11: Frequency Responses

- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary
If $x(t)$ is a sine wave, then $y(t)$ will also be a sine wave but with a different amplitude and phase shift. $X$ is an input phasor and $Y$ is the output phasor.
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\frac{Y}{X} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1}
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**Magnitude Response**
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**Sine Wave Response**

\[ \frac{Y}{X} = \frac{1}{j\omega RC + 1} = \frac{1}{0.01j\omega + 1} \]

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\begin{align*}
\omega &= 50 \Rightarrow \frac{Y}{X} = 0.89 \angle -27^\circ \\
\omega &= 100 \Rightarrow \frac{Y}{X} = 0.71 \angle -45^\circ \\
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The output, \( y(t) \), lags the input, \( x(t) \), by up to \( 90^\circ \).
Logarithmic axes

We usually use logarithmic axes for frequency and gain (but not phase) because % differences are more significant than absolute differences. E.g. 5 kHz versus 5.005 kHz is less significant than 10 Hz versus 15 Hz even though both differences equal 5 Hz.
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Common voltage ratios:

| \( \frac{|V_2|}{|V_1|} \) | dB |
|---|---|
| 1 | 0 |

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Common voltage ratios:

\[
\begin{array}{c|c|c|c|c|c}
\frac{|V_2|}{|V_1|} & 0.1 & 1 & 10 & 100 \\
\text{dB} & -20 & 0 & 20 & 40 \\
\end{array}
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Common voltage ratios:

<table>
<thead>
<tr>
<th>(\frac{V_2}{V_1})</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>10</th>
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</tr>
</thead>
<tbody>
<tr>
<td>dB</td>
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| \( \frac{|V_2|}{|V_1|} \) | 0.1 | 0.5 | \( \sqrt{0.5} \) | 1 | \( \sqrt{2} \) | 2 | 10 | 100 |
|---------------------|-----|-----|-------------|---|---------|---|----|-----|
| dB                  | -20 | -6  | -3          | 0 | 3       | 6 | 20 | 40  |

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Note: \( P \propto V^2 \Rightarrow \text{decibel power ratios are given by } 10 \log_{10} \frac{P_2}{P_1} \).
Suppose we plot the magnitude and phase of $H = c (j\omega)^r$
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**Magnitude (log-log graph):**

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![Graph showing a straight line with logarithmic axes and the equation $(j\omega)^2$ plotted.](image)
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If \( |H| \) is measured in decibels, a slope of \( r \) is called \( 6r \text{ dB/octave} \) or \( 20r \text{ dB/decade} \).

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**Note:** Phase angles are modulo $360^\circ$, i.e. $+180^\circ \equiv -180^\circ$ and $450^\circ \equiv 90^\circ$. 
Logs of Powers

\[ H = c (j\omega)^r \] has a straight-line magnitude graph and a constant phase.

Magnitude (log-log graph):
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Key idea: \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\)
Key idea: $(a j\omega + b) \approx \begin{cases} a j\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}$

Gain: $H(j\omega) = \frac{1}{j\omega RC + 1}$
Key idea: 

\[ (a j \omega + b) \approx \begin{cases} 
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\end{cases} \]

Gain: 

\[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

Low frequencies (\( \omega \ll \frac{1}{RC} \)): 

\[ H(j\omega) \approx 1 \]
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**Straight Line Approximations**

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Approximate the magnitude response as two straight lines.
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Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC}\)

Approximate the magnitude response as two straight lines.
Straight Line Approximations

Key idea: 
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(a j \omega + b) \approx \begin{cases} 
  a j \omega & \text{for } |a \omega| \gg |b| \\
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H(j \omega) = \frac{1}{j \omega RC + 1}
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Low frequencies \((\omega \ll \frac{1}{RC})\): 
\[
H(j \omega) \approx 1 \Rightarrow |H(j \omega)| \approx 1
\]

High frequencies \((\omega \gg \frac{1}{RC})\): 
\[
H(j \omega) \approx \frac{1}{j \omega RC} \Rightarrow |H(j \omega)| \approx \frac{1}{RC \omega^{-1}}
\]

Approximate the magnitude response as two straight lines
Key idea: \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies (\(\omega \ll \frac{1}{RC}\)): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

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Approximate the magnitude response as two straight lines
**Straight Line Approximations**

Key idea: \((aj\omega + b) \approx \begin{cases} \frac{aj\omega}{b} & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

High frequencies \((\omega \gg \frac{1}{RC})\): \(H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC} \omega^{-1}\)

Approximate the magnitude response as two straight lines intersecting at the **corner frequency**, \(\omega_c = \frac{1}{RC}\).
Key idea: \((aj\omega + b) \approx \begin{cases} a\omega & \text{for } |a\omega| \gg |b| \\ b & \text{for } |a\omega| \ll |b| \end{cases}\)

Gain: \(H(j\omega) = \frac{1}{j\omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1\)

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Approximate the magnitude response as two straight lines intersecting at the **corner frequency** \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1\) \((-6 \text{ dB/octave} = -20 \text{ dB/decade})\).
Straight Line Approximations

Key idea: \((a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a| \omega \gg |b| \\ b & \text{for } |a| \omega \ll |b| \end{cases}\)

Gain: \(H(j \omega) = \frac{1}{j \omega RC + 1}\)

Low frequencies \((\omega \ll \frac{1}{RC})\): \(H(j \omega) \approx 1 \Rightarrow |H(j \omega)| \approx 1\)

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Approximate the magnitude response as two straight lines intersecting at the corner frequency, \(\omega_c = \frac{1}{RC}\).

At the corner frequency:

(a) the gradient changes by \(-1 \approx -6 \text{ dB/octave} = -20 \text{ dB/decade}\).
(b) \(|H(j \omega_c)| = \left| \frac{1}{1+j} \right| = \frac{1}{\sqrt{2}} = -3 \text{ dB} \) (worst-case error).
### Straight Line Approximations

#### Key idea:

\( (a j \omega + b) \approx \begin{cases} a j \omega & \text{for } |a \omega| \gg |b| \\ b & \text{for } |a \omega| \ll |b| \end{cases} \)

#### Gain:

\[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

- **Low frequencies** \((\omega \ll \frac{1}{RC})\): \( H(j\omega) \approx 1 \Rightarrow |H(j\omega)| \approx 1 \)
- **High frequencies** \((\omega \gg \frac{1}{RC})\): \( H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow |H(j\omega)| \approx \frac{1}{RC} \omega^{-1} \)

Approximate the magnitude response as two straight lines intersecting at the **corner frequency**, \( \omega_c = \frac{1}{RC} \).

At the corner frequency:

1. **(a)** the gradient changes by \(-1\) \((-6 \text{ dB/octave} = -20 \text{ dB/decade})\).
2. **(b)** \(|H(j\omega_c)| = \left| \frac{1}{1+j} \right| = \frac{1}{\sqrt{2}} = -3 \text{ dB} \) (worst-case error).

A linear factor \((a j \omega + b)\) has a corner frequency of \(\omega_c = \left| \frac{b}{a} \right|\).
The gain of a linear circuit is always a rational polynomial in $j\omega$ and is called the transfer function of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$$
The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

**Step 1:** Factorize the polynomials
The gain of a linear circuit is always a *rational polynomial* in \( j\omega \) and is called the *transfer function* of the circuit. For example:

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H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)^3}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}
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**Step 1:** Factorize the polynomials

**Step 2:** Sort corner freqs: 1, 4, 12, 50
The gain of a linear circuit is always a **rational polynomial** in $j\omega$ and is called the **transfer function** of the circuit. For example:

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**Step 1:** Factorize the polynomials
**Step 2:** Sort corner freqs: 1, 4, 12, 50
**Step 3:** For $\omega < 1$ all linear factors equal their constant terms:

$$|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.$$
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**Step 4:** For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so

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The gain of a linear circuit is always a \textit{rational polynomial} in $j\omega$ and is called the \textit{transfer function} of the circuit. For example:

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$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$
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H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}
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|H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1.
\]

**Step 4:** For \( 1 < \omega < 4 \), the factor \((j\omega + 1) \approx j\omega\) so

\[
|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.
\]

**Step 5:** For \( 4 < \omega < 12 \), \( |H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1} \).
Plot Magnitude Response

The gain of a linear circuit is always a *rational polynomial* in $j\omega$ and is called the *transfer function* of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

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**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.

**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}$. 

![Magnitude Response Graph](image-url)
The gain of a linear circuit is always a rational polynomial in $j\omega$ and is called the transfer function of the circuit. For example:

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$$|H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}.$$  

**Step 5:** For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.  

**Step 6:** For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}$.  

**Step 7:** For $\omega > 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times \omega} = 20\omega^{-1}$. 

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**Plot Magnitude Response**

- Frequency Response
- Logarithmic axes
- Logs of Powers
- Sine Wave Response
- Straight Line Approximations
- Low and High Frequency Asymptotes
- Phase Approximation
- RCR Circuit
- Summary
The gain of a linear circuit is always a \textit{rational polynomial} in $j\omega$ and is called the \textit{transfer function} of the circuit. For example:

$$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$$

\begin{itemize}
  \item \textbf{Step 1:} Factorize the polynomials
  \item \textbf{Step 2:} Sort corner freqs: 1, 4, 12, 50
  \item \textbf{Step 3:} For $\omega < 1$ all linear factors equal their constant terms:
    \[ |H| \approx \frac{20\omega \times 12}{1 \times 4 \times 50} = 1.2\omega^1. \]
  \item \textbf{Step 4:} For $1 < \omega < 4$, the factor $(j\omega + 1) \approx j\omega$ so
    \[ |H| \approx \frac{20\omega \times 12}{\omega \times 4 \times 50} = 1.2\omega^0 = +1.58 \text{ dB}. \]
  \item \textbf{Step 5:} For $4 < \omega < 12$, $|H| \approx \frac{20\omega \times 12}{\omega \times \omega \times 50} = 4.8\omega^{-1}$.\]
  \item \textbf{Step 6:} For $12 < \omega < 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times 50} = 0.4\omega^0 = -7.96 \text{ dB}$. \]
  \item \textbf{Step 7:} For $\omega > 50$, $|H| \approx \frac{20\omega \times \omega}{\omega \times \omega \times \omega} = 20\omega^{-1}$. \]
\end{itemize}

At each corner frequency, the graph is continuous but its gradient changes abruptly by $+1$ (numerator factor) or $-1$ (denominator factor).
You can find the low and high frequency asymptotes without factorizing:

\[
H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}
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Low and High Frequency Asymptotes

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Low Frequency Asymptote:
Low and High Frequency Asymptotes

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\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Low Frequency Asymptote:**

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Low Frequency Asymptote:**
From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

**Lowest** power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Low Frequency Asymptote:**

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

**Lowest** power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)

**High Frequency Asymptote:**
You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Low Frequency Asymptote:**

From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

The **lowest** power of \( j\omega \) on top and bottom: \( H(j\omega) \approx \frac{720(j\omega)}{600} = 1.2j\omega \)

**High Frequency Asymptote:**

From factors: \( H_{HF}(j\omega) = \frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)} = 20(j\omega)^{-1} \)
Low and High Frequency Asymptotes

You can find the low and high frequency asymptotes without factorizing:

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Low Frequency Asymptote:**
From factors: \( H_{LF}(j\omega) = \frac{20j\omega(12)}{(1)(4)(50)} = 1.2j\omega \)

Lowest power of \( j\omega \) on top and bottom: \( H(j\omega) \sim \frac{720(j\omega)}{600} = 1.2j\omega \)

**High Frequency Asymptote:**
From factors: \( H_{HF}(j\omega) = \frac{20j\omega(j\omega)}{(j\omega)(j\omega)(j\omega)} = 20 (j\omega)^{-1} \)

Highest power of \( j\omega \) on top and bottom: \( H(j\omega) \sim \frac{60(j\omega)^2}{3(j\omega)^3} = 20 (j\omega)^{-1} \)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
\( H(j\omega) \approx 1 \)
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies \((\omega \ll \frac{1}{RC})\):
\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

High frequencies \((\omega \gg \frac{1}{RC})\): \( H(j\omega) \approx \frac{1}{j\omega RC} \)
Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

**Low frequencies** \( (\omega \ll \frac{1}{RC}) \):

\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

**High frequencies** \( (\omega \gg \frac{1}{RC}) \):

\[ H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2} \]
Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies \((\omega \ll \frac{1}{RC})\):
\[
H(j\omega) \approx 1 \Rightarrow \angle 1 = 0
\]

High frequencies \((\omega \gg \frac{1}{RC})\):
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H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2}
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Approximate the phase response as three straight lines.
Phase Approximation

Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
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H(j\omega) \approx 1 \Rightarrow \angle 1 = 0
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High frequencies (\( \omega \gg \frac{1}{RC} \)):
\[
H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2}
\]

Approximate the phase response as three straight lines.

By chance, they intersect close to 0.1\(\omega_c\) and 10\(\omega_c\) where \(\omega_c = \frac{1}{RC}\).
Phase Approximation

Gain: $H(j\omega) = \frac{1}{j\omega RC + 1}$

Low frequencies ($\omega \ll \frac{1}{RC}$):

$H(j\omega) \approx 1 \Rightarrow \angle 1 = 0$

High frequencies ($\omega \gg \frac{1}{RC}$): $H(j\omega) \approx \frac{1}{j\omega RC} \Rightarrow \angle j^{-1} = -\frac{\pi}{2}$

Approximate the phase response as three straight lines.

By chance, they intersect close to $0.1\omega_c$ and $10\omega_c$ where $\omega_c = \frac{1}{RC}$.

Between $0.1\omega_c$ and $10\omega_c$ the phase changes by $-\frac{\pi}{2}$ over two decades. This gives a gradient $= -\frac{\pi}{4}$ radians/decade.
Phase Approximation

Gain: \[ H(j\omega) = \frac{1}{j\omega RC + 1} \]

Low frequencies (\( \omega \ll \frac{1}{RC} \)):
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Between \( 0.1\omega_c \) and \( 10\omega_c \) the phase changes by \( -\frac{\pi}{2} \) over two decades. This gives a gradient = \( -\frac{\pi}{4} \) radians/decade.

\( (aj\omega + b) \) in denominator
\[ \Rightarrow \Delta \text{gradient} = \mp \frac{\pi}{4}/\text{decade} \text{ at } \omega = 10^\mp 1 \left| \frac{b}{a} \right|. \]
Gain: \( H(j\omega) = \frac{1}{j\omega RC + 1} \)

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\[ H(j\omega) \approx 1 \Rightarrow \angle 1 = 0 \]

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By chance, they intersect close to \(0.1\omega_c\) and \(10\omega_c\) where \(\omega_c = \frac{1}{RC}\).

Between \(0.1\omega_c\) and \(10\omega_c\) the phase changes by \(-\frac{\pi}{2}\) over two decades.
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\((aj\omega + b)\) in denominator
\[ \Rightarrow \Delta \text{gradient} = \mp \frac{\pi}{4} / \text{decade} \text{ at } \omega = 10 \mp 1 \left| \frac{b}{a} \right|. \]

The sign of \(\Delta \text{gradient}\) is reversed for (a) numerator factors and (b) \(\frac{b}{a} < 0\).
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} \]
Plot Phase Response

\[
H(j\omega) = \frac{60(j\omega)^2+720(j\omega)}{3(j\omega)^3+165(j\omega)^2+762(j\omega)+600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}
\]

Step 1: Factorize the polynomials

Step 2: List corner freqs: \( \pm = \frac{\text{num}}{\text{den}} \)

\( \omega_c = \{1^-, 4^-, 12^+, 50^-\} \)

\[
\begin{array}{c}
\omega (\text{rad/s}) \\
0.1 & 1 & 10 & 100 & 1000 \\
\end{array}
\]

\[
\begin{array}{c}
\Delta H (\text{rad}) \\
0.5\pi & 0 & -0.5\pi \\
\end{array}
\]
\( H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \)

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm \frac{\text{num}}{\text{den}} \)
\[
\omega_c = \{1^-, 4^-, 12^+, 50^-\}
\]

**Step 3:** Gradient changes at \( 10^{\pm1}\omega_c \).
Sign depends on num/den and \( \text{sgn}\left(\frac{b}{a}\right)\):
\[.1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+\]
$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)}$

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: $\pm = \frac{\text{num}}{\text{den}}$

$\omega_c = \{1^- , 4^- , 12^+ , 50^- \}$

**Step 3:** Gradient changes at $10^{\pm 1}\omega_c$.
Sign depends on num/den and $\text{sgn} \left( \frac{b}{a} \right)$:

$1^- , 10^+ ; 4^- , 40^+ ; 1.2^+ , 120^- ; 5^- , 500^+$

**Step 4:** Put in ascending order and calculate gaps as $\log_{10} \frac{\omega_2}{\omega_1}$ decades:

$1^- (.6) .4^- (.48) 1.2^+ (.62) 5^- (.3) 10^+ (.6) 40^+ (.48) 120^- (.62) 500^+$. 
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**Step 5:** Find phase of LF asymptote: $\angle 1.2j\omega = +\frac{\pi}{2}$.
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

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Step 1: Factorize the polynomials

Step 2: List corner freqs: $\pm = \text{num/den}$

$$\omega_c = \{1^-, 4^-, 12^+, 50^-\}$$

Step 3: Gradient changes at $10^{\pm 1}\omega_c$.

Sign depends on num/den and $\text{sgn} \left( \frac{b}{a} \right)$:

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Step 6: At $\omega = 0.1$ gradient becomes $-\frac{\pi}{4}$ rad/decade. $\phi$ is still $\frac{\pi}{2}$. 
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

Step 1: Factorize the polynomials
Step 2: List corner freqs: \( \pm = \frac{\text{num}}{\text{den}} \)
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Step 7: At \( \omega = 0.4, \phi = \frac{\pi}{2} - 0.6 \frac{\pi}{4} = 0.35\pi \). New gradient is \( -\frac{\pi}{2} \).
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**Step 1**: Factorize the polynomials

**Step 2**: List corner freqs: \( \pm = \text{num/den} \)
\( \omega_c = \{1^-, 4^-, 12^+, 50^-\} \)

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**Step 8**: At \( \omega = 1.2 \), \( \phi = 0.35\pi - 0.48 \frac{\pi}{2} = 0.11\pi \). New gradient is \( -\frac{\pi}{4} \).
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]

**Step 3:** Gradient changes at \( 10^{\pm1}\omega_c \).

Sign depends on \( \text{num/den} \) and \( \text{sgn} \left( \frac{b}{a} \right) \):

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**Steps 9-13:** Repeat for each gradient change.
Plot Phase Response

\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)} \]

**Step 1:** Factorize the polynomials
**Step 2:** List corner freqs: \( \pm = \text{num/den} \) \n\[ \omega_c = \{1^-, 4^-, 12^+, 50^-\} \]
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**Steps 9-13:** Repeat for each gradient change.
Step 1: Factorize the polynomials

Step 2: List corner freqs: \(\pm = \text{num/den}\)
\[
\omega_c = \{1^-, 4^-, 12^+, 50^-\}
\]

Step 3: Gradient changes at \(10^{+1}\omega_c\).
Sign depends on num/den and sgn \(\left(\frac{b}{a}\right)\):
\[
.1^-, 10^+; .4^-, 40^+; 1.2^+, 120^-; 5^-, 500^+
\]

Step 4: Put in ascending order and calculate gaps as \(\log_{10} \frac{\omega_2}{\omega_1}\) decades:
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Sign depends on num/den and sgn \((\frac{b}{a})\):

- \(1^-\), \(10^+\); \(4^-\), \(40^+\); \(1.2^+\), \(120^-\); \(5^-\), \(500^+\)

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- \(.1^-\) \(.6^-\) \(.4^-\) \(.48^-\) \(.12^+\) \(.62^-\) \(.5^-\) \(.3^-\) \(.10^+\) \(.6^-\) \(.40^+\) \(.48^-\) \(.120^-\) \(.62^-\) \(.500^+\)

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$H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega + 12)}{(j\omega + 1)(j\omega + 4)(j\omega + 50)}$

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**Steps 9-13:** Repeat for each gradient change. Final gradient is always 0.
\[ H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600} = \frac{20j\omega(j\omega+12)}{(j\omega+1)(j\omega+4)(j\omega+50)} \]

**Step 1:** Factorize the polynomials

**Step 2:** List corner freqs: \( \pm = \text{num/den} \)

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**Steps 9-13:** Repeat for each gradient change. Final gradient is always 0.

At 0.1 and 10 times each corner frequency, the graph is continuous but its gradient changes abruptly by \( \pm \frac{\pi}{4} \) rad/decade.
11: Frequency Responses
- Frequency Response
- Sine Wave Response
- Logarithmic axes
- Logs of Powers
- Straight Line Approximations
- Plot Magnitude Response
- Low and High Frequency Asymptotes
- Phase Approximation
- Plot Phase Response
- RCR Circuit
- Summary

RCR Circuit

\[ X \quad 3R \quad Y \]

C
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}}
\]
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

\[
X \quad 3R \quad Y
\]

\[
\begin{align*}
0 & & 0 & & 0 \\
-10 & & -5 & & -0.1 \pi \\
-10 & & -5 & & -0.2 \pi \\
0 & & 0 & & 0
\end{align*}
\]

\[
\begin{align*}
0 & & 0 & & 0 \\
-10 & & -5 & & -0.1 \pi \\
-10 & & -5 & & -0.2 \pi \\
0 & & 0 & & 0
\end{align*}
\]

\[
\begin{align*}
0 & & 0 & & 0 \\
-10 & & -5 & & -0.1 \pi \\
-10 & & -5 & & -0.2 \pi \\
0 & & 0 & & 0
\end{align*}
\]
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \)

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E1.1 Analysis of Circuits (2017-10213)

Frequency Responses: 11 – 11 / 12
Y \over X = \frac{R+{1 \over j\omega C}}{3R+R+{1 \over j\omega C}} = \frac{j\omega RC+1}{4j\omega RC+1}

Corner freqs: \frac{0.25}{RC}, \frac{1}{RC} +

LF Asymptote: H(j\omega) = 1
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25\), \(\frac{1}{RC}\) +

LF Asymptote: \(H(j\omega) = 1\)

Magnitude Response:
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) +

LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC} \) and \(+20 \text{ at } \omega = \frac{1}{RC} \)
### RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

**Corner freqs:** \( \frac{0.25}{RC} \), \( \frac{1}{RC} \)  
**LF Asymptote:** \( H(j\omega) = 1 \)

**Magnitude Response:**  
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC} \) and \(+20 \) at \( \omega = \frac{1}{RC} \)
RCR Circuit

\[ \frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1} \]

Corner freqs: \[ \frac{0.25}{RC}, \frac{1}{RC} \] +

LF Asymptote: \[ H(j\omega) = 1 \]

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC} \text{ and } +20 \text{ at } \omega = \frac{1}{RC} \)

Line equations: \[ H(j\omega) = (a) 1, \quad (b) \frac{1}{4j\omega RC}, \quad (c) \frac{j\omega RC}{4j\omega RC} = 0.25 \]
**RCR Circuit**

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25\frac{R}{C}\), \( \frac{1}{R C}\) +

LF Asymptote: \(H(j\omega) = 1\)

*Magnitude Response:*

Gradient Changes: \(-20\) dB/dec at \(\omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\)

Line equations: \(H(j\omega) = (a) 1, (b) \frac{1}{4j\omega RC}, (c) \frac{j\omega RC}{4j\omega RC} = 0.25\)

*Phase Response:*

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**Summary**
RCR Circuit

Frequency Responses

Corner freqs: \( \frac{0.25}{RC}, \frac{1}{RC} \) + LF Asymptote: \( H(j\omega) = 1 \)

Magnitude Response:
Gradient Changes: \(-20 \text{ dB/dec at } \omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\)

Line equations: \( H(j\omega) = (a) \ 1, \ (b) \ \frac{1}{4j\omega RC}, \ (c) \ \frac{j\omega RC}{4j\omega RC} = 0.25 \)

Phase Response:
LF asymptote: \( \phi = \angle 1 = 0 \)
**RCR Circuit**

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \(0.25\frac{1}{RC}, \frac{1}{RC}\)

**LF Asymptote:** \(H(j\omega) = 1\)

**Magnitude Response:**
Gradient Changes: \(-20\, \text{dB/dec}\) at \(\omega = \frac{0.25}{RC}\) and \(+20\) at \(\omega = \frac{1}{RC}\)

Line equations: \(H(j\omega) = (a) 1, \quad (b) \frac{1}{4j\omega RC}, \quad (c) \frac{j\omega RC}{4j\omega RC} = 0.25\)

**Phase Response:**
LF asymptote: \(\phi = \angle 1 = 0\)

Gradient changes of \(\pm \frac{\pi}{4}\) /decade at: \(\omega = \frac{0.025}{RC}, \frac{0.1}{RC}, \frac{2.5}{RC}, \frac{10}{RC}\).
RCR Circuit

\[
\frac{Y}{X} = \frac{R + \frac{1}{j\omega C}}{3R + R + \frac{1}{j\omega C}} = \frac{j\omega RC + 1}{4j\omega RC + 1}
\]

Corner freqs: \( \frac{0.25}{RC} \), \( \frac{1}{RC} \) 

LF Asymptote: \( H(j\omega) = 1 \)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{magnitude_response.png}
\end{figure}

**Magnitude Response:**

Gradient Changes: \(-20\, \text{dB/dec}\) at \( \omega = \frac{0.25}{RC} \) and \(+20\) at \( \omega = \frac{1}{RC} \)

Line equations: \( H(j\omega) = (a) \, 1, \ (b) \, \frac{1}{4j\omega RC}, \ (c) \, \frac{j\omega RC}{4j\omega RC} = 0.25 \)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{phase_response.png}
\end{figure}

**Phase Response:**

LF asymptote: \( \phi = \angle 1 = 0 \)

Gradient changes of \( \pm \frac{\pi}{4} / \text{decade} \) at: \( \omega = \frac{0.025}{RC}, \frac{0.1}{RC}, \frac{2.5}{RC}, \frac{10}{RC} \)

At \( \omega = \frac{0.1}{RC} \), \( \phi = 0 - \frac{\pi}{4} \log_{10} \frac{0.1}{0.025} = -\frac{\pi}{4} \times 0.602 = -0.15\pi \)
Summary

- **Frequency response**: magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
Summary

- **Frequency response**: magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
  - Use **log axes** for frequency and gain but **linear** for phase
    - Decibels $= 20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1}$
11: Frequency Responses

- Frequency response: magnitude and phase of $\frac{Y}{X}$ as a function of $\omega$
  - Only applies to sine waves
  - Use log axes for frequency and gain but linear for phase
  - Decibels = $20 \log_{10} \frac{V_2}{V_1} = 10 \log_{10} \frac{P_2}{P_1}$

- Linear factor $(a j \omega + b)$ gives corner frequency at $\omega = \left| \frac{b}{a} \right|$.
  - Magnitude plot gradient changes by $\pm 20$ dB/decade @ $\omega = \left| \frac{b}{a} \right|$.
Summary

- **Frequency response**: magnitude and phase of \( \frac{Y}{X} \) as a function of \( \omega \)
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  - Magnitude plot gradient changes by \( \pm 20 \text{ dB/decade} \) @ \( \omega = \left| \frac{b}{a} \right| \).
  - Phase gradient changes in two places by:
    - \( \pm \frac{\pi}{4} \text{ rad/decade} @ \omega = 0.1 \times \left| \frac{b}{a} \right| \)
    - \( \mp \frac{\pi}{4} \text{ rad/decade} @ \omega = 10 \times \left| \frac{b}{a} \right| \)
Summary

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- **LF/HF asymptotes**: keep only the terms with the lowest/highest power of $j\omega$ in numerator and denominator polynomials
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For further details see Hayt Ch 16 or Irwin Ch 12.