12: Resonance

Quadratic Factors

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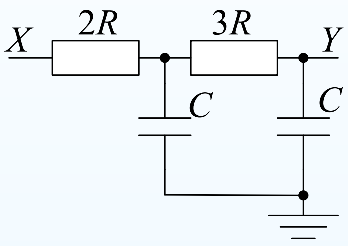

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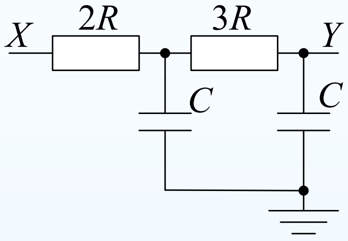

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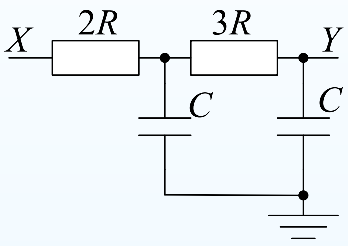

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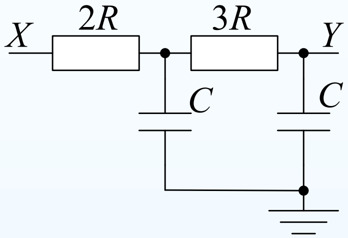

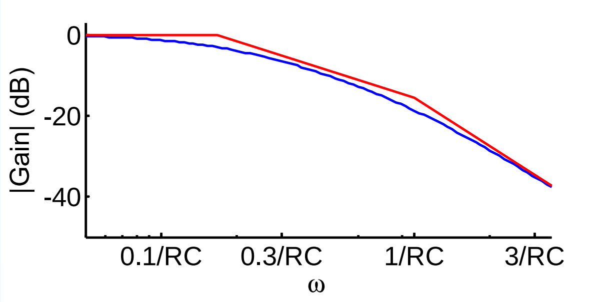

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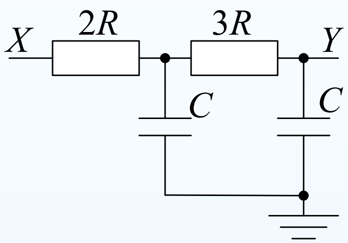

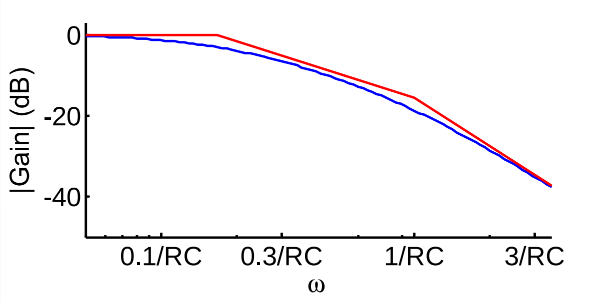

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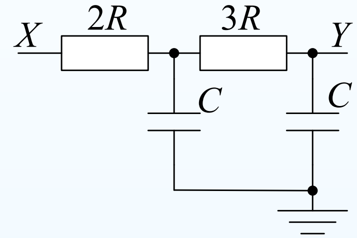

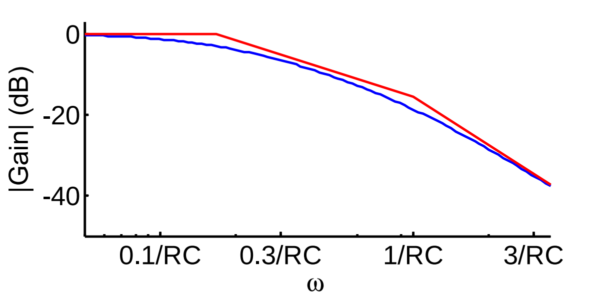

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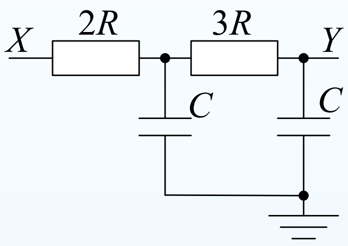

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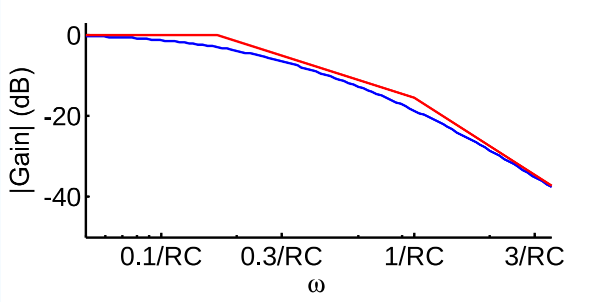

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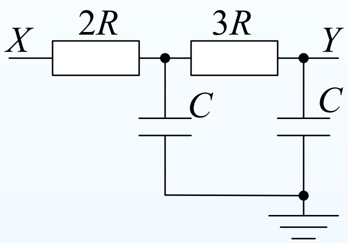

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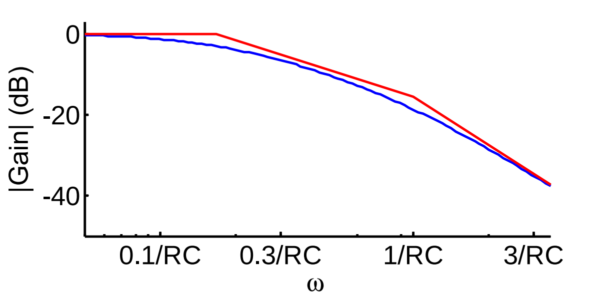

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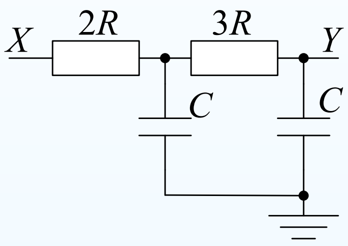

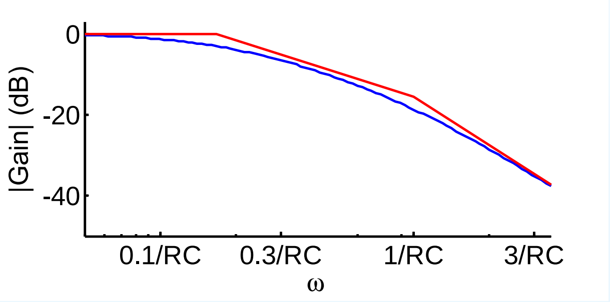

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Any polynomial with real coefficients can be factored into linear and quadratic factors $\Rightarrow$ a quadratic factor is as complicated as it gets.

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Alternatively, we sometimes use the *quality factor*, $Q \approx \frac{1}{2\zeta} = \frac{a\omega_c}{b}$.

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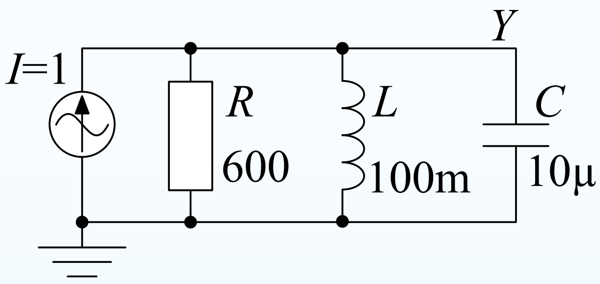

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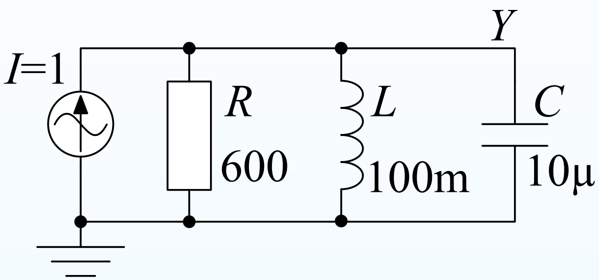

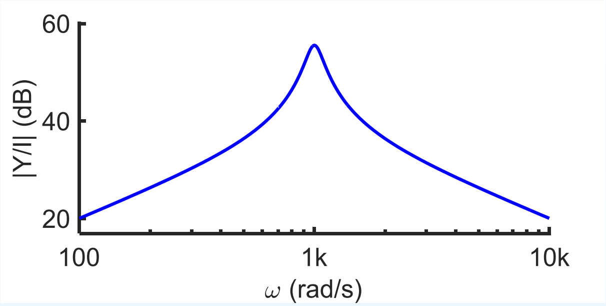

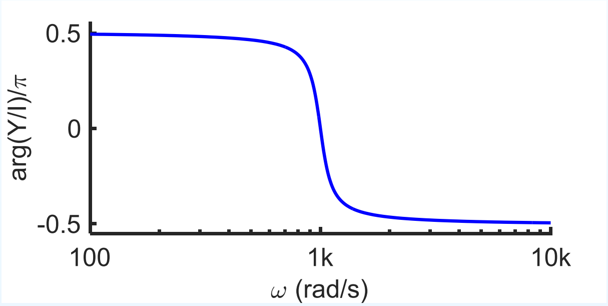

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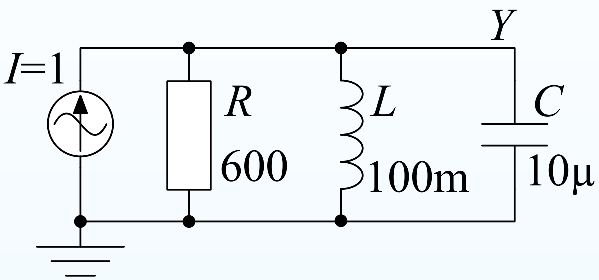
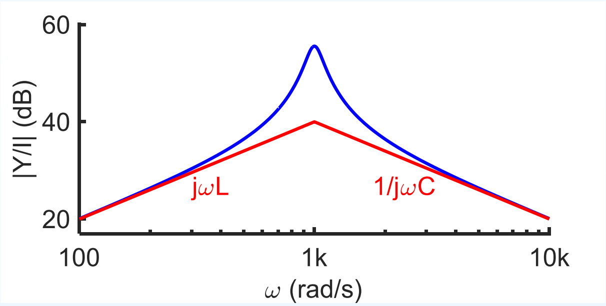
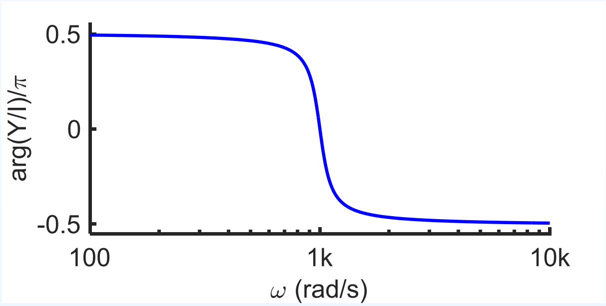


Asymptotes: $j\omega L$ and $\frac{1}{j\omega C}$.





Parallel RLC

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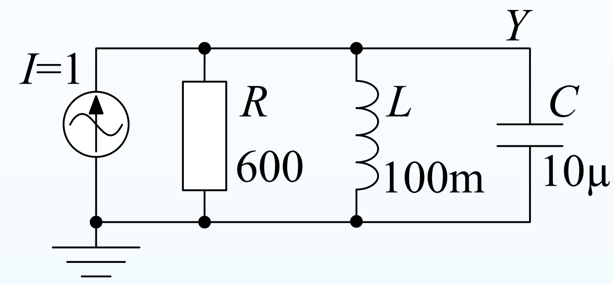

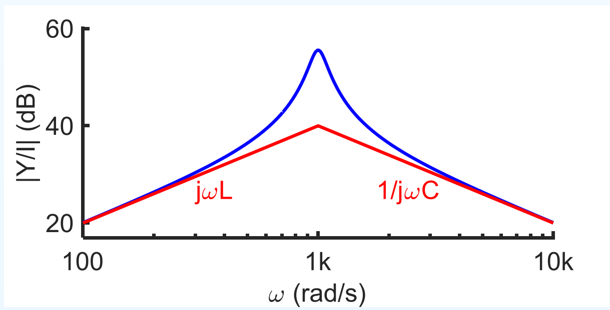

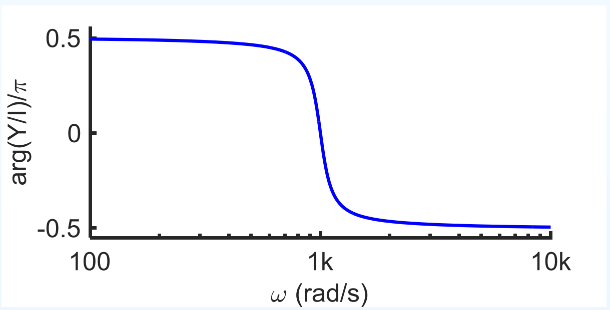

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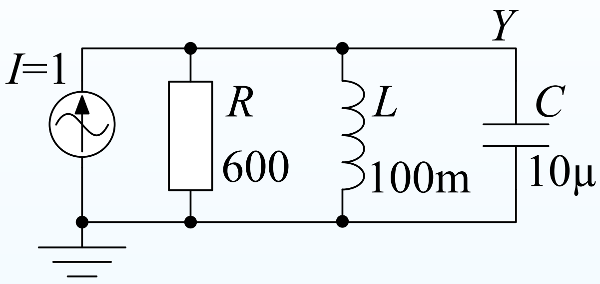

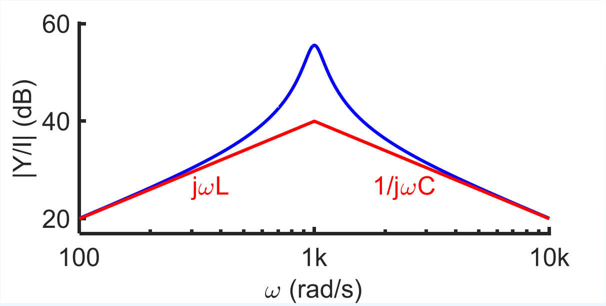

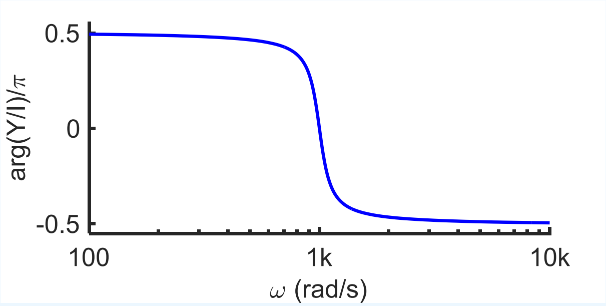

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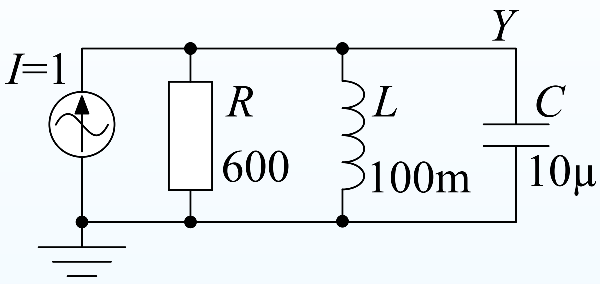

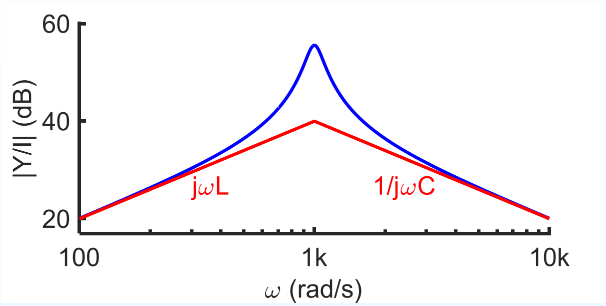

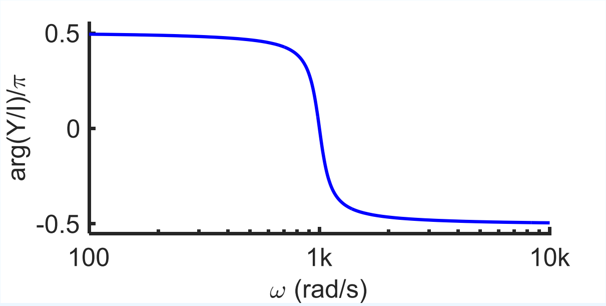

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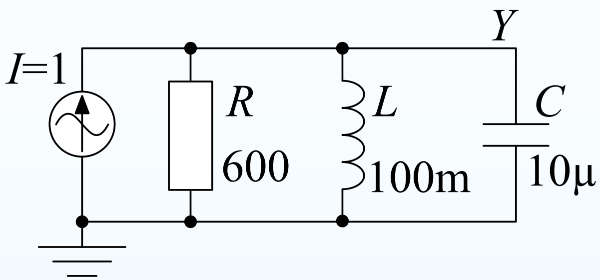

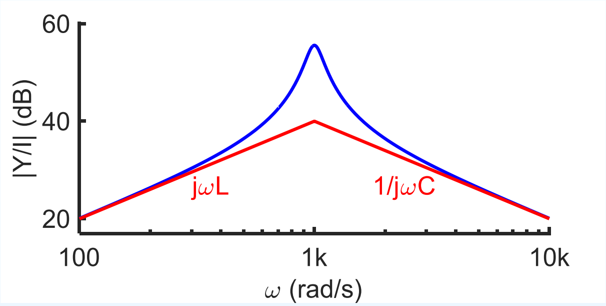

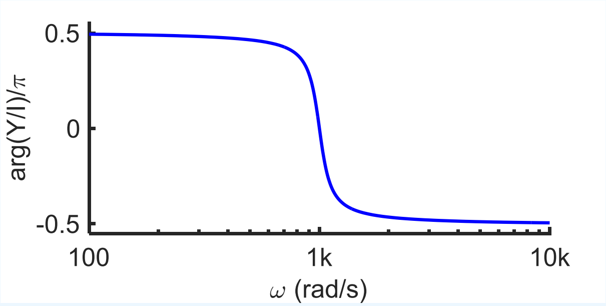

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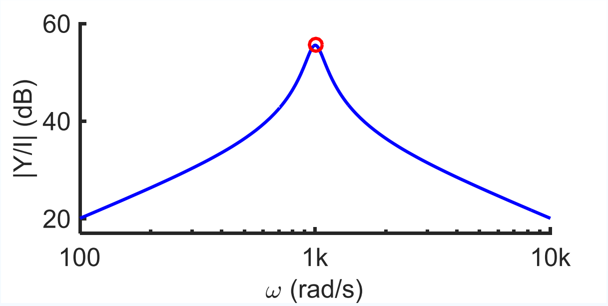

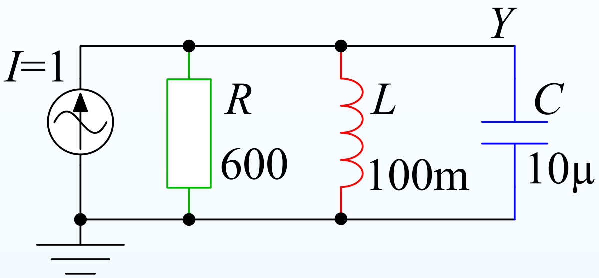

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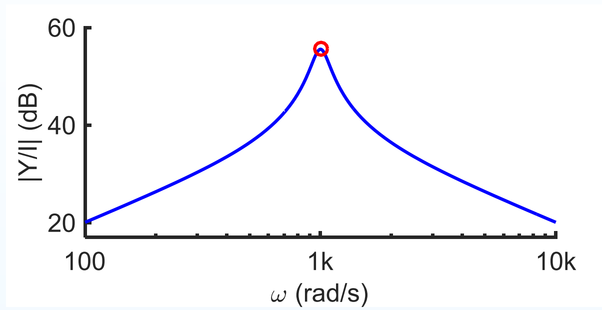
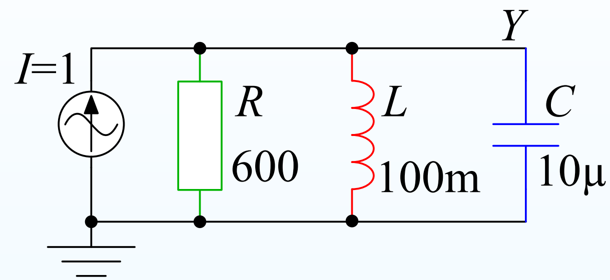
Behaviour at Resonance





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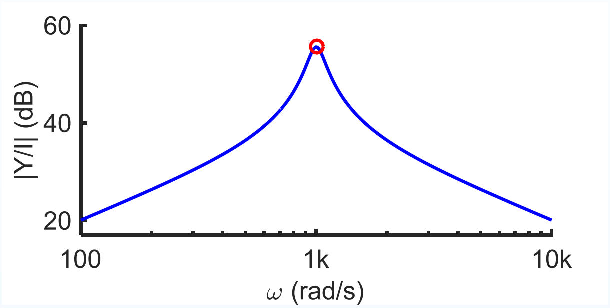

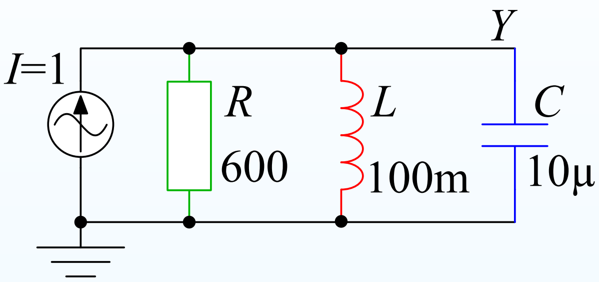

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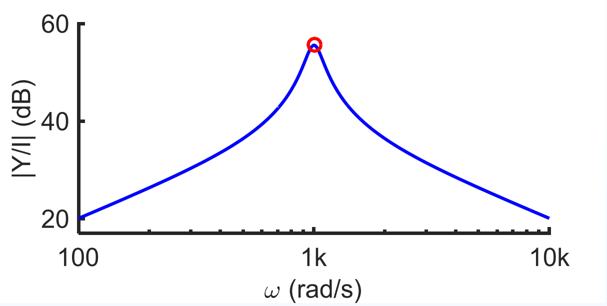

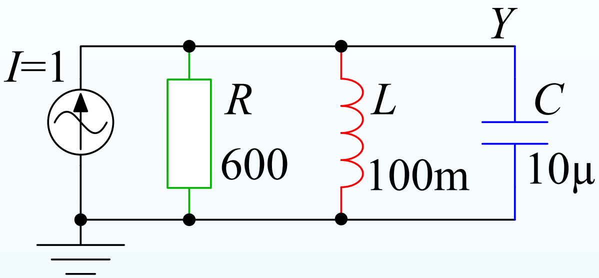

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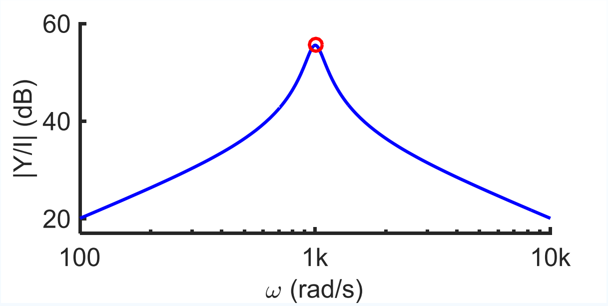

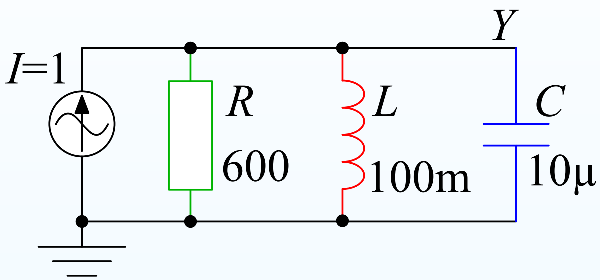

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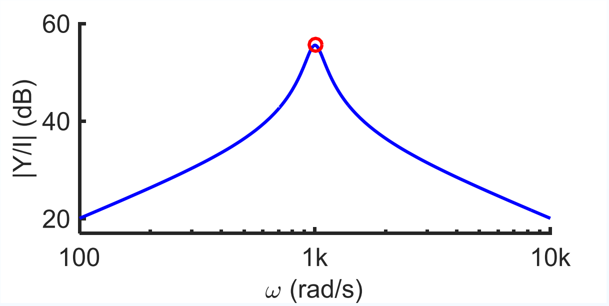

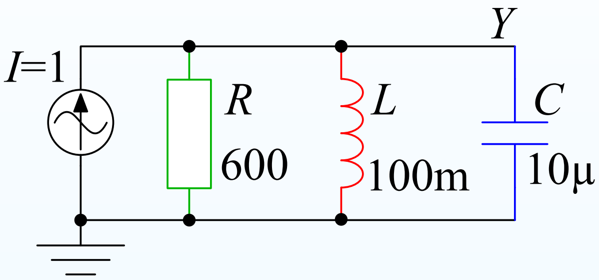

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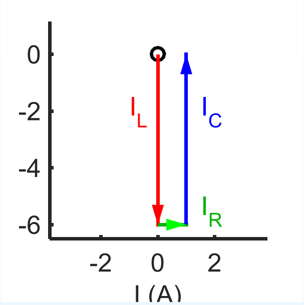

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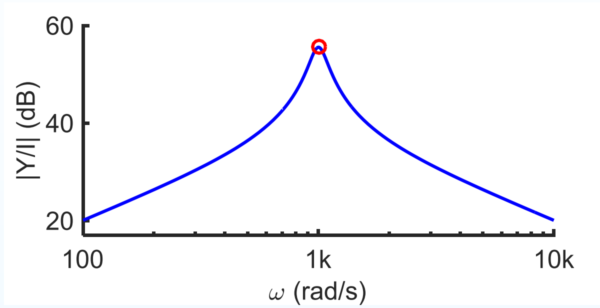

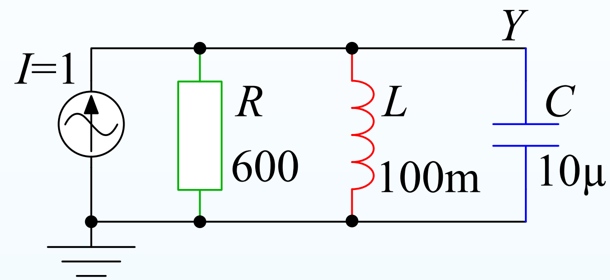

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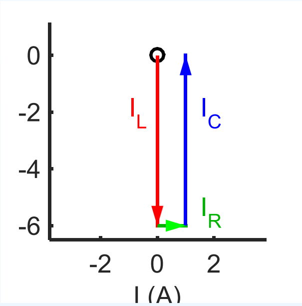

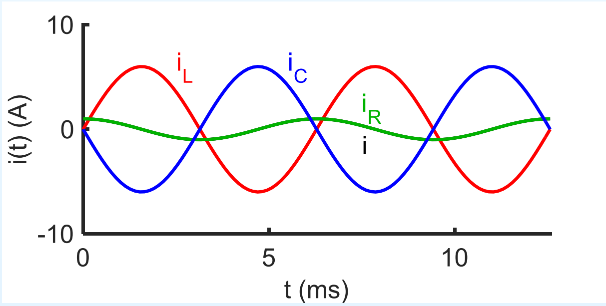

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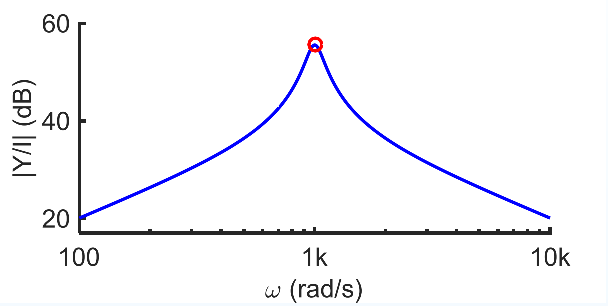

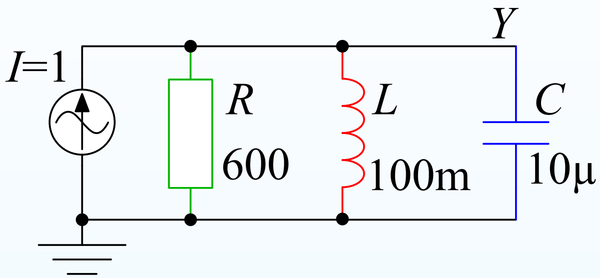

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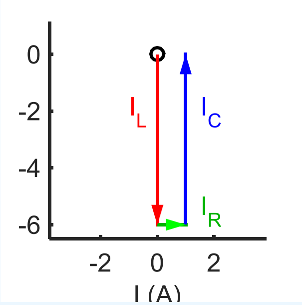

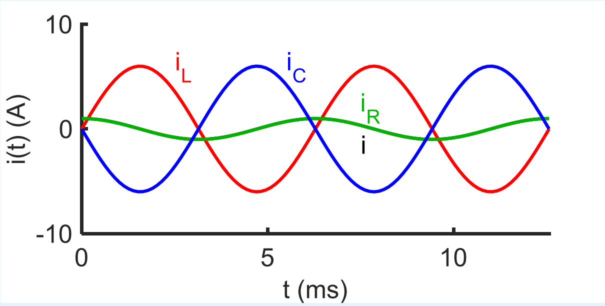

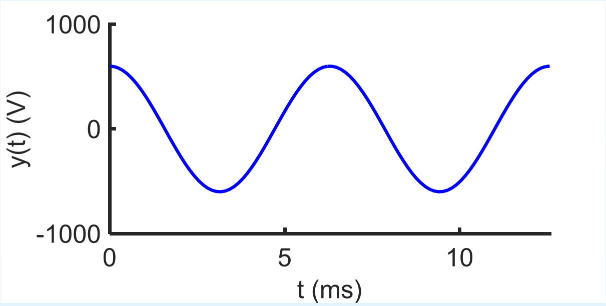

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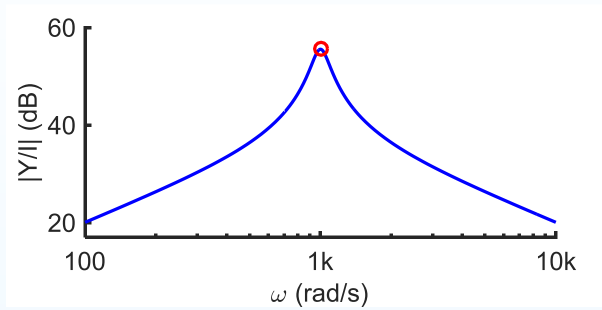

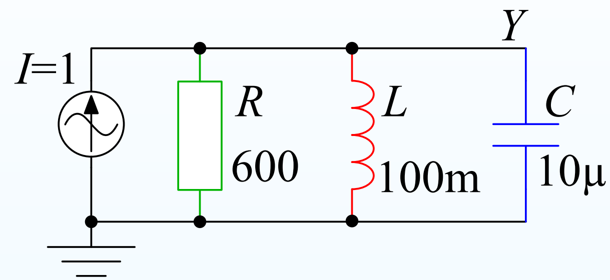

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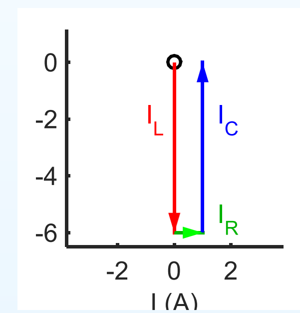
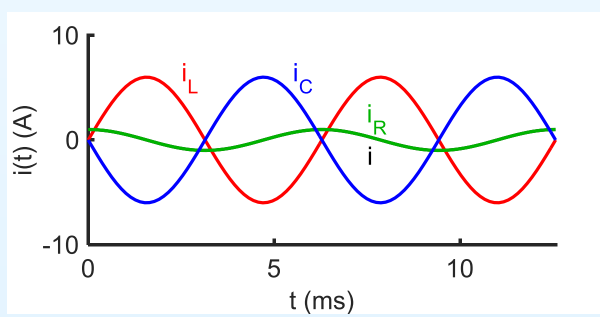
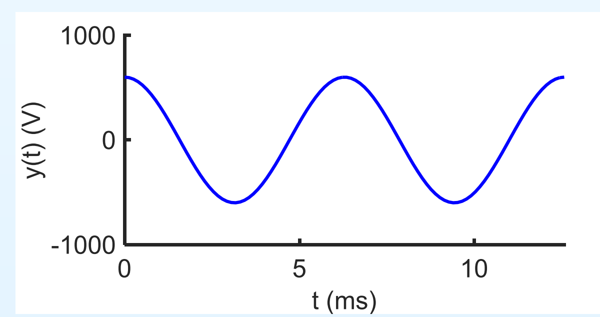
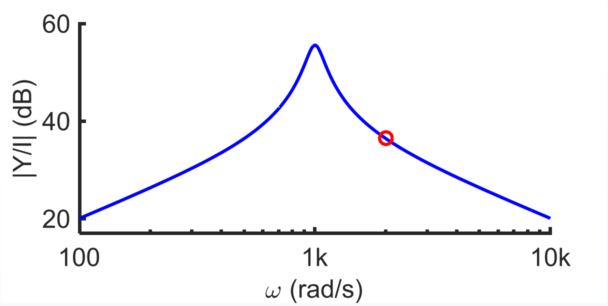


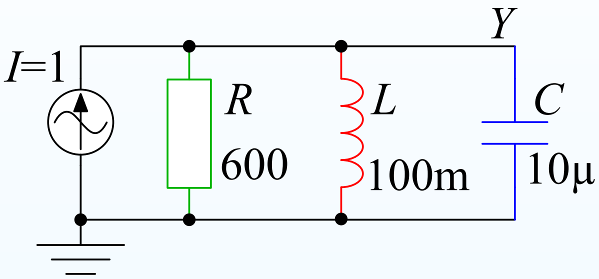




Large currents in L and C exactly cancel out $\Rightarrow I_R = I$ and $Z = R$ (real)

Away from resonance





$$\omega = 2000 \Rightarrow Z_L = 200j,\ Z_C = -50j$$

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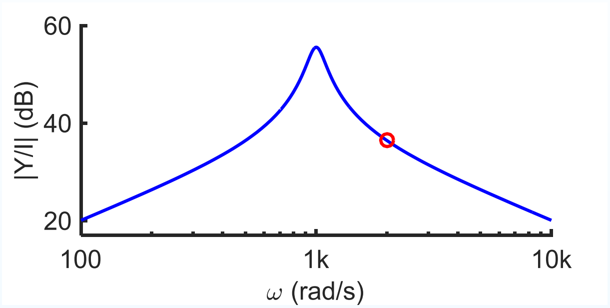

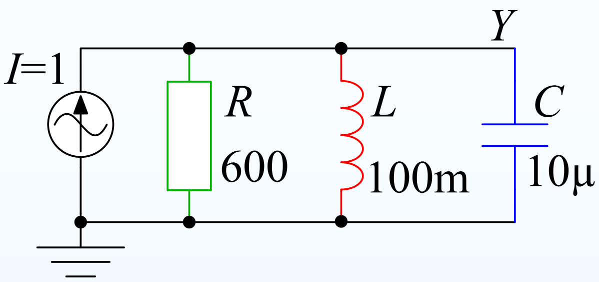

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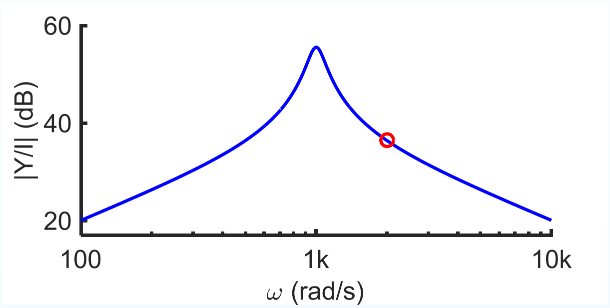

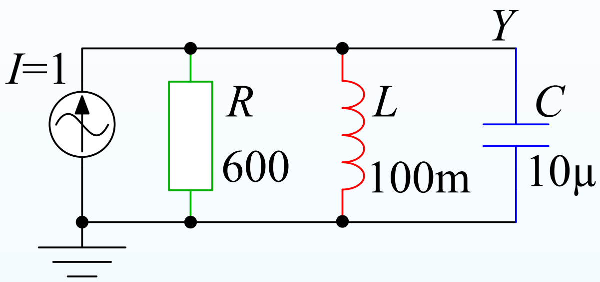

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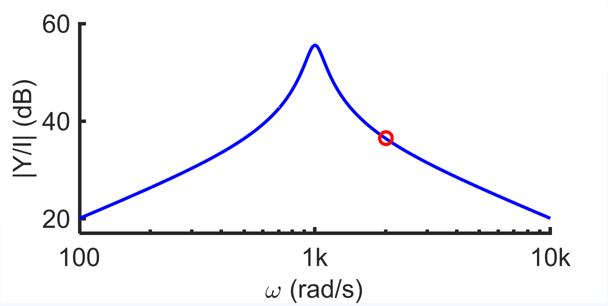

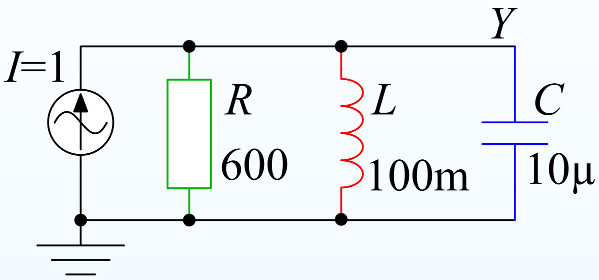

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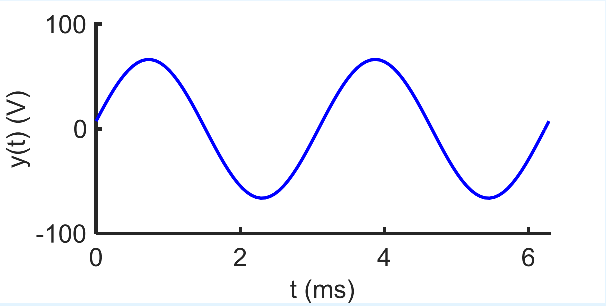

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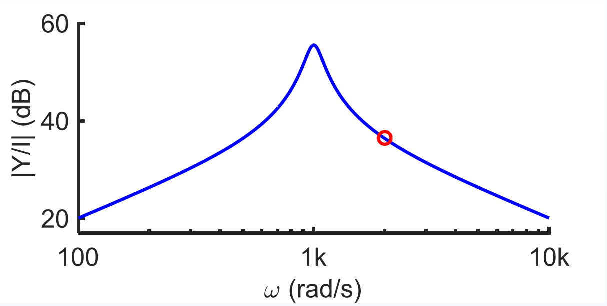

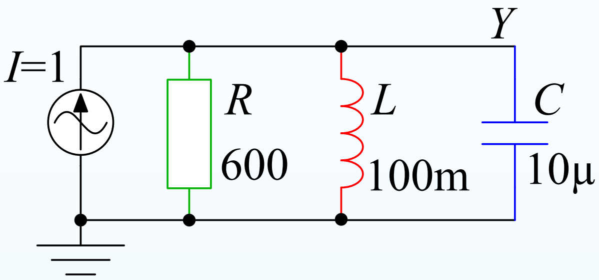

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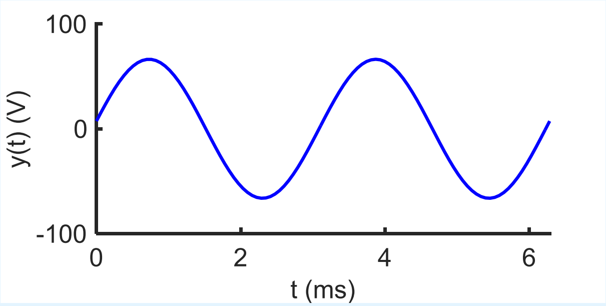

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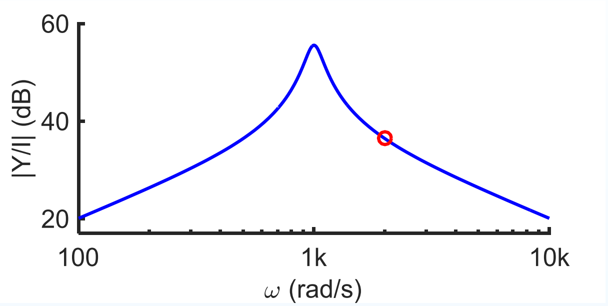

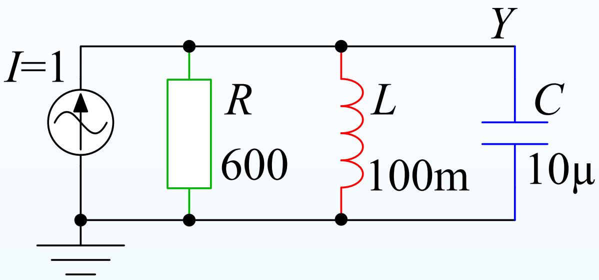

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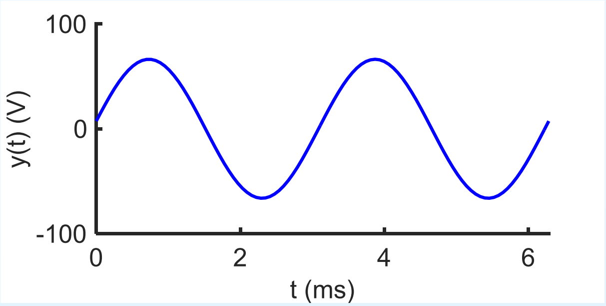
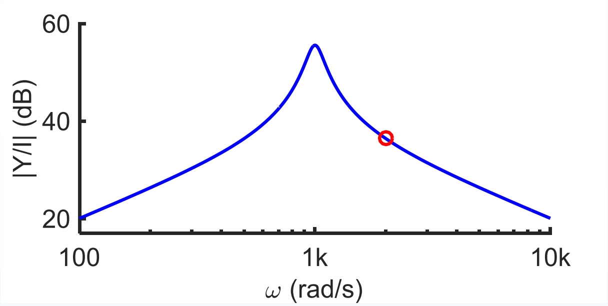
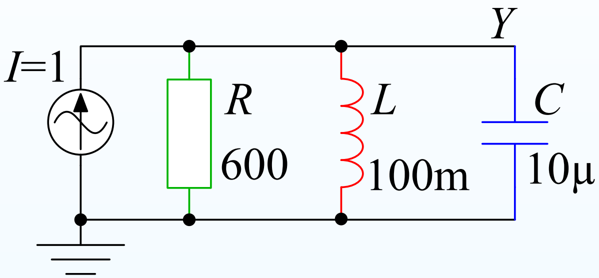
$$I_L = \frac{Y}{Z_L} = 0.33\angle -174^\circ$$

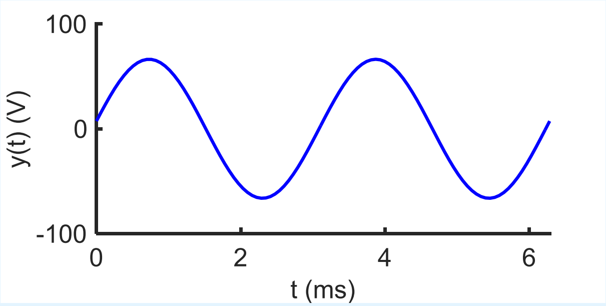


Away from resonance





$$\omega = 2000 \Rightarrow Z_L = 200j,\ Z_C = -50j$$
$$Z = \left(\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}\right)^{-1} = 66\angle - 84°$$
$$Y = I \times Z = 66\angle - 84° = 36\,\text{dBV}$$
$$I_R = \frac{Y}{R} = 0.11\angle - 84°$$
$$I_L = \frac{Y}{Z_L} = 0.33\angle - 174°,\ I_C = 1.33\angle + 6°$$



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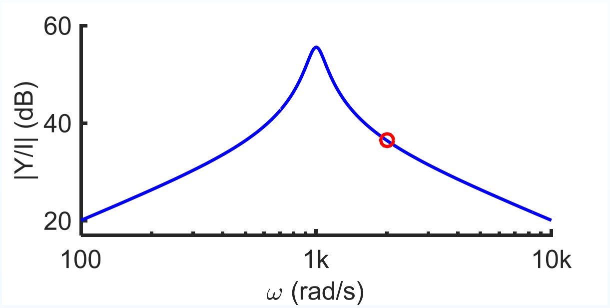

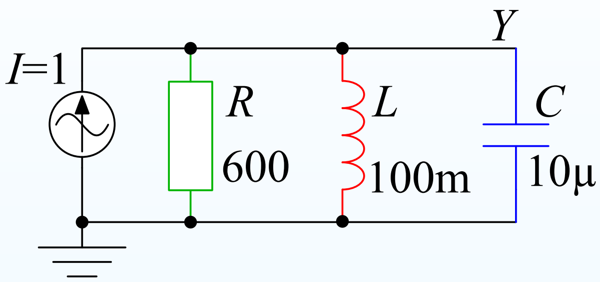

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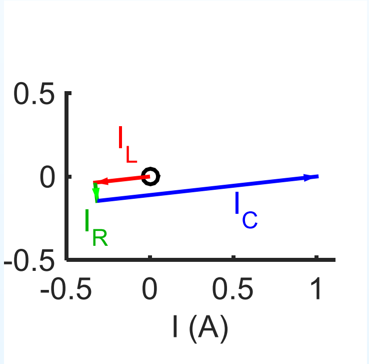

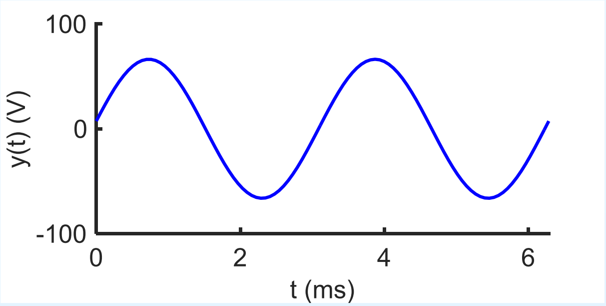

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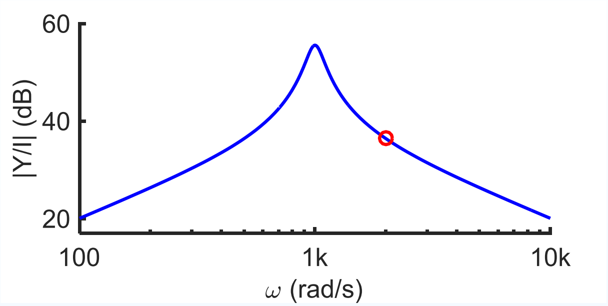

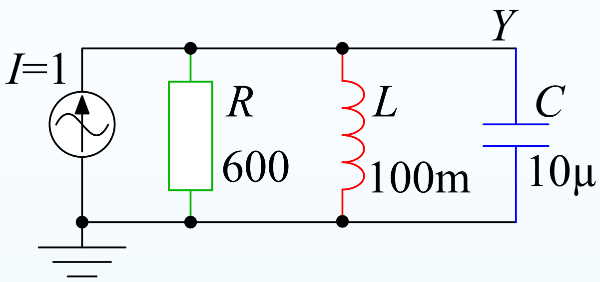

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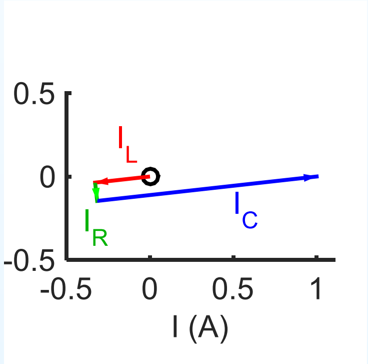

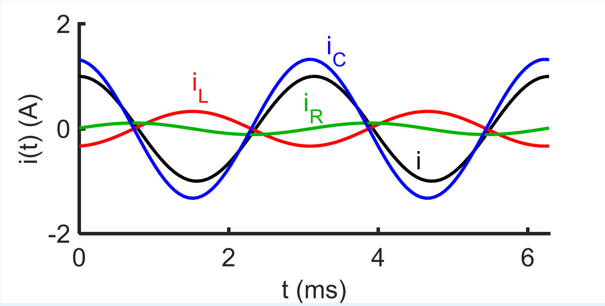

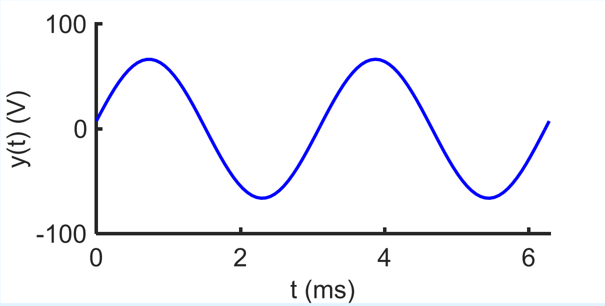

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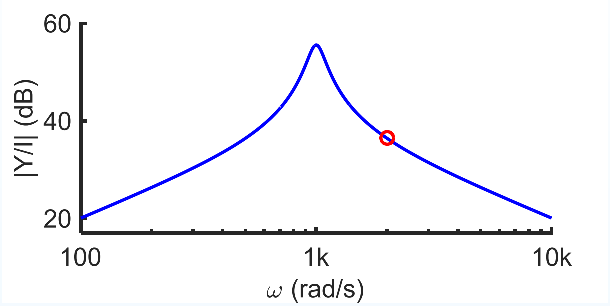

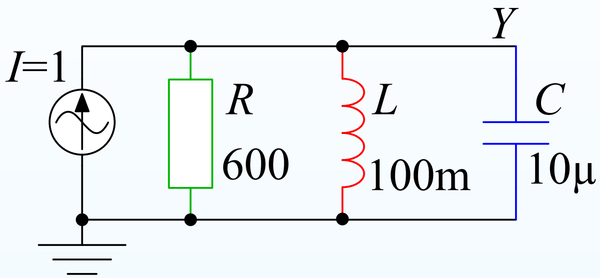

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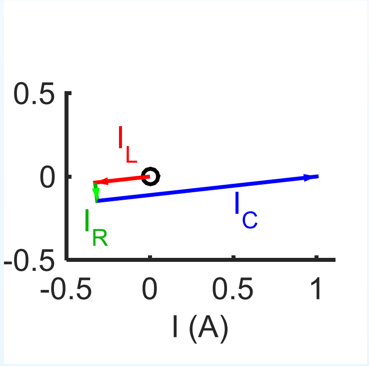
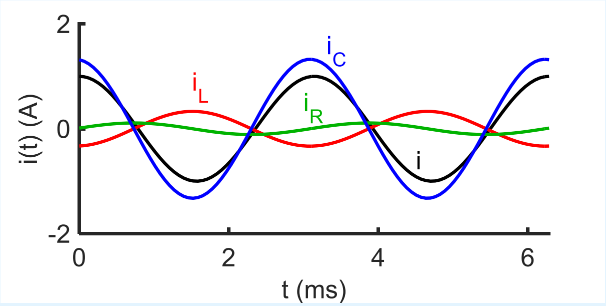
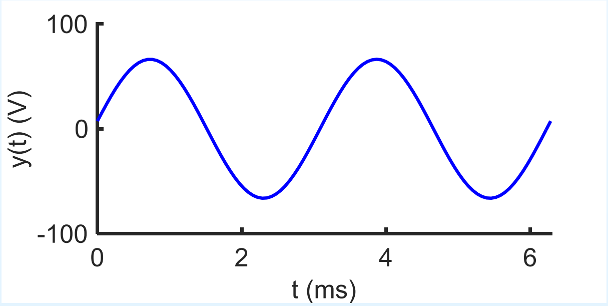


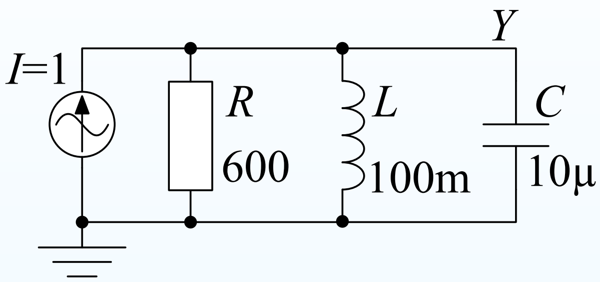




Most current now flows through C, only 0.11 through R.

Bandwidth and Q

$$\frac{Y}{I} = \frac{1}{1/R + j\left(\omega C - 1/\omega L\right)}$$



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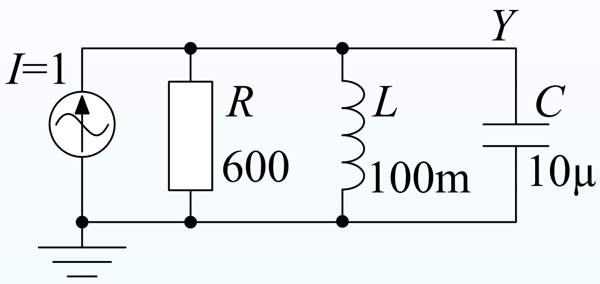

Bandwidth is the range of frequencies for which $\left|\frac{Y}{I}\right|^2$ is greater than half its peak.

Also called *half-power bandwidth* or *3dB bandwidth*.

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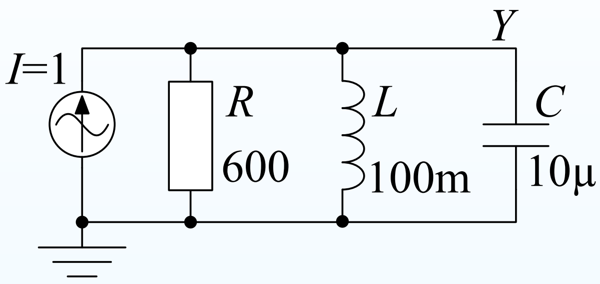

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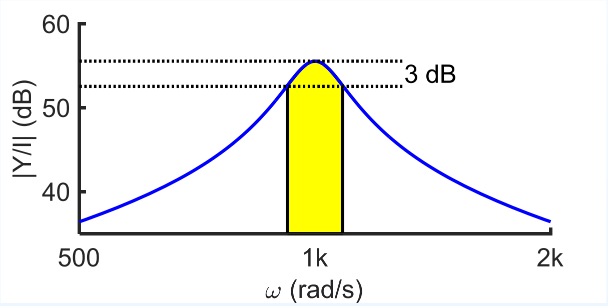

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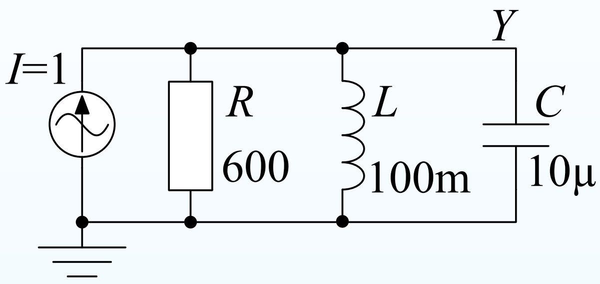

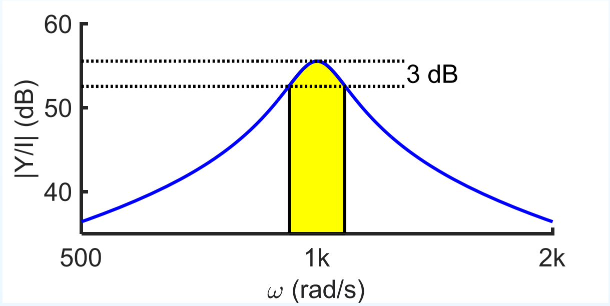

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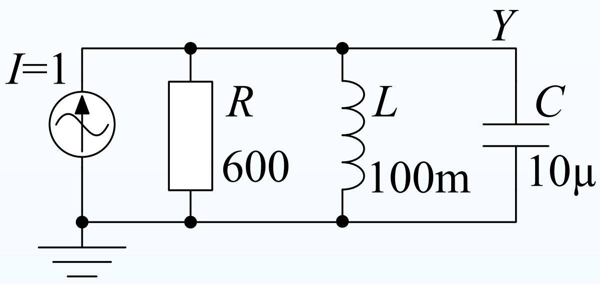

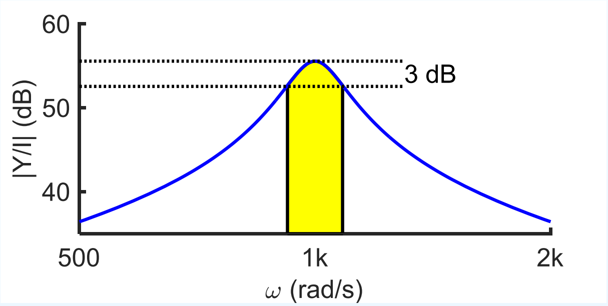

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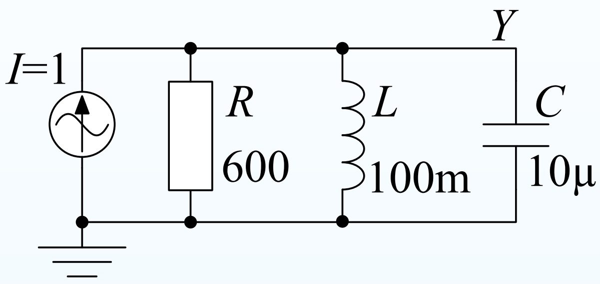

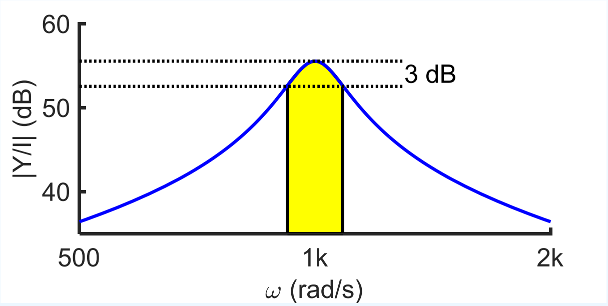

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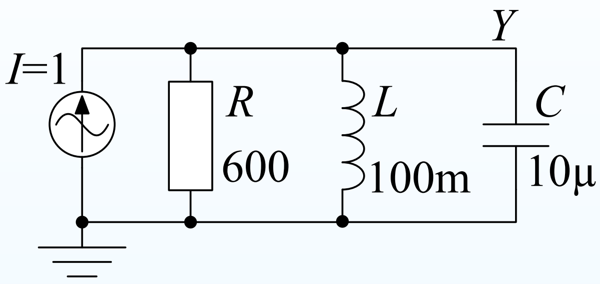

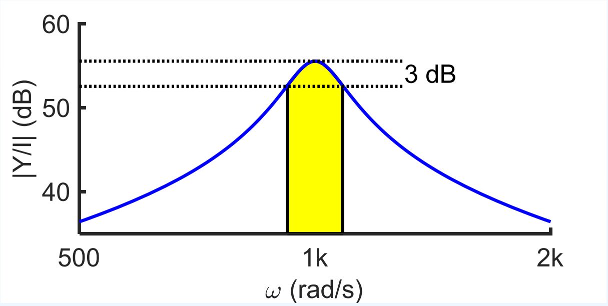

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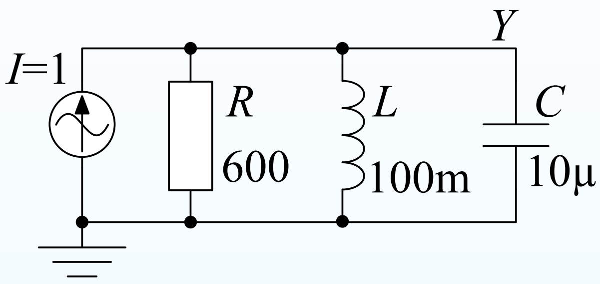

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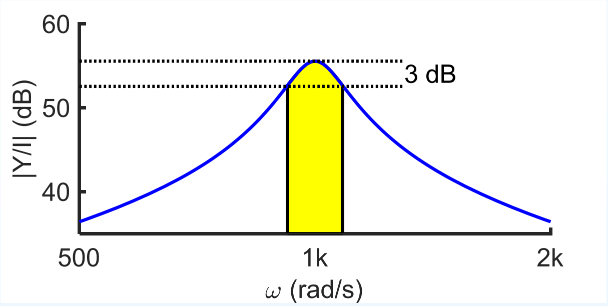

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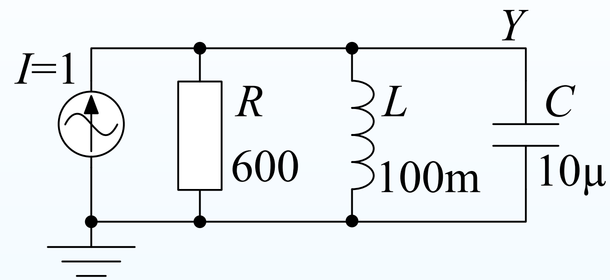

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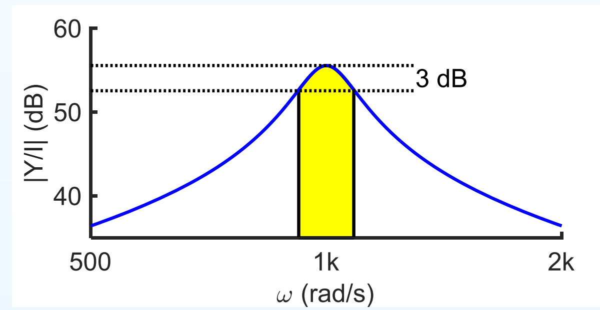

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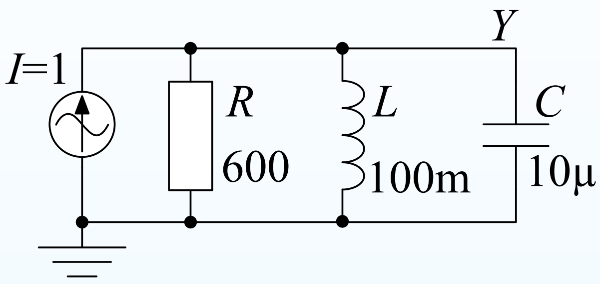

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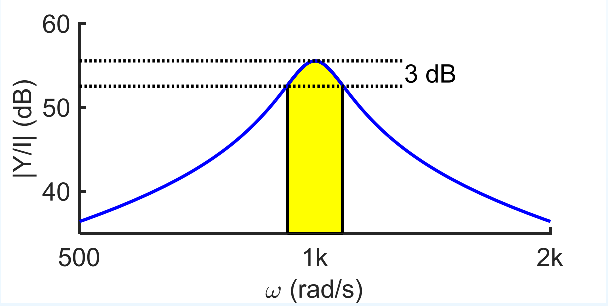

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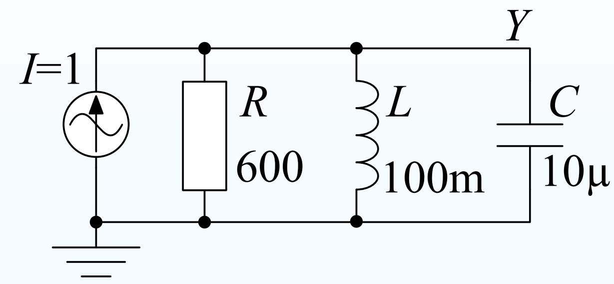

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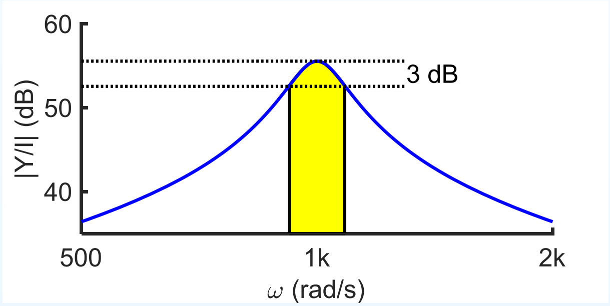

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Positive roots: $\omega_{3\text{dB}} = \frac{\pm L + \sqrt{L^2 + 4R^2 LC}}{2RLC} = \{920,\ 1086\}$ rad/s

Bandwidth and Q

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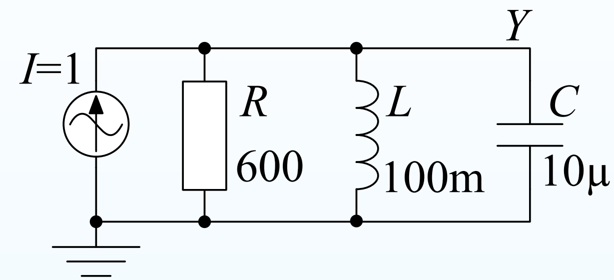

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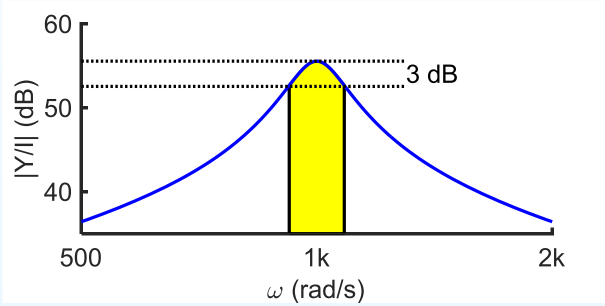

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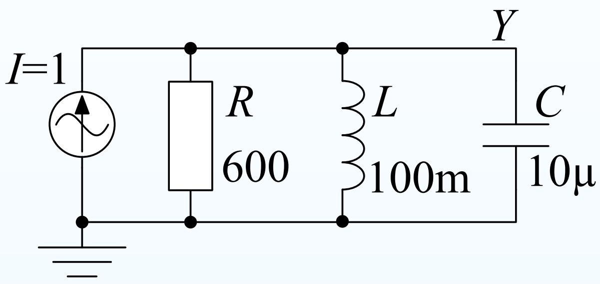

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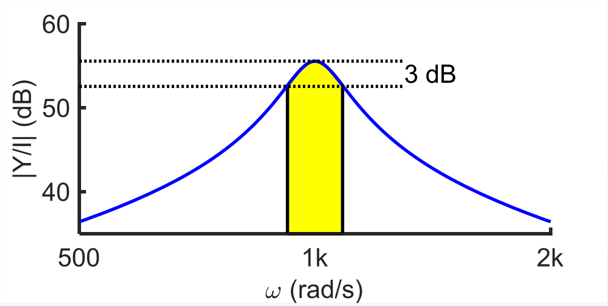

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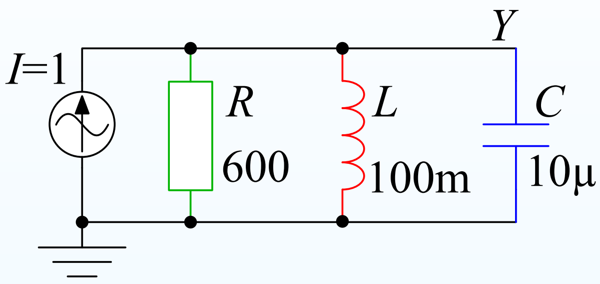
Bandwidth: $B = 1086 - 920 = 167\,\text{rad/s}.$

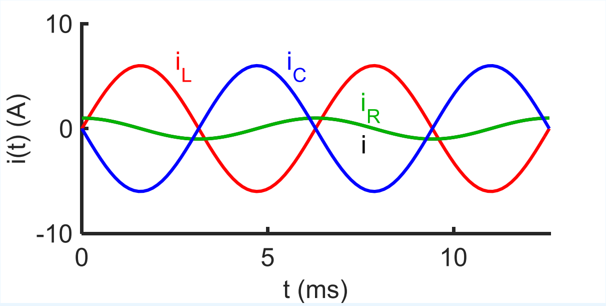
Q factor $\approx \frac{\omega_0}{B} = \frac{1}{2\zeta} = 6.$ (Q = "Quality")

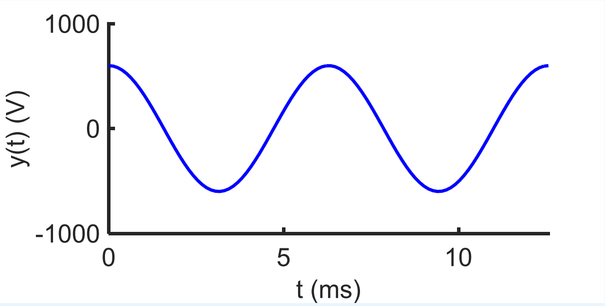
Power and Energy at Resonance



@$\omega = 1000$: $Y = 600$,

$I_R = 1$, $I_L = -6j$, $I_C = +6j$





Power and Energy at Resonance

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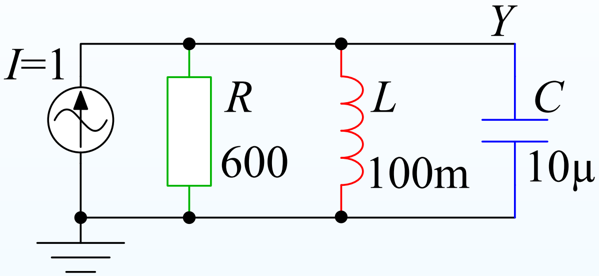

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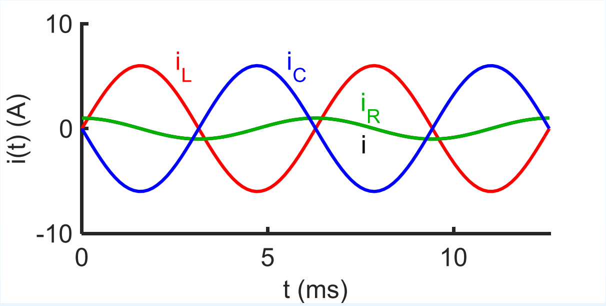

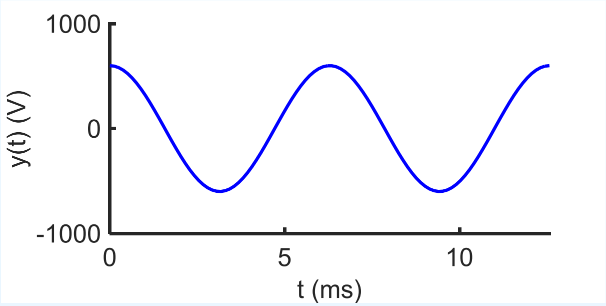

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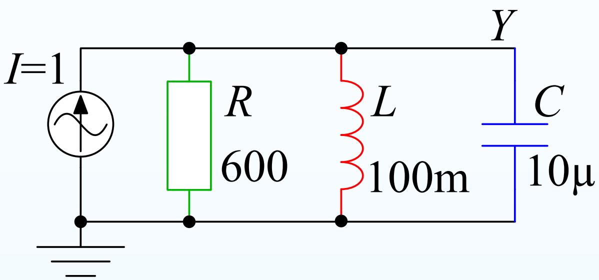

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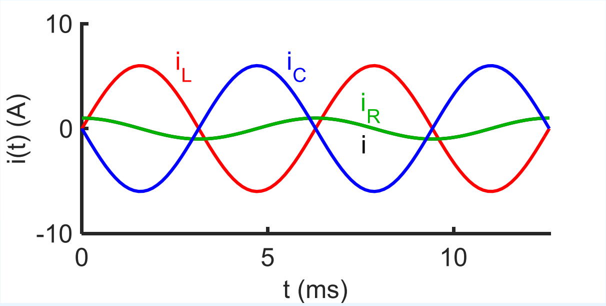

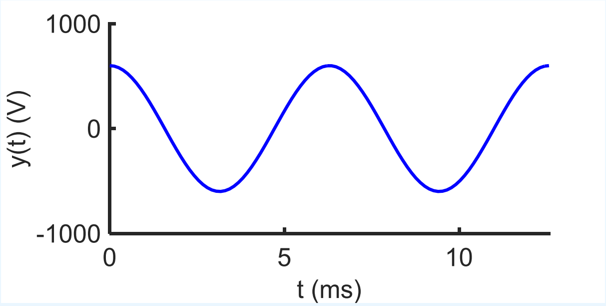

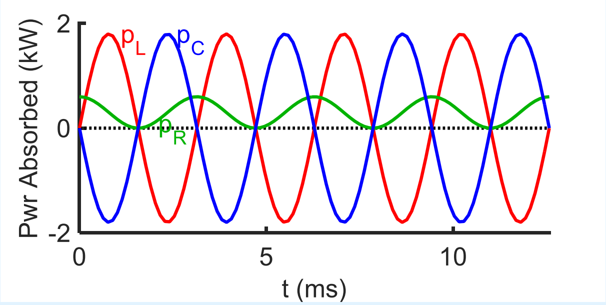

Power and Energy at Resonance

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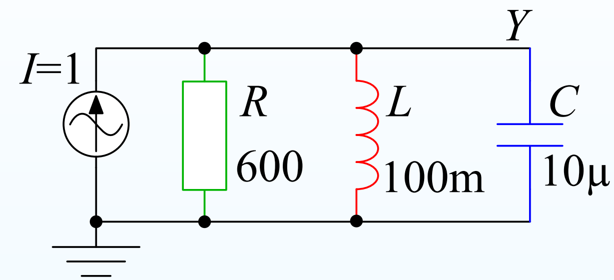

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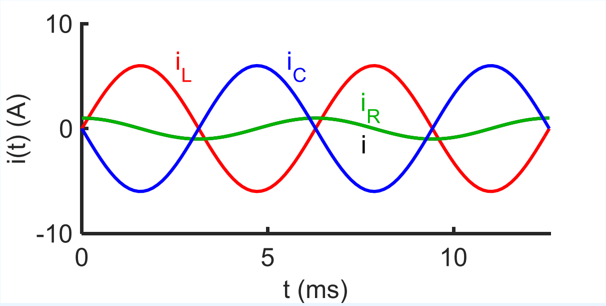

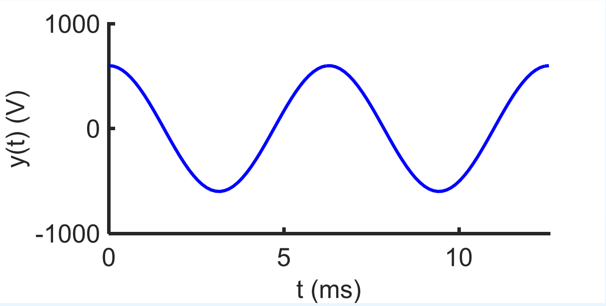

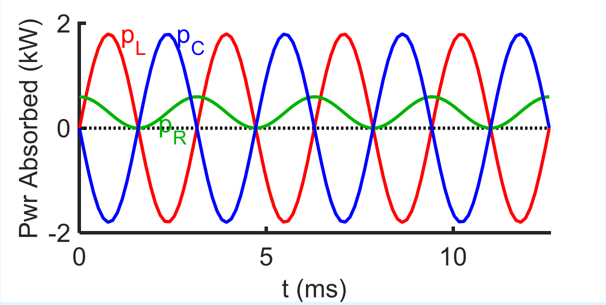

Power and Energy at Resonance

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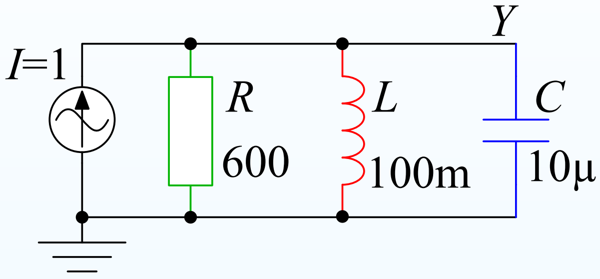

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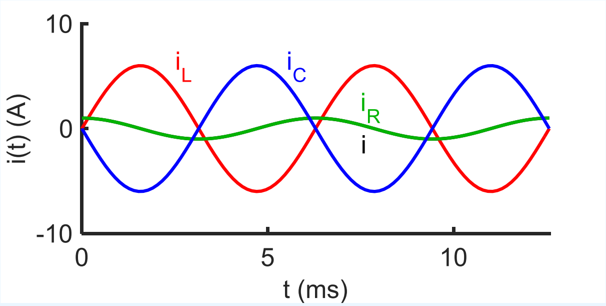

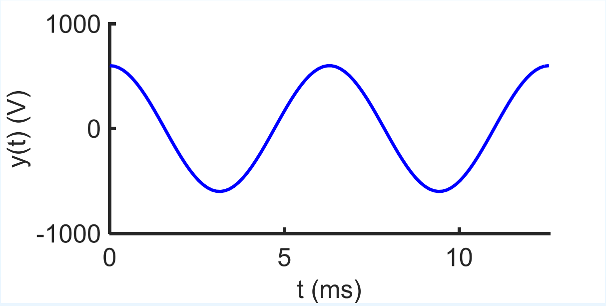

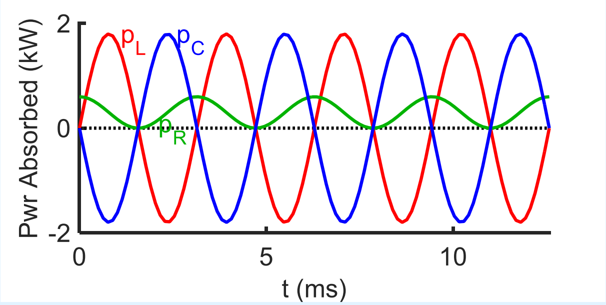

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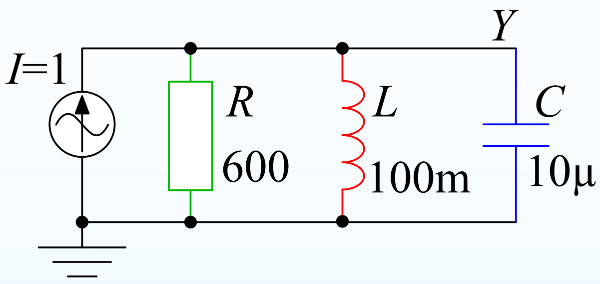

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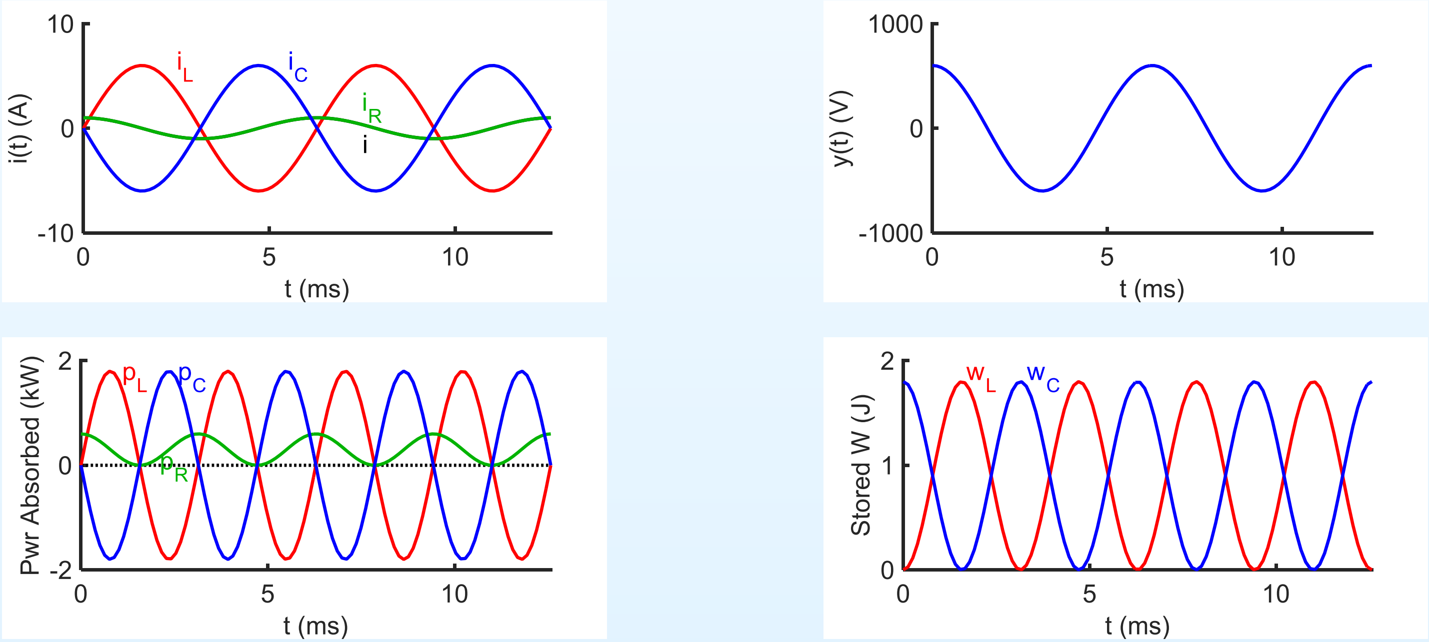

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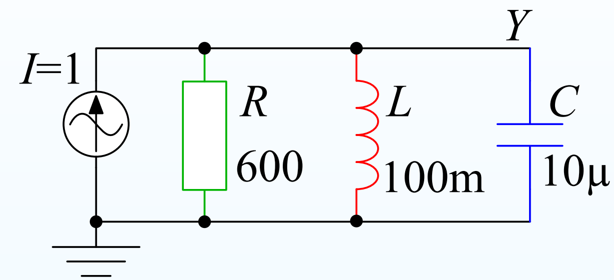

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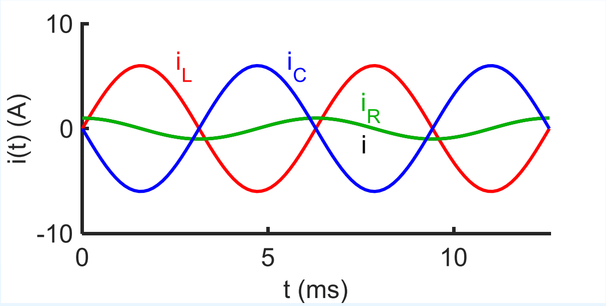

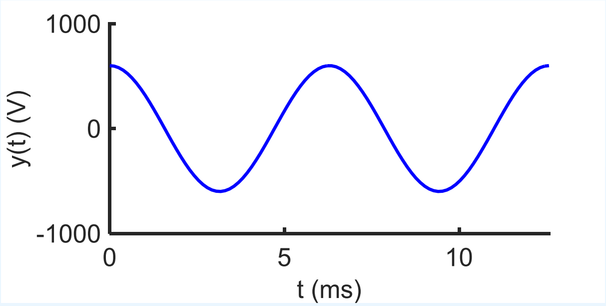

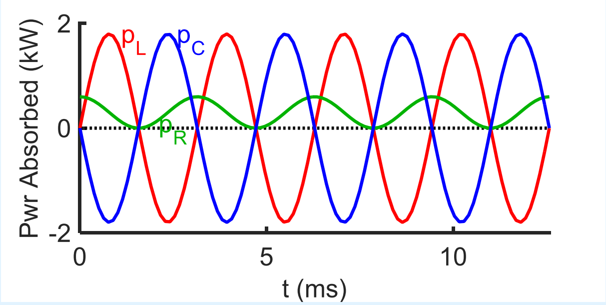

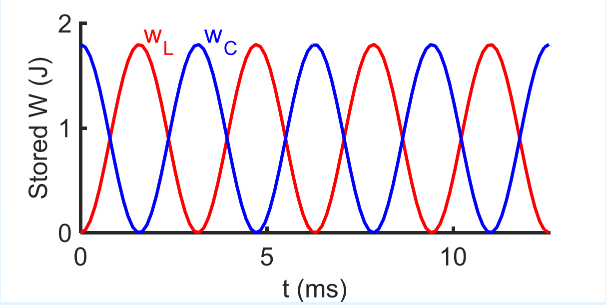

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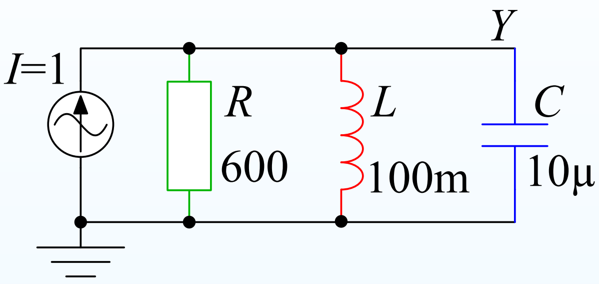
$Q \triangleq \omega \times W_{\text{stored}} \div \overline{P}_R$



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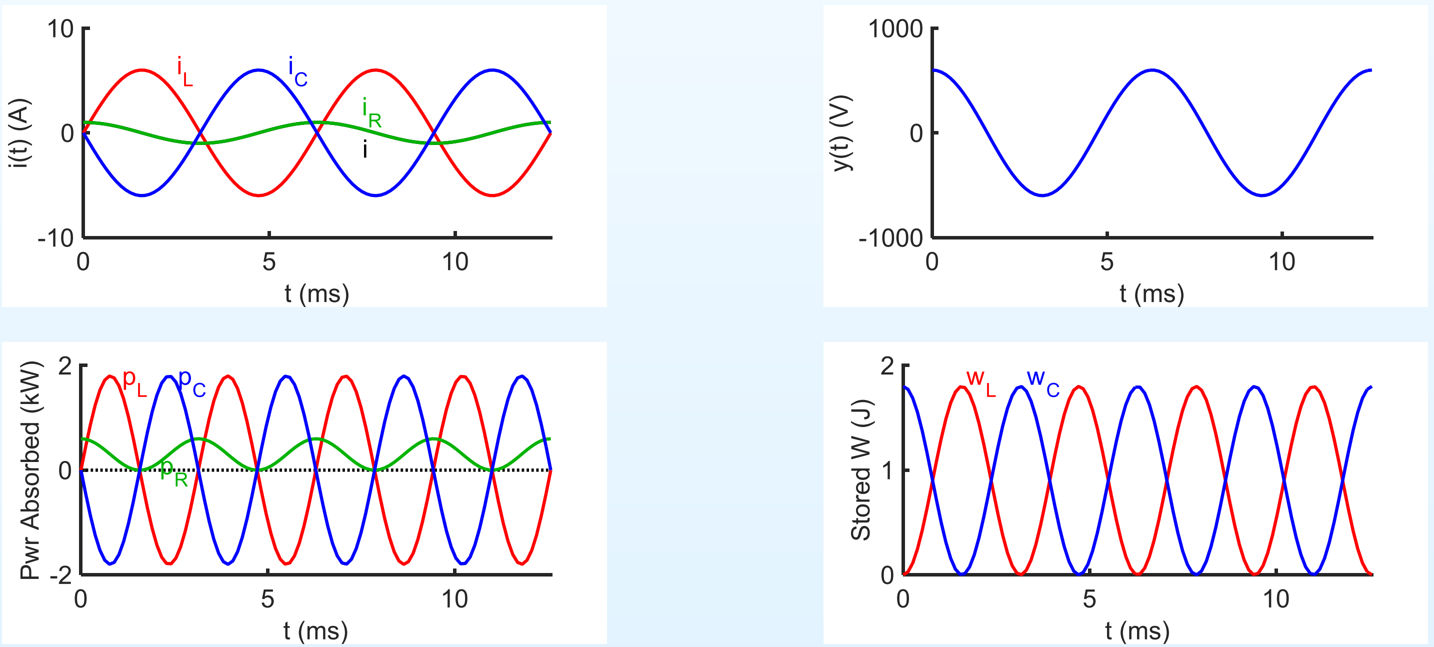

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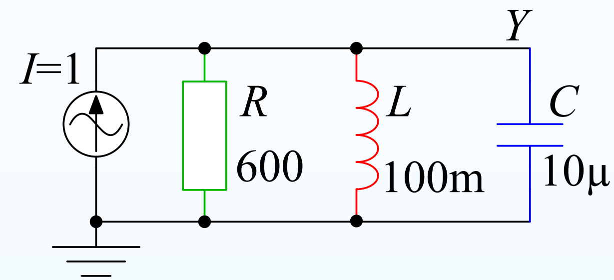

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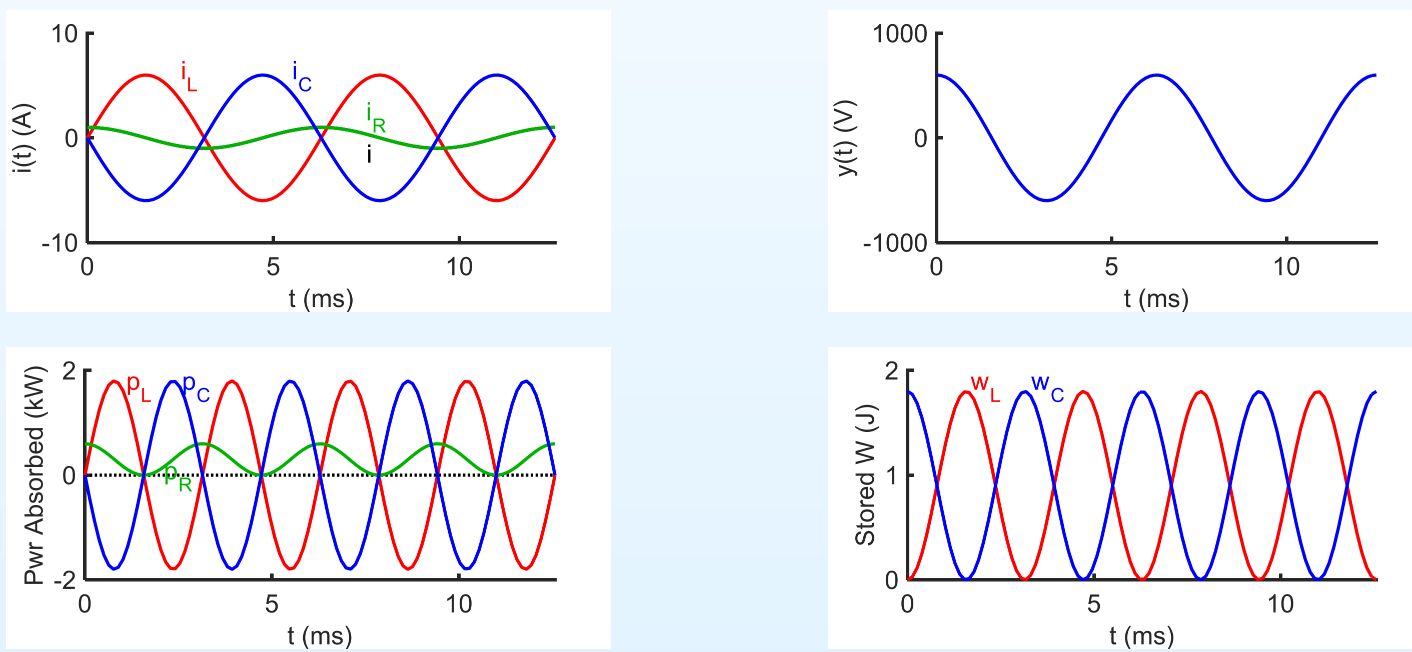

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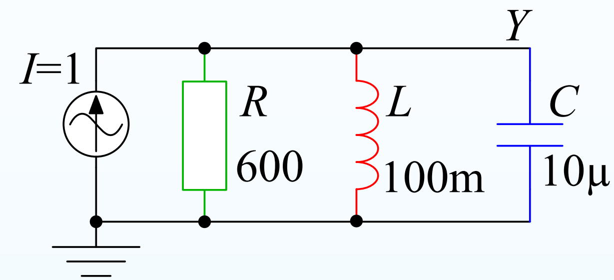

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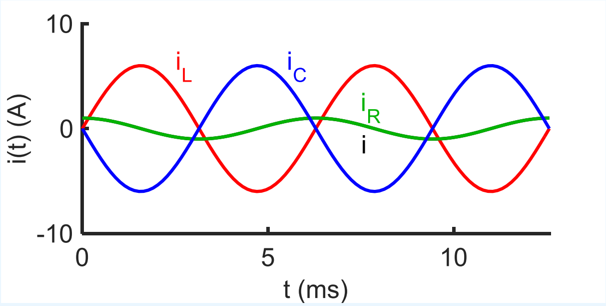
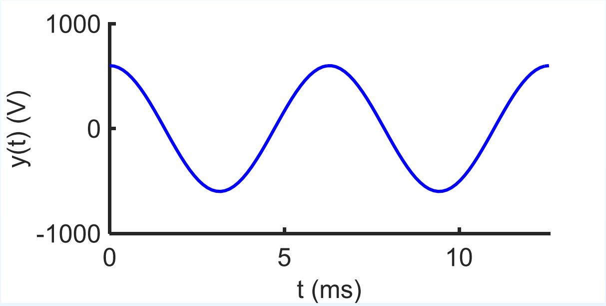
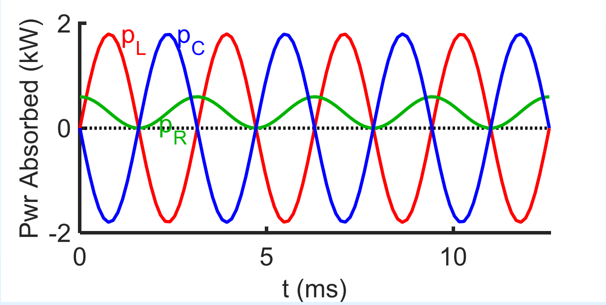
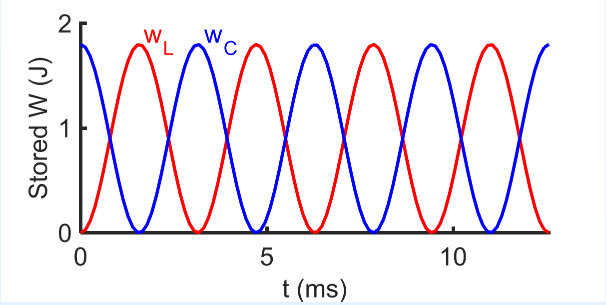




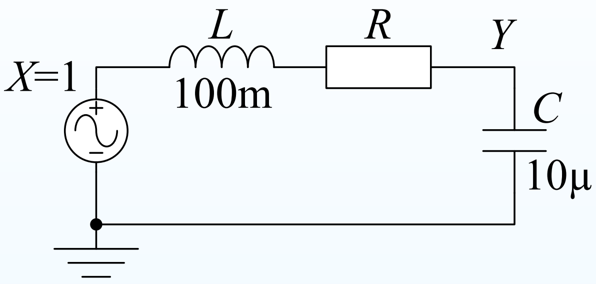




$Q \triangleq \omega \times$ peak stored energy $\div$ average power loss.

Low Pass Filter

$$\frac{Y}{X} = \frac{1/j\omega C}{R+j\omega L+\frac{1}{j\omega C}} = \frac{1}{LC(j\omega)^2+RCj\omega+1}$$



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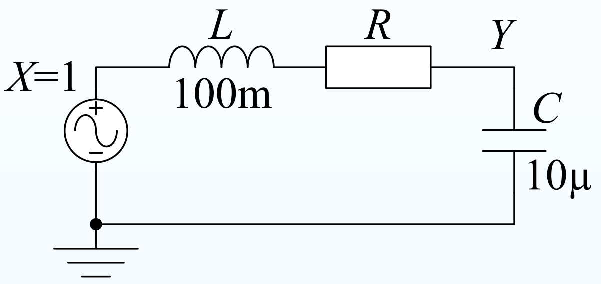
Asymptotes: 1 and $\frac{1}{LC}(j\omega)^{-2}$.



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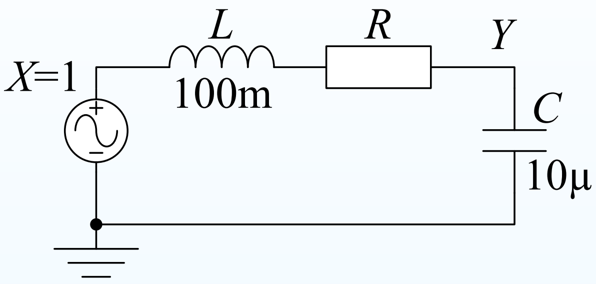

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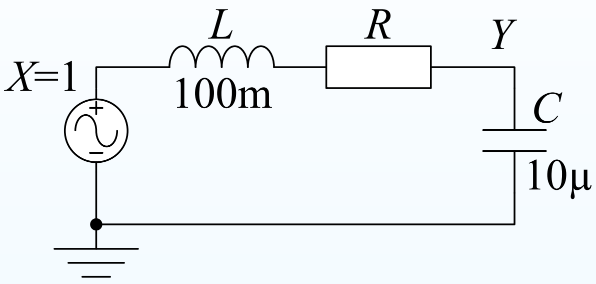

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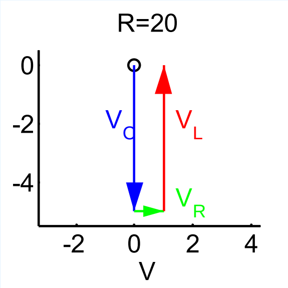

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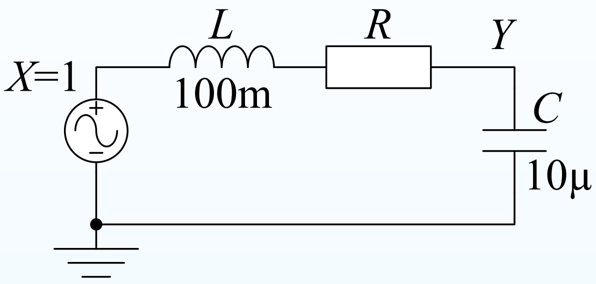

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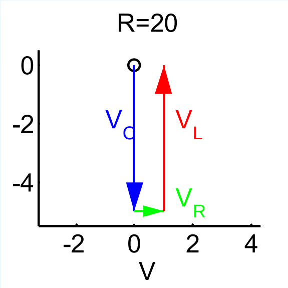

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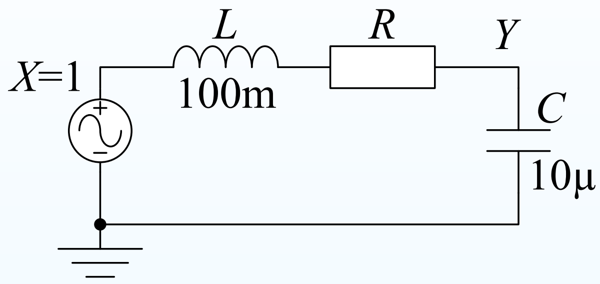

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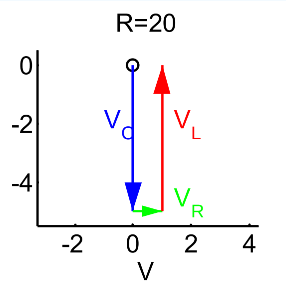

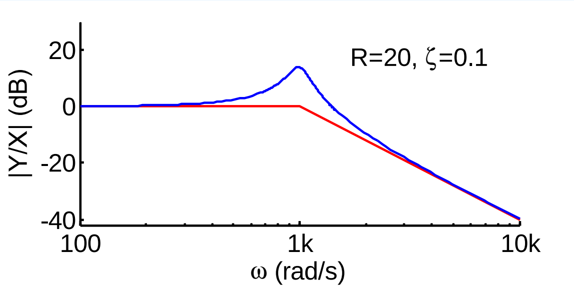

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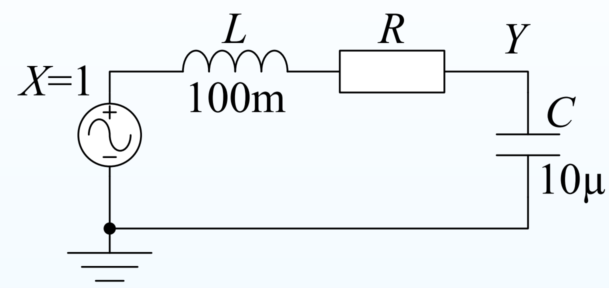

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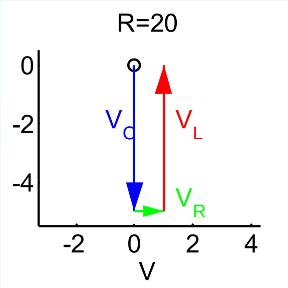
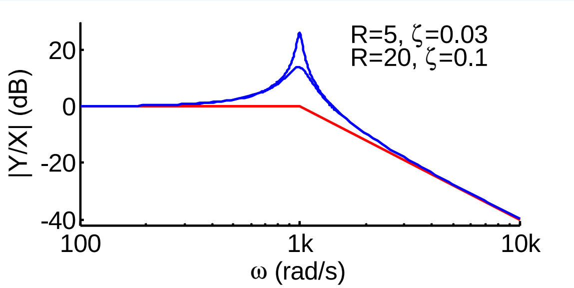
Small $\zeta \Rightarrow$ less loss, higher peak, smaller bandwidth.





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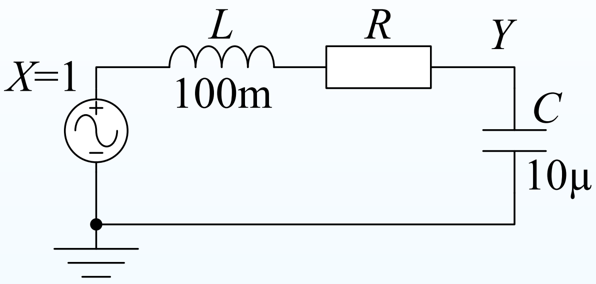

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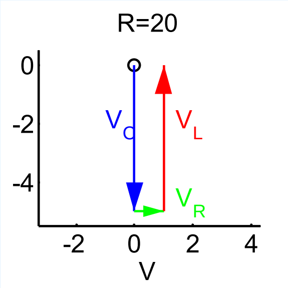

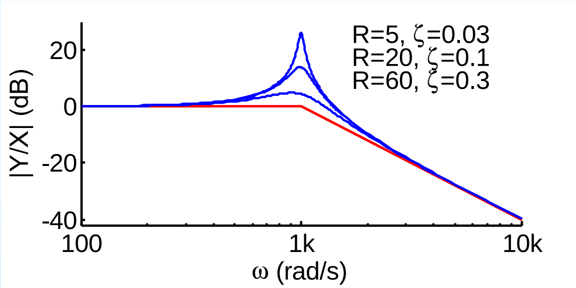

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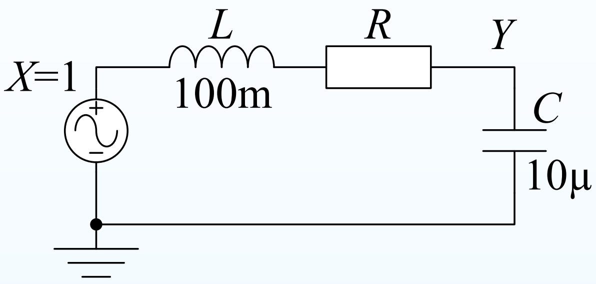

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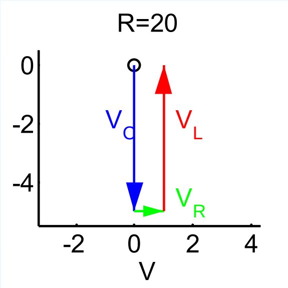

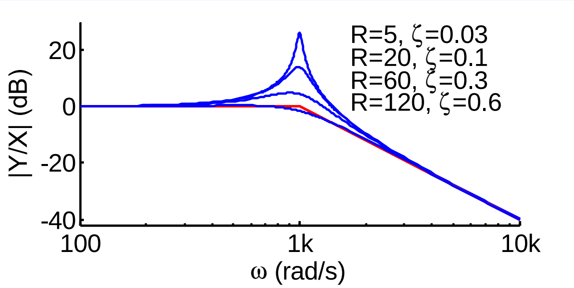

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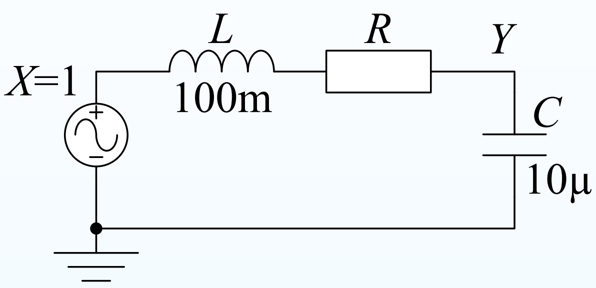

Asymptotes: 1 and $\frac{1}{LC}(j\omega)^{-2}$.

$\omega_c=\sqrt{\frac{c}{a}}=1000,\ \zeta=\frac{b}{2a\omega_c}=\frac{R}{200}$

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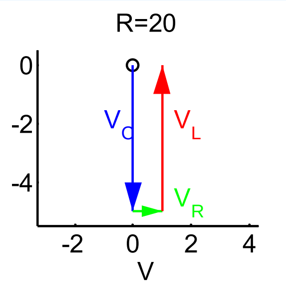

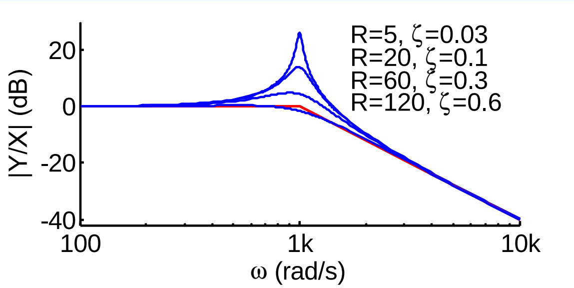

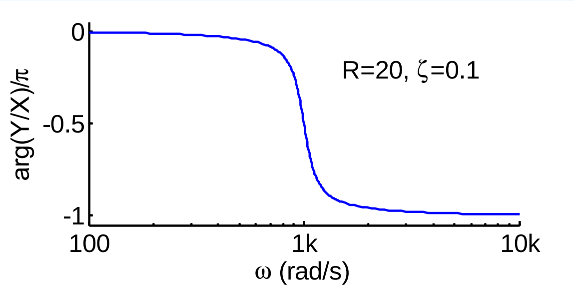

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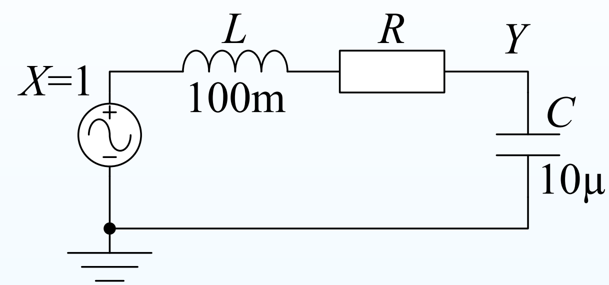

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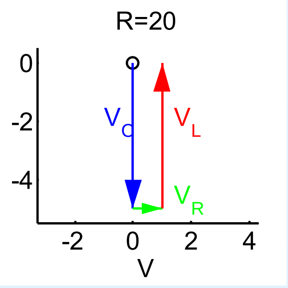

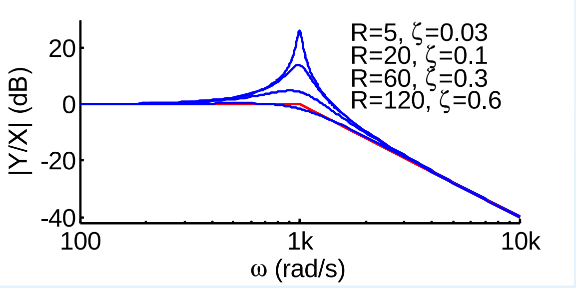

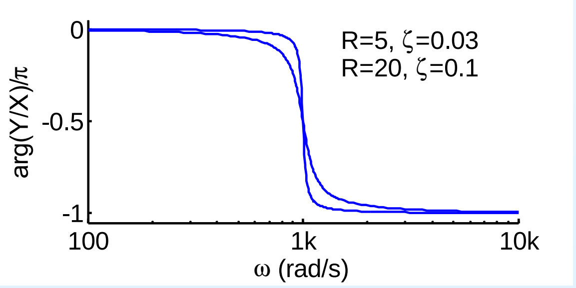

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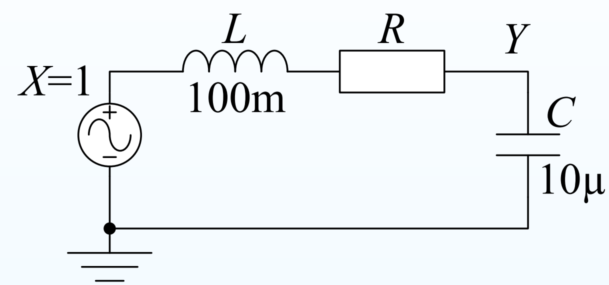

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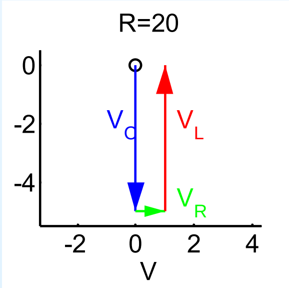

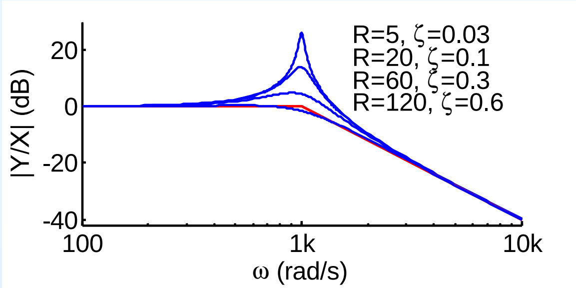

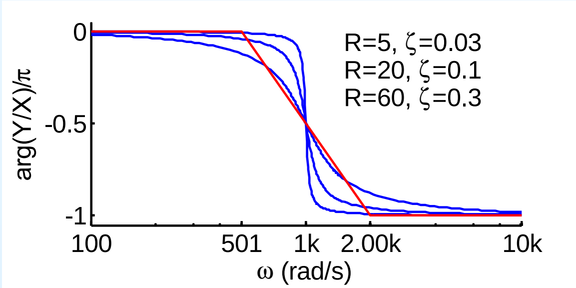

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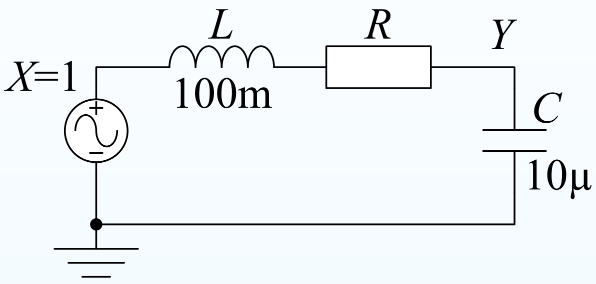

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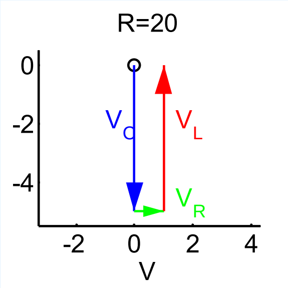

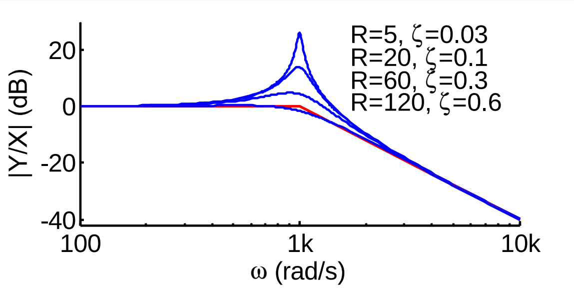

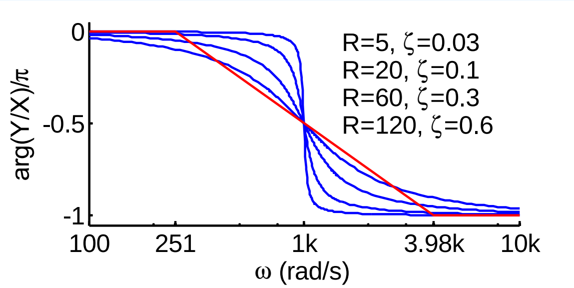

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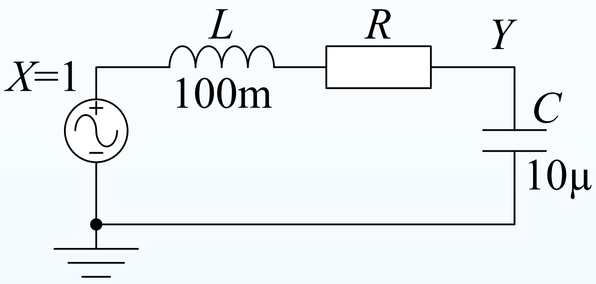

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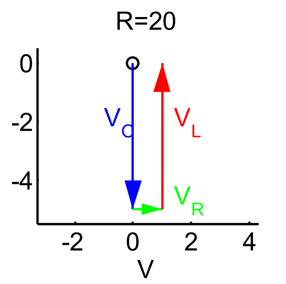
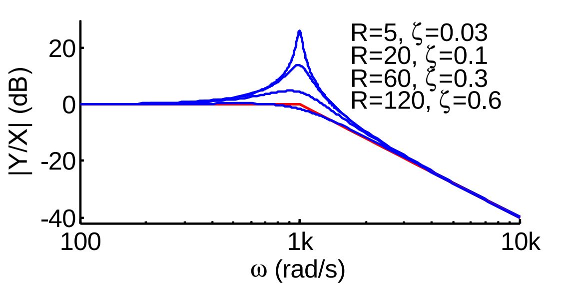
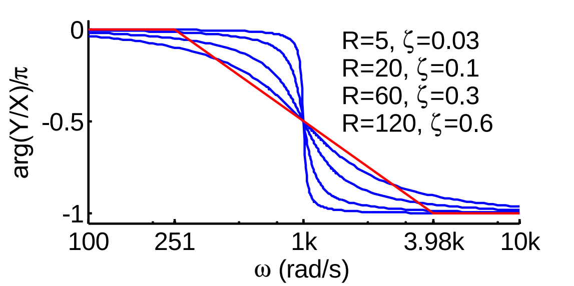
$\angle\frac{Y}{X} \approx \frac{-\pi}{2}\left(1+\frac{1}{\zeta}\log_{10}\frac{\omega}{\omega_c}\right)$ for $10^{-\zeta} < \frac{\omega}{\omega_c} < 10^{+\zeta}$

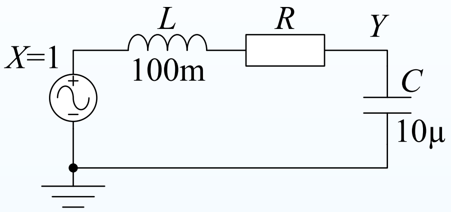






Resonance Peak for LP filter

$$\frac{Y}{X} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1}$$



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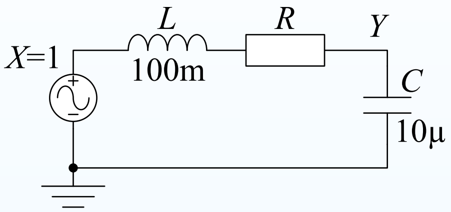

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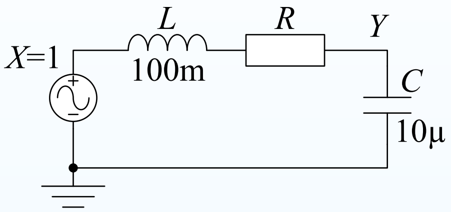

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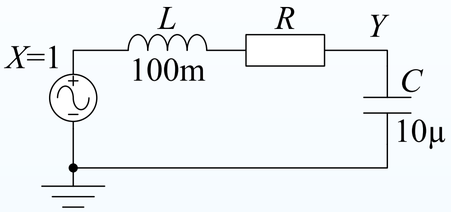

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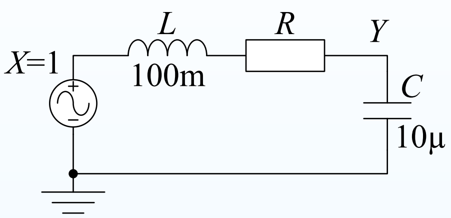

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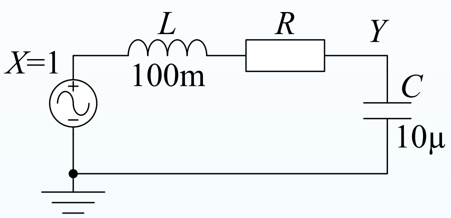

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Peak frequency:

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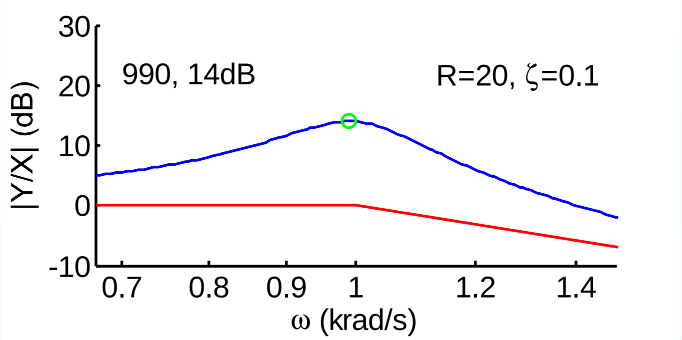

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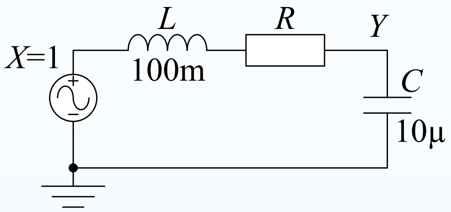

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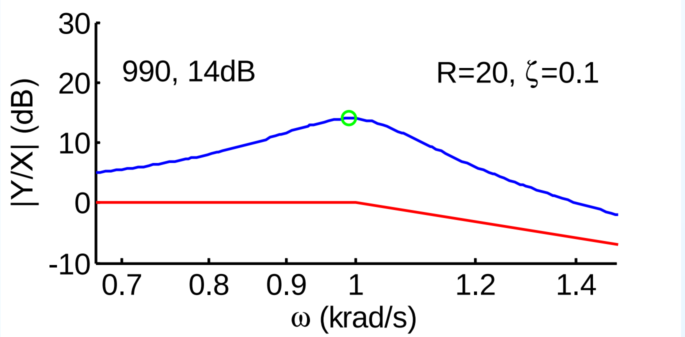

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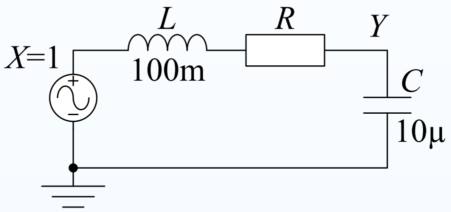

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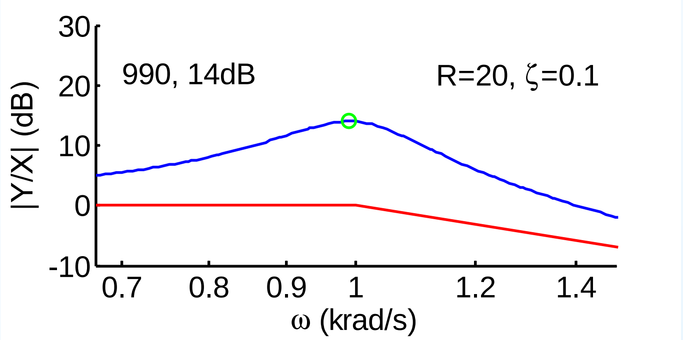

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Resonance Peak for LP filter

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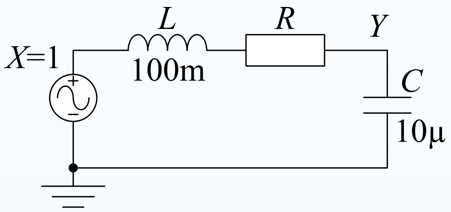

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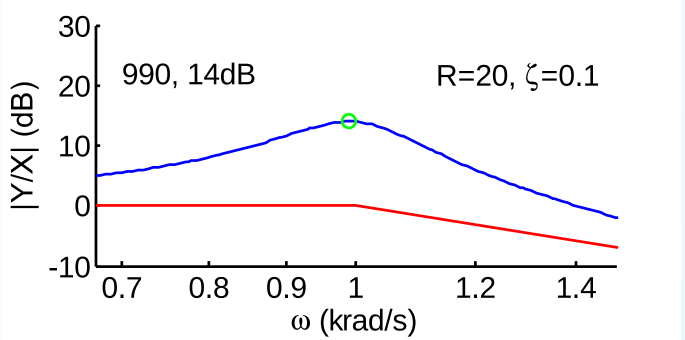

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Gain relative to asymptote: @ ω_p: $\frac{1}{2\zeta\sqrt{1-\zeta^2}}$ @ ω_c: $\frac{1}{2\zeta} \approx Q$

Resonance Peak for LP filter

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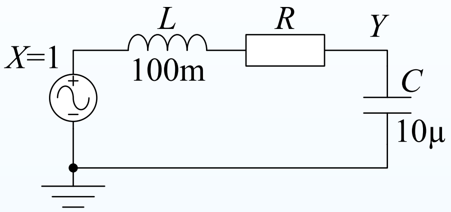

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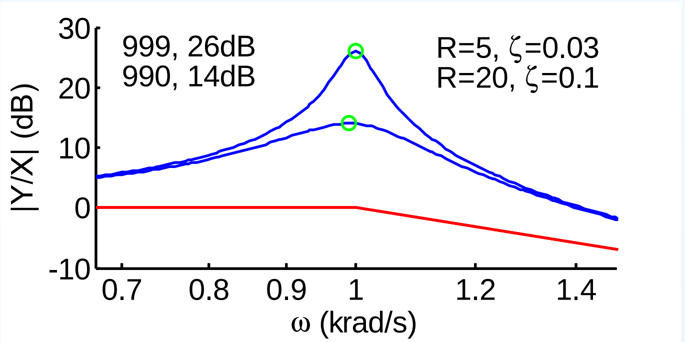

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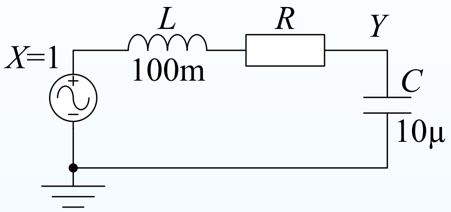

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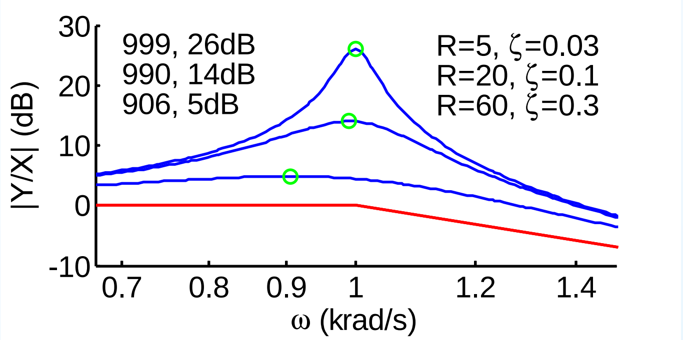

$\zeta \geq 0.71 \Rightarrow$ no peak,
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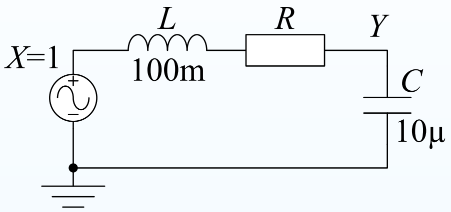

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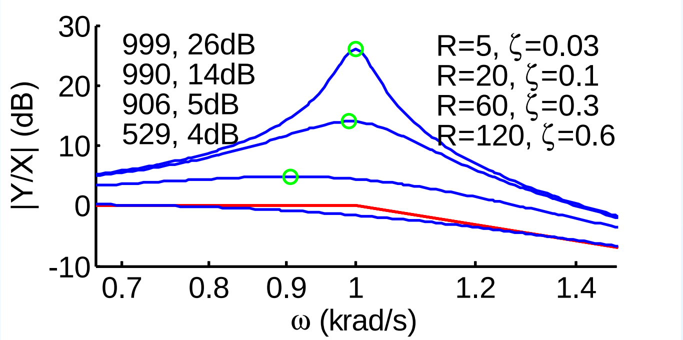
The *damping factor*, ζ, ("zeta") determines the shape of the peak.

Peak frequency:

$$\omega_p = \omega_c\sqrt{1 - 2\zeta^2}$$

$\zeta \geq 0.5 \Rightarrow$ passes under corner,
$\zeta \geq 0.71 \Rightarrow$ no peak,
$\zeta \geq 1 \Rightarrow$ can factorize



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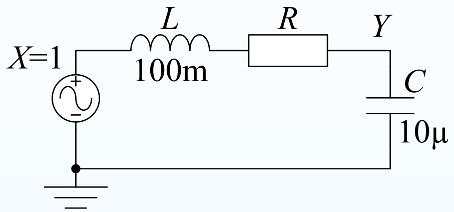
Resonance Peak for LP filter

$$\frac{Y}{X} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} \quad = \frac{1}{\left(j\frac{\omega}{\omega_c}\right)^2 + 2\zeta j\frac{\omega}{\omega_c} + 1}$$



$$\omega_c = \sqrt{\frac{c}{a}} = 1000,\ \zeta = \frac{b}{2a\omega_c} = \frac{b\omega_c}{2c} = \frac{R}{200}$$

$\frac{Y}{X}$ is a function of $\frac{\omega}{\omega_c}$ so ω_c just scales frequency axis (= shift on log axis).
The *damping factor*, ζ, ("zeta") determines the shape of the peak.

Peak frequency:

$$\omega_p = \omega_c\sqrt{1 - 2\zeta^2}$$

$\zeta \geq 0.5 \Rightarrow$ passes under corner,
$\zeta \geq 0.71 \Rightarrow$ no peak,
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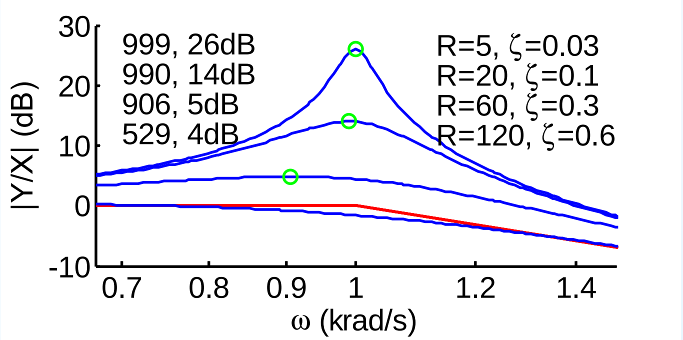

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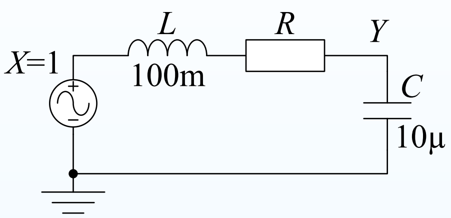

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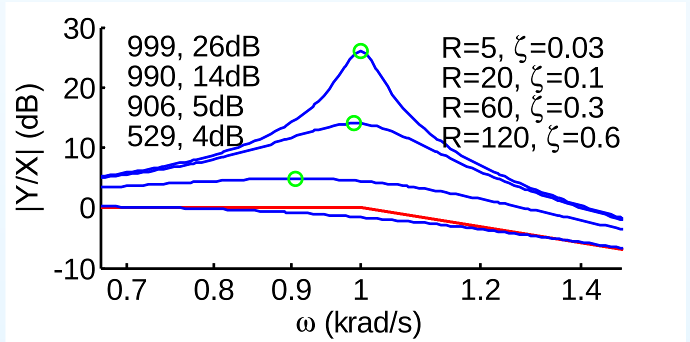

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The exact relationship between ω_p, ω_c and ω_r and the gain at these frequencies is affected by any other corner frequencies in the response.

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For further details see Hayt Ch 16 or Irwin Ch 12.