11: Frequency Responses

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary
If \( x(t) \) is a sine wave, then \( y(t) \) will also be a sine wave but with a different amplitude and phase shift. \( X \) is an input phasor and \( Y \) is the output phasor.
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\frac{Y}{X} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1}
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**Sine Wave Response**

\[ RC = 10 \text{ ms} \]

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\frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1}
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\[ \omega = 50 \Rightarrow \frac{Y}{X} = 0.89\angle -27^\circ \]

\[ \omega = 100 \Rightarrow \frac{Y}{X} = 0.71\angle -45^\circ \]

\[ \omega = 300 \Rightarrow \frac{Y}{X} = 0.32\angle -72^\circ \]

\[ R = 10k \quad C = 1\mu \]
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\[ X \quad R = 10k \quad Y \]

\[ C = 1\mu \]

\[ 0 \quad 0.5 \quad 1 \]

\[ 0 \quad -0.2 \quad -0.4 \]

\[ \omega = 100 \]

\[ 0 \quad 0.5 \quad 1 \]

\[ \text{Real} \]

\[ \text{Imag} \]

\[ w = 100 \text{ rad/s}, \text{Gain} = 0.71, \text{Phase} = -45^\circ \]

\[ x = \text{blue}, y = \text{red} \]

\[ 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100 \quad 120 \]

\[ 0 \quad 0.5 \quad 1 \]

\[ -0.5 \quad -1 \]

\[ 0 \quad 100 \quad 200 \quad 300 \quad 400 \quad 500 \]

\[ \omega \text{ (rad/s)} \]

\[ |Y/X| \]

\[ 0 \quad 0.5 \quad 1 \]

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\[ \text{Phase (°)} \]
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$$w = 300 \text{ rad/s}, \text{ Gain} = 0.32, \text{ Phase} = -72^\circ$$

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The output, \( y(t) \), lags the input, \( x(t) \), by up to \( 90^\circ \).
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.
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![RCR Circuit Diagram]

![Magnitude Response Graph]

![Phase Response Graph]
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Logarithmic voltage ratios are specified in \textit{decibels (dB)} = 20 \log_{10} \left| \frac{V_2}{V_1} \right|.

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Common voltage ratios:

$$\begin{array}{c|c|c|c}
|V_2|/|V_1| & \text{dB} & 1 & 0 \\
\hline
1 \\
\end{array}$$

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| \( \frac{|V_2|}{|V_1|} \) | 0.1   | 1     | 10    | 100   |
|----------------------|-------|-------|-------|-------|
| dB                   | −20   | 0     | 20    | 40    |

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Common voltage ratios:

| \( \frac{|V_2|}{|V_1|} \) | 0.1 | 0.5 | 1 | 2 | 10 | 100 |
|---|---|---|---|---|---|---|
| dB | −20 | −6 | 0 | 6 | 20 | 40 |

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Common voltage ratios:

| \( \frac{|V_2|}{|V_1|} \) | 0.1 | 0.5 | \( \sqrt{0.5} \) | 1 | \( \sqrt{2} \) | 2 | 10 | 100 |
|----------------|-----|-----|----------|---|--------|---|----|-----|
| dB             | -20 | -6  | -3       |  0|  3     |  6| 20 |  40 |

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Common voltage ratios:

| \( \frac{|V_2|}{|V_1|} \) | 0.1  | 0.5  | \( \sqrt{0.5} \) | 1     | \( \sqrt{2} \) | 2     | 10    | 100   |
| dB    | -20  | -6   | -3            | 0     | 3            | 6     | 20    | 40    |

Note that 0 does not exist on a log axis and so the starting point of the axis is arbitrary.

\[ P \propto V^2 \implies \text{decibel power ratios are given by } 10 \log_{10} \frac{P_2}{P_1} \]
Suppose we plot the magnitude and phase of $H = c(j\omega)^r$.
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**Magnitude (log-log graph):**

$$|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega$$
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\[ \angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} (+\pi \text{ if } c < 0) \]
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The phase is constant $\forall \omega$. 

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**Logs of Powers**

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- Sine Wave Response
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- Straight Line Approximations
- Log of Magnitude Response
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This is a straight line with a slope of $r$. $c$ only affects the line’s vertical position.

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$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2}$ ($+\pi$ if $c < 0$)

The phase is constant $\forall \omega$.

If $c > 0$, phase = $90^\circ \times$ magnitude slope.
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$

**Magnitude (log-log graph):**

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If $|H|$ is measured in decibels, a slope of $r$ is called $6r$ dB/octave or $20r$ dB/decade.

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The phase is constant $\forall \omega$.

If $c > 0$, phase $= 90^\circ \times$ magnitude slope.

Negative $c$ adds $\pm 180^\circ$ to the phase.
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$.

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**Note:** Phase angles are modulo $360^\circ$, i.e. $+180^\circ \equiv -180^\circ$ and $450^\circ \equiv 90^\circ$. 

- \( (j\omega)^2 \)
- \( 0.2(j\omega)^2 \)
- \( 8(j\omega)^0 \)
- \( -9(j\omega)^{-1} \)
- \( 65(j\omega)^{-1} \)
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- **Summary**

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\[ H = c (j \omega)^r \] has a straight-line magnitude graph and a constant phase.

**Magnitude (log-log graph):**

\[
|H| = c\omega^r \Rightarrow \log |H| = \log |c| + r \log \omega
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This is a straight line with a slope of \( r \).

\( c \) only affects the line's vertical position.

If \( |H| \) is measured in decibels, a slope of \( r \) is called \( 6r \text{ dB/octave} \) or \( 20r \text{ dB/decade} \).

**Phase (log-lin graph):**

\[
\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2} \ ( +\pi \text{ if } c < 0)
\]

The phase is constant \( \forall \omega \).

If \( c > 0 \), phase = \( 90^\circ \times \) magnitude slope.

Negative \( c \) adds \( \pm 180^\circ \) to the phase.

**Note:** Phase angles are modulo \( 360^\circ \), i.e. \( +180^\circ \equiv -180^\circ \) and \( 450^\circ \equiv 90^\circ \).
Key idea: 

\[(a j \omega + b) \approx \begin{cases} 
  a j \omega & \text{for } |a \omega| \gg |b| \\
  b & \text{for } |a \omega| \ll |b| 
\end{cases}\]
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1\)
Key idea: \((a \cdot j\omega + b) \approx \begin{cases} a \cdot j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1\)
High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC}\)
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC}\)

Approximate the magnitude response as two straight lines
**Straight Line Approximations**

### Key idea:

\[ (a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases} \]

### Gain:

\[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

#### Low frequencies

(\( \omega \ll \frac{1}{RC} \)):

\[ H(j\omega) \approx 1 \implies |H(j\omega)| \approx 1 \]

#### High frequencies

(\( \omega \gg \frac{1}{RC} \)):

\[ H(j\omega) \approx \frac{1}{j\omega RC} \]

Approximate the magnitude response as two straight lines.

---

**Diagram:**

A circuit diagram showing a resistor (R) and a capacitor (C) in series, with nodes X and Y. The magnitude response graph shows a curve that approximates two straight lines over different frequency ranges.
Key idea: \( (a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases} \)

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \( H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1 \)

High frequencies (\( \omega \gg \frac{1}{RC} \)): \( H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC} \omega^{-1} \)

Approximate the magnitude response as two straight lines
Key idea: 

\[ (a j \omega + b) \approx \begin{cases} 
    a j \omega & \text{for } |a \omega| \gg |b| \\
    b & \text{for } |a \omega| \ll |b| 
\end{cases} \]

Gain: 

\[ H(j \omega) = \frac{1}{j \omega RC + 1} \]

Low frequencies \((\omega \ll \frac{1}{RC})\): 

\[ H(j \omega) \approx 1 \Rightarrow |H(j \omega)| \approx 1 \]

High frequencies \((\omega \gg \frac{1}{RC})\): 

\[ H(j \omega) \approx \frac{1}{j \omega RC} \Rightarrow |H(j \omega)| \approx \frac{1}{RC} \omega^{-1} \]

Approximate the magnitude response as two straight lines
**Key idea:** \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\)

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Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC \omega^{-1}}\)

Approximate the magnitude response as two straight lines intersecting at the **corner frequency**, \(\omega_c = \frac{1}{RC}\).
Key idea: \((a\,j\omega + b) \approx \begin{cases} a\,j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

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Approximate the magnitude response as two straight lines intersecting at the corner frequency \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) \((-6\,\text{dB/octave} = -20\,\text{dB/decade})\).
Key idea: \((a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

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Approximate the magnitude response as two straight lines intersecting at the corner frequency, \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) (\(= -6 \text{ dB/octave} = -20 \text{ dB/decade}\)).

(b) \(|H(j\omega_c)| = \left| \frac{1}{1+j} \right| = \frac{1}{\sqrt{2}} = -3 \text{ dB} \) (worst-case error).
**Key idea:** \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\)

**Gain:** \(H(j \omega) = \frac{1}{j \omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j \omega) \approx 1 \Rightarrow |H(j \omega)| \approx 1\)

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Approximate the magnitude response as two straight lines intersecting at the **corner frequency**, \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) (\(= -6\) dB/octave = \(-20\) dB/decade).

(b) \(|H(j \omega_c)| = \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}} = -3\) dB (worst-case error).

A linear factor \((a j \omega + b)\) has a corner frequency of \(\omega_c = \left|\frac{b}{a}\right|\).
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$$
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

The graph shows the magnitude response of the transfer function $H(j\omega)$ over the frequency range from 0.1 to 1000 rad/s.
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50
The gain of a linear circuit is always a **rational polynomial** in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials  
**Step 2:** Sort corner freqs: 1, 4, 12, 50  
**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:  
$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials  
**Step 2:** Sort corner freqs: 1, 4, 12, 50  
**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:  
$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$  
**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so  
$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0.$$
The gain of a linear circuit is always a *rational polynomial* in \( j\omega \) and is called the *transfer function* of the circuit. For example:

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\]

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For \( \omega < 1 \) all linear factors equal their constant terms:

\[
|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.
\]

**Step 4:** For \( 1 < \omega < 4 \), the factor \((j\omega + 1) \approx j\omega\) so

\[
|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58\,\text{dB}.
\]
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}$$

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**Step 2:** Sort corner freqs: 1, 4, 12, 50  
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$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58\text{ dB}.$$  
**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$. 


The gain of a linear circuit is always a rational polynomial in \( j\omega \) and is called the transfer function of the circuit. For example:

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H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}
\]

**Step 1:** Factorize the polynomials  
**Step 2:** Sort corner freqs: 1, 4, 12, 50  
**Step 3:** For \( \omega < 1 \) all linear factors equal their constant terms:  
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|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.
\]

**Step 4:** For \( 1 < \omega < 4 \), the factor \( (j\omega + 1) \approx j\omega \) so  
\[
|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.
\]

**Step 5:** For \( 4 < \omega < 12 \),  
\[
|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}.
\]

**Step 6:** For \( 12 < \omega < 50 \),  
\[
|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}.
\]
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

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**Step 2:** Sort corner freqs: 1, 4, 12, 50  
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**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.  
**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}$.  
**Step 7:** For $\omega > 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times \omega} = 20\omega^{-1}$.  

The gain of a linear circuit is always a **rational polynomial** in $j\omega$ and is called the **transfer function** of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50

**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$

**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so

$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$

**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.

**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}.$

**Step 7:** For $\omega > 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times \omega} = 20\omega^{-1}$.

At each corner frequency, the graph is continuous but its gradient changes abruptly by $+1$ (numerator factor) or $-1$ (denominator factor).
You can find the low and high frequency asymptotes without factorizing:

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}
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**Low Frequency Asymptote:**

[Graph showing the magnitude response with a peak around \( \omega = 2 \) rad/s and a steeper slope at lower frequencies.]

[Graph showing the phase response with a phase shift of \(-90\) degrees at \( \omega = 1 \) rad/s and a phase derivative with a slope of \(-10\) rad/s at lower frequencies.]
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

Low Frequency Asymptote:

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

Low Frequency Asymptote:
From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

Lowest power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Low Frequency Asymptote:**
From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

**Lowest** power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)

**High Frequency Asymptote:**
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}$$

Low Frequency Asymptote:
From factors: $$H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega$$

Lowest power of $$j\omega$$ on top and bottom: $$H(j\omega) \sim \frac{720(j\omega)}{600} = 1.2j\omega$$

High Frequency Asymptote:
From factors: $$H_{HF}(j\omega) = \frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)} = 20(j\omega)^{-1}$$
You can find the low and high frequency asymptotes without factorizing:

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}
\]

**Low Frequency Asymptote:**
From factors: \(H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega\)

**Lowest** power of \(j\omega\) on top and bottom: \(H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega\)

**High Frequency Asymptote:**
From factors: \(H_{HF}(j\omega) = \frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)} = 20(j\omega)^{-1}\)

**Highest** power of \(j\omega\) on top and bottom: \(H(j\omega) \approx \frac{60(j\omega)^2}{3(j\omega)^3} = 20(j\omega)^{-1}\)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC+1} \)

[Diagram of an RC circuit]
Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies \( (\omega \ll \frac{1}{RC}) \):

\[ H(j\omega) \approx 1 \]
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies \((\omega \ll \frac{1}{RC})\):

\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \( H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \)

High frequencies (\( \omega \gg \frac{1}{RC} \)): \( H(j\omega) \approx \frac{1}{j\omega RC} \)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \( H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \)

High frequencies (\( \omega \gg \frac{1}{RC} \)): \( H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \)
**Phase Approximation**

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
\[
H(j\omega) \approx 1 \Rightarrow \angle 1 = 0
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H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2}
\]

Approximate the phase response as three straight lines.
Phase Approximation

Gain: \[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

High frequencies (\( \omega \gg \frac{1}{RC} \)): \[ H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \]

Approximate the phase response as three straight lines.

By chance, they intersect close to 0.1\( \omega_c \) and 10\( \omega_c \) where \( \omega_c = \frac{1}{RC} \).
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \( H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \)

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Approximate the phase response as three straight lines.

By chance, they intersect close to \( 0.1\omega_c \) and \( 10\omega_c \) where \( \omega_c = \frac{1}{RC} \).

Between \( 0.1\omega_c \) and \( 10\omega_c \) the phase changes by \(-\frac{\pi}{2}\) over two decades. This gives a gradient = \(-\frac{\pi}{4}\) radians/decade.
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC+1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):

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Approximate the phase response as three straight lines.

By chance, they intersect close to 0.1\( \omega_c \) and 10\( \omega_c \) where \( \omega_c = \frac{1}{RC} \).

Between 0.1\( \omega_c \) and 10\( \omega_c \) the phase changes by \(-\frac{\pi}{2}\) over two decades. This gives a gradient = \(-\frac{\pi}{4}\) radians/decade.

\((aj\omega + b)\) in denominator

\[ \Rightarrow \Delta \text{gradient} = \mp \frac{\pi}{4} / \text{decade} \text{ at } \omega = 10^{\mp1} \left| \frac{b}{a} \right|. \]
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)): \( H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \)

High frequencies (\( \omega \gg \frac{1}{RC} \)): \( H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \)

Approximate the phase response as three straight lines.

By chance, they intersect close to 0.1\( \omega_c \) and 10\( \omega_c \) where \( \omega_c = \frac{1}{RC} \).

Between 0.1\( \omega_c \) and 10\( \omega_c \) the phase changes by \( -\frac{\pi}{2} \) over two decades. This gives a gradient = \( -\frac{\pi}{4} \) radians/decade.

\( (aj\omega + b) \) in denominator

\( \Rightarrow \Delta \text{gradient} = \mp \frac{\pi}{4} / \text{decade} \) at \( \omega = 10^{\mp 1} \left| \frac{b}{a} \right| \).

The sign of \( \Delta \text{gradient} \) is reversed for (a) numerator factors and (b) \( \frac{b}{a} < 0 \).
11: Frequency Responses

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary

Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} \]
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

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**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{\pm1}\omega_c \).

Sign depends on \( \text{num/den} \) and \( \text{sgn} \left( \frac{b}{a} \right) \):

\[ .1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+ \]
$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}$

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: $\pm \frac{\text{num}}{\text{den}}$

$\omega_c = \{1^-, 4^-, 12^+, 50^-\}$

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Sign depends on $\frac{\text{num}}{\text{den}}$ and $\text{sgn}\left(\frac{b}{a}\right)$:

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**Step 4:** Put in ascending order and calculate gaps as $\log_{10}\frac{\omega_2}{\omega_1}$ decades:

$.1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+$.
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

Step 1: Factorize the polynomials
Step 2: List corner freqs: \( \pm \frac{\text{num}}{\text{den}} = \omega_c \)
\( \omega_c = \{1^{-}, 4^{-}, 12^{+}, 50^{-}\} \)
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Sign depends on \( \text{num}/\text{den} \) and \( \text{sgn}\left(\frac{b}{a}\right) \):
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Step 4: Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:
\( 1^{-}(.6), 4^{-}(.48), 1.2^{+}(.62), 5^{-}(.3), 10^{+}(.6), 40^{+}(.48), 120^{-}(.62), 500^{+}. \)
Step 5: Find phase of LF asymptote: \( \angle 1.2j\omega = \frac{\pi}{2} \).
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\( \omega_c = \{1^{-}, 4^{-}, 12^{+}, 50^{-}\} \)

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \).

Sign depends on \( \frac{b}{a} \):

\( .1^{-}, 10^{+}; .4^{-}, 40^{+}; 1.2^{+}, 120^{-}; 5^{-}, 500^{+} \)

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\( .1^{-} (.6) .4^{-} (.48) 1.2^{+} (.62) 5^{-} (.3) 10^{+} (.6) 40^{+} (.48) 120^{-} (.62) 500^{+} \).

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).
**Plot Phase Response**

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}
\]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \(\pm \frac{\text{num}}{\text{den}}\)

\(\omega_c = \{1^-\}, 4^-, 12^+, 50^-\}

**Step 3:** Gradient changes at \(10^{\pm 1}\omega_c\).

Sign depends on \(\frac{\text{num}}{\text{den}}\) and \(\text{sgn}\left(\frac{b}{a}\right)\):

\(.1^-\), \(10^+\); \(4^-\), \(40^+\); \(1.2^+\), \(120^-\); \(5^-\), \(500^+\)

**Step 4:** Put in ascending order and calculate gaps as \(\log_{10} \frac{\omega_2}{\omega_1}\) decades:

\(.1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+\).

**Step 5:** Find phase of LF asymptote: \(\angle 1.2j\omega = +\frac{\pi}{2}\).

**Step 6:** At \(\omega = 0.1\) gradient becomes \(-\frac{\pi}{4}\) rad/decade. \(\phi\) is still \(\frac{\pi}{2}\).
### Plot Phase Response

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}
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**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)
\[
\omega_c = \{ 1^-, 4^-, 12^+, 50^- \}
\]

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \).
Sign depends on num/den and \( \text{sgn} \left( \frac{b}{a} \right) \):
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### Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{+1}\omega_c \).

Sign depends on num/den and \( \text{sgn} \left( \frac{b}{a} \right) \):

\[ .1^-, 10^+; .4^-, 40^+; 1.2^+; 120^-; 5^-; 500^+ \]

**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\[ .1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+ \]

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

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\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm \frac{\text{num}}{\text{den}} \)

\[ \omega_c = \{1^{-}, 4^{-}, 12^{+}, 50^{-}\} \]

**Step 3:** Gradient changes at \( 10^{\pm 1}\omega_c \). Sign depends on \( \frac{\text{num}}{\text{den}} \) and \( \text{sgn} \left( \frac{b}{a} \right) \):

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**Step 4:** Put in ascending order and calculate gaps as \( \log_{10} \frac{\omega_2}{\omega_1} \) decades:

\[ 1^{-} (0.6) 4^{-} (0.48) 1.2^{+} (0.62) 5^{-} (0.3) 10^{+} (0.6) 40^{+} (0.48) 120^{-} (0.62) 500^{+} \]

**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

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**Steps 9-13:** Repeat for each gradient change.
11: Frequency Responses

- Frequency Response
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Sign depends on num/den and \( \text{sgn}(\frac{b}{a}) \): 

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**Step 5:** Find phase of LF asymptote: \( \angle 1.2j\omega = +\frac{\pi}{2} \).

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**Steps 9-13:** Repeat for each gradient change. Final gradient is always 0.
$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: $\pm \frac{\text{num}}{\text{den}}$

$\omega_c = \{1^- , 4^- , 12^+ , 50^-\}$

**Step 3:** Gradient changes at $10^{\frac{\pm 1}{\text{num}}} \omega_c$.

Sign depends on $\text{num}/\text{den}$ and $\text{sgn} \left( \frac{b}{a} \right)$:

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**Step 4:** Put in ascending order and calculate gaps as $\log_{10} \frac{\omega_2}{\omega_1}$ decades:

$1^- (0.6) 4^- (0.48) 1.2^+ (0.62) 5^- (0.3) 10^+ (0.6) 40^+ (0.48) 120^- (0.62) 500^+$

**Step 5:** Find phase of LF asymptote: $\angle 1.2j\omega = +\frac{\pi}{2}$.

**Step 6:** At $\omega = 0.1$ gradient becomes $-\frac{\pi}{4}$ rad/decade. $\phi$ is still $\frac{\pi}{2}$.

**Step 7:** At $\omega = 0.4$, $\phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi$. New gradient is $-\frac{\pi}{2}$.

**Step 8:** At $\omega = 1.2$, $\phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi$. New gradient is $-\frac{\pi}{4}$.

**Steps 9-13:** Repeat for each gradient change. Final gradient is always 0.

At 0.1 and 10 times each corner frequency, the graph is continuous but its gradient changes abruptly by $\pm \frac{\pi}{4}$ rad/decade.
## RCR Circuit

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![RCR Circuit Diagram]

E1.1 Analysis of Circuits (2018-10340)
\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} \]
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

**Sine Wave Response**

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### RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(\frac{0.25}{RC}\), \(\frac{1}{RC}\)

Graphs showing magnitude and phase responses.

---

**11: Frequency Responses**
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\[
\frac{Y}{X} = \frac{R + \frac{1}{jωC}}{3R + R + \frac{1}{jωC}} = \frac{jωRC + 1}{4jωRC + 1}
\]

Corner freqs: \(0.25 \cdot \frac{RC}{RC}\), \(\frac{1}{RC}\) +

LF Asymptote: \(H(jω) = 1\)
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25 \frac{R}{C}, \frac{1}{RC}\) + LF Asymptote: \(H(j\omega) = 1\)

Magnitude Response:
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25 \frac{RC}{R}
\), \(\frac{1}{RC}\) +

LF Asymptote: \(H(j\omega) = 1\)

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC}\) and \(+20 \text{ at } \omega = \frac{1}{RC}\)
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25\frac{R}{C}\) \(-\), \(\frac{1}{RC}\) \(+\)  

LF Asymptote: \(H(j\omega) = 1\)

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Gradient Changes: \(-20\, \text{dB/dec at } \omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\)
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(\frac{0.25}{RC}\), \(\frac{1}{RC}\) +

LF Asymptote: \(H(j\omega) = 1\)

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC}\) and \(+20 \text{ at } \omega = \frac{1}{RC}\)

Line equations: 
(a) \(H(j\omega) = 1\),  
(b) \(\frac{1}{4j\omega RC}\),  
(c) \(\frac{j\omega RC}{4j\omega RC} = 0.25\)
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) +

LF Asymptote: \( H(j\omega) = 1 \)

**Magnitude Response:**
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC} \) and \(+20 \text{ at } \omega = \frac{1}{RC} \)

Line equations: \( H(j\omega) = \) (a) 1, (b) \( \frac{1}{4j\omega RC} \), (c) \( \frac{j\omega RC}{4j\omega RC} = 0.25 \)

**Phase Response:**

\[ X \quad 3R \quad \frac{R}{R} \quad C \quad Y \]
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(\frac{0.25}{RC}\), \(\frac{1}{RC}\) 

LF Asymptote: \(H(j\omega) = 1\)

Magnitude Response:
Gradient Changes: \(-20\) dB/dec at \(\omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\)

Line equations: \(H(j\omega) = (a)\ 1, \ (b)\ \frac{1}{4j\omega RC}, \ (c)\ \frac{j\omega RC}{4j\omega RC} = 0.25\)

Phase Response:
LF asymptote: \(\phi = \angle 1 = 0\)
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} - \), \( \frac{1}{RC} + \)  
LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20\) dB/dec at \( \omega = \frac{0.25}{RC} \) and \(+20\) at \( \omega = \frac{1}{RC} \)

Line equations: \( H(j\omega) = (a) 1, \)  
\( (b) \frac{1}{4j\omega RC}, \)  
\( (c) \frac{j\omega RC}{4j\omega RC} = 0.25 \)

Phase Response:
LF asymptote: \( \phi = \angle 1 = 0 \)
Gradient changes of \( \pm \frac{\pi}{4} \) /decade at: \( \omega = \frac{0.025}{RC}, \frac{0.1}{RC}, \frac{2.5}{RC}, \frac{10}{RC} \).
RCR Circuit

\[
\frac{Y}{X} = \frac{\frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}}}{\frac{j\omega RC + \frac{1}{4j\omega RC + 1}}{4j\omega RC + 1}}
\]

Corner freqs: \(\frac{0.25}{RC}\), \(\frac{1}{RC}\)

**LF Asymptote:** \(H(j\omega) = 1\)

**Magnitude Response:**
Gradient Changes: \(-20\) dB/dec at \(\omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\)

Line equations: \(H(j\omega) = (a) 1\), \(b) \frac{1}{4j\omega RC}\), \(c) \frac{j\omega RC}{4j\omega RC} = 0.25\)

**Phase Response:**
LF asymptote: \(\phi = \angle 1 = 0\)
Gradient changes of \(\pm \frac{\pi}{4}\) /decade at: \(\omega = \frac{0.025}{RC}, \frac{0.1}{RC}, \frac{2.5}{RC}, \frac{10}{RC}\).

At \(\omega = \frac{0.1}{RC}\), \(\phi = 0 - \frac{\pi}{4} \log_{10} \frac{0.1}{0.025} = -\frac{\pi}{4} \times 0.602 = -0.15\pi\)

E1.1 Analysis of Circuits (2018-10340)
Summary

- **Frequency response:** magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
Summary

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  - Use log axes for frequency and gain but linear for phase
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  - Magnitude plot gradient changes by $\pm 20 \text{ dB/decade}$ @ $\omega = \left| \frac{b}{a} \right|$. 
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  - Phase gradient changes in two places by:
    - \( \pm \frac{\pi}{4} \text{ rad/decade} \) @ \( \omega = 0.1 \times \left| \frac{b}{a} \right| \)
    - \( \mp \frac{\pi}{4} \text{ rad/decade} \) @ \( \omega = 10 \times \left| \frac{b}{a} \right| \)
Summary

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- **LF/HF asymptotes**: keep only the terms with the lowest/highest power of $j\omega$ in numerator and denominator polynomials
Summary

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For further details see Hayt Ch 16 or Irwin Ch 12.