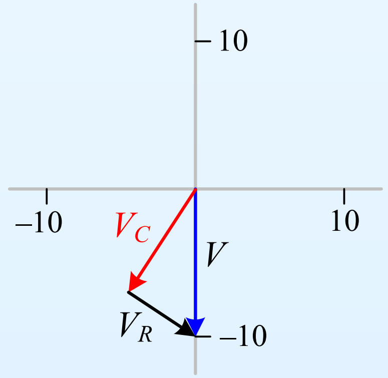

Phasors add like vectors

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Casio fx-991 (available in all exams except Maths) will do complex arithmetic $(+, -, \times, \div, x^2, \frac{1}{x}, |x|, x^*)$ in CMPLX mode.

Learn how to use this: it will save lots of time and errors.

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Beware: $G \neq \frac{1}{R}$ unless $X = 0$.

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