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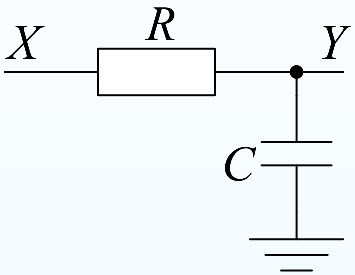

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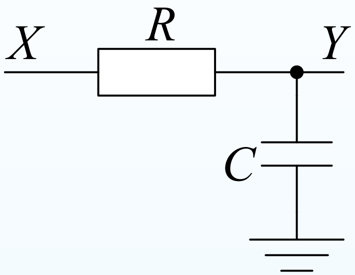

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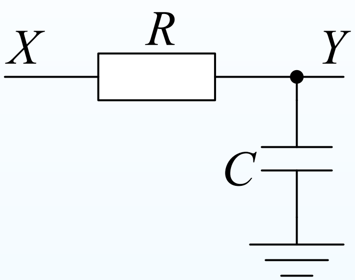

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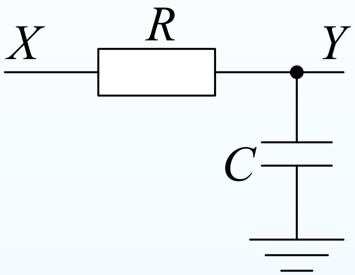

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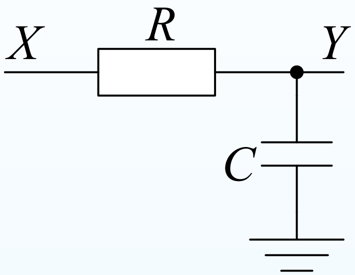

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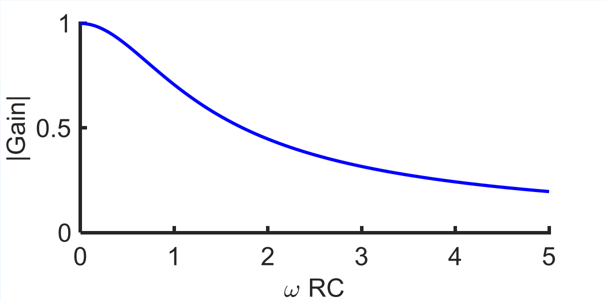

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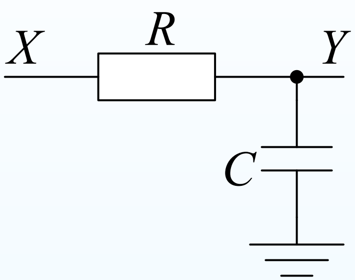

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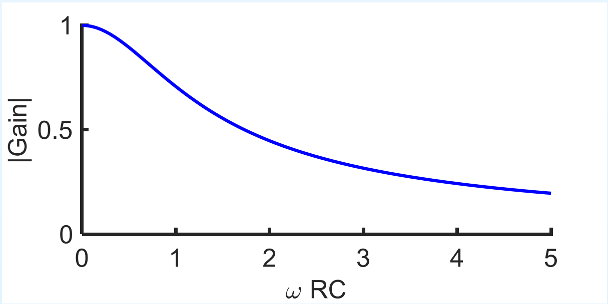

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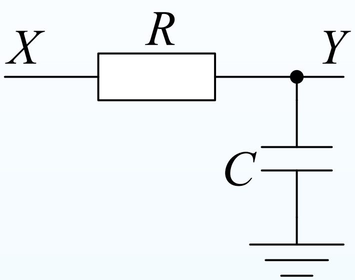

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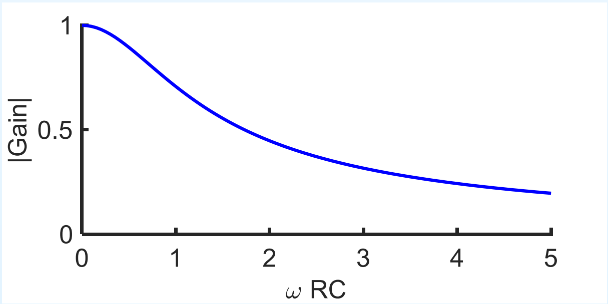

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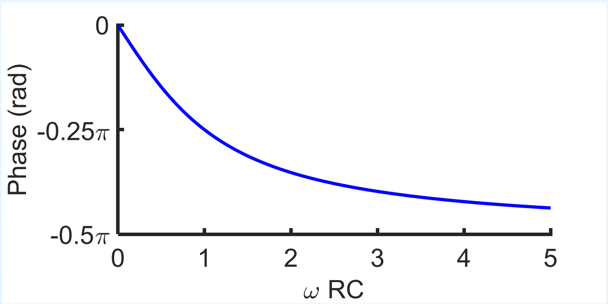

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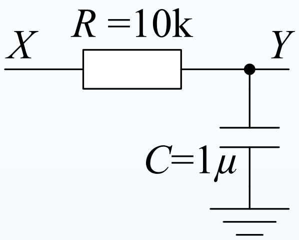

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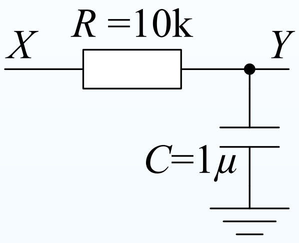

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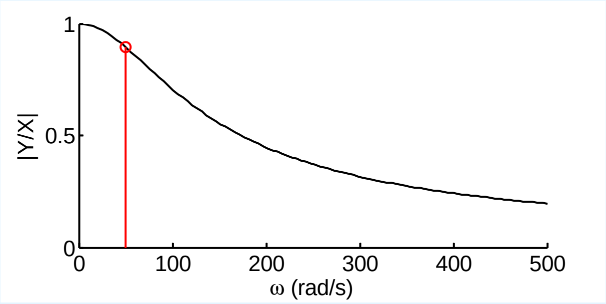

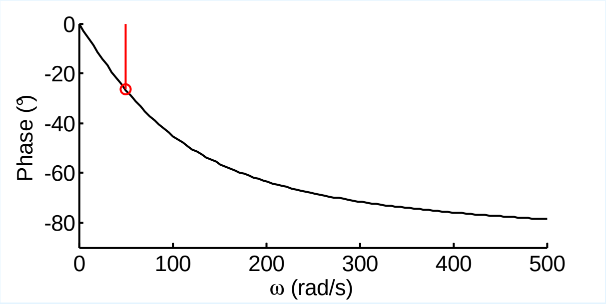

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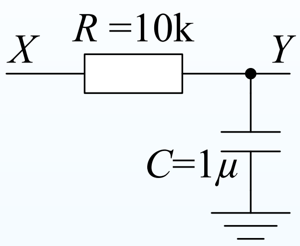

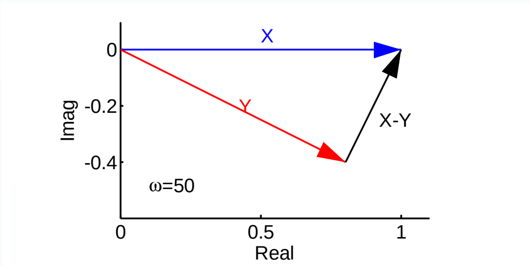

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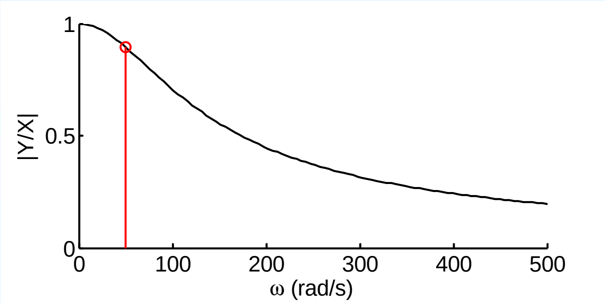

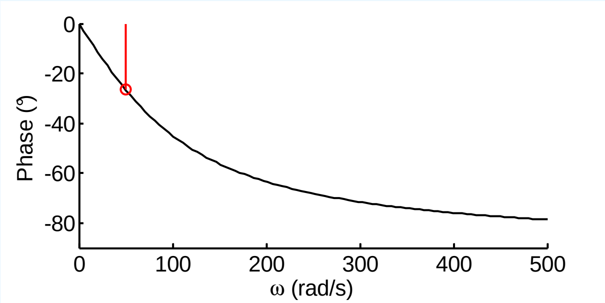

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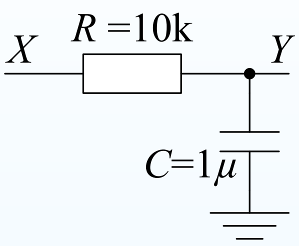

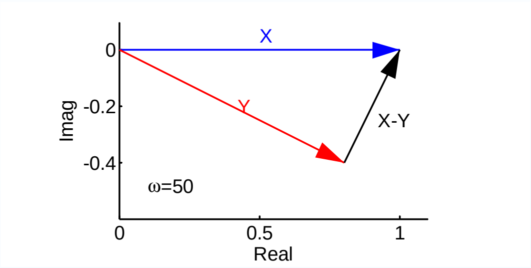

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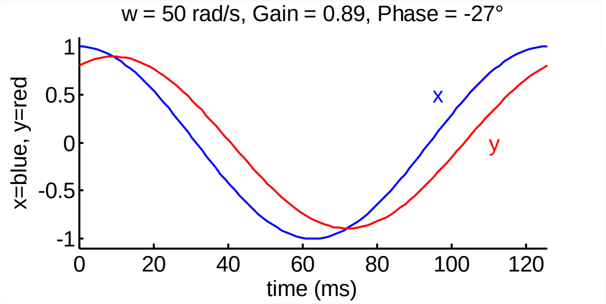

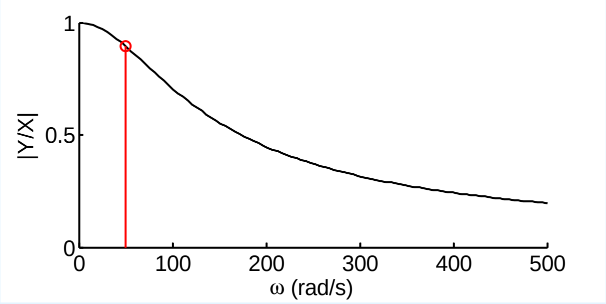

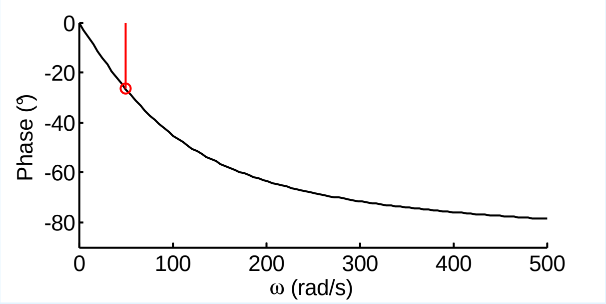

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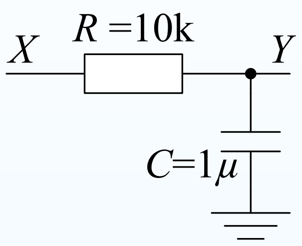

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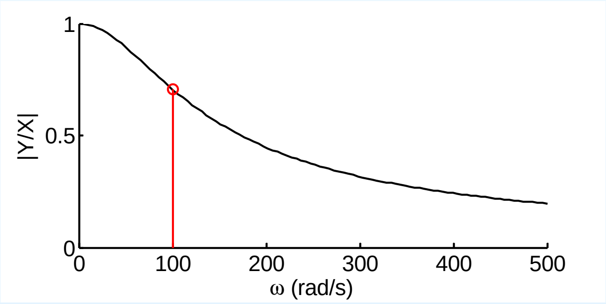

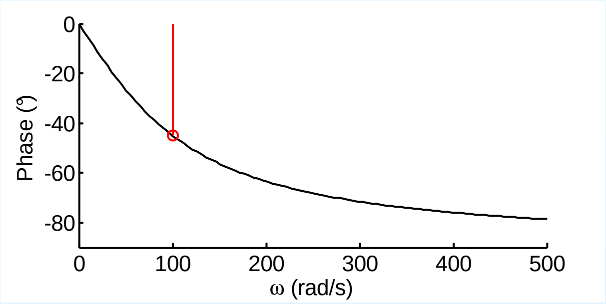

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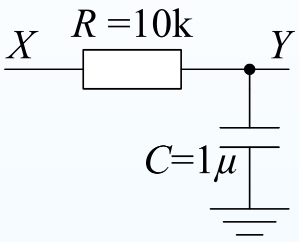

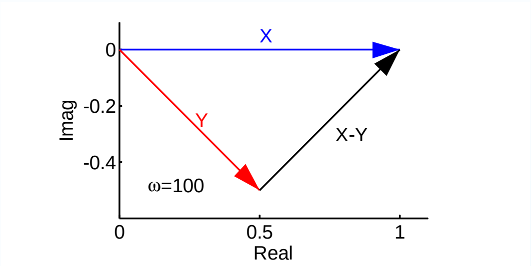

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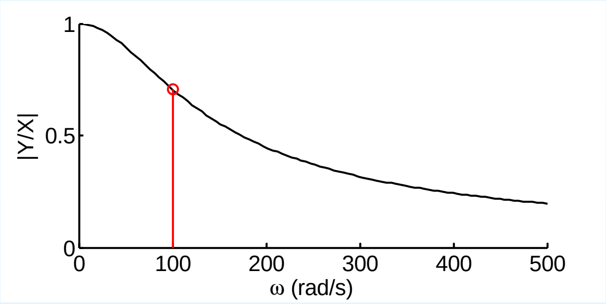

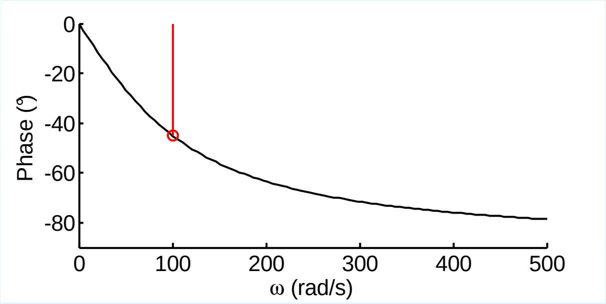

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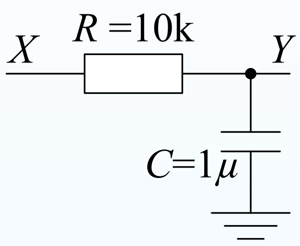

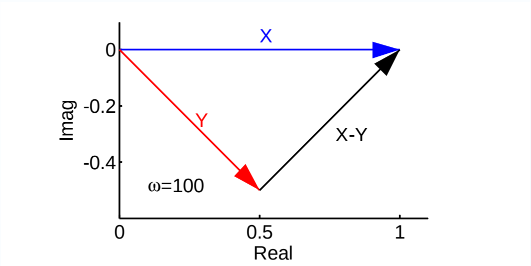

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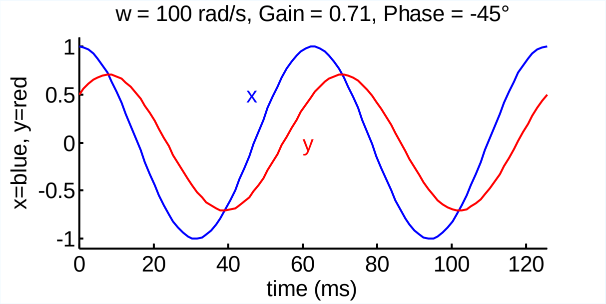

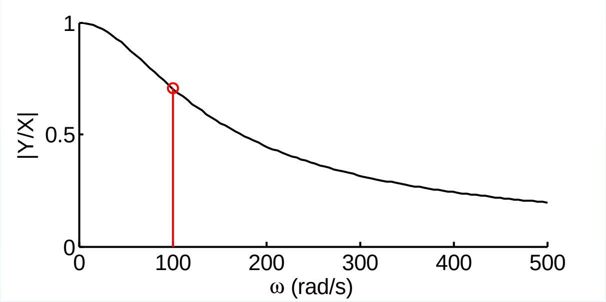

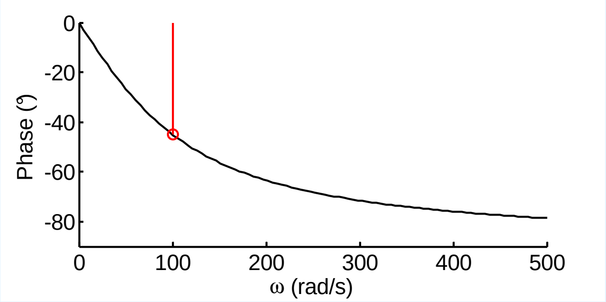

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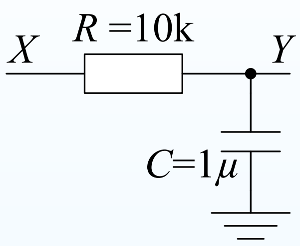

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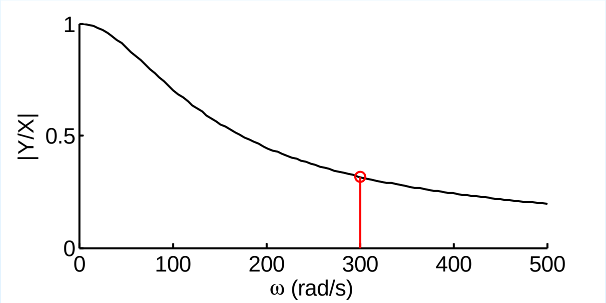

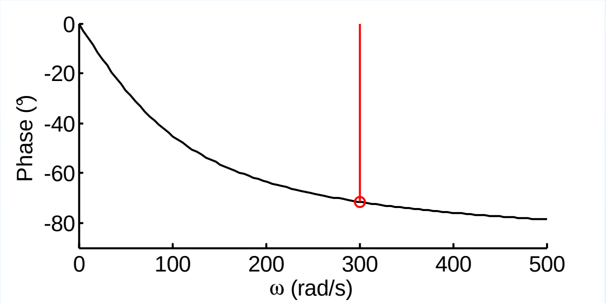

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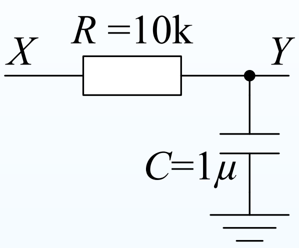

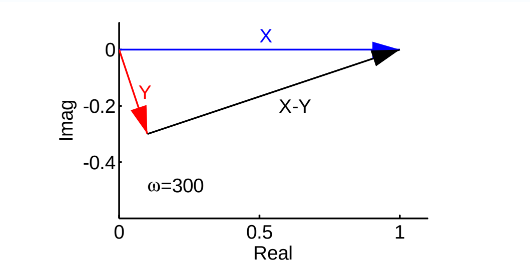

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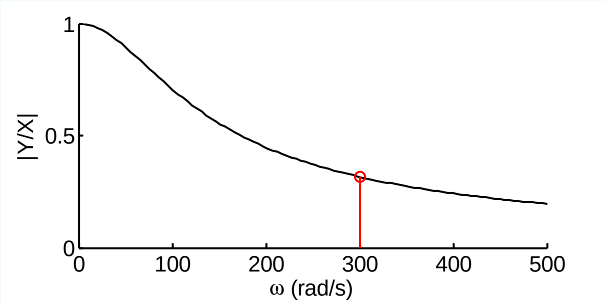
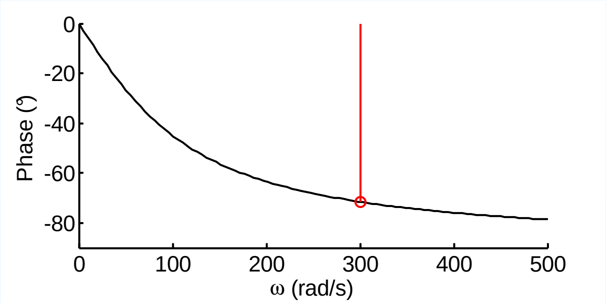
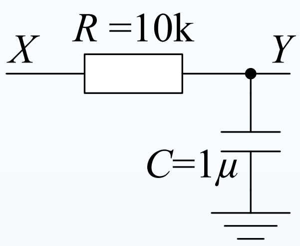


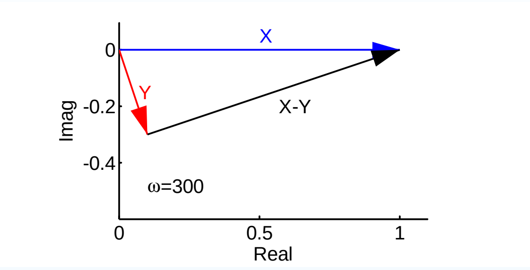


Sine Wave Response

$$RC = 10\,\text{ms}$$

$$\frac{Y}{X} = \frac{1}{j\omega RC+1} = \frac{1}{0.01j\omega+1}$$

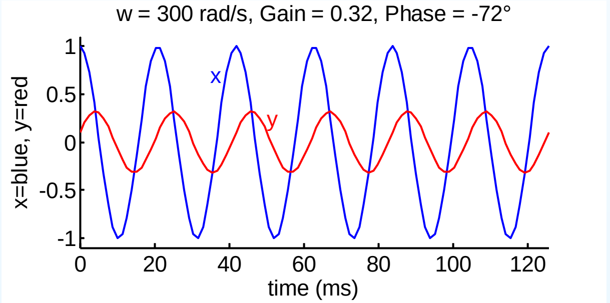


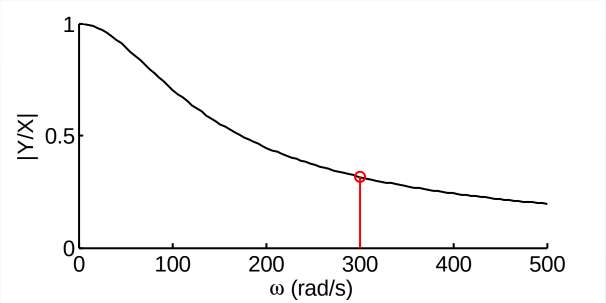


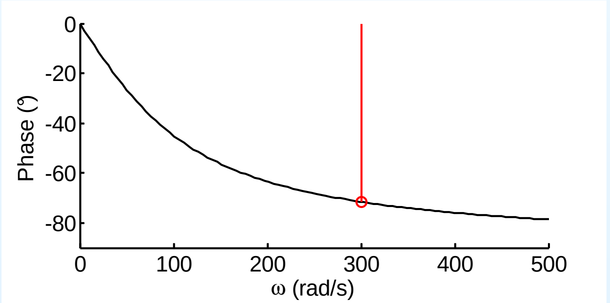
$$\omega = 50 \Rightarrow \frac{Y}{X} = 0.89\angle-27°$$

$$\omega = 100 \Rightarrow \frac{Y}{X} = 0.71\angle-45°$$

$$\omega = 300 \Rightarrow \frac{Y}{X} = 0.32\angle-72°$$







Sine Wave Response

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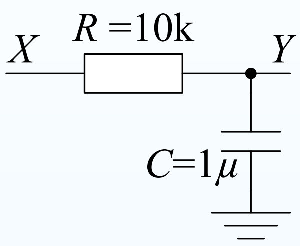

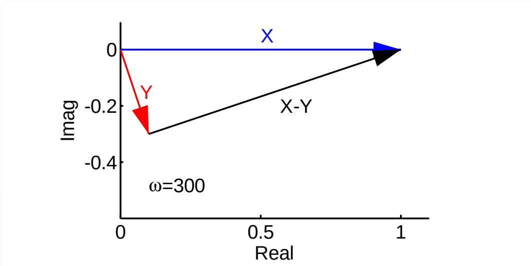

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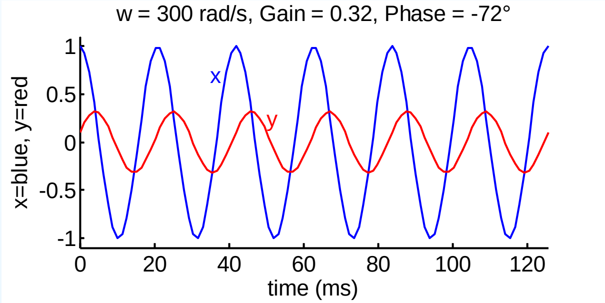

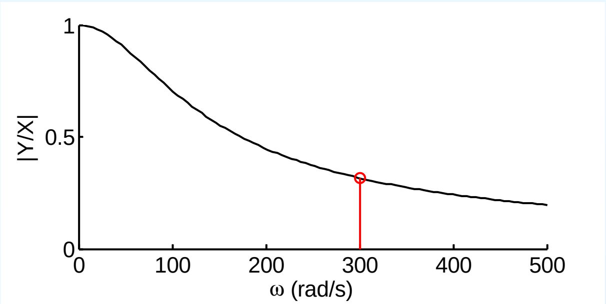

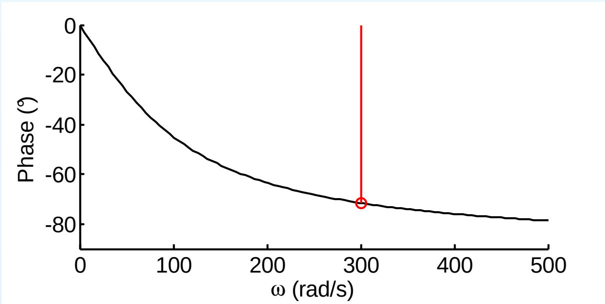

The output, $y(t)$, *lags* the input, $x(t)$, by up to $90°$.

Logarithmic axes

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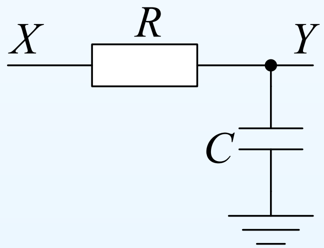

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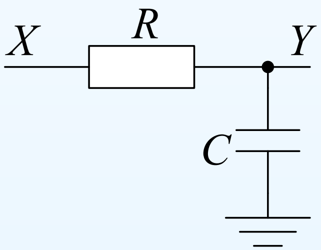

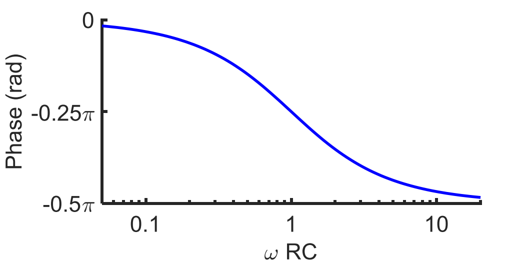

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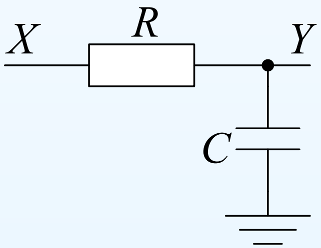

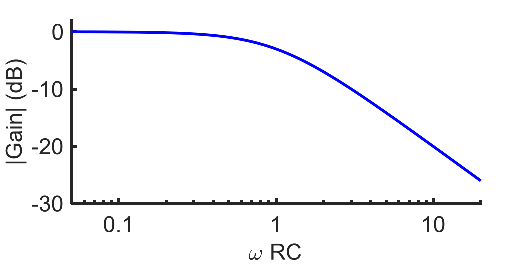

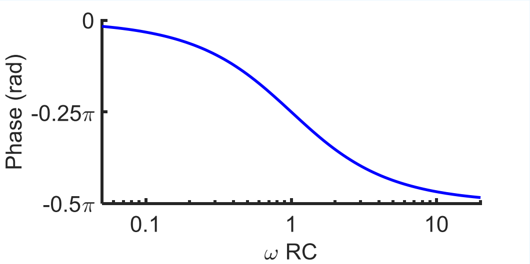

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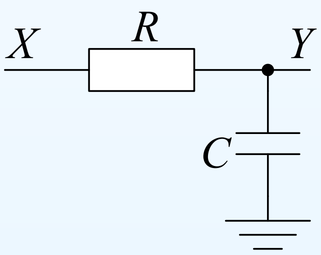

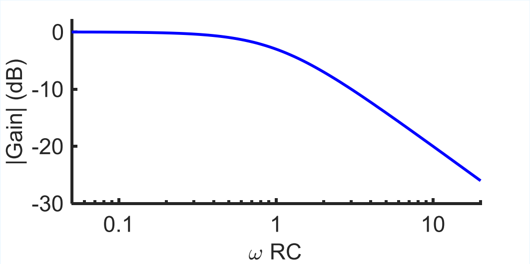

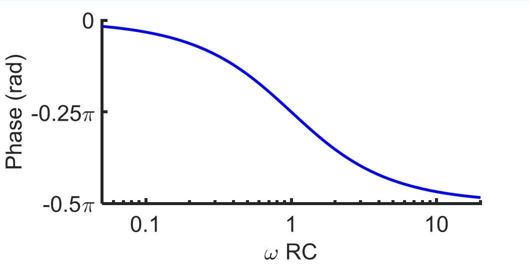

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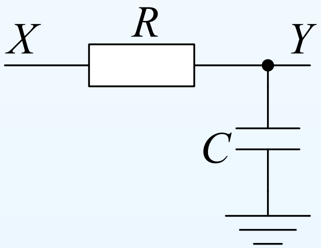

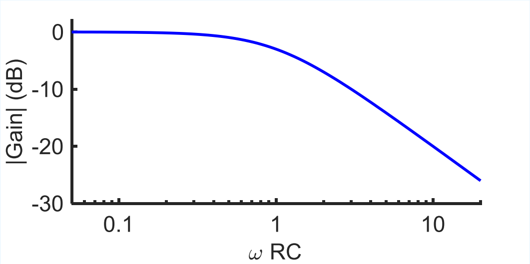

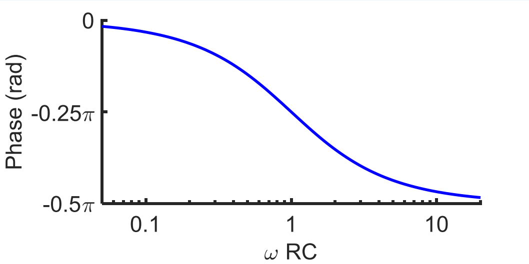

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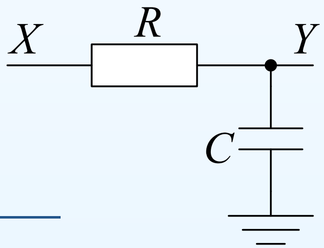
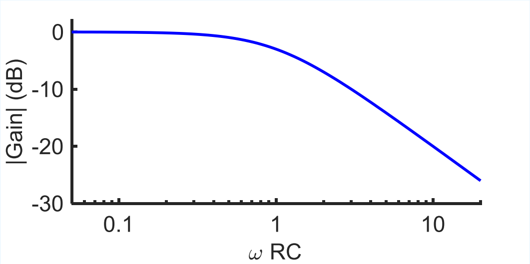
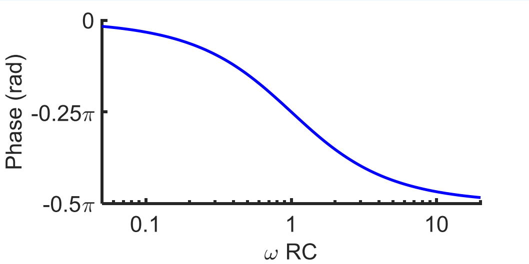
Common voltage ratios:



$\frac{\|V_2\|}{\|V_1\|}$				1				
dB				0				





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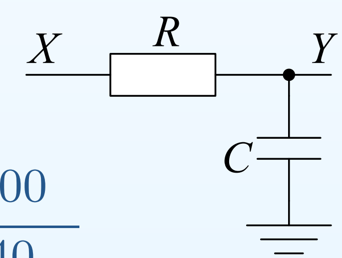
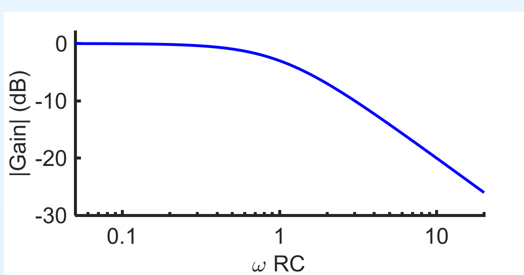
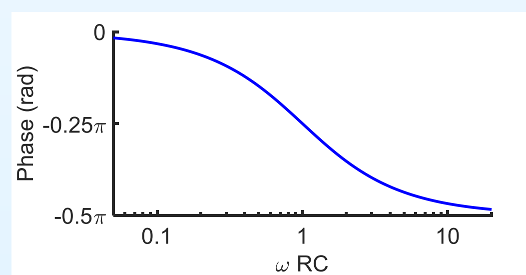


Common voltage ratios:

$\frac{\|V_2\|}{\|V_1\|}$	0.1			1			10	100
dB	-20			0			20	40





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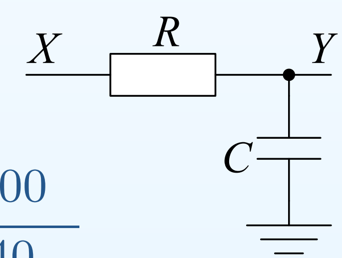
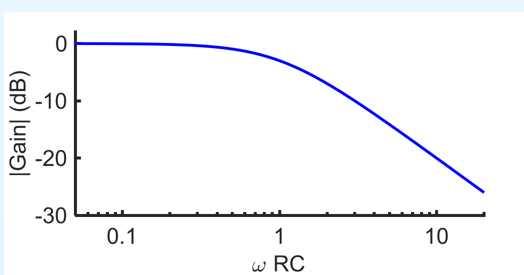
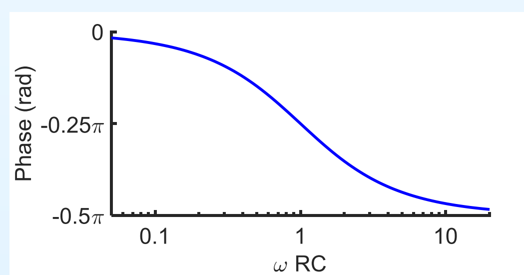


$\frac{\lvert V_2\rvert}{\lvert V_1\rvert}$	0.1	0.5		1		2	10	100
dB	-20	-6		0		6	20	40





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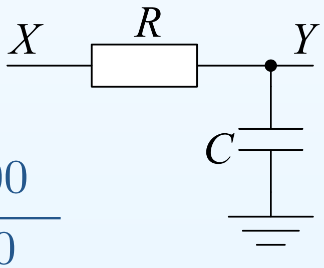
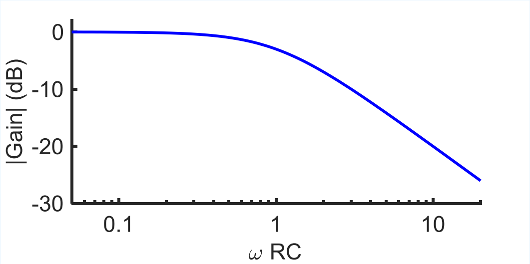
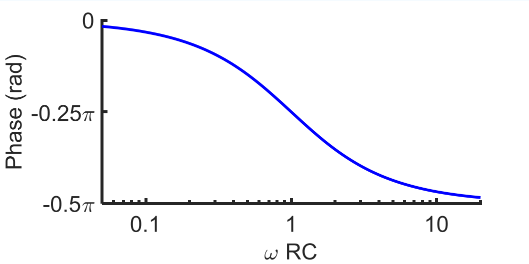


$\frac{\lvert V_2\rvert}{\lvert V_1\rvert}$	0.1	0.5	$\sqrt{0.5}$	1	$\sqrt{2}$	2	10	100
dB	-20	-6	-3	0	3	6	20	40





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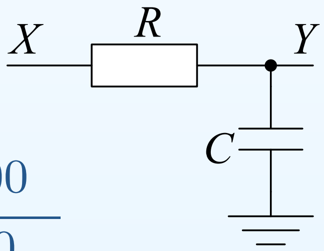

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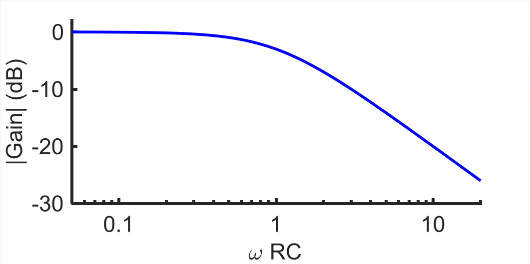

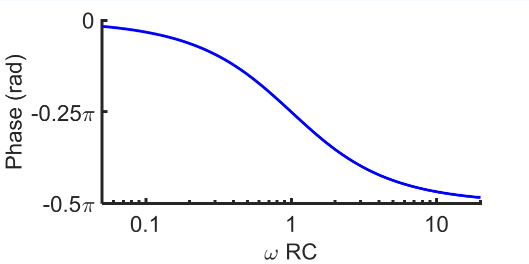

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Note: $P \propto V^2 \Rightarrow$ decibel power ratios are given by $10\log_{10}\frac{P_2}{P_1}$

Logs of Powers

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Suppose we plot the magnitude and phase of $H = c\,(j\omega)^r$

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Magnitude (log-log graph):

$|H| = c\omega^r \Rightarrow \log|H| = \log|c| + r\log\omega$

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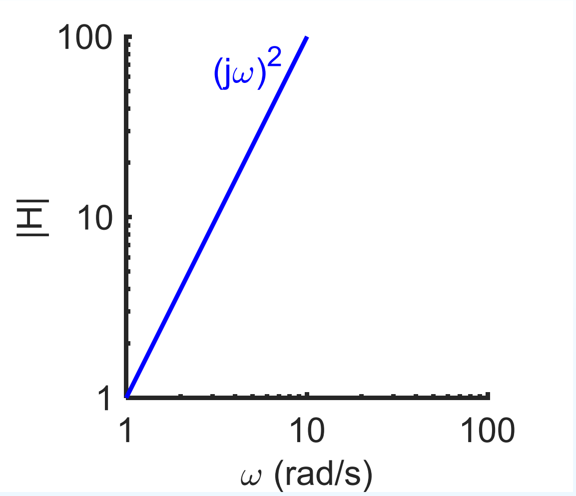

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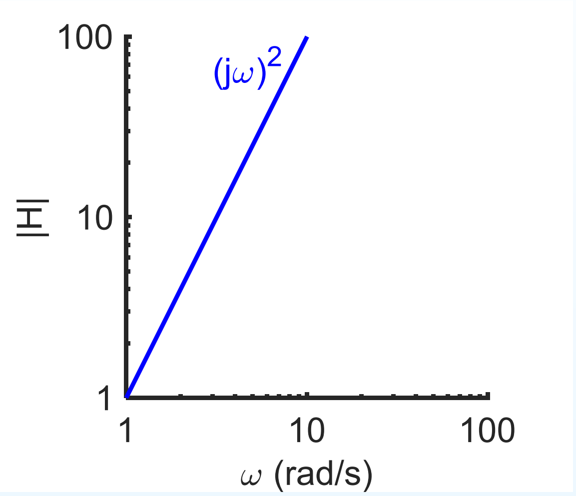

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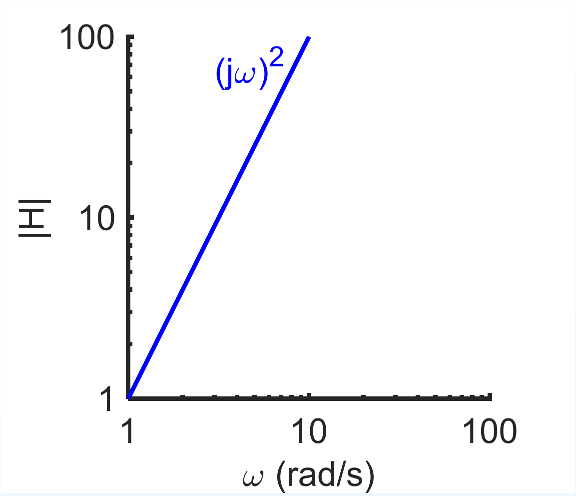

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$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2}$ ($+\pi$ if $c < 0$)

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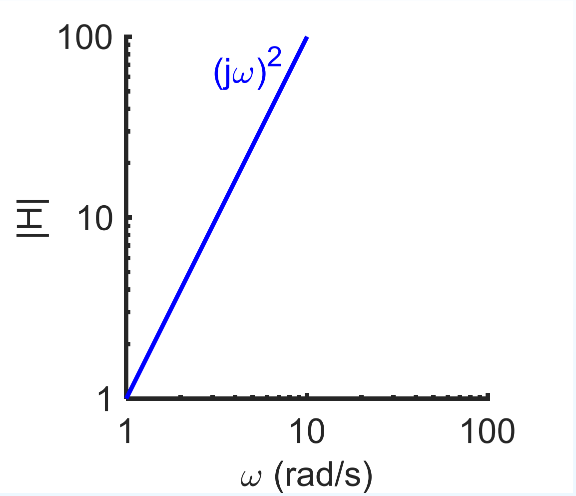

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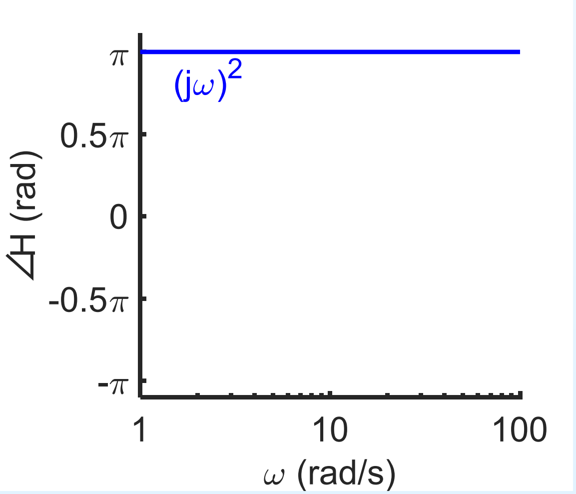

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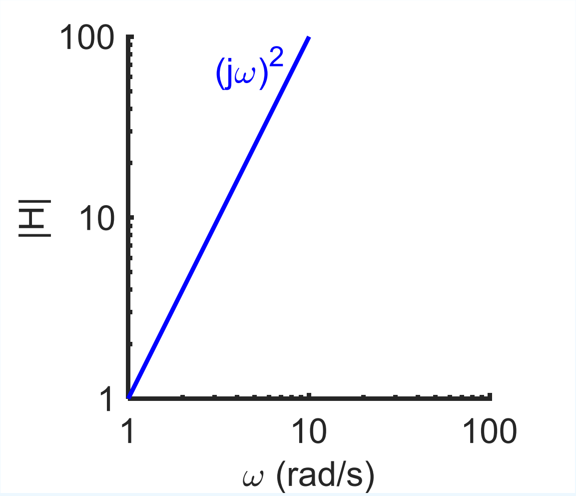

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$\angle H = \angle j^r + \angle c = r \times \frac{\pi}{2}$ ($+\pi$ if $c < 0$)

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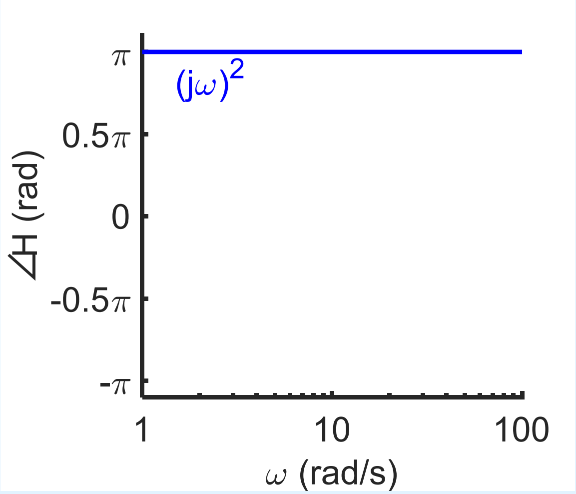

Logs of Powers

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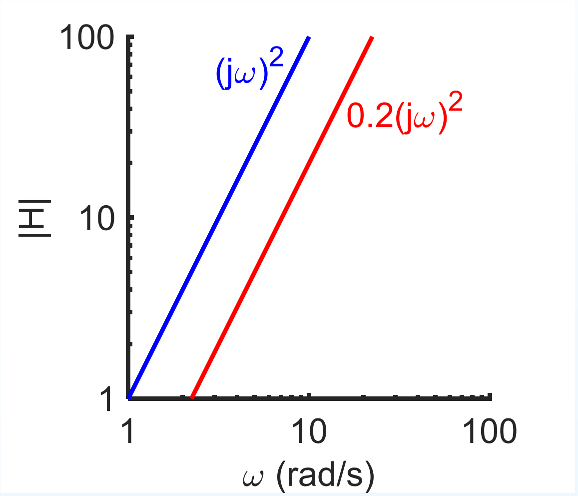

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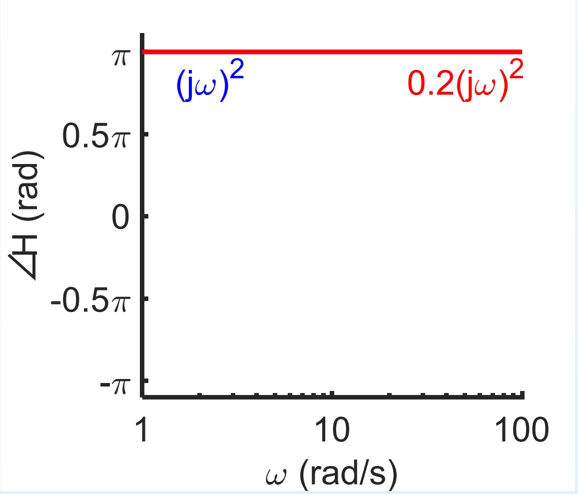

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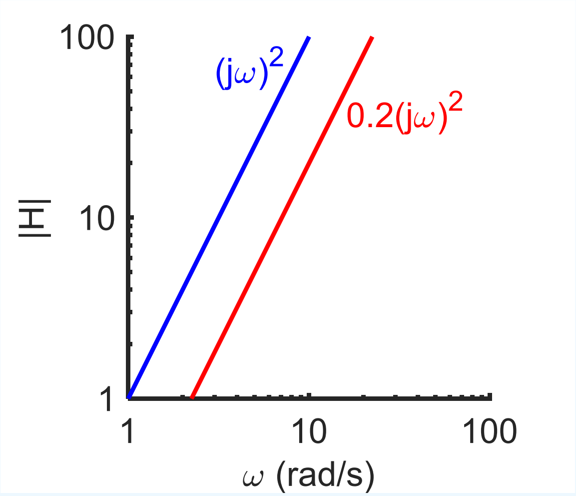

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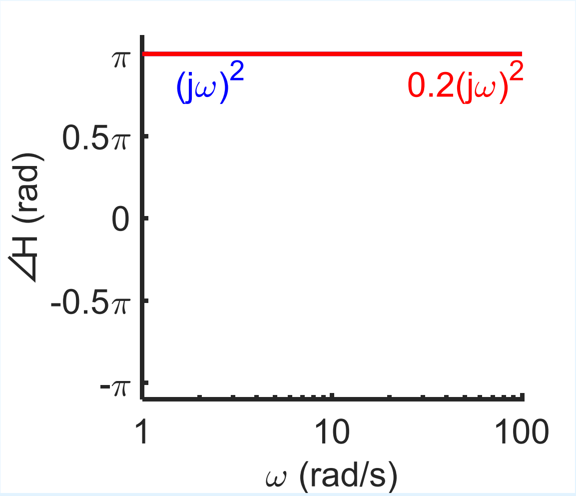

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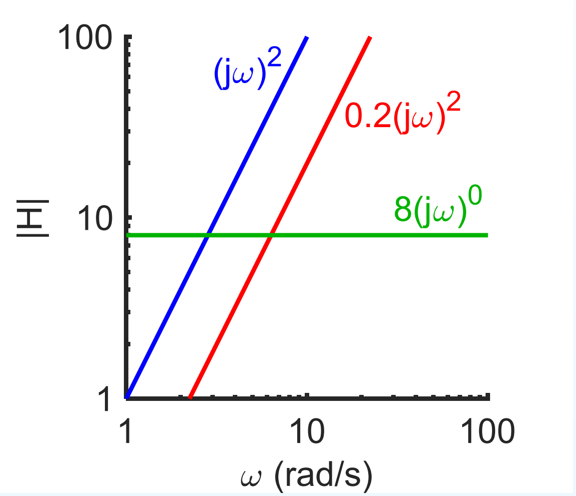

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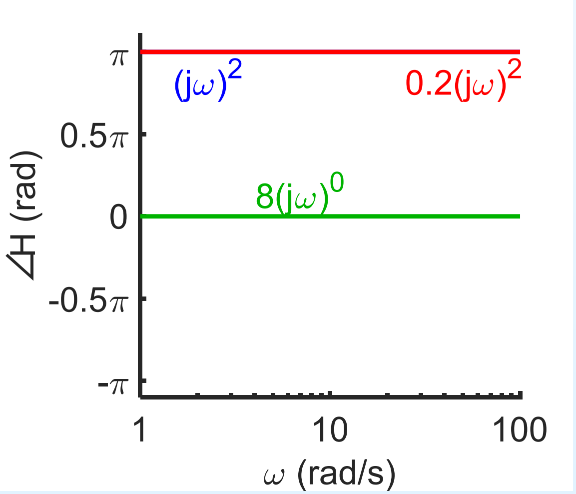

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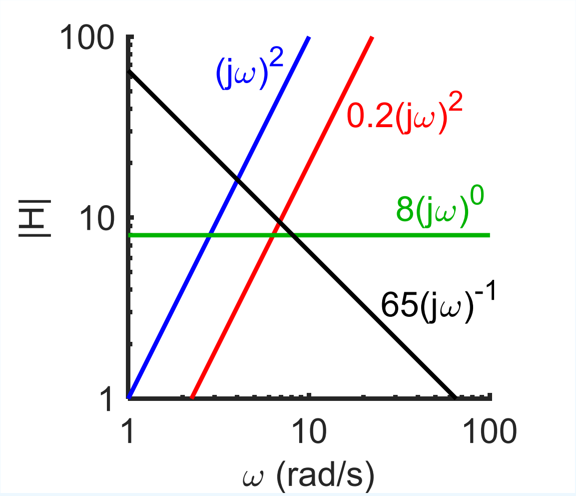

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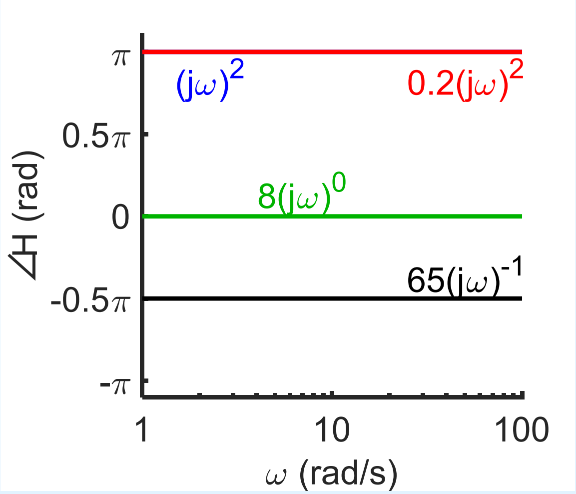

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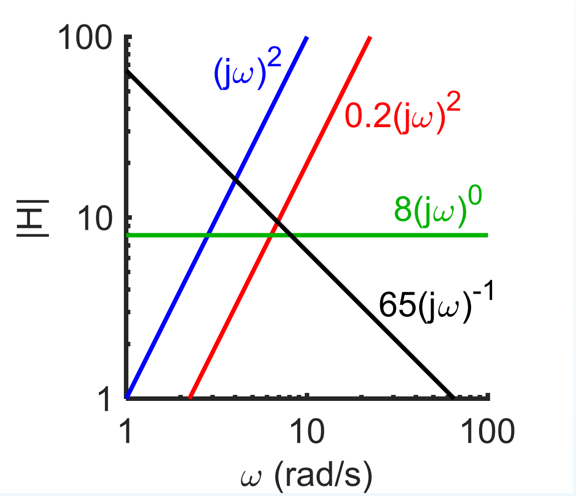

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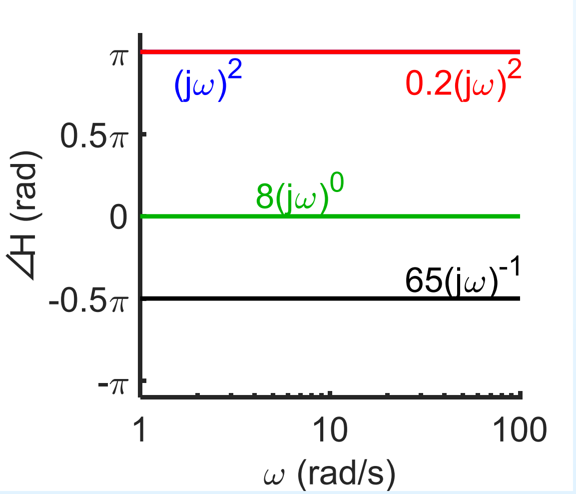

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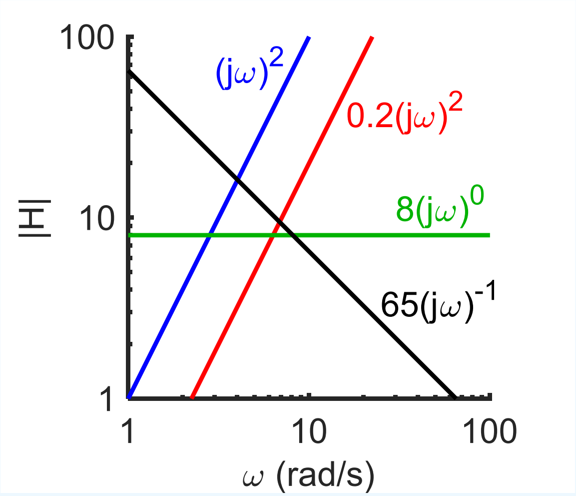

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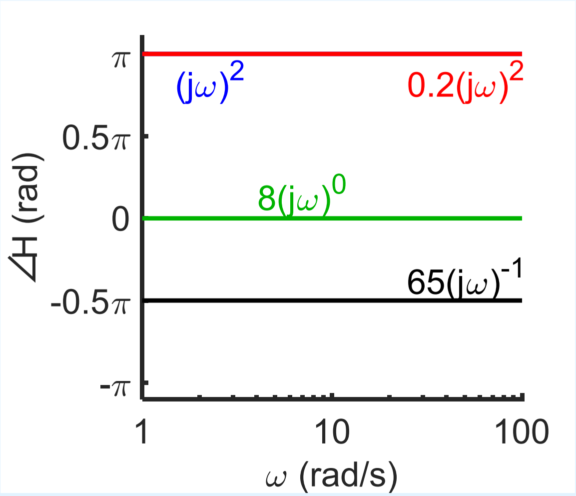

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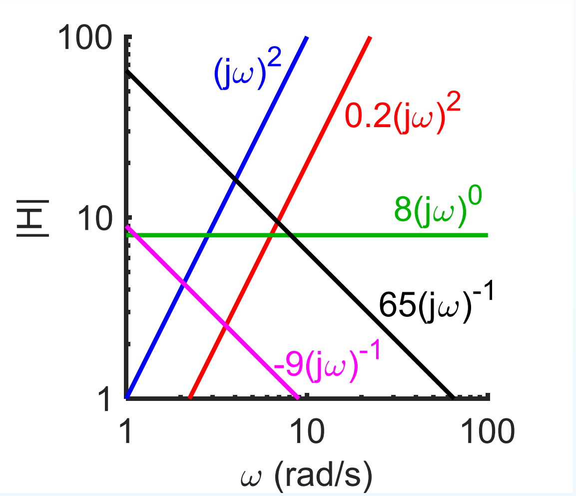

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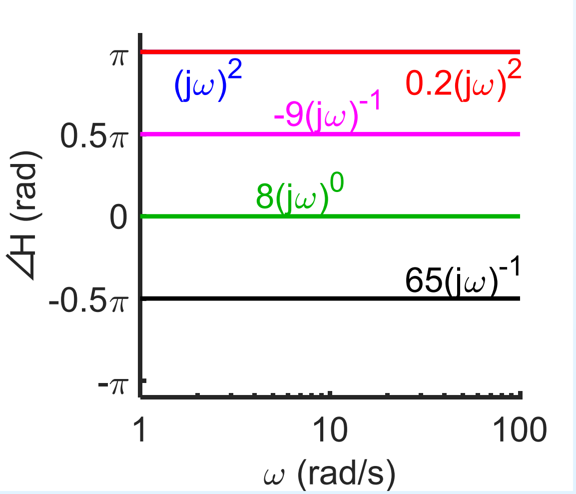

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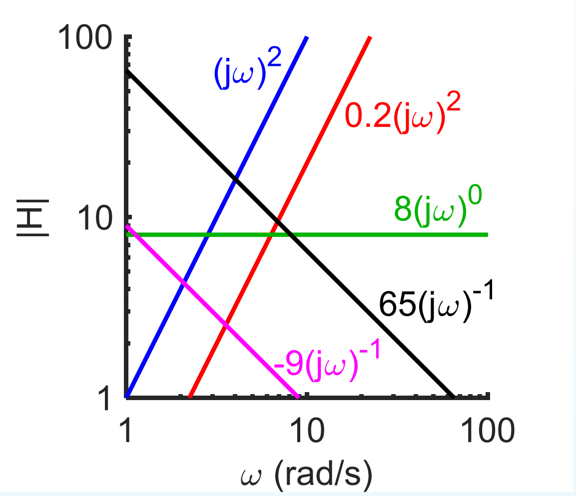

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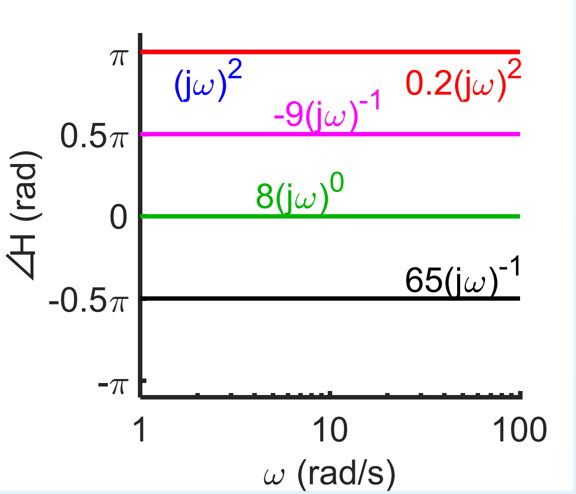

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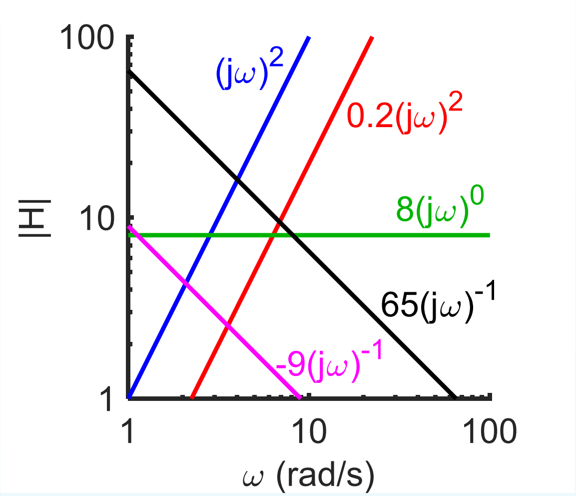

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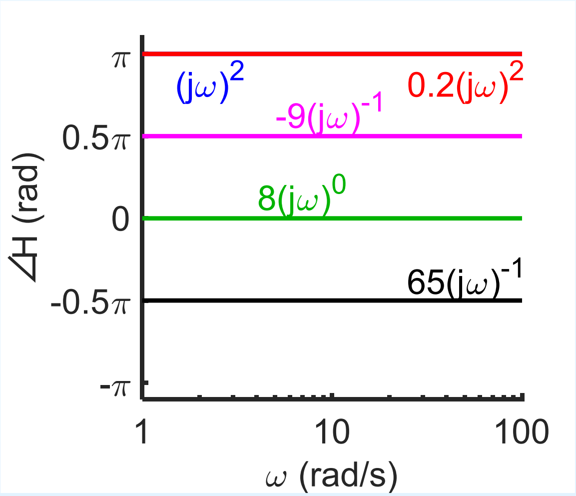

Straight Line Approximations

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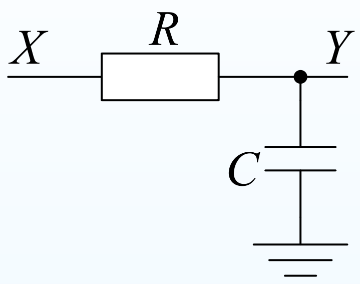

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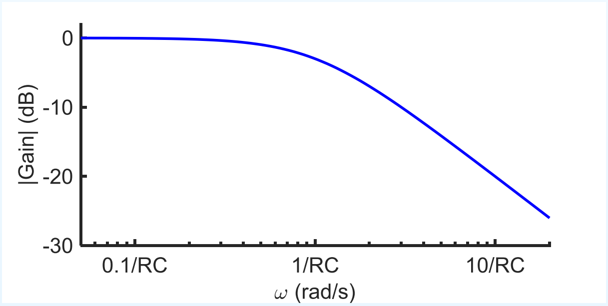

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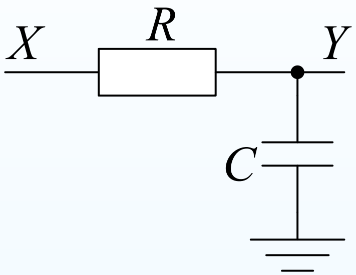

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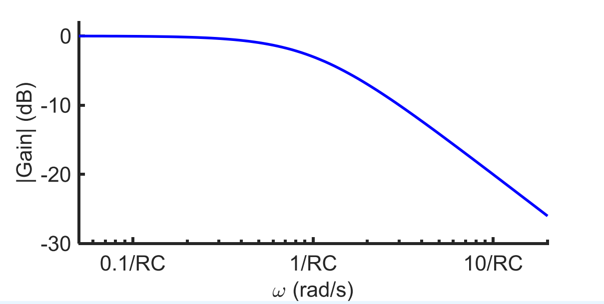

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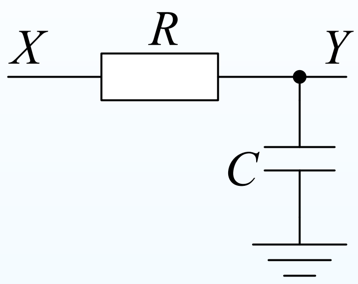

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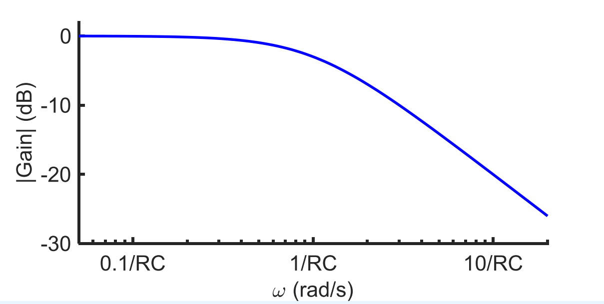

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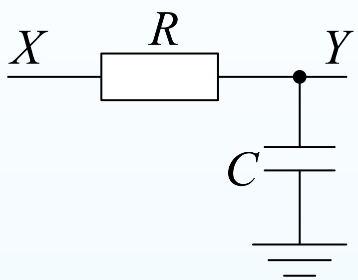

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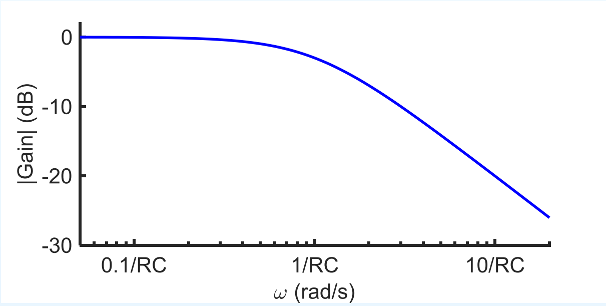

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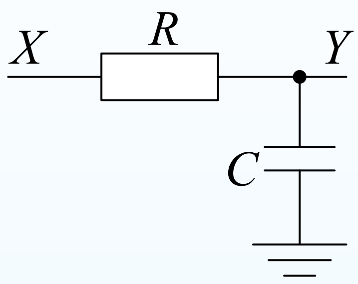

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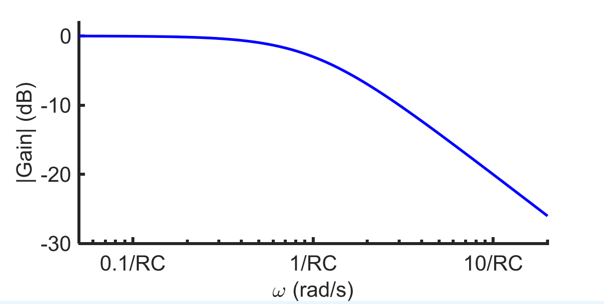

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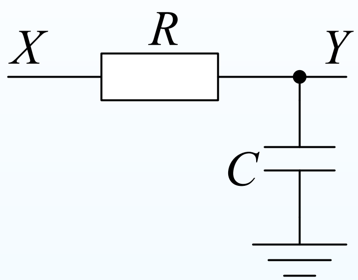

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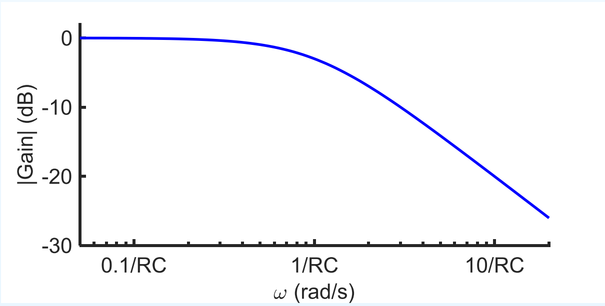

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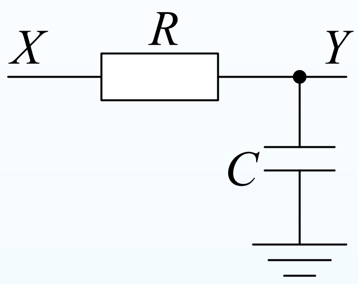

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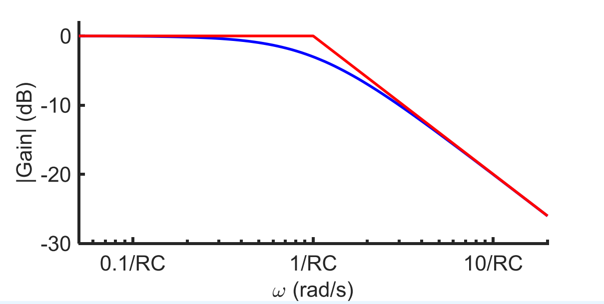

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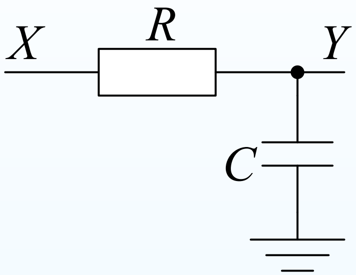

Gain: $H(j\omega) = \frac{1}{j\omega RC+1}$

Low frequencies ($\omega \ll \frac{1}{RC}$): $H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1$
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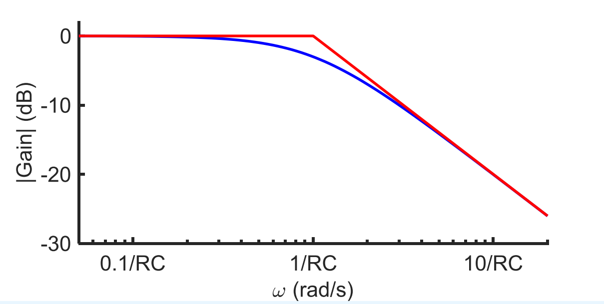
Approximate the magnitude response as two straight lines intersecting at the *corner frequency*, $\omega_c = \frac{1}{RC}$.



Straight Line Approximations

11: Frequency Responses
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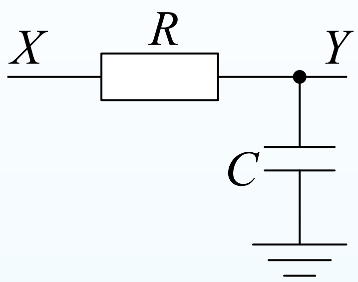

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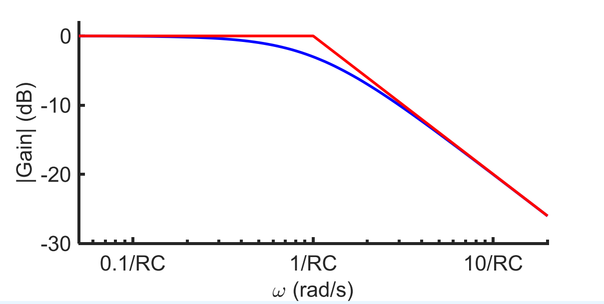

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(a) the gradient changes by -1 ($= -6$ dB/octave $= -20$ dB/decade).

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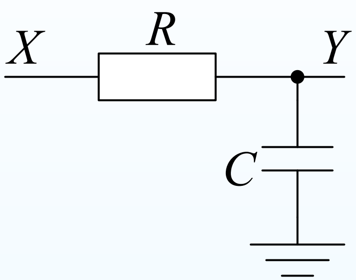

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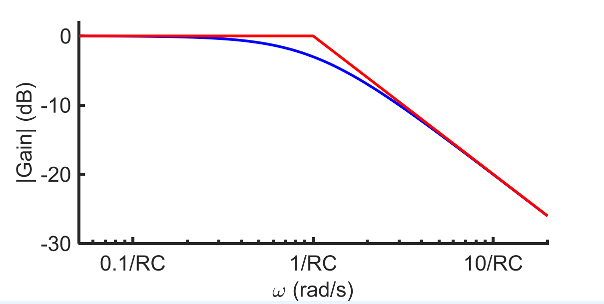

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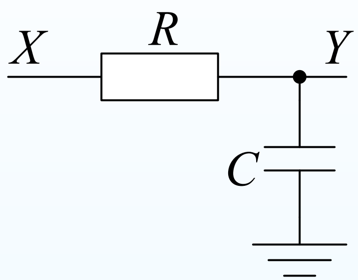

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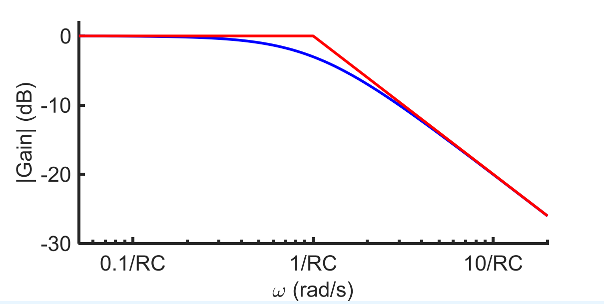

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A linear factor $(aj\omega + b)$ has a corner frequency of $\omega_c = \left|\frac{b}{a}\right|$.

Plot Magnitude Response

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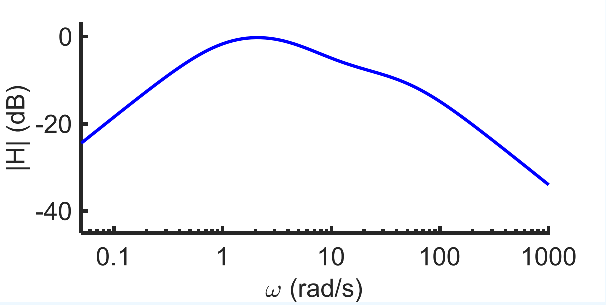
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$$



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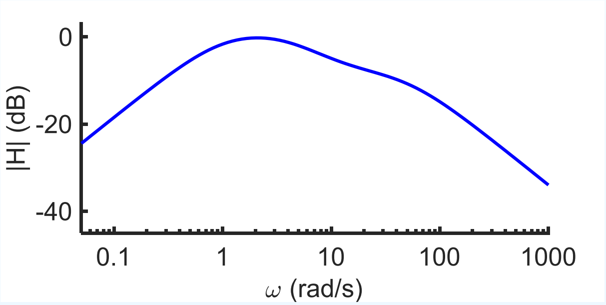

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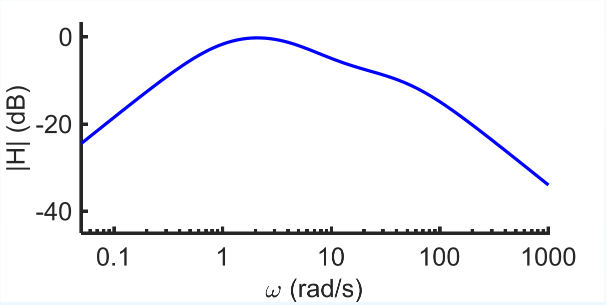

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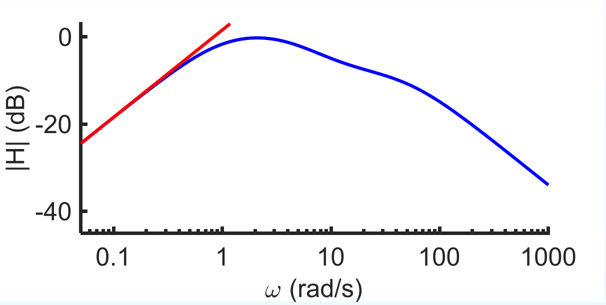

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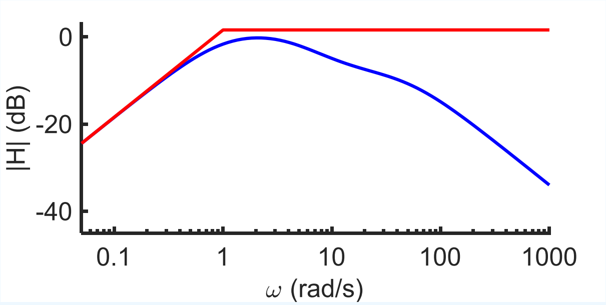

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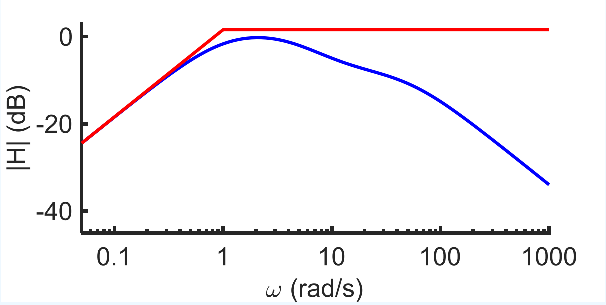

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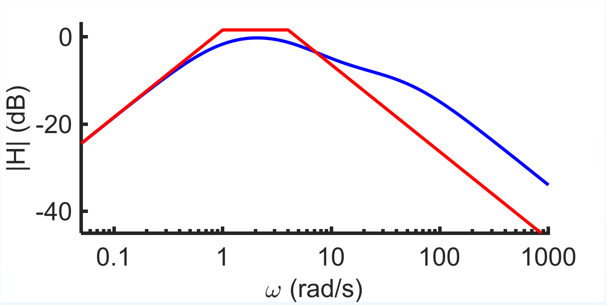

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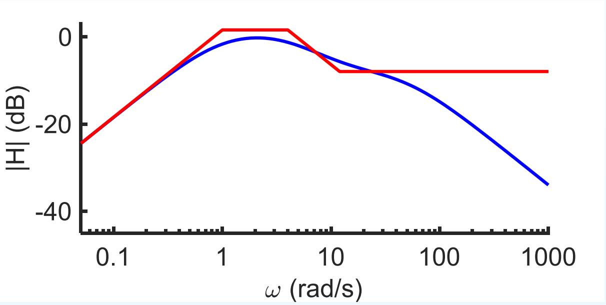

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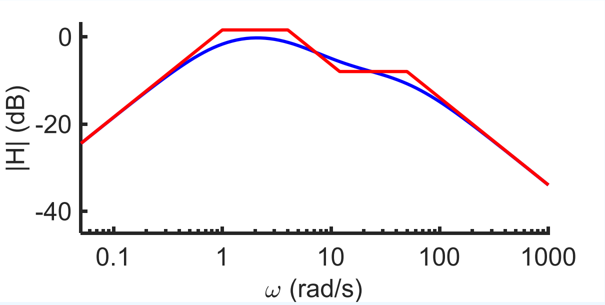

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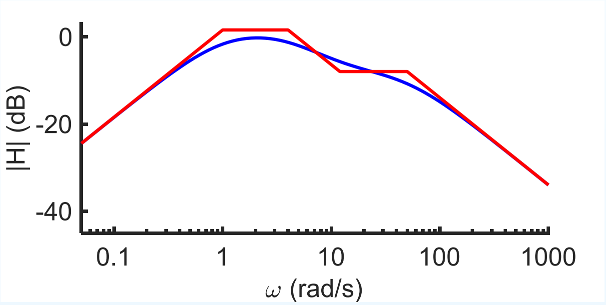

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At each corner frequency, the graph is continuous but its gradient changes abruptly by $+1$ (numerator factor) or -1 (denominator factor).

Low and High Frequency Asymptotes

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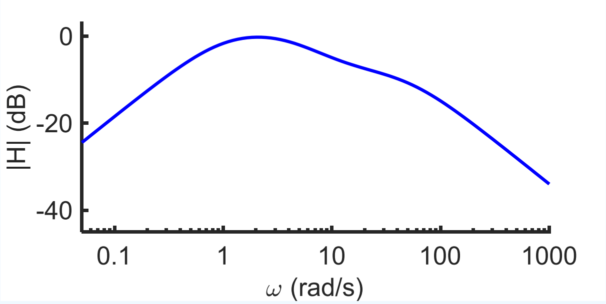

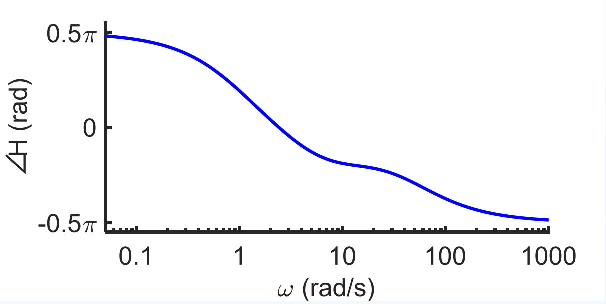

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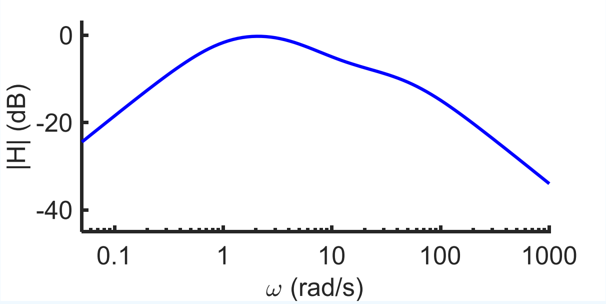

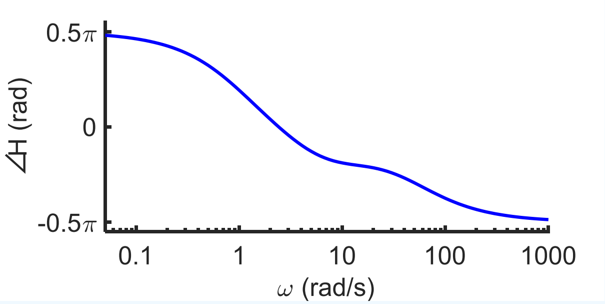

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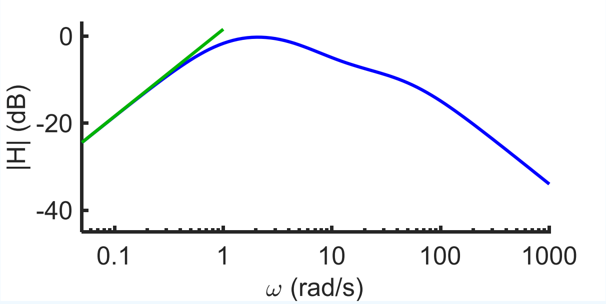

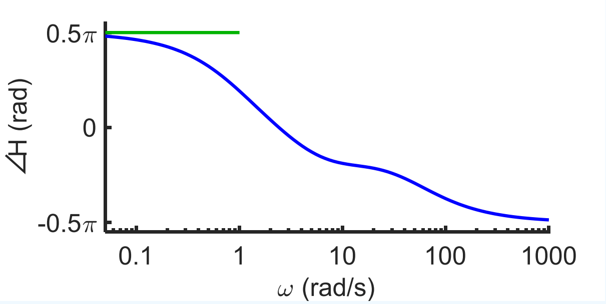

Low Frequency Asymptote:

From factors: $H_{\text{LF}}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega$

Low and High Frequency Asymptotes

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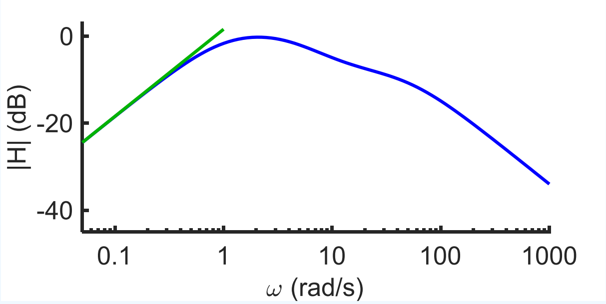
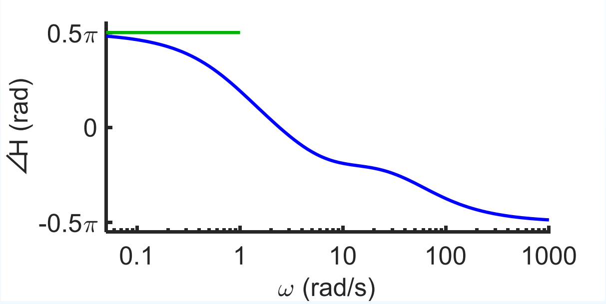
You can find the low and high frequency asymptotes without factorizing:

$$H(j\omega) = \frac{60(j\omega)^2+720(j\omega)}{3(j\omega)^3+165(j\omega)^2+762(j\omega)+600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$





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Lowest power of $j\omega$ on top and bottom: $H(j\omega) \simeq \frac{720(j\omega)}{600} = 1.2j\omega$

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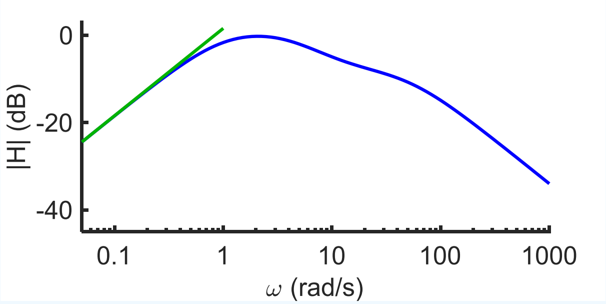

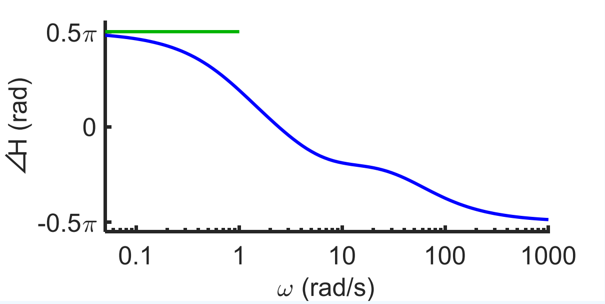

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Low and High Frequency Asymptotes

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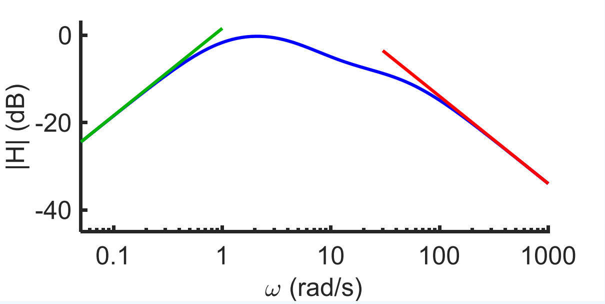

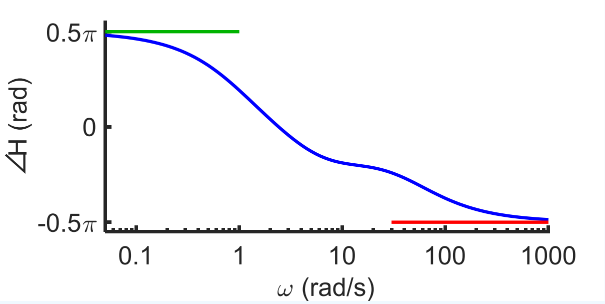

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High Frequency Asymptote:

From factors: $H_{\text{HF}}(j\omega)=\frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)}=20(j\omega)^{-1}$

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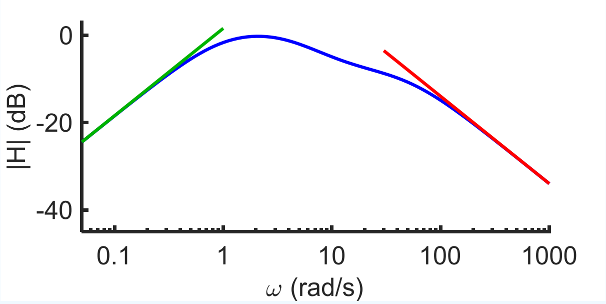

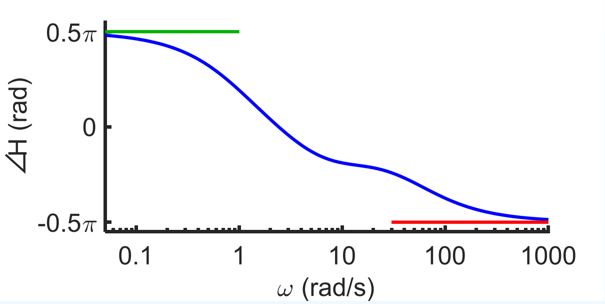

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High Frequency Asymptote:

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Highest power of $j\omega$ on top and bottom: $H\left(j\omega\right) \simeq \frac{60(j\omega)^2}{3(j\omega)^3} = 20\left(j\omega\right)^{-1}$

Phase Approximation

+

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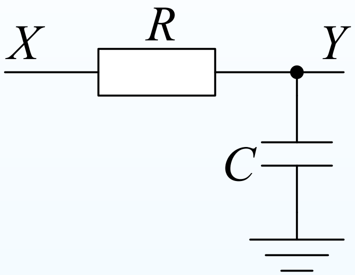
Gain: $H(j\omega) = \frac{1}{j\omega RC+1}$



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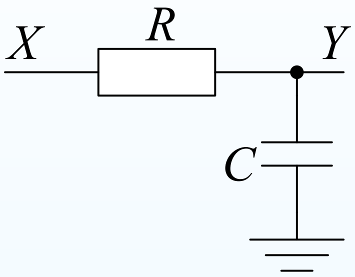
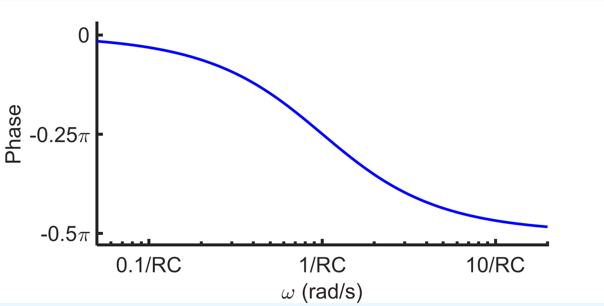
Low frequencies ($\omega \ll \frac{1}{RC}$):

$$H(j\omega) \approx 1$$





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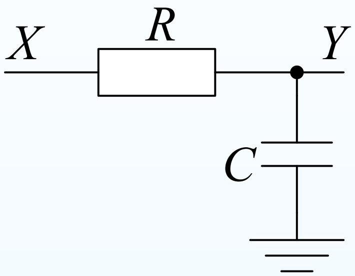
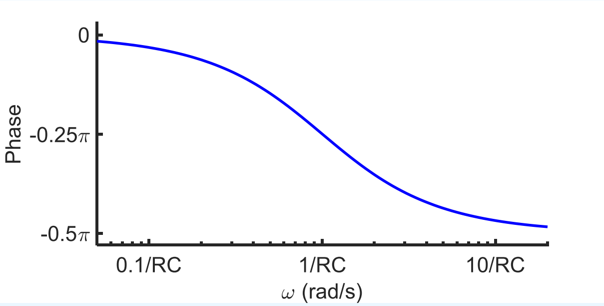


Low frequencies ($\omega \ll \frac{1}{RC}$):

$$H(j\omega) \approx 1 \Rightarrow \angle 1 = 0$$



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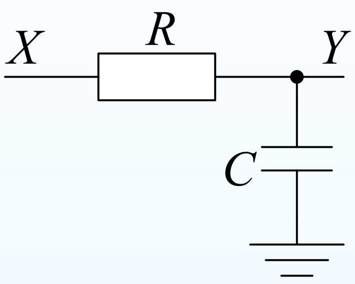

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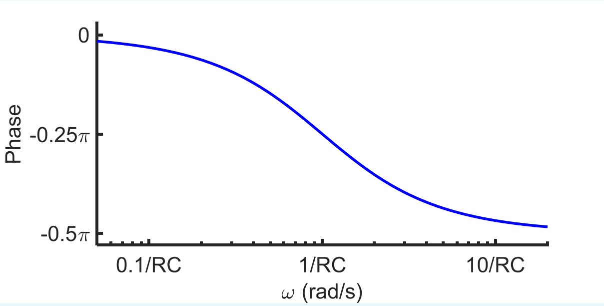

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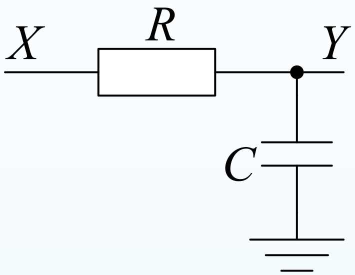

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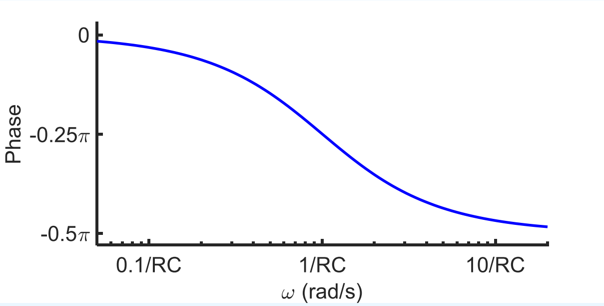

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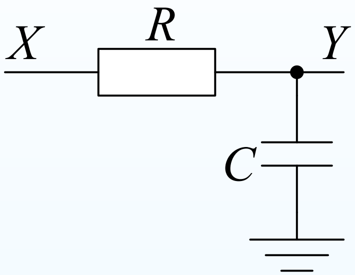

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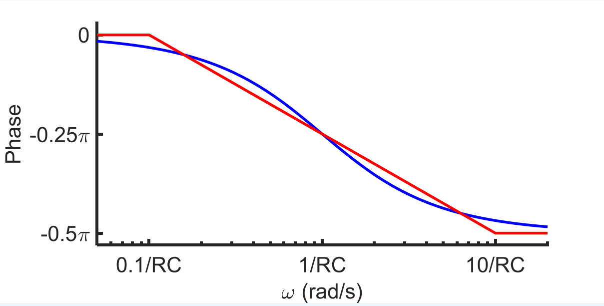
Approximate the phase response as three straight lines.



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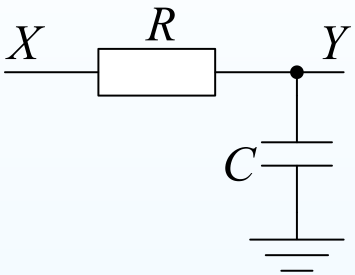

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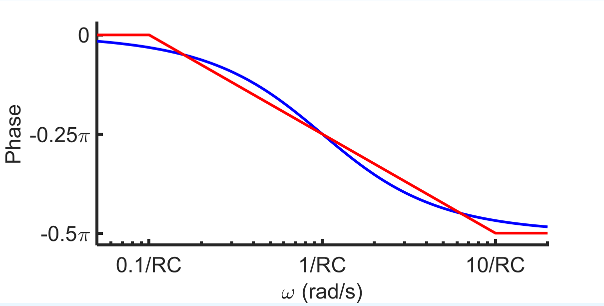

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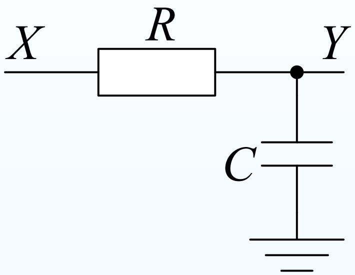

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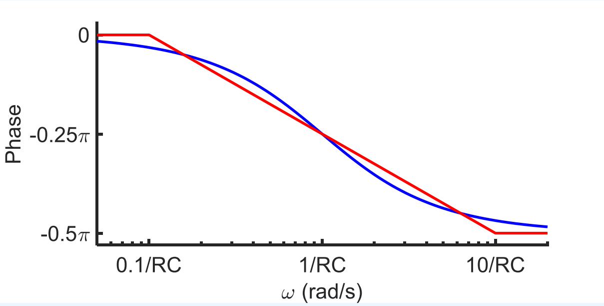

Between $0.1\omega_c$ and $10\omega_c$ the phase changes by $-\frac{\pi}{2}$ over two decades. This gives a gradient = $-\frac{\pi}{4}$ radians/decade.

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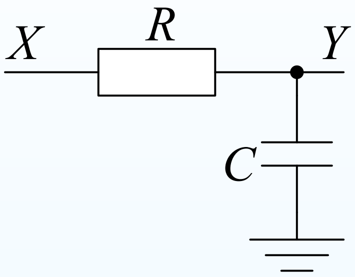

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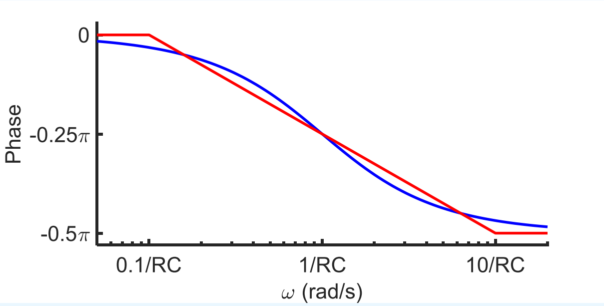

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$(aj\omega + b)$ in denominator

$\Rightarrow \Delta$gradient $= \mp\frac{\pi}{4}$/decade at $\omega = 10^{\mp 1}\left|\frac{b}{a}\right|$.

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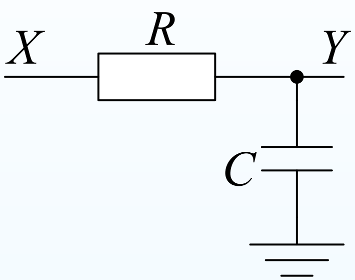

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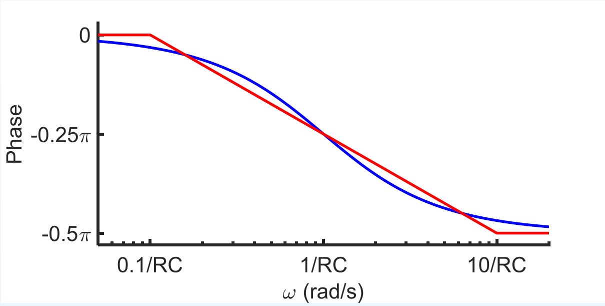

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The sign of Δgradient is reversed for (a) numerator factors and (b) $\frac{b}{a} < 0$.

Plot Phase Response

+

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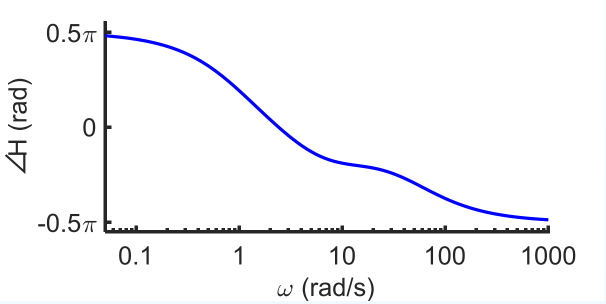

Plot Phase Response

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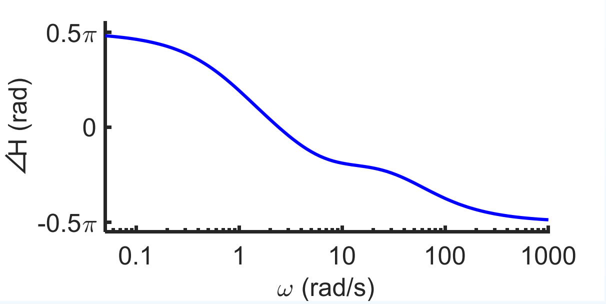
Step 1: Factorize the polynomials

Step 2: List corner freqs: $\pm =$ num/den

$$\omega_c = \{1^-, 4^-, 12^+, 50^-\}$$



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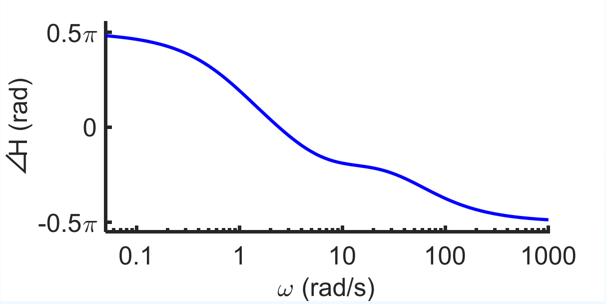
Step 3: Gradient changes at $10^{\mp 1}\omega_c$.

Sign depends on num/den and sgn $\left(\frac{b}{a}\right)$:

$.1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+$



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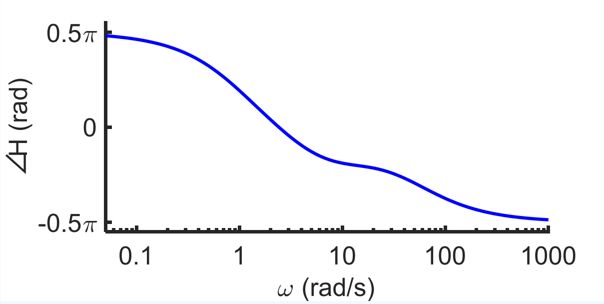

Step 4: Put in ascending order and calculate gaps as $\log_{10}\frac{\omega_2}{\omega_1}$ decades:

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Plot Phase Response

11: Frequency Responses
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- Phase Approximation +
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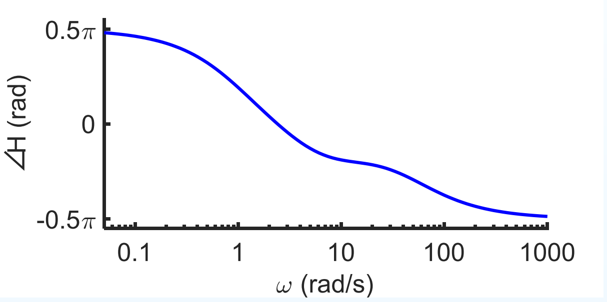
- Summary

$$H(j\omega) = \frac{60(j\omega)^2+720(j\omega)}{3(j\omega)^3+165(j\omega)^2+762(j\omega)+600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$



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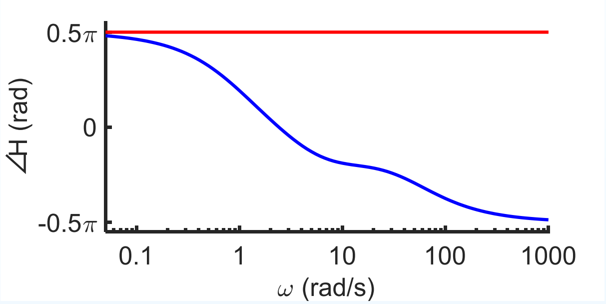

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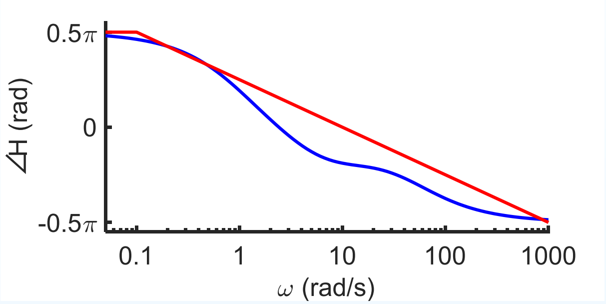

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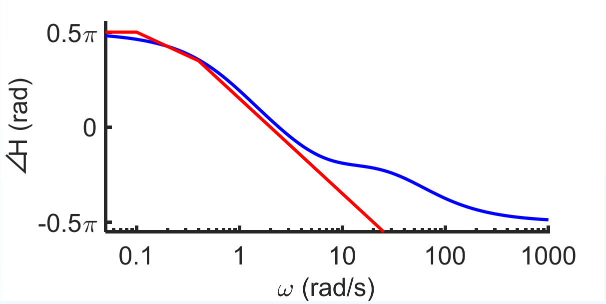

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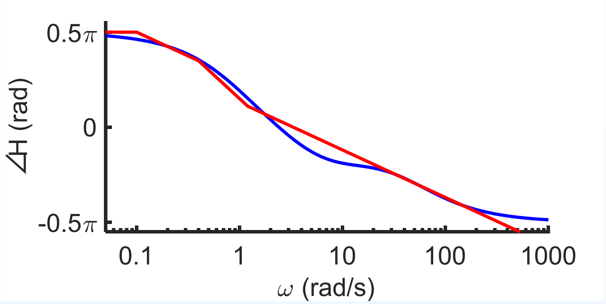

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Plot Phase Response

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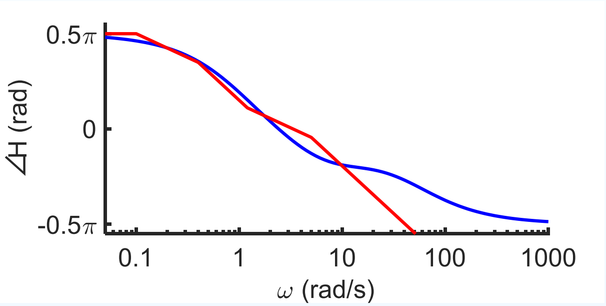

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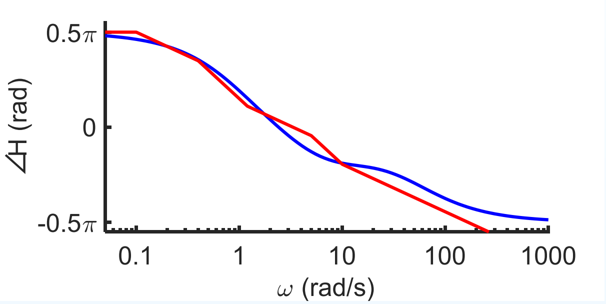

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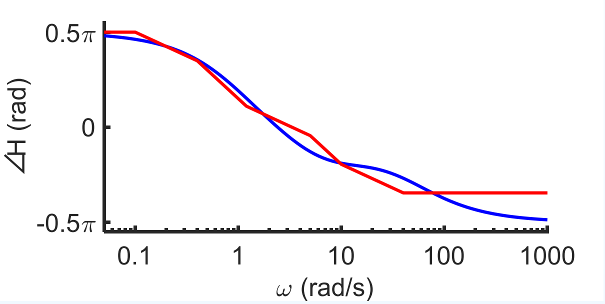

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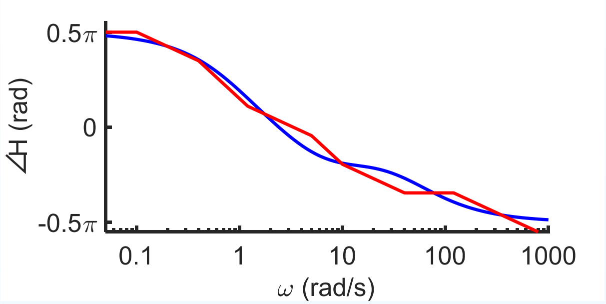

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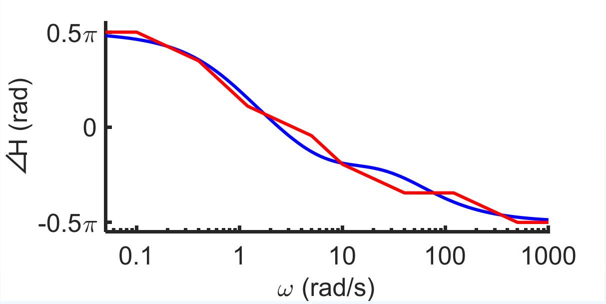

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Steps 9-13: Repeat for each gradient change. Final gradient is always 0.

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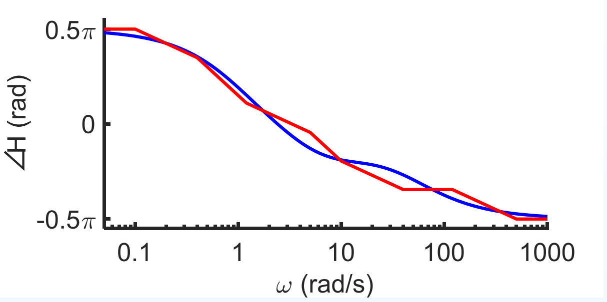

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Steps 9-13: Repeat for each gradient change. Final gradient is always 0.

At 0.1 and 10 times each corner frequency, the graph is continuous but its gradient changes abruptly by $\pm\frac{\pi}{4}$ rad/decade.

RCR Circuit

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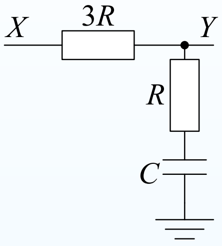

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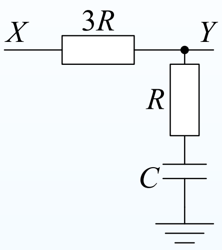
$$\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}}$$



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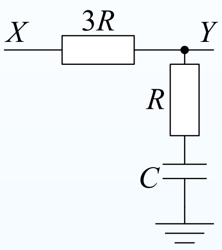

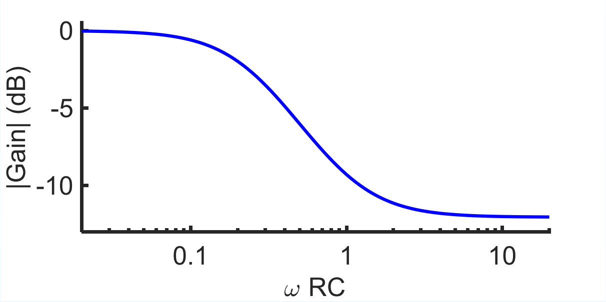

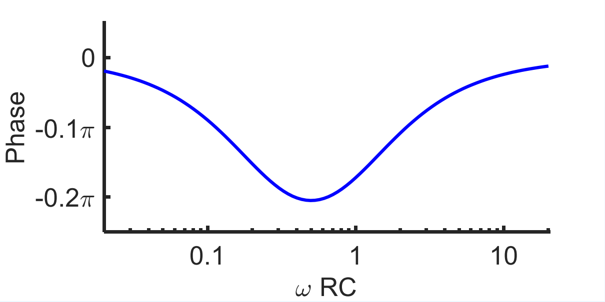

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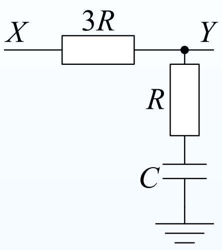
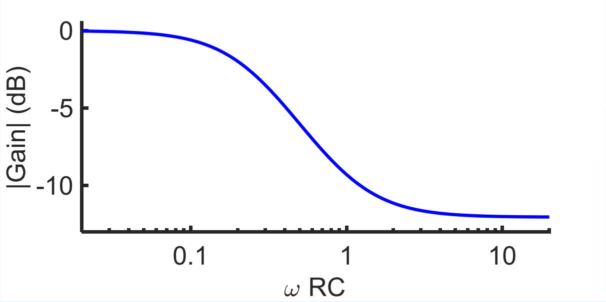
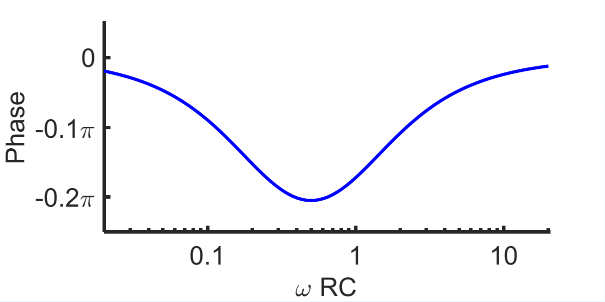


Corner freqs: $\frac{0.25}{RC}^{-}, \frac{1}{RC}^{+}$





RCR Circuit

11: Frequency Responses

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers +
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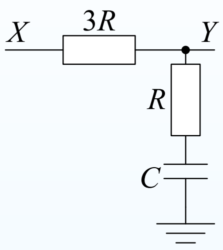
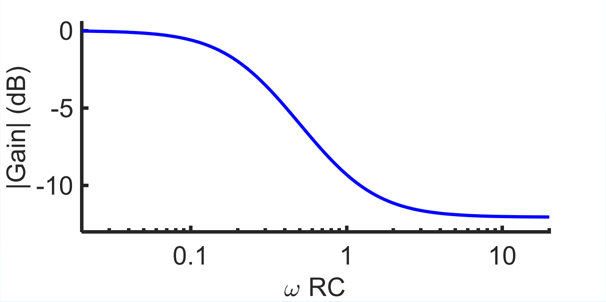
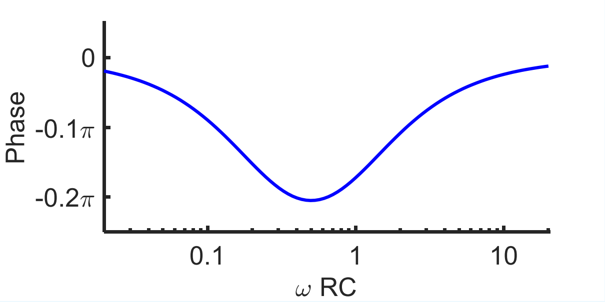
$$\frac{Y}{X} = \frac{R+\frac{1}{j\omega C}}{3R+R+\frac{1}{j\omega C}} = \frac{j\omega RC+1}{4j\omega RC+1}$$



Corner freqs: $\frac{0.25}{RC}^{-}, \frac{1}{RC}^{+}$ LF Asymptote: $H(j\omega) = 1$





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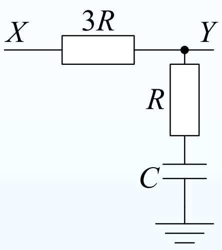

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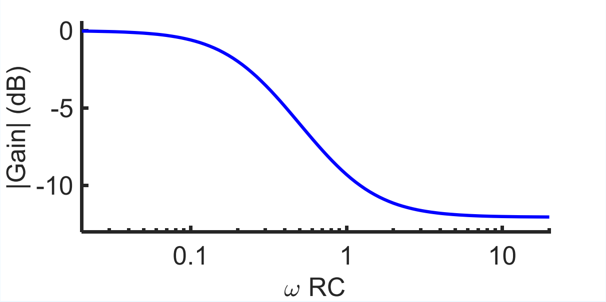

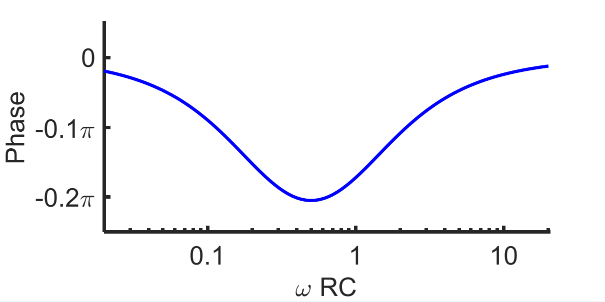

Magnitude Response:

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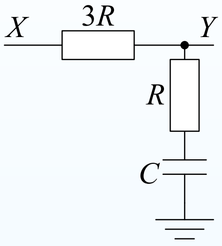

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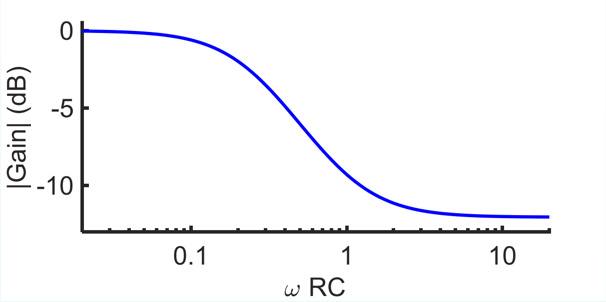

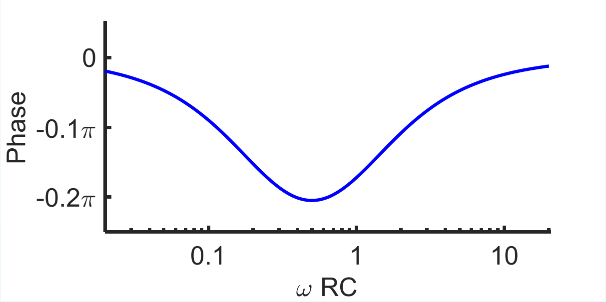

Magnitude Response:

Gradient Changes: -20 dB$/$dec at $\omega = \frac{0.25}{RC}$ and $+20$ at $\omega = \frac{1}{RC}$

RCR Circuit

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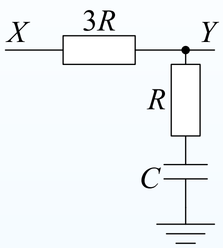

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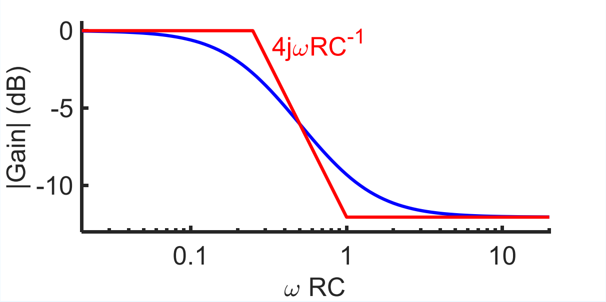

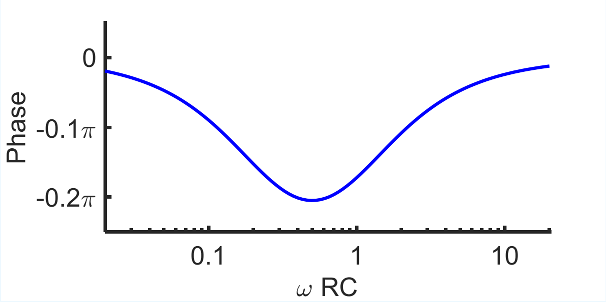

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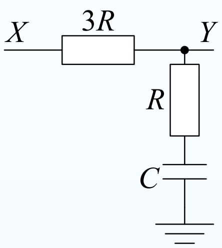

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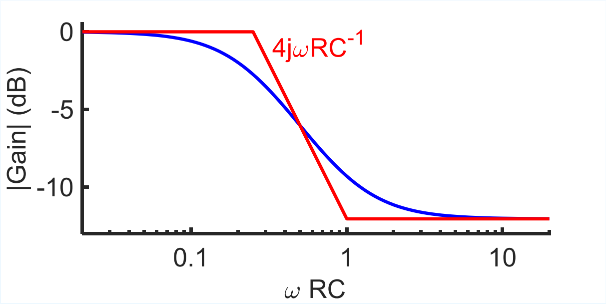

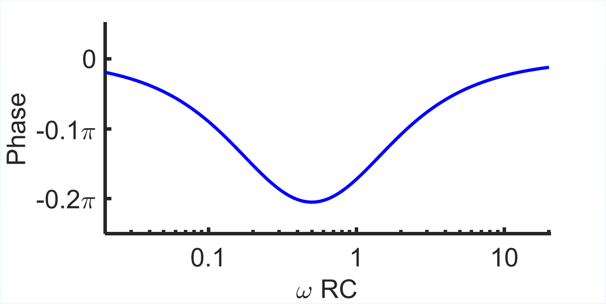

Magnitude Response:

Gradient Changes: -20 dB$/$dec at $\omega = \frac{0.25}{RC}$ and $+20$ at $\omega = \frac{1}{RC}$

Line equations: $H(j\omega) =$ (a) 1, (b) $\frac{1}{4j\omega RC}$, (c) $\frac{j\omega RC}{4j\omega RC} = 0.25$

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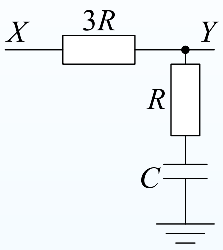

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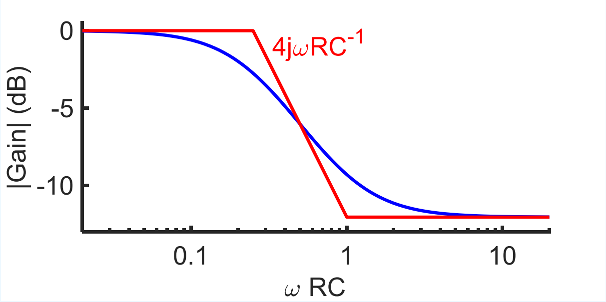

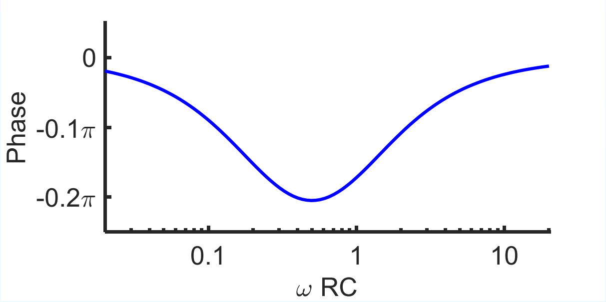

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Phase Response:

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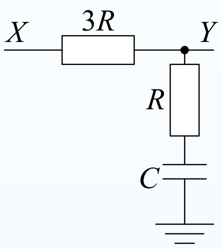

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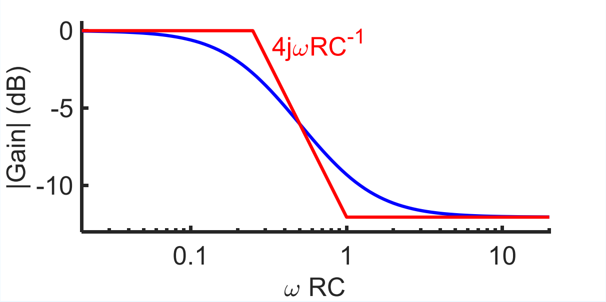

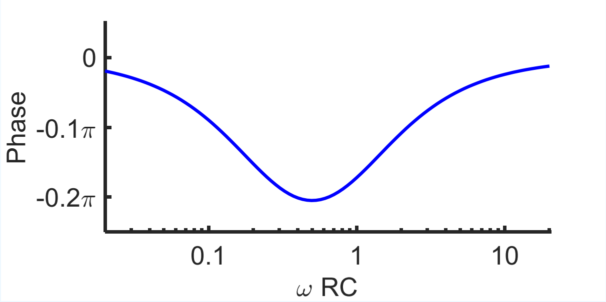

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Phase Response:

LF asymptote: $\phi = \angle 1 = 0$

RCR Circuit

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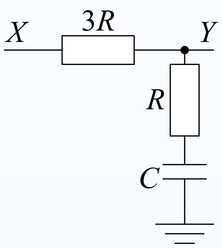
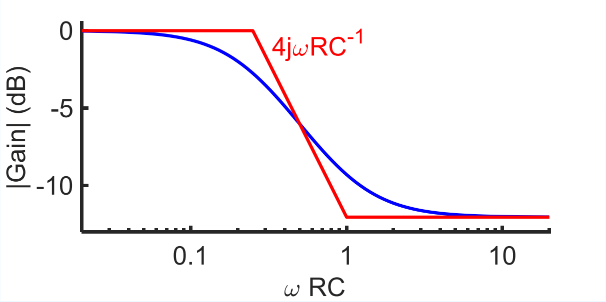
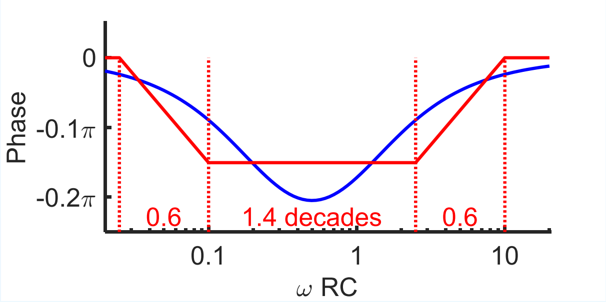


Corner freqs: $\frac{0.25}{RC}^-, \frac{1}{RC}^+$ LF Asymptote: $H(j\omega) = 1$





Magnitude Response:

Gradient Changes: -20 dB$/$dec at $\omega = \frac{0.25}{RC}$ and $+20$ at $\omega = \frac{1}{RC}$

Line equations: $H(j\omega) =$ (a) 1, (b) $\frac{1}{4j\omega RC}$, (c) $\frac{j\omega RC}{4j\omega RC} = 0.25$

Phase Response:

LF asymptote: $\phi = \angle 1 = 0$

Gradient changes of $\pm\frac{\pi}{4}/$decade at: $\omega = \frac{0.025}{RC}^-, \frac{0.1}{RC}^+, \frac{2.5}{RC}^+, \frac{10}{RC}^-$.

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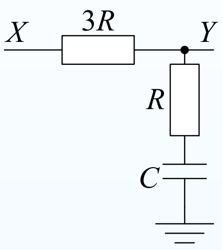

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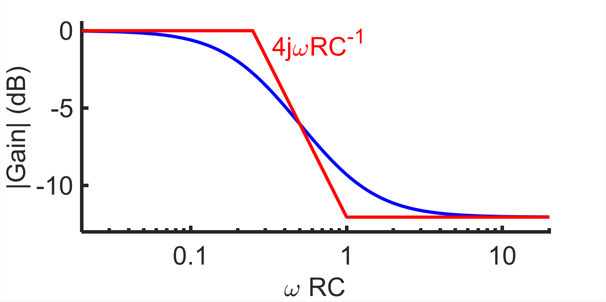

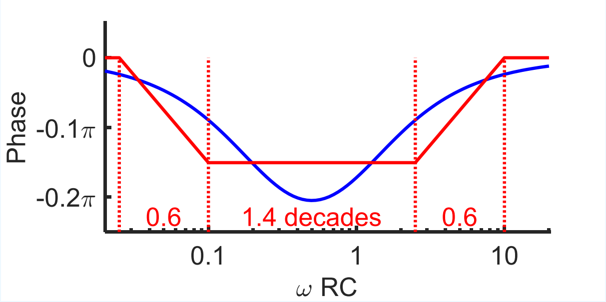

Magnitude Response:

Gradient Changes: $-20\,\text{dB}/\text{dec}$ at $\omega=\frac{0.25}{RC}$ and $+20$ at $\omega=\frac{1}{RC}$

Line equations: $H(j\omega)=$ (a) 1, (b) $\frac{1}{4j\omega RC}$, (c) $\frac{j\omega RC}{4j\omega RC}=0.25$

Phase Response:

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Gradient changes of $\pm\frac{\pi}{4}$/decade at: $\omega=\frac{0.025}{RC}^{-}, \frac{0.1}{RC}^{+}, \frac{2.5}{RC}^{+}, \frac{10}{RC}^{-}$.

At $\omega=\frac{0.1}{RC}$, $\phi=0-\frac{\pi}{4}\log_{10}\frac{0.1}{0.025}=-\frac{\pi}{4}\times 0.602=-0.15\pi$

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Summary

- **Frequency response:** magnitude and phase of $\frac{Y}{X}$ as a function of ω
  - Only applies to sine waves

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Summary

- Frequency response: magnitude and phase of $\frac{Y}{X}$ as a function of ω
  - Only applies to sine waves
  - Use log axes for frequency and gain but linear for phase
    - Decibels = $20\log_{10}\frac{V_2}{V_1} = 10\log_{10}\frac{P_2}{P_1}$

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- Linear factor $(aj\omega + b)$ gives corner frequency at $\omega = \left|\frac{b}{a}\right|$.
  - Magnitude plot gradient changes by ± 20 dB/decade @ $\omega = \left|\frac{b}{a}\right|$.

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    - ▷ $\pm\frac{\pi}{4}$ rad/decade $@\,\omega = 0.1 \times \left|\frac{b}{a}\right|$
    - ▷ $\mp\frac{\pi}{4}$ rad/decade $@\,\omega = 10 \times \left|\frac{b}{a}\right|$

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- LF/HF asymptotes: keep only the terms with the lowest/highest power of $j\omega$ in numerator and denominator polynomials

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For further details see Hayt Ch 16 or Irwin Ch 12.