12: Resonance

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- Summary
A quadratic factor in a transfer function is: \( F(j\omega) = a(j\omega)^2 + b(j\omega) + c. \)
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Any polynomial with real coefficients can be factored into linear and quadratic factors \( \Rightarrow \) a quadratic factor is as complicated as it gets.
Damping Factor and Q

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**Low/High freq asymptotes:** $F_{LF}(j\omega) = c$
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\Rightarrow F(j\omega) = c \left( (j \frac{\omega}{\omega_c})^2 + 2\zeta \left( j \frac{\omega}{\omega_c} \right) + 1 \right)
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Suppose $b^2 < 4ac$ in $F(j\omega) = a (j\omega)^2 + b (j\omega) + c$.

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Properties to notice in this expression:

**a** $c$ is just an overall scale factor.
Suppose \( b^2 < 4ac \) in \( F(j\omega) = a(j\omega)^2 + b(j\omega) + c \).

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Properties to notice in this expression:
(a) \( c \) is just an overall scale factor.
(b) \( \omega_c \) just scales the frequency axis since \( F(j\omega) \) is a function of \( \frac{\omega}{\omega_c} \).
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(e) At $\omega = \omega_c$, asymptote gain = $c$ but $F(j\omega) = c \times 2j\zeta$. 
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Alternatively, we sometimes use the *quality factor*, $Q \approx \frac{1}{2\zeta} = \frac{a\omega_c}{b}$. 
Parallel RLC

\[ \frac{Y}{T} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{LC(j\omega)^2 + \frac{1}{R}j\omega + 1} \]
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\frac{Y}{I} = \frac{1}{R + \frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{LC(j\omega)^2 + \frac{R}{L}j\omega + 1}
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\[\omega_c = \sqrt{\frac{C}{L}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = 0.083\]

\[\frac{|Y|}{I} \text{ (dB)}\]

\[\arg(Y)/\pi\]

\[\begin{align*}
I &= 1 \\
R &= 600 \\
L &= 100 \text{ mH} \\
C &= 10 \mu\text{F}
\end{align*}\]
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Power absorbed by resistor \( \propto Y^{-2} \). It peaks quite sharply at \( \omega = 1000 \).
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Power absorbed by resistor \(\propto Y^2\). It peaks quite sharply at \(\omega = 1000\). The resonant frequency, \(\omega_r\), is when the impedance is purely real:

at \(\omega_r = 1000\), \(Z_{RLC} = \frac{Y}{I} = R\).
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A system with a strong peak in power absorption is a resonant system.
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\[ \omega = 1000 \Rightarrow Z_L = 100j, \quad Z_C = -100j. \]
**Behaviour at Resonance**

$$\omega = 1000 \Rightarrow Z_L = 100j, \quad Z_C = -100j.$$  
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\( \Rightarrow Y = I_RR = 600\angle0^\circ = 56 \text{ dBV} \)
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Large currents in \( L \) and \( C \) exactly cancel out \( \Rightarrow I_R = I \) and \( Z = R \) (real)
Away from resonance

\[ \omega = 2000 \Rightarrow Z_L = 200j, \ Z_C = -50j \]
Away from resonance

$$\omega = 2000 \Rightarrow Z_L = 200j, \quad Z_C = -50j$$

$$Z = \left(\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}\right)^{-1} = 66 \angle -84^\circ$$
Away from resonance

\[ \omega = 2000 \Rightarrow Z_L = 200j, \quad Z_C = -50j \]

\[ Z = \left( \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C} \right)^{-1} = 66 \angle -84^\circ \]

\[ Y = I \times Z = 66 \angle -84^\circ = 36 \text{ dBV} \]
Away from resonance

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Away from resonance

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\[ I_R = \frac{Y}{R} = 0.11 \angle -84^\circ \]
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\[ I_R = \frac{Y}{R} = 0.11 \angle -84^\circ \]

\[ I_L = \frac{Y}{Z_L} = 0.33 \angle -174^\circ \]
Away from resonance

\[ \omega = 2000 \Rightarrow Z_L = 200j, \ Z_C = -50j \]

\[ Z = \left( \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C} \right)^{-1} = 66 \angle -84^\circ \]

\[ Y = I \times Z = 66 \angle -84^\circ = 36 \text{ dBV} \]

\[ I_R = \frac{Y}{R} = 0.11 \angle -84^\circ \]

\[ I_L = \frac{Y}{Z_L} = 0.33 \angle -174^\circ, \ I_C = 1.33 \angle +6^\circ \]
Away from resonance

\[ \omega = 2000 \Rightarrow Z_L = 200j, Z_C = -50j \]

\[ Z = \left( \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C} \right)^{-1} = 66 \angle -84^\circ \]

\[ Y = I \times Z = 66 \angle -84^\circ = 36 \text{ dBV} \]

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\[ I_L = \frac{Y}{Z_L} = 0.33 \angle -174^\circ, \quad I_C = 1.33 \angle +6^\circ \]

Most current now flows through \( C \), only 0.11 through \( R \).
Bandwidth and Q

\[ \frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)} \]

- Quadratic Factors
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Bandwidth and Q

\[
\frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)}
\]

**Bandwidth** is the range of frequencies for which \(|\frac{Y}{I}|^2\) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

![Diagram of RLC circuit](image)
**Bandwidth and Q**

\[
\frac{Y}{I} = \frac{1}{\frac{1}{R} + j(\omega C - \frac{1}{\omega L})}
\]

*Bandwidth* is the range of frequencies for which \(\left|\frac{Y}{I}\right|^2\) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.
**Bandwidth and Q**

\[
\frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)}
\]

*Bandwidth* is the range of frequencies for which \( \left| \frac{Y}{I} \right|^2 \) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

\[
\left| \frac{Y}{I} \right|^2 = \frac{1}{(1/R)^2 + (\omega C - 1/\omega L)^2}
\]
Bandwidth and Q

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\frac{Y}{I} = \frac{1}{\frac{1}{R} + j(\omega C - \frac{1}{\omega L})}
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**Bandwidth** is the range of frequencies for which \( |\frac{Y}{I}|^2 \) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

\[
|\frac{Y}{I}|^2 = \frac{1}{(\frac{1}{R})^2 + (\omega C - \frac{1}{\omega L})^2}
\]

Peak is \( |\frac{Y}{I}(\omega_0)|^2 = R^2 \) @ \( \omega_0 = 1000 \)
### Bandwidth and Q

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\frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)}
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Peak is \(|\frac{Y}{I}(\omega_0)|^2 = R^2 @ \omega_0 = 1000

At \omega_{3dB} : \quad |\frac{Y}{I}(\omega_{3dB})|^2 = \frac{1}{2} |\frac{Y}{I}(\omega_0)|^2
Bandwidth and Q

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\frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)}
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Peak is \(|\frac{Y}{I}(\omega_0)|^2 = R^2 @ \omega_0 = 1000\)

At \(\omega_{3dB}\): \(|\frac{Y}{I}(\omega_{3dB})|^2 = \frac{1}{2} |\frac{Y}{I}(\omega_0)|^2\)

\[
\frac{1}{(1/R)^2 + (\omega_{3dB} C - 1/\omega_{3dB} L)^2} = \frac{R^2}{2}
\]
Bandwidth and Q

\[ \frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)} \]

**Bandwidth** is the range of frequencies for which \( |\frac{Y}{I}|^2 \) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

Peak is \( |\frac{Y}{I}(\omega_0)|^2 = R^2 @ \omega_0 = 1000 \)

At \( \omega_{3\text{dB}} \):
\[
|\frac{Y}{I}(\omega_{3\text{dB}})|^2 = \frac{1}{2} |\frac{Y}{I}(\omega_0)|^2
\]
\[
\frac{1}{(1/R)^2 + (\omega_{3\text{dB}} C - 1/\omega_{3\text{dB}} L)^2} = \frac{R^2}{2} \Rightarrow 1 + \left(\omega_{3\text{dB}} RC - \frac{R}{\omega_{3\text{dB}} L}\right)^2 = 2
\]
Bandwidth and Q

\[ \frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)} \]

**Bandwidth** is the range of frequencies for which \( |\frac{Y}{I}|^2 \) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

\[ |\frac{Y}{I}|^2 = \frac{1}{(1/R)^2 + (\omega C - 1/\omega L)^2} \]

Peak is \( |\frac{Y}{I}(\omega_0)|^2 = R^2 \) @ \( \omega_0 = 1000 \)

At \( \omega_{3dB} \):

\[ \left| \frac{Y}{I}(\omega_{3dB}) \right|^2 = \frac{1}{2} \left| \frac{Y}{I}(\omega_0) \right|^2 \]

\[ \frac{1}{(1/R)^2 + (\omega_{3dB} C - 1/\omega_{3dB} L)^2} = \frac{R^2}{2} \Rightarrow 1 + \left( \omega_{3dB} RC - \frac{R}{\omega_{3dB} L} \right)^2 = 2 \]

\[ \omega_{3dB} RC - \frac{R}{\omega_{3dB} L} = \pm 1 \]
Bandwidth and Q

\[ \frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)} \]

**Bandwidth** is the range of frequencies for which \( |\frac{Y}{I}|^2 \) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

\[ |\frac{Y}{I}|^2 = \frac{1}{(1/R)^2 + (\omega C - 1/\omega L)^2} \]

Peak is \( |\frac{Y}{I}(\omega_{0})|^2 = R^2 \) @ \( \omega_{0} = 1000 \)

At \( \omega_{3dB} \): \( |\frac{Y}{I}(\omega_{3dB})|^2 = \frac{1}{2} |\frac{Y}{I}(\omega_{0})|^2 \)

\[ \frac{1}{(1/R)^2 + (\omega_{3dB} C - 1/\omega_{3dB} L)^2} = \frac{R^2}{2} \Rightarrow 1 + \left( \frac{\omega_{3dB} RC - R}{\omega_{3dB} L} \right)^2 = 2 \]

\( \omega_{3dB} RC - \frac{R}{\omega_{3dB} L} = \pm 1 \Rightarrow \omega_{3dB}^2 RLC \pm \omega_{3dB} L - R = 0 \)
Bandwidth and Q

\[ \frac{Y}{I} = \frac{1}{1/R+j(\omega C - 1/\omega L)} \]

**Bandwidth** is the range of frequencies for which \(|\frac{Y}{I}|^2\) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

\[ |\frac{Y}{I}|^2 = \frac{1}{(1/R)^2 + (\omega C - 1/\omega L)^2} \]

Peak is 

\[ |\frac{Y}{I}(\omega_0)|^2 = R^2 \quad \text{at} \quad \omega_0 = 1000 \]

At \(\omega_{3dB}\):

\[ |\frac{Y}{I}(\omega_{3dB})|^2 = \frac{1}{2} |\frac{Y}{I}(\omega_0)|^2 \]

\[ \frac{1}{(1/R)^2 + (\omega_{3dB} C - 1/\omega_{3dB} L)^2} = \frac{R^2}{2} \Rightarrow 1 + \left( \frac{\omega_{3dB} RC - \frac{R}{\omega_{3dB} L}}{\omega_{3dB} L} \right)^2 = 2 \]

\[ \omega_{3dB} RC - \frac{R}{\omega_{3dB} L} = \pm 1 \quad \Rightarrow \quad \omega_{3dB}^2 RLC \pm \omega_{3dB} L - R = 0 \]

Positive roots:

\[ \omega_{3dB} = \frac{\pm L + \sqrt{L^2 + 4R^2LC}}{2RLC} = \{920, 1086\} \text{ rad/s} \]
**Bandwidth and Q**

\[
\frac{Y}{I} = \frac{1}{1/R+j(\omega C-1/\omega L)}
\]

**Bandwidth** is the range of frequencies for which \( |\frac{Y}{I}|^2 \) is greater than half its peak. Also called *half-power bandwidth* or *3dB bandwidth*.

\[
|\frac{Y}{I}|^2 = \frac{1}{(1/R)^2+(\omega C-1/\omega L)^2}
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Peak is \( |\frac{Y}{I}(\omega_0)|^2 = R^2 \) @ \( \omega_0 = 1000 \)

At \( \omega_{3dB} \) : \( |\frac{Y}{I}(\omega_{3dB})|^2 = \frac{1}{2} |\frac{Y}{I}(\omega_0)|^2 \)

\[
\frac{1}{(1/R)^2+(\omega_{3dB} C-1/\omega_{3dB} L)^2} = \frac{R^2}{2} \Rightarrow 1 + \left( \omega_{3dB} RC - \frac{R}{\omega_{3dB} L} \right)^2 = 2
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\omega_{3dB} RC - \frac{R}{\omega_{3dB} L} = \pm 1 \quad \Rightarrow \quad \omega_{3dB}^2 RLC \pm \omega_{3dB} L - R = 0
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Positive roots:

\[
\omega_{3dB} = \frac{\pm L+\sqrt{L^2+4R^2LC}}{2RLC} = \{920, 1086\} \text{ rad/s}
\]

**Bandwidth:** \( B = 1086 - 920 = 167 \text{ rad/s} \).
Bandwidth and Q

\[
\frac{Y}{I} = \frac{1}{1/R + j(\omega C - 1/\omega L)}
\]

Bandwidth is the range of frequencies for which \[\left| \frac{Y}{I} \right|^2\] is greater than half its peak. Also called \textit{half-power bandwidth} or \textit{3dB bandwidth}.

\[
\left| \frac{Y}{I} \right|^2 = \frac{1}{(1/R)^2 + (\omega C - 1/\omega L)^2}
\]

Peak is \[\left| \frac{Y}{I} (\omega_0) \right|^2 = R^2 @ \omega_0 = 1000\]

At \(\omega_{3dB}\): \[\left| \frac{Y}{I} (\omega_{3dB}) \right|^2 = \frac{1}{2} \left| \frac{Y}{I} (\omega_0) \right|^2\]

\[
\frac{1}{(1/R)^2 + (\omega_{3dB} C - 1/\omega_{3dB} L)^2} = \frac{R^2}{2} \Rightarrow 1 + \left( \omega_{3dB} RC - \frac{R}{\omega_{3dB} L} \right)^2 = 2
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Positive roots: \(\omega_{3dB} = \pm L + \sqrt{L^2 + 4R^2 LC} \frac{2RL}{2RLC} = \{920, 1086\} \text{ rad/s}\)

**Bandwidth:** \(B = 1086 - 920 = 167 \text{ rad/s.}\)

**Q factor** \(\approx \frac{\omega_0}{B} = \frac{1}{2\zeta} = 6. \) (\(Q = \text{“Quality”}\))
Power and Energy at Resonance

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@\omega = 1000: Y = 600,

\[ I_R = 1, \quad I_L = -6j, \quad I_C = +6j \]
Absorbed Power $= v(t)i(t)$:

@$\omega = 1000$: $Y = 600$, $I_R = 1$, $I_L = -6j$, $I_C = +6j$
Power and Energy at Resonance

Absorbed Power $= v(t)i(t)$:

@\omega = 1000: Y = 600,
$I_R = 1, I_L = -6j, I_C = +6j$
Absorbed Power = \( v(t)i(t) \):

\[ P_L \text{ and } P_C \text{ opposite and } \gg P_R. \]
Absorbed Power \( = v(t)i(t) \):

\[ P_L \text{ and } P_C \text{ opposite and } \gg P_R. \]

Stored Energy = \( \frac{1}{2} L i_L^2 + \frac{1}{2} C y^2 \):

\[ @\omega = 1000: Y = 600, \quad I_R = 1, \; I_L = -6j, \; I_C = +6j \]
Power and Energy at Resonance

Absorbed Power = \( v(t)i(t) \):

\[ P_L \text{ and } P_C \text{ opposite and } \gg P_R. \]

Stored Energy = \( \frac{1}{2}L_i_L^2 + \frac{1}{2}Cy^2 \):

\(@\omega = 1000: Y = 600,\)

\[ I_R = 1, I_L = -6j, I_C = +6j \]
Absorbed Power \( = v(t)i(t) \):

\[ P_L \text{ and } P_C \text{ opposite and } \gg P_R. \]

Stored Energy \( = \frac{1}{2} L i_L^2 + \frac{1}{2} C y^2 \):
sloshes between \( L \) and \( C \).

\[ @\omega = 1000: Y = 600, \]
\[ I_R = 1, I_L = -6j, I_C = +6j \]
Absorbed Power $= v(t)i(t)$:

$P_L$ and $P_C$ opposite and $\gg P_R$.

Stored Energy $= \frac{1}{2} Li_L^2 + \frac{1}{2} Cy^2$:

sloshes between $L$ and $C$.

$Q \triangleq \omega \times \frac{W_{\text{stored}}}{P_R}$

$@\omega = 1000: Y = 600$,

$I_R = 1, I_L = -6j, I_C = +6j$
Absorbed Power \(=v(t)i(t)\):

\[ P_L \text{ and } P_C \text{ opposite and } \gg P_R. \]

Stored Energy \(=\frac{1}{2}L\dot{i}_L^2 + \frac{1}{2}Cy^2\):
sloshes between \(L\) and \(C\).

\[ Q \triangleq \omega \times \frac{W_{\text{stored}}}{P_R} = \omega \times \frac{1}{2}C|IR|^2 \div \frac{1}{2}|I|^2 R = \omega RC \]

@\(\omega = 1000\): \(Y = 600\),
\(I_R = 1, I_L = -6j, I_C = +6j\)
Absorbed Power $= v(t)i(t)$:

- $P_L$ and $P_C$ opposite and $\gg P_R$.

Stored Energy $= \frac{1}{2} L i_L^2 + \frac{1}{2} C y^2$:

- sloshes between $L$ and $C$.

$$Q \triangleq \omega \times \frac{W_{\text{stored}}}{P_R} = \omega \times \frac{1}{2} C |IR|^2 \div \frac{1}{2} |I|^2 R = \omega R C$$

@ $\omega = 1000: Y = 600$, $I_R = 1, I_L = -6j, I_C = +6j$

$Q \triangleq \omega \times \text{peak stored energy} \div \text{average power loss}$. 
Low Pass Filter

\[
\frac{Y}{X} = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1}
\]

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E1.1 Analysis of Circuits (2017-10213)
Low Pass Filter

\[
\frac{Y}{X} = \frac{1/j\omega C}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{LC(j\omega)^2 + R C j\omega + 1}
\]

Asymptotes: 1 and \(\frac{1}{LC(j\omega)^{-2}}\).
Low Pass Filter

\[ \frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} \]

Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).

\( \omega_c = \sqrt{\frac{c}{a}} = 1000 \), \( \zeta = \frac{b}{2a\omega_c} = \frac{R}{200} \)
Low Pass Filter

\[
\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1}
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Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).

\[
\omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200}
\]

@\( \omega_c \):

\[
Z_L = -Z_C = 100j, \quad I = \frac{X}{R}
\]
Low Pass Filter

\[
\frac{Y}{X} = \frac{1/jωC}{R+jωL+1/jωC} = \frac{1}{LC(jω)^2+RCjω+1}
\]

Asymptotes: 1 and \( \frac{1}{LC} (jω)^{-2} \).

\[ \omega_c = \sqrt{\frac{C}{a}} = 1000, \quad ζ = \frac{b}{2aω_c} = \frac{R}{200} \]

@\( ω_c \) : \( Z_L = -Z_C = 100j \), \( I = \frac{X}{R} \), \( \left| \frac{Y}{X} \right| = \frac{1}{RCω} = \frac{1}{2ζ}, \quad \angle \frac{Y}{X} = -\frac{π}{2} \)
Low Pass Filter

\[ \frac{Y}{X} = \frac{1/j\omega C}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} \]

Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).

\[ \omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200} \]

@\( \omega_c \): \( Z_L = -Z_C = 100j \), \( I = \frac{X}{R} \), \( \left| \frac{Y}{X} \right| = \frac{1}{RC\omega} = \frac{1}{2\zeta} \), \( \angle \frac{Y}{X} = -\frac{\pi}{2} \)

Magnitude Plot:
Low Pass Filter

\[
\frac{Y}{X} = \frac{1/\omega_C}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1}
\]

Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).
\[
\omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200}
\]

@\( \omega_c \): \( Z_L = -Z_C = 100j \), \( I = \frac{X}{R} \), \( |\frac{Y}{X}| = \frac{1}{RC\omega} = \frac{1}{2\zeta} \), \( \angle \frac{Y}{X} = -\frac{\pi}{2} \)

Magnitude Plot:

Small \( \zeta \) ⇒ less loss, higher peak, smaller bandwidth.
**Low Pass Filter**

\[
\frac{Y}{X} = \frac{1/j\omega C}{R+j\omega L+j\omega C} = \frac{1}{LC(j\omega)^2+RCj\omega+1}
\]

Asymptotes: 1 and \(\frac{1}{LC} (j\omega)^{-2}\).

\[
\omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200}
\]

@\(\omega_c\) : \(Z_L = -Z_C = 100j, I = \frac{X}{R}, |\frac{Y}{X}| = \frac{1}{RC\omega} = \frac{1}{2\zeta}, \angle \frac{Y}{X} = -\frac{\pi}{2}\)

**Magnitude Plot:**
- Small \(\zeta \Rightarrow\) less loss, higher peak, smaller bandwidth.
- Large \(\zeta \Rightarrow\) more loss, smaller peak at a lower \(\omega\), larger bandwidth.
Low Pass Filter

\[
\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R+j\omega L+j\omega C} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1}
\]

Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).

\[
\omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200}
\]

\( \omega_c \): \( Z_L = -Z_C = 100j, \quad I = \frac{X}{R}, \quad \left| \frac{Y}{X} \right| = \frac{1}{RC\omega} = \frac{1}{2\zeta}, \quad \angle \frac{Y}{X} = -\frac{\pi}{2} \)

**Magnitude Plot:**

- Small \( \zeta \) ⇒ less loss, higher peak, smaller bandwidth.
- Large \( \zeta \) more loss, smaller peak at a lower \( \omega \), larger bandwidth.
Low Pass Filter

\[ \frac{Y}{X} = \frac{1/j\omega C}{R+j\omega L+1/j\omega C} = \frac{1}{LC(j\omega)^2+RCj\omega+1} \]

Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).

\[ \omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200} \]

@\( \omega_c \) : \( Z_L = -Z_C = 100j \), \( I = \frac{X}{R} \), \( |Y/X| = \frac{1}{RC\omega} = \frac{1}{2\zeta} \), \( \angle \frac{Y}{X} = -\frac{\pi}{2} \)

Magnitude Plot:
- Small \( \zeta \) \( \Rightarrow \) less loss, higher peak, smaller bandwidth.
- Large \( \zeta \) more loss, smaller peak at a lower \( \omega \), larger bandwidth.

Phase Plot:
Low Pass Filter

\[
\frac{Y}{X} = \frac{1/j\omega C}{R+j\omega L+1/j\omega C} = \frac{1}{LC(j\omega)^2+RCj\omega+1}
\]

Asymptotes: 1 and \( \frac{1}{LC} (j\omega)^{-2} \).

\[
\omega_c = \sqrt{\frac{C}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{R}{200}
\]

@\( \omega_c \) : \( Z_L = -Z_C = 100j \), \( I = \frac{X}{R} \), \( \left| \frac{Y}{X} \right| = \frac{1}{RC\omega} = \frac{1}{2\zeta} \), \( \angle \frac{Y}{X} = -\frac{\pi}{2} \)

Magnitude Plot:
Small \( \zeta \Rightarrow \) less loss, higher peak, smaller bandwidth.
Large \( \zeta \) more loss, smaller peak at a lower \( \omega \), larger bandwidth.

Phase Plot:
Small \( \zeta \Rightarrow \) fast phase change: \( \pi \) over 2\( \zeta \) decades.
**Low Pass Filter**

\[
\frac{Y}{X} = \frac{1/j\omega C}{R+j\omega L+1/j\omega C} = \frac{1}{LC(j\omega)^2+RCj\omega+1}
\]

Asymptotes: 1 and \(\frac{1}{LC} (j\omega)^{-2}\).

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Large \( \zeta \) more loss, smaller peak at a lower \( \omega \), larger bandwidth.

Phase Plot:
Small \( \zeta \) \( \Rightarrow \) fast phase change: \( \pi \) over \( 2\zeta \) decades.
\[ \angle \frac{Y}{X} \approx -\frac{\pi}{2} \left( 1 + \frac{1}{\zeta} \log_{10} \frac{\omega}{\omega_c} \right) \text{ for } 10^{-\zeta} < \frac{\omega}{\omega_c} < 10^{+\zeta} \]
Resonance Peak for LP filter

\[ \frac{Y}{X} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} \]

\[ X = 1 \]

\[ Y \]

\[ R \]

\[ 100\text{m} \]

\[ C \]

\[ 10\mu \]
Resonance Peak for LP filter

\[
\frac{Y}{X} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1}
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\omega_c = \sqrt{\frac{c}{a}} = 1000, \quad \zeta = \frac{b}{2a\omega_c} = \frac{b\omega_c}{2c} = \frac{R}{200}
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Resonance Peak for LP filter

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**Peak frequency:**

\[
\omega_p = \omega_c \sqrt{1 - 2\zeta^2}
\]
Resonance Peak for LP filter

\[ \frac{Y}{X} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} = \frac{1}{(j\frac{\omega}{\omega_c})^2 + 2\zeta j\frac{\omega}{\omega_c} + 1} \]

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The damping factor, \( \zeta \), ("zeta") determines the shape of the peak.

Peak frequency:
\[ \omega_p = \omega_c \sqrt{1 - 2\zeta^2} \]

\( \zeta \geq 0.71 \Rightarrow \text{no peak,} \)
Resonance Peak for LP filter

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\( \zeta \geq 0.71 \Rightarrow \) no peak, 
\( \zeta \geq 1 \Rightarrow \) can factorize

Gain relative to asymptote: 
\[ @ \omega_p: \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad \text{and} \quad @ \omega_c: \frac{1}{2\zeta} \approx Q \]
### Resonance Peak for LP filter

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Resonance Peak for LP filter

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Peak frequency:

\[ \omega_p = \omega_c \sqrt{1 - 2\zeta^2} \]

\( \zeta \geq 0.5 \Rightarrow \) passes under corner,
\( \zeta \geq 0.71 \Rightarrow \) no peak,
\( \zeta \geq 1 \Rightarrow \) can factorize

Gain relative to asymptote: \( @ \omega_p: \frac{1}{2\zeta \sqrt{1 - \zeta^2}} \) \( @ \omega_c: \frac{1}{2\zeta} \approx Q \)
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\[\@ \omega_p: \frac{1}{2\zeta \sqrt{1-\zeta^2}} \quad \@ \omega_c: \frac{1}{2\zeta} \approx Q\]

**Three frequencies:** \(\omega_p = \) peak, \(\omega_c = \) asymptotes cross, \(\omega_r = \) real impedance  
For \(\zeta < 0.3\), \(\omega_p \approx \omega_c \approx \omega_r\). All get called the **resonant frequency**.
Resonance Peak for LP filter

\[ \frac{Y}{X} = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} = \frac{1}{(j\frac{\omega}{\omega_c})^2 + 2\zeta j\frac{\omega}{\omega_c} + 1} \]

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**Peak frequency:**

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Three frequencies: \( \omega_p = \text{peak,} \quad \omega_c = \text{asymptotes cross,} \quad \omega_r = \text{real impedance} \)

For \( \zeta < 0.3, \omega_p \approx \omega_c \approx \omega_r \). All get called the resonant frequency.

The exact relationship between \( \omega_p, \omega_c \) and \( \omega_r \) and the gain at these frequencies is affected by any other corner frequencies in the response.
Summary

- Resonance is a peak in energy absorption
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- Parallel or series circuit has a real impedance at $\omega_r$.
  - Peak response may be at a slightly different frequency.
Summary

- **Resonance** is a peak in energy absorption
  - Parallel or series circuit has a **real impedance** at $\omega_r$
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  - The quality factor, $Q$, of the resonance is
    \[ Q \triangleq \frac{\omega_0 \times \text{stored energy}}{\text{power in } R} \approx \frac{\omega_0}{3 \text{ dB bandwidth}} \approx \frac{1}{2\zeta} \]
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- **Quadratic factor:**
  \[ \left( \frac{j\omega}{\omega_c} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_c} \right) + 1 \]
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  - $a(j\omega)^2 + b(j\omega) + c \Rightarrow \omega_c = \sqrt{\frac{c}{a}}$ and $\zeta = \frac{b}{2a\omega_c} = \frac{bsgn(a)}{\sqrt{4ac}}$
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  - $\pm 40$ dB/decade slope change in magnitude response.
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For further details see Hayt Ch 16 or Irwin Ch 12.