11: Frequency Responses

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary
If \( x(t) \) is a sine wave, then \( y(t) \) will also be a sine wave but with a different amplitude and phase shift. \( X \) is an input phasor and \( Y \) is the output phasor.
If \( x(t) \) is a sine wave, then \( y(t) \) will also be a sine wave but with a different amplitude and phase shift. \( X \) is an input phasor and \( Y \) is the output phasor.

The gain of the circuit is

\[
\frac{Y}{X} = \frac{1/j\omega C}{R+1/j\omega C} = \frac{1}{j\omega RC+1}
\]
If $x(t)$ is a sine wave, then $y(t)$ will also be a sine wave but with a different amplitude and phase shift. $X$ is an input phasor and $Y$ is the output phasor.

The *gain* of the circuit is

$$\frac{Y}{X} = \frac{1/j\omega C}{R+1/j\omega C} = \frac{1}{j\omega RC+1}$$

This is a complex function of $\omega$ so we plot separate graphs for:
If $x(t)$ is a sine wave, then $y(t)$ will also be a sine wave but with a different amplitude and phase shift. $X$ is an input phasor and $Y$ is the output phasor.

The *gain* of the circuit is

$$\frac{Y}{X} = \frac{1/jωC}{R+1/jωC} = \frac{1}{jωRC+1}$$

This is a complex function of $ω$ so we plot separate graphs for:

**Magnitude:**  
$$\left|\frac{Y}{X}\right| = \frac{1}{|jωRC+1|} = \frac{1}{\sqrt{1+(ωRC)^2}}$$
If $x(t)$ is a sine wave, then $y(t)$ will also be a sine wave but with a different amplitude and phase shift. $X$ is an input phasor and $Y$ is the output phasor.

The gain of the circuit is

$$\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{\frac{R}{1} + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1}$$

This is a complex function of $\omega$ so we plot separate graphs for:

**Magnitude:**

$$|\frac{Y}{X}| = \frac{1}{|j\omega RC + 1|} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$
If \( x(t) \) is a sine wave, then \( y(t) \) will also be a sine wave but with a different amplitude and phase shift. \( X \) is an input phasor and \( Y \) is the output phasor.

The **gain** of the circuit is

\[
\frac{Y}{X} = \frac{1/j \omega C}{R+1/j \omega C} = \frac{1}{j \omega RC + 1}
\]

This is a complex function of \( \omega \) so we plot separate graphs for:

**Magnitude:**

\[
|\frac{Y}{X}| = \frac{1}{|j \omega RC + 1|} = \frac{1}{\sqrt{1+(\omega RC)^2}}
\]

**Phase Shift:**

\[
\angle \left( \frac{Y}{X} \right) = -\angle (j \omega RC + 1) = -\arctan \left( \frac{\omega RC}{1} \right)
\]
If $x(t)$ is a sine wave, then $y(t)$ will also be a sine wave but with a different amplitude and phase shift. $X$ is an input phasor and $Y$ is the output phasor.

The gain of the circuit is

$$\frac{Y}{X} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1}$$

This is a complex function of $\omega$ so we plot separate graphs for:

**Magnitude:**

$$|\frac{Y}{X}| = \frac{1}{|j\omega RC + 1|} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

**Phase Shift:**

$$\angle (\frac{Y}{X}) = -\angle (j\omega RC + 1) = -\arctan (\frac{\omega RC}{1})$$

**Magnitude Response**

**Phase Response**
**Sine Wave Response**

Given \( RC = 10 \text{ ms} \),

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

For \( \omega = 50 \):

\[\frac{Y}{X} = 0.89 \angle -27^\circ\]

For \( \omega = 100 \):

\[\frac{Y}{X} = 0.71 \angle -45^\circ\]

For \( \omega = 300 \):

\[\frac{Y}{X} = 0.32 \angle -72^\circ\]
Sine Wave Response

\[ RC = 10 \text{ ms} \]

\[ \frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1} \]

\[ X \quad R = 10k \quad Y \]

\[ C = \frac{1}{\mu} \]

\[ \omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ \]

\[ \omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ \]

\[ \omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ \]
\[ RC = 10 \text{ ms} \]

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

\[ \omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ \]

\[ \omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ \]

\[ \omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ \]
\[ RC = 10 \text{ ms} \]

\[ \frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1} \]

\[ X \quad R = 10k \]

\[ C = 1\mu \]

\[ Y \]

\[ \omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ \]

\[ \omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ \]

\[ \omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ \]
\[ RC = 10 \text{ ms} \]

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

\[
\omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ
\]

\[
\omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ
\]

\[
\omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ
\]
**Sine Wave Response**

\[ RC = 10 \text{ ms} \]

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

\[
\begin{align*}
\omega &= 50 \Rightarrow \frac{Y}{X} &= 0.89 \angle -27^\circ \\
\omega &= 100 \Rightarrow \frac{Y}{X} &= 0.71 \angle -45^\circ \\
\omega &= 300 \Rightarrow \frac{Y}{X} &= 0.32 \angle -72^\circ
\end{align*}
\]
**Sine Wave Response**

\[ RC = 10 \text{ ms} \]

\[ \frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1} \]

- \( \omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ \)
- \( \omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ \)
- \( \omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ \)
Sine Wave Response

\[ RC = 10 \, \text{ms} \]

\[ \frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1} \]

\[ \omega = 50 \Rightarrow \frac{Y}{X} = 0.89\angle -27^\circ \]

\[ \omega = 100 \Rightarrow \frac{Y}{X} = 0.71\angle -45^\circ \]

\[ \omega = 300 \Rightarrow \frac{Y}{X} = 0.32\angle -72^\circ \]
Sine Wave Response

\[ RC = 10 \text{ ms} \]

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

\[
\omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ
\]

\[
\omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ
\]

\[
\omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ
\]
**Sine Wave Response**

\[ RC = 10 \text{ ms} \]

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

\[ R = 10k \]

\[ C = 1 \mu \]

\[ \omega = 50 \Rightarrow \frac{Y}{X} = 0.89\angle -27^\circ \]

\[ \omega = 100 \Rightarrow \frac{Y}{X} = 0.71\angle -45^\circ \]

\[ \omega = 300 \Rightarrow \frac{Y}{X} = 0.32\angle -72^\circ \]
### Sine Wave Response

**RC** = 10 ms

\[
\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
\]

\[
\omega = 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ
\]

\[
\omega = 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ
\]

\[
\omega = 300 \Rightarrow \frac{Y}{X} = 0.32 \angle -72^\circ
\]

The output, \(y(t)\), *lags* the input, \(x(t)\), by up to \(90^\circ\).
We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.
We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.
We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Logarithmic voltage ratios are specified in \( \text{decibels (dB)} = 20 \log_{10} \frac{|V_2|}{|V_1|} \).

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Logarithmic voltage ratios are specified in \( \text{decibels (dB)} = 20 \log_{10} \frac{|V_2|}{|V_1|} \).

Common voltage ratios:

\[
\begin{array}{c|c|c|c|c|c}
\text{dB} & 0 & 1 & 2 & 3 & 4 \\
\hline
\frac{|V_2|}{|V_1|} & 0 & 1 & \ldots & \ldots & \ldots \\
\end{array}
\]

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Logarithmic voltage ratios are specified in *decibels (dB)* = \(20 \log_{10} \frac{|V_2|}{|V_1|}\).

| | | | | | |
|---|---|---|---|---|
| \(\frac{|V_2|}{|V_1|}\) | 0.1 | 1 | 10 | 100 |
| dB | -20 | 0 | 20 | 40 |

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Logarithmic voltage ratios are specified in *decibels* (dB) = \(20 \log_{10} \frac{|V_2|}{|V_1|}\).

**Common voltage ratios:**

| \(\frac{|V_2|}{|V_1|}\) | 0.1 | 0.5 | 1 | 2 | 10 | 100 |
|---|---|---|---|---|---|---|
| dB | -20 | -6 | 0 | 6 | 20 | 40 |

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.
We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Logarithmic voltage ratios are specified in \( \text{decibels (dB)} = 20 \log_{10} \frac{|V_2|}{|V_1|} \).

Common voltage ratios:

| \( \frac{|V_2|}{|V_1|} \) | 0.1 | 0.5 | \( \sqrt{0.5} \) | 1 | \( \sqrt{2} \) | 2 | 10 | 100 |
|-------------------|--|--|--|--|--|--|--|--|
| dB                | -20 | -6 | -3 | 0 | 3 | 6 | 20 | 40 |

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.

Logarithmic voltage ratios are specified in \( \text{decibels (dB)} = 20 \log_{10} \left| \frac{V_2}{V_1} \right| \).

Common voltage ratios:

\[
\begin{array}{cccccccc}
\frac{|V_2|}{|V_1|} & 0.1 & 0.5 & \sqrt{0.5} & 1 & \sqrt{2} & 2 & 10 & 100 \\
dB & -20 & -6 & -3 & 0 & 3 & 6 & 20 & 40 \\
\end{array}
\]

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.

Note: \( P \propto V^2 \Rightarrow \text{decibel power ratios are given by } 10 \log_{10} \frac{P_2}{P_1} \).
Suppose we plot the magnitude and phase of \( H = c (j\omega)^r \).
Suppose we plot the magnitude and phase of $H = c(j\omega)^r$

**Magnitude (log-log graph):**

$$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$$
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$

**Magnitude (log-log graph):**

$$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$$
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$

**Magnitude (log-log graph):**

$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$

This is a straight line with a slope of $r$. 
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$

**Magnitude (log-log graph):**

$$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$$

This is a straight line with a slope of $r$.

**Phase (log-lin graph):**

$$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} \ (\pm \pi \text{ if } c < 0)$$
Suppose we plot the magnitude and phase of $H = c(j\omega)^r$

**Magnitude (log-log graph):**

$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$

This is a straight line with a slope of $r$.

**Phase (log-lin graph):**

$\angle H = \angle j^r + \angle c = r\times\frac{\pi}{2} (+\pi \text{ if } c < 0)$
Suppose we plot the magnitude and phase of \( H = c (j\omega)^r \)

**Magnitude (log-log graph):**
\[
|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega
\]
This is a straight line with a slope of \( r \).

**Phase (log-lin graph):**
\[
\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} (+\pi \text{ if } c < 0)
\]
The phase is constant \( \forall \omega \).
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$

**Magnitude (log-log graph):**

$$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$$

This is a straight line with a slope of $r$.

**Phase (log-lin graph):**

$$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} (+\pi \text{ if } c < 0)$$

The phase is constant $\forall \omega$. 

---

**Logs of Powers**

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- **Logs of Powers**
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary
Suppose we plot the magnitude and phase of \( H = c(j\omega)^r \)

**Magnitude (log-log graph):**

\[ |H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega \]

This is a straight line with a slope of \( r \).

\( c \) only affects the line’s vertical position.

**Phase (log-lin graph):**

\[ \angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} \quad (+\pi \text{ if } c < 0) \]

The phase is constant \( \forall \omega \).
Suppose we plot the magnitude and phase of $H = c(j\omega)^r$

**Magnitude (log-log graph):**

$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$

This is a straight line with a slope of $r$. $c$ only affects the line’s vertical position.

**Phase (log-lin graph):**

$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} (+\pi$ if $c < 0)$

The phase is constant $\forall \omega$. 
Suppose we plot the magnitude and phase of \( H = c (j\omega)^r \)

**Magnitude (log-log graph):**
\[
|H| = c\omega^r 
\Rightarrow \log |H| = \log |c| + r \log \omega
\]
This is a straight line with a slope of \( r \).
\( c \) only affects the line’s vertical position.

**Phase (log-lin graph):**
\[
\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} (+\pi \text{ if } c < 0)
\]
The phase is constant \( \forall \omega \).
Suppose we plot the magnitude and phase of \( H = c (j\omega)^r \).

**Magnitude (log-log graph):**

\[
|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega
\]

This is a straight line with a slope of \( r \).

\( c \) only affects the line’s vertical position.

**Phase (log-lin graph):**

\[
\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} \quad (+\pi \text{ if } c < 0)
\]

The phase is constant \( \forall \omega \).

If \( c > 0 \), phase = \( 90^\circ \times \) magnitude slope.
Suppose we plot the magnitude and phase of \( H = c (j\omega)^r \)

**Magnitude (log-log graph):**
\[
|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega
\]
This is a straight line with a slope of \( r \). 
\( c \) only affects the line’s vertical position.

If \( |H| \) is measured in decibels, a slope of \( r \) is called \( 6r \) dB/octave or \( 20r \) dB/decade.

**Phase (log-lin graph):**
\[
\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} (+\pi \text{ if } c < 0)
\]
The phase is constant \( \forall \omega \).
If \( c > 0 \), phase = \( 90^\circ \times \) magnitude slope.
Suppose we plot the magnitude and phase of $H = c(j\omega)^r$

**Magnitude (log-log graph):**

$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$

This is a straight line with a slope of $r$. $c$ only affects the line’s vertical position.

If $|H|$ is measured in decibels, a slope of $r$ is called $6r$ dB/octave or $20r$ dB/decade.

**Phase (log-lin graph):**

$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2}$ ($+\pi$ if $c < 0$)

The phase is constant $\forall \omega$.

If $c > 0$, phase $= 90^\circ \times$ magnitude slope.
Negative $c$ adds $\pm 180^\circ$ to the phase.
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$

**Magnitude (log-log graph):**

$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$

This is a straight line with a slope of $r$. $c$ only affects the line’s vertical position.

If $|H|$ is measured in decibels, a slope of $r$ is called $6r$ dB/octave or $20r$ dB/decade.

**Phase (log-lin graph):**

$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2}$ ($+\pi$ if $c < 0$)

The phase is constant $\forall \omega$.

If $c > 0$, phase = $90^\circ \times$ magnitude slope.

Negative $c$ adds $\pm 180^\circ$ to the phase.

**Note:** Phase angles are modulo $360^\circ$, i.e. $+180^\circ \equiv -180^\circ$ and $450^\circ \equiv 90^\circ$. 
Logs of Powers

\[ H = c (j\omega)^r \] has a straight-line magnitude graph and a constant phase.

Magnitude (log-log graph):
\[
|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega
\]
This is a straight line with a slope of \( r \).
\( c \) only affects the line’s vertical position.

If \(|H|\) is measured in decibels, a slope of \( r \) is called \( 6r \text{ dB/octave} \) or \( 20r \text{ dB/decade} \).

Phase (log-lin graph):
\[
\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} \ (\pm \pi \text{ if } c < 0)
\]
The phase is constant \( \forall \omega \).
If \( c > 0 \), phase = \( 90^\circ \times \) magnitude slope.
Negative \( c \) adds \( \pm 180^\circ \) to the phase.

Note: Phase angles are modulo \( 360^\circ \), i.e. \( +180^\circ \equiv -180^\circ \) and \( 450^\circ \equiv 90^\circ \).
Key idea: \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)
**Straight Line Approximations**

Key idea: \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)
Key idea: \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\)

Gain: \(H(j \omega) = \frac{1}{j \omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j \omega) \approx 1\)
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC}\)
Straight Line Approximations

Key idea: \((a j\omega + b) \approx \begin{cases} \frac{a j\omega}{b} & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC+1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC}\)

Approximate the magnitude response as two straight lines
Key idea: \((aj\omega + b) \approx \begin{cases} aj\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC}\)

Approximate the magnitude response as two straight lines
Key idea: \( (a j\omega + b) \approx \begin{cases} a \cdot j\omega & \text{for } |\omega| \gg |b| \\ b & \text{for } |\omega| \ll |b| \end{cases} \)

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \( H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1 \)

High frequencies (\( \omega \gg \frac{1}{RC} \)): \( H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC} \omega^{-1} \)

Approximate the magnitude response as two straight lines
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \(\omega \ll \frac{1}{RC}\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \(\omega \gg \frac{1}{RC}\): \(H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC\omega^{-1}}\)

Approximate the magnitude response as two straight lines
Key idea: \((a j \omega + b) \approx \begin{cases} a \omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC \omega^{-1}}\)

Approximate the magnitude response as two straight lines intersecting at the corner frequency, \(\omega_c = \frac{1}{RC}\).
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC} \omega^{-1}\)

Approximate the magnitude response as two straight lines intersecting at the corner frequency, \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) \((= -6 \text{ dB/octave} = -20 \text{ dB/decade})\).
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC} \omega^{-1}\)

Approximate the magnitude response as two straight lines intersecting at the corner frequency, \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) (= \(-6\) dB/octave = \(-20\) dB/decade).

(b) \(|H(j\omega_c)| = \left| \frac{1}{1+j} \right| = \frac{1}{\sqrt{2}} = -3\) dB (worst-case error).
Key idea: 

\[(a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\]

Gain: 

\[H(j \omega) = \frac{\frac{1}{j \omega RC} + 1}{j \omega RC + 1}\]

Low frequencies \((\omega \ll \frac{1}{RC})\): \[H(j \omega) \approx 1 \Rightarrow |H(j \omega)| \approx 1\]

High frequencies \((\omega \gg \frac{1}{RC})\): \[H(j \omega) \approx \frac{1}{j \omega RC} \Rightarrow |H(j \omega)| \approx \frac{1}{RC} \omega^{-1}\]

Approximate the magnitude response as two straight lines intersecting at the corner frequency, \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) \((-6 \text{ dB/octave} = -20 \text{ dB/decade})\).

(b) \[|H(j \omega_c)| = \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}} = -3 \text{ dB} \text{ (worst-case error)}\].

A linear factor \((a j \omega + b)\) has a corner frequency of \(\omega_c = \left|\frac{b}{a}\right|\).
The gain of a linear circuit is always a *rational polynomial* in \( j\omega \) and is called the *transfer function* of the circuit. For example:

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}
\]
The gain of a linear circuit is always a **rational polynomial** in $j\omega$ and is called the **transfer function** of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the **transfer function** of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials  
**Step 2:** Sort corner freqs: 1, 4, 12, 50
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$
The gain of a linear circuit is always a **rational polynomial** in $j\omega$ and is called the **transfer function** of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials  
**Step 2:** Sort corner freqs: 1, 4, 12, 50  
**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:  
$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$  
**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so  
$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0.$$
The gain of a linear circuit is always a rational polynomial in $j\omega$ and is called the transfer function of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20 \omega \times 12}{1 \times 4 \times 50} = 1.2 \omega^1.$$

**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so

$$|H| \approx \frac{20 \omega \times 12}{\omega \times 4 \times 50} = 1.2 \omega^0 = +1.58 \text{ dB}.$$
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$

**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so

$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$

**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$. 
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$

**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so

$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$

**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.

**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}.$
The gain of a linear circuit is always a **rational polynomial** in $j\omega$ and is called the **transfer function** of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$  

**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so

$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$  

**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.  

**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}$.  

**Step 7:** For $\omega > 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times \omega} = 20\omega^{-1}$.  

![Magnitude Response Plot](image-url)
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials  
**Step 2:** Sort corner freqs: 1, 4, 12, 50  
**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:  
$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$  
**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so  
$$|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$  
**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}.$  
**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}.$  
**Step 7:** For $\omega > 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times \omega} = 20\omega^{-1}.$

At each corner frequency, the graph is continuous but its gradient changes abruptly by $+1$ (numerator factor) or $-1$ (denominator factor).
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Low Frequency Asymptote:**
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

Low Frequency Asymptote:

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Low Frequency Asymptote:**

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

Lowest power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

Low Frequency Asymptote:

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

Lowest power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)

High Frequency Asymptote:
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Low Frequency Asymptote:**
From factors: \[ H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \]

**Lowest** power of \( j\omega \) on top and bottom: \[ H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \]

**High Frequency Asymptote:**
From factors: \[ H_{HF}(j\omega) = \frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)} = 20(j\omega)^{-1} \]
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}
\]

**Low Frequency Asymptote:**
From factors: \(H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega\)

Lowest power of \(j\omega\) on top and bottom: \(H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega\)

**High Frequency Asymptote:**
From factors: \(H_{HF}(j\omega) = \frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)} = 20(j\omega)^{-1}\)

Highest power of \(j\omega\) on top and bottom: \(H(j\omega) \approx \frac{60(j\omega)^2}{3(j\omega)^3} = 20(j\omega)^{-1}\)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies \( \omega \ll \frac{1}{RC} \):
\[ H(j\omega) \approx 1 \]
Gain: $H(j\omega) = \frac{1}{j\omega RC + 1}$

Low frequencies ($\omega \ll \frac{1}{RC}$):

$H(j\omega) \approx 1 \Rightarrow \angle 1 = 0$
Phase Approximation

Gain: \[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

High frequencies (\( \omega \gg \frac{1}{RC} \)): \[ H(j\omega) \approx \frac{1}{j\omega RC} \]
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):

\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

High frequencies (\( \omega \gg \frac{1}{RC} \)):

\[ H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \]
Phase Approximation

Gain: \[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

Low frequencies (\(\omega \ll \frac{1}{RC}\)): \[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

High frequencies (\(\omega \gg \frac{1}{RC}\)): \[ H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \]

Approximate the phase response as three straight lines.

\[ -0.5\pi \quad 0 \quad -0.25\pi \]

\[ 0.1/RC \quad 1/RC \quad 10/RC \quad \omega \text{ (rad/s)} \]
Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

High frequencies (\( \omega \gg \frac{1}{RC} \)):
\[ H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \]

Approximate the phase response as three straight lines.

By chance, they intersect close to \( 0.1\omega_c \) and \( 10\omega_c \) where \( \omega_c = \frac{1}{RC} \).
Gain: \[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

**Low frequencies** \( (\omega \ll \frac{1}{RC}) \):

\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

**High frequencies** \( (\omega \gg \frac{1}{RC}) \):

\[ H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \]

Approximate the phase response as three straight lines.

By chance, they intersect close to \( 0.1\omega_c \) and \( 10\omega_c \) where \( \omega_c = \frac{1}{RC} \).

Between \( 0.1\omega_c \) and \( 10\omega_c \) the phase changes by \(-\frac{\pi}{2}\) over two decades. This gives a gradient = \(-\frac{\pi}{4}\) radians/decade.
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
\[
H(j\omega) \approx 1 \Rightarrow \angle 1 = 0
\]

High frequencies (\( \omega \gg \frac{1}{RC} \)):
\[
H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2}
\]

Approximate the phase response as three straight lines.

By chance, they intersect close to 0.1\( \omega_c \) and 10\( \omega_c \) where \( \omega_c = \frac{1}{RC} \).

Between 0.1\( \omega_c \) and 10\( \omega_c \) the phase changes by \( -\frac{\pi}{2} \) over two decades. This gives a gradient = \( -\frac{\pi}{4} \) radians/decade.

\((aj\omega + b)\) in denominator
\[
\Rightarrow \Delta\text{gradient} = \mp\frac{\pi}{4}/\text{decade at } \omega = 10^{\mp1} \left| \frac{b}{a} \right|.
\]
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
\[
H(j\omega) \approx 1 \Rightarrow \angle 1 = 0
\]

High frequencies (\( \omega \gg \frac{1}{RC} \)):
\[
H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2}
\]

Approximate the phase response as three straight lines.

By chance, they intersect close to 0.1\( \omega_c \) and 10\( \omega_c \) where \( \omega_c = \frac{1}{RC} \).

Between 0.1\( \omega_c \) and 10\( \omega_c \) the phase changes by \( -\frac{\pi}{2} \) over two decades. This gives a gradient = \( -\frac{\pi}{4} \) radians/decade.

\((aj\omega + b)\) in denominator
\[
\Rightarrow \Delta \text{gradient} = \mp \frac{\pi}{4} \text{/decade at } \omega = 10^\mp1 \left| \frac{b}{a} \right|.
\]

The sign of \( \Delta \text{gradient} \) is reversed for (a) numerator factors and (b) \( \frac{b}{a} < 0 \).
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} \]
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm \frac{\text{num}}{\text{den}} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \).

Sign depends on num/den and \( \text{sgn}\left(\frac{b}{a}\right): 1^-; 10^+; 4^-; 40^+; 1.2^+; 120^-; 5^-; 500^+ \)
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

Step 1: Factorize the polynomials
Step 2: List corner freqs: \( \pm = \text{num/den} \)
\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]
Step 3: Gradient changes at \( 10^\pm 1 \omega_c \).
Sign depends on num/den and \( \text{sgn} \left( \frac{b}{a} \right) \):
\[ .1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+ \]
Step 4: Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:
\[ .1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+. \]
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \).

Sign depends on \( \text{num/den} \) and \( \text{sgn} \left( \frac{b}{a} \right) \):

- \( 1^- \), \( 10^+ \); \( 4^- \), \( 40^+ \);
- \( 1.2^+ \), \( 120^- \); \( 5^- \), \( 500^+ \)

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

- \( 1^- (0.6) \), \( 4^- (0.48) \), \( 1.2^+ (0.62) \), \( 5^- (0.3) \), \( 10^+ (0.6) \), \( 40^+ (0.48) \), \( 120^- (0.62) \), \( 500^+ \).

**Step 5:** Find phase of LF asymptote: \( \angle 1.2 j\omega = \pm \frac{\pi}{2} \).
Body text in natural format...
**Plot Phase Response**

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)
\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{+1}\omega_c \).
Sign depends on num/den and \( \text{sgn}\left(\frac{b}{a}\right)\):
\( .1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+ \)

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10}\frac{\omega_2}{\omega_1} \) decades:
\( .1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+ \).

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

**Step 6:** At \( \omega = 0.1 \) gradient becomes \( -\frac{\pi}{4} \) rad/decade. \( \phi \) is still \( \frac{\pi}{2} \).
$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$

Step 1: Factorize the polynomials
Step 2: List corner freqs: $\pm = \frac{\text{num}}{\text{den}}$
\[\omega_c = \{1^-, 4^-, 12^+, 50^-\}\]
Step 3: Gradient changes at $10^{\pm1}\omega_c$.
Sign depends on $\text{num/\text{den}}$ and $\text{sgn} \left(\frac{b}{a}\right)$:
\[.1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+\]
Step 4: Put in ascending order and calculate gaps as $\log_{10} \frac{\omega_2}{\omega_1}$ decades:
\[.1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+.\]
Step 5: Find phase of LF asymptote: $\angle 1.2j\omega = +\frac{\pi}{2}$.
Step 6: At $\omega = 0.1$ gradient becomes $-\frac{\pi}{4}$ rad/decade. $\phi$ is still $\frac{\pi}{2}$.
Step 7: At $\omega = 0.4$, $\phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi$. New gradient is $-\frac{\pi}{2}$. 
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm \frac{\text{num}}{\text{den}} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \).

Sign depends on \( \text{num}/\text{den} \) and \( \text{sgn} \left( \frac{b}{a} \right) \):

\[ .1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+ \]

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\[ .1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+. \]

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

**Step 6:** At \( \omega = 0.1 \) gradient becomes \( -\frac{\pi}{4} \) rad/decade. \( \phi \) is still \( \frac{\pi}{2} \).

**Step 7:** At \( \omega = 0.4, \phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi \). New gradient is \( -\frac{\pi}{2} \).

**Step 8:** At \( \omega = 1.2, \phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi \). New gradient is \( -\frac{\pi}{4} \).
$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \(\pm = \text{num/den}\)
\[\omega_c = \{1^-, 4^-, 12^+, 50^-\}\]

**Step 3:** Gradient changes at \(10^\pm 1\omega_c\).
Sign depends on num/den and \(\text{sgn} \left(\frac{b}{a}\right)\):
\[1^-, 10^+; 4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+\]

**Step 4:** Put in ascending order and calculate gaps as \(\log_{10} \frac{\omega_2}{\omega_1}\) decades:
\[1^- (.6) 4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+\]

**Step 5:** Find phase of LF asymptote: \(\angle 1.2 j\omega = +\frac{\pi}{2}\).

**Step 6:** At \(\omega = 0.1\) gradient becomes \(-\frac{\pi}{4}\) rad/decade. \(\phi\) is still \(\frac{\pi}{2}\).

**Step 7:** At \(\omega = 0.4\), \(\phi = \frac{\pi}{2} - 0.6 \cdot \frac{\pi}{4} = 0.35\pi\). New gradient is \(-\frac{\pi}{2}\).

**Step 8:** At \(\omega = 1.2\), \(\phi = 0.35\pi - 0.48 \cdot \frac{\pi}{2} = 0.11\pi\). New gradient is \(-\frac{\pi}{4}\).

**Steps 9-13:** Repeat for each gradient change.
**Plot Phase Response**

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[
\omega_c = \{1^-, 4^-, 12^+, 50^-\}
\]

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \).

Sign depends on \( \text{num/den} \) and \( \text{sgn} \left( \frac{b}{a} \right) \):

\[.1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+\]

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\[.1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+.\]

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = + \frac{\pi}{2} \).

**Step 6:** At \( \omega = 0.1 \) gradient becomes \( -\frac{\pi}{4} \) rad/decade. \( \phi \) is still \( \frac{\pi}{2} \).

**Step 7:** At \( \omega = 0.4, \phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi \). New gradient is \( -\frac{\pi}{2} \).

**Step 8:** At \( \omega = 1.2, \phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi \). New gradient is \( -\frac{\pi}{4} \).

**Steps 9-13:** Repeat for each gradient change.
$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: $\pm = \text{num/den}$

$\omega_c = \{1^-, 4^-, 12^+, 50^-\}$

**Step 3:** Gradient changes at $10^{\pm 1} \omega_c$.

Sign depends on num/den and $\text{sgn} \left(\frac{b}{a}\right)$:

$1^- , 10^+ ; 4^- , 40^+ ; 1.2^+ , 120^- ; 5^- , 500^+$

**Step 4:** Put in ascending order and calculate gaps as $\log_{10} \frac{\omega_2}{\omega_1}$ decades:

$1^- (.6) 4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+.$

**Step 5:** Find phase of LF asymptote: $\angle 1.2 j\omega = +\frac{\pi}{2}$.  

**Step 6:** At $\omega = 0.1$ gradient becomes $-\frac{\pi}{4}$ rad/decade. $\phi$ is still $\frac{\pi}{2}$.  

**Step 7:** At $\omega = 0.4$, $\phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi$. New gradient is $-\frac{\pi}{2}$.  

**Step 8:** At $\omega = 1.2$, $\phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi$. New gradient is $-\frac{\pi}{4}$.  

**Steps 9-13:** Repeat for each gradient change.
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}

Step 1: Factorize the polynomials

Step 2: List corner freqs: ± = num/den
ω_c = \{1^-, 4^-, 12^+, 50^-\}

Step 3: Gradient changes at 10\(^{\pm 1}\)ω_c.
Sign depends on num/den and sgn \(\frac{b}{a}\):
\begin{align*}
.1^- & , 10^+; .4^- & , 40^+; 1.2^+ & , 120^-; 5^- & , 500^+
\end{align*}

Step 4: Put in ascending order and calculate gaps as \(\log_{10} \frac{\omega_2}{\omega_1}\) decades:
\begin{align*}
.1^- & , (.6) .4^- & , (.48) 1.2^+ & , (.62) 5^- & , (.3) 10^+ & , (.6) 40^+ & , (.48) 120^- & , (.62) 500^+.
\end{align*}

Step 5: Find phase of LF asymptote: \(\angle 1.2j\omega = + \frac{\pi}{2}\).

Step 6: At \(\omega = 0.1\) gradient becomes \(-\frac{\pi}{4}\) rad/decade. \(\phi\) is still \(\frac{\pi}{2}\).

Step 7: At \(\omega = 0.4\), \(\phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi\). New gradient is \(-\frac{3\pi}{4}\).

Step 8: At \(\omega = 1.2\), \(\phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi\). New gradient is \(-\frac{\pi}{4}\).

Steps 9-13: Repeat for each gradient change.
\( H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \)

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\( \omega_c = \{1^- , 4^- , 12^+ , 50^- \} \)

**Step 3:** Gradient changes at \( 10^{\pm1} \omega_c \).

Sign depends on num/den and \( \text{sgn} \left( \frac{b}{a} \right) \):

\( .1^- , 10^+ ; .4^- , 40^+ ; 1.2^+ , 120^- ; 5^- , 500^+ \)

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\( .1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+ \).

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

**Step 6:** At \( \omega = 0.1 \) gradient becomes \( -\frac{\pi}{4} \) rad/decade. \( \phi \) is still \( \frac{\pi}{2} \).

**Step 7:** At \( \omega = 0.4, \phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi \). New gradient is \( -\frac{\pi}{2} \).

**Step 8:** At \( \omega = 1.2, \phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi \). New gradient is \( -\frac{\pi}{4} \).

**Steps 9-13:** Repeat for each gradient change. Final gradient is always 0.
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{\pm1}\omega_c \).

Sign depends on num/den and \( \text{sgn} \left( \frac{b}{a} \right) \):

\( .1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+ \)

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\( .1^-(.6) .4^-(.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+ \)

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

**Step 6:** At \( \omega = 0.1 \) gradient becomes \( -\frac{\pi}{4} \) rad/decade. \( \phi \) is still \( \frac{\pi}{2} \).

**Step 7:** At \( \omega = 0.4, \phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi \). New gradient is \( -\frac{\pi}{2} \).

**Step 8:** At \( \omega = 1.2, \phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi \). New gradient is \( -\frac{\pi}{4} \).

**Steps 9-13:** Repeat for each gradient change. Final gradient is always 0.

At 0.1 and 10 times each corner frequency, the graph is continuous but its gradient changes abruptly by \( \pm \frac{\pi}{4} \) rad/decade.
11: Frequency Responses

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}}
\]
RCR Circuit

\[ Y \frac{X}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

![Diagram of RCR Circuit with Y and X connected through R and C](image)

![Graph showing magnitude response with dB gain vs \( \omega RC \)](image)

![Graph showing phase response with phase vs \( \omega RC \)](image)
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(\frac{0.25}{RC}\), \(\frac{1}{RC}\)

---

Frequency Responses: 11 – 11 / 12
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \( \frac{0.25}{RC}, \frac{1}{RC} \) + LF Asymptote: \( H(j\omega) = 1 \)
### RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(\frac{0.25}{RC}, \frac{1}{RC}\) +

LF Asymptote: \(H(j\omega) = 1\)

**Magnitude Response:**
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \)  
LF Asymptote: \( H(j\omega) = 1 \)

**Magnitude Response:**
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC}\) and \(+20 \text{ at } \omega = \frac{1}{RC}\)
**RCR Circuit**

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) + LF Asymptote: \( H(j\omega) = 1 \)

**Magnitude Response:**
Gradient Changes: \(-20\) dB/dec at \( \omega = \frac{0.25}{RC} \) and \(+20\) at \( \omega = \frac{1}{RC} \)
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) +

LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20\) dB/dec at \( \omega = \frac{0.25}{RC} \) and \(+20\) at \( \omega = \frac{1}{RC} \)

Line equations: \( H(j\omega) = \)
(a) \( 1 \),
(b) \( \frac{1}{4j\omega RC} \),
(c) \( \frac{j\omega RC}{4j\omega RC} = 0.25 \)
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \)

LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20\) dB/dec at \( \omega = \frac{0.25}{RC} \) and \(+20\) at \( \omega = \frac{1}{RC} \)

Line equations: \( H(j\omega) = (a) 1, \) \( (b) \frac{1}{4j\omega RC}, \) \( (c) \frac{j\omega RC}{4j\omega RC} = 0.25 \)

Phase Response:
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25 \frac{RC}{\omega} - \), \(1 \frac{RC}{\omega} + \)

LF Asymptote: \(H(j\omega) = 1\)

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\).

Line equations: \(H(j\omega) = (a) 1, (b) \frac{1}{4j\omega RC}, (c) \frac{j\omega RC}{4j\omega RC} = 0.25\)

Phase Response:
LF asymptote: \(\phi = \angle 1 = 0\)
11: Frequency Responses
- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary

RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) +
LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec} \) at \( \omega = \frac{0.25}{RC} \) and \(+20 \) at \( \omega = \frac{1}{RC} \)

Line equations: \( H(j\omega) = (a) 1 \), \( (b) \frac{1}{4j\omega RC} \), \( (c) \frac{j\omega RC}{4j\omega RC} = 0.25 \)

Phase Response:
LF asymptote: \( \phi = \angle 1 = 0 \)
Gradient changes of \( \pm \frac{\pi}{4} / \text{decade} \) at: \( \omega = \frac{0.025}{RC} \), \( \frac{0.1}{RC} \), \( \frac{2.5}{RC} \), \( \frac{10}{RC} \).

E1.1 Analysis of Circuits (2017-10213)
**RCR Circuit**

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) + LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20\) dB/dec at \( \omega = \frac{0.25}{RC} \) and \(+20\) at \( \omega = \frac{1}{RC} \)
Line equations: \( H(j\omega) = (a) 1, \) (b) \( \frac{1}{4j\omega RC} \), (c) \( \frac{j\omega RC}{4j\omega RC} = 0.25 \)

Phase Response:
LF asymptote: \( \phi = \angle 1 = 0 \)
Gradient changes of \( \pm \frac{\pi}{4} \)/decade at: \( \omega = \frac{0.025}{RC} \), \( \frac{0.1}{RC} \), \( \frac{2.5}{RC} \), \( \frac{10}{RC} \)
At \( \omega = \frac{0.1}{RC} \), \( \phi = 0 - \frac{\pi}{4} \log_{10} \frac{0.1}{0.025} = -0.15\pi \)
Summary

- **Frequency response:** magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
**Summary**

- **Frequency response**: magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
  - Use log axes for frequency and gain but linear for phase
  - Decibels = $20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1}$
Summary

- **Frequency response**: magnitude and phase of \( \frac{Y}{X} \) as a function of \( \omega \)
  - Only applies to sine waves
  - Use **log axes** for frequency and gain but **linear** for phase
    - Decibels = \( 20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1} \)

- Linear factor \((aj\omega + b)\) gives corner frequency at \( \omega = \left| \frac{b}{a} \right| \).
  - Magnitude plot gradient changes by \( \pm 20 \text{ dB/decade} \) @ \( \omega = \left| \frac{b}{a} \right| \).
### Summary

- **Frequency response**: magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
  - Use log axes for frequency and gain but linear for phase
    - Decibels = $20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1}$

- Linear factor $(a j\omega + b)$ gives corner frequency at $\omega = \left| \frac{b}{a} \right|$.
  - Magnitude plot gradient changes by $\pm 20$ dB/decade @ $\omega = \left| \frac{b}{a} \right|$.
  - Phase gradient changes in two places by:
    - $\pm \frac{\pi}{4}$ rad/decade @ $\omega = 0.1 \times \left| \frac{b}{a} \right|$.
    - $\mp \frac{\pi}{4}$ rad/decade @ $\omega = 10 \times \left| \frac{b}{a} \right|$.
**Summary**

- **Frequency response**: magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
  - Use log axes for frequency and gain but linear for phase
    - Decibels = $20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1}$

- **Linear factor** $(a j\omega + b)$ gives corner frequency at $\omega = \frac{|b|}{a}$.
  - Magnitude plot gradient changes by $\pm 20$ dB/decade @ $\omega = \frac{|b|}{a}$.
  - Phase gradient changes in two places by:
    - $\pm \frac{\pi}{4}$ rad/decade @ $\omega = 0.1 \times \frac{|b|}{a}$
    - $\mp \frac{\pi}{4}$ rad/decade @ $\omega = 10 \times \frac{|b|}{a}$

- **LF/HF asymptotes**: keep only the terms with the lowest/highest power of $j\omega$ in numerator and denominator polynomials
Summary

- **Frequency response**: magnitude and phase of \( \frac{Y}{X} \) as a function of \( \omega \)
  - Only applies to sine waves
  - Use log axes for frequency and gain but linear for phase
    - Decibels = \( 20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1} \)
- **Linear factor** \( (a j \omega + b) \) gives corner frequency at \( \omega = \left| \frac{b}{a} \right| \).
  - Magnitude plot gradient changes by \( \pm 20 \text{ dB/decade} @ \omega = \left| \frac{b}{a} \right| \).
  - Phase gradient changes in two places by:
    - \( \pm \frac{\pi}{4} \text{ rad/decade} @ \omega = 0.1 \times \left| \frac{b}{a} \right| \)
    - \( \mp \frac{\pi}{4} \text{ rad/decade} @ \omega = 10 \times \left| \frac{b}{a} \right| \)
- **LF/HF asymptotes**: keep only the terms with the lowest/highest power of \( j \omega \) in numerator and denominator polynomials

For further details see Hayt Ch 16 or Irwin Ch 12.