13: Filters

Filters

A filter is a circuit whose gain varies with frequency. Often a filter aims to allow some frequencies to pass while blocking others.

- Radio/TV: a “tuning” filter blocks all frequencies except the wanted channel
- Loudspeaker: “crossover” filters send the right frequencies to different drive units
- Sampling: an “anti-aliasing filter” eliminates all frequencies above half the sampling rate
  - Phones: Sample rate = 8 kHz : filter eliminates frequencies above 3.4 kHz.
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1st Order Low-Pass Filter

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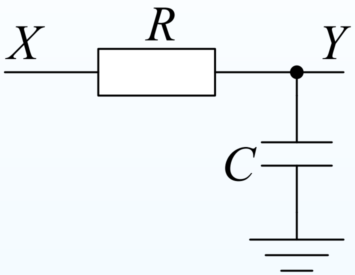

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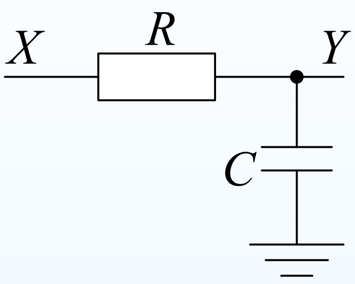

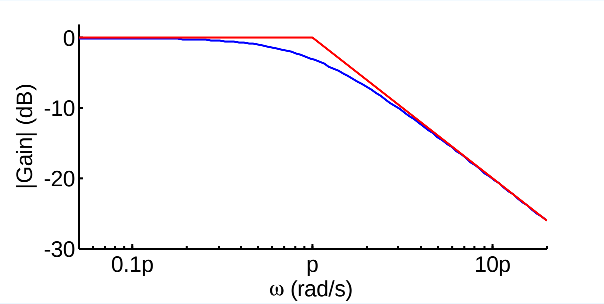

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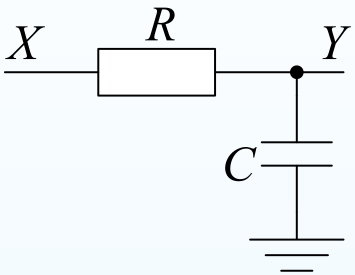

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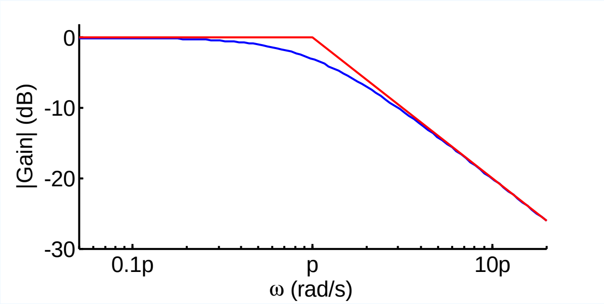

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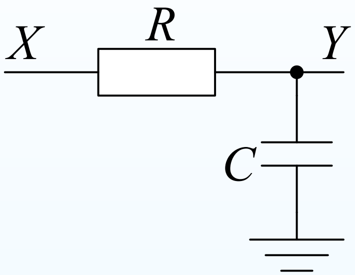

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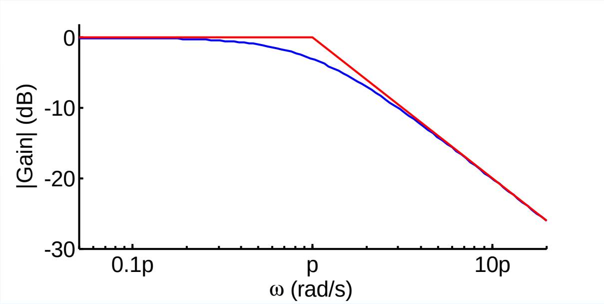

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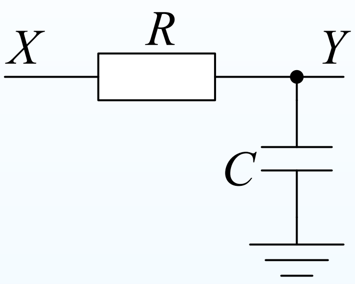

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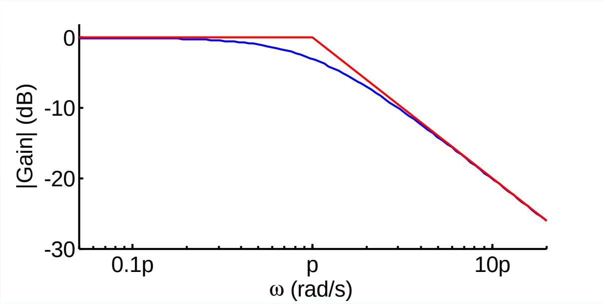

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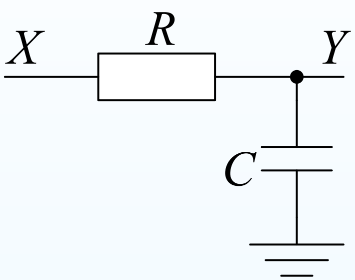

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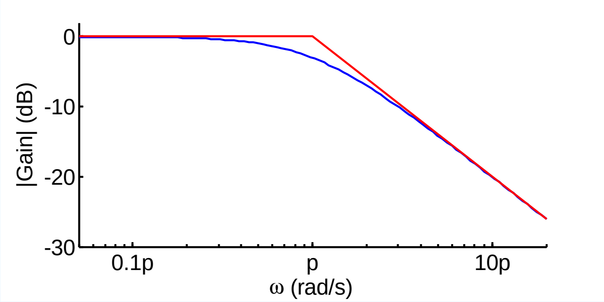

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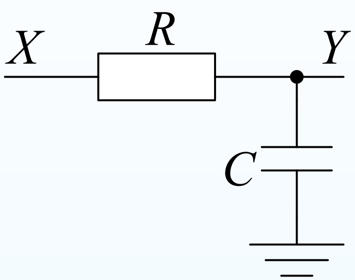

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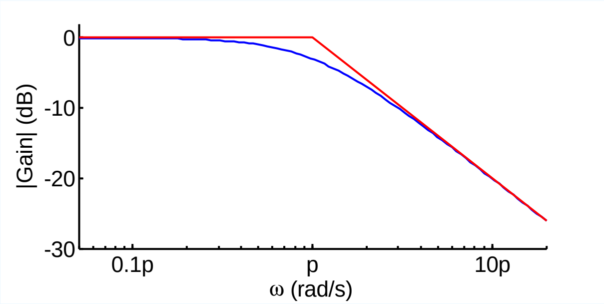

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The *order* of a filter: highest power of $j\omega$ in the denominator.
Almost always equals the total number of L and/or C.

Low-Pass with Gain Floor

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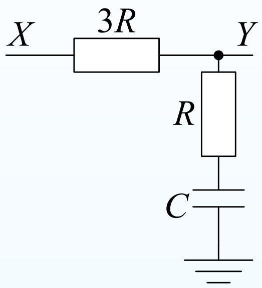

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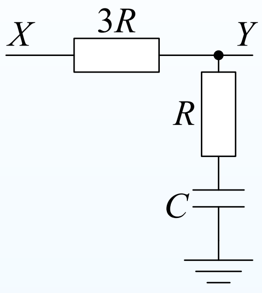

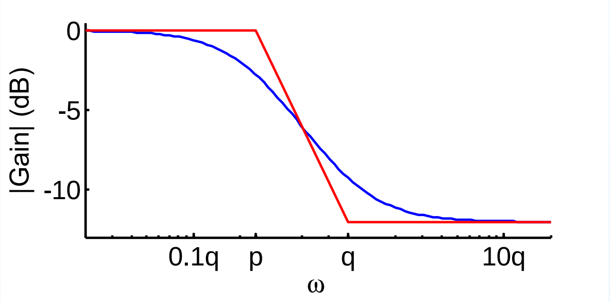

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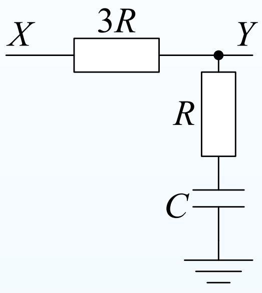
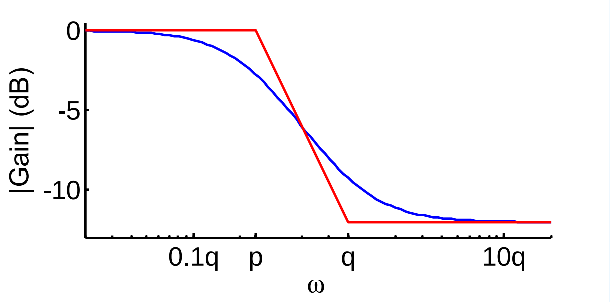


Corner frequencies: $p = \frac{1}{4RC}$, $q = \frac{1}{RC}$

Asymptotes: 1 and $\frac{1}{4}$



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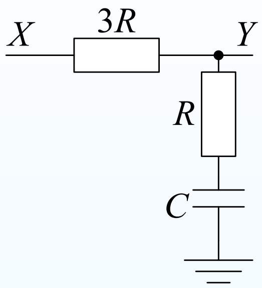

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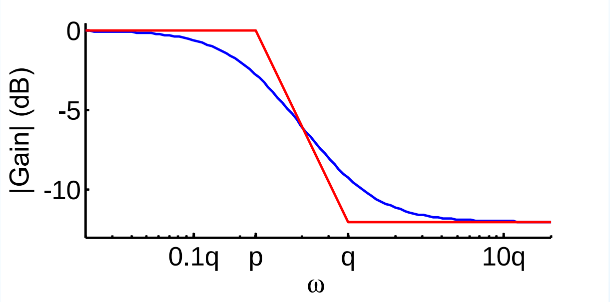
Resistor R unattached. Gain $= 1$



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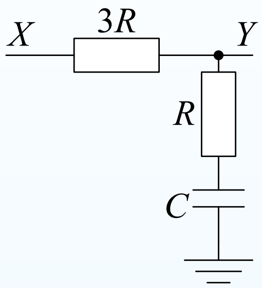

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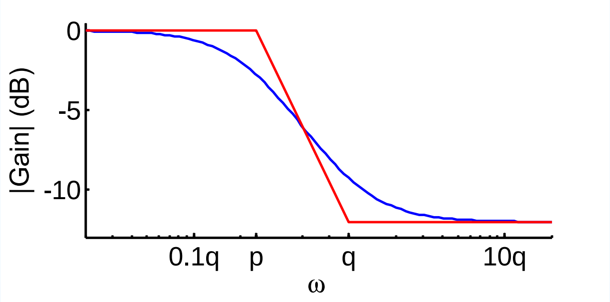

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Very high ω:

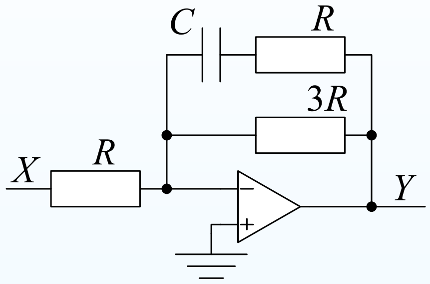
Capacitor short circuit

Circuit is potential divider with gain $20 \log_{10} \frac{1}{4} = -12$ dB.

Opamp filter

Inverting amplifier so

$$\frac{Y}{X} = -\frac{3R||(R+{}^1/_{j\omega C})}{R}$$



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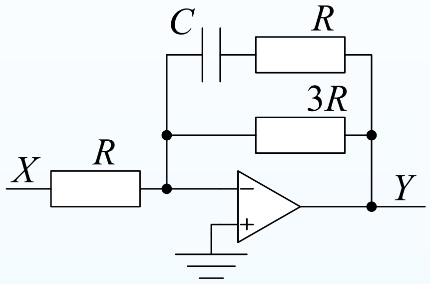

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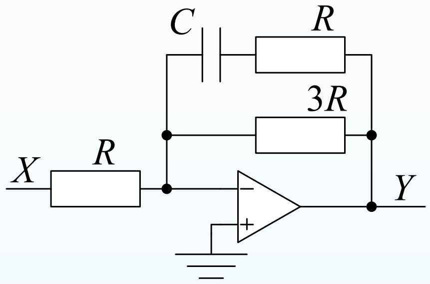

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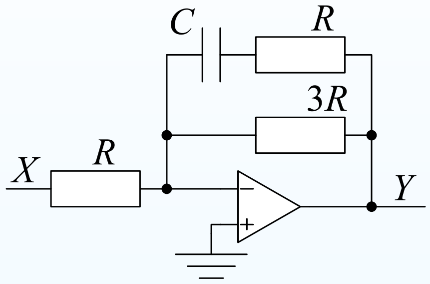

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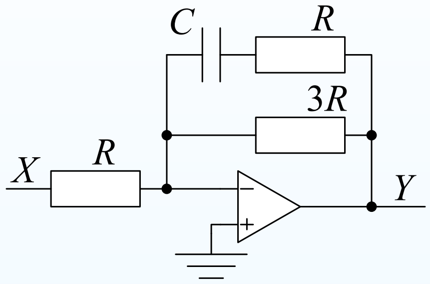

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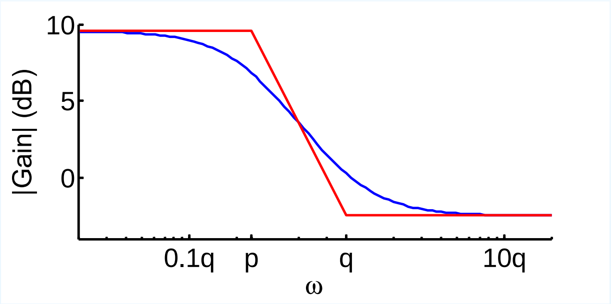

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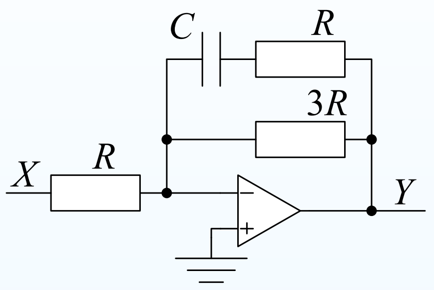

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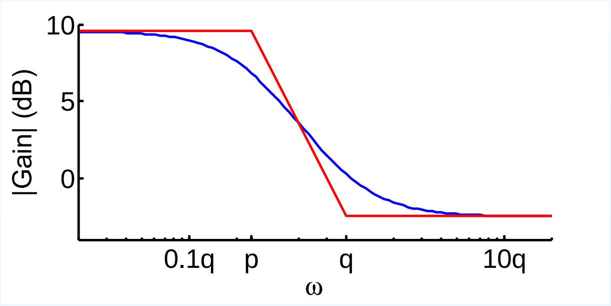

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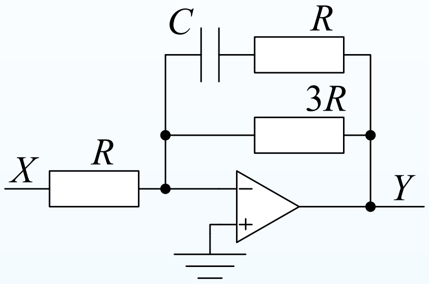

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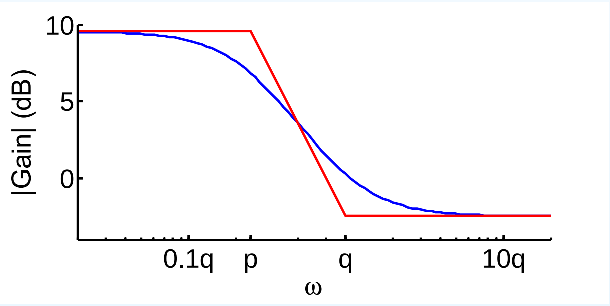

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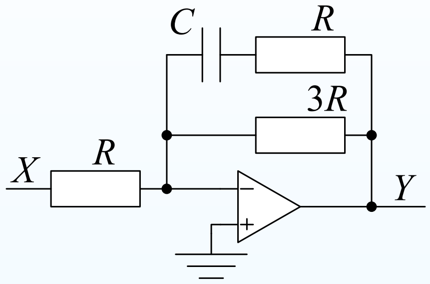

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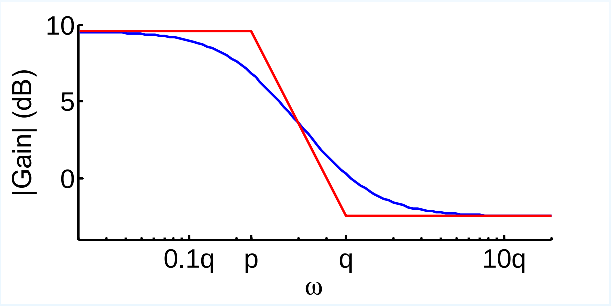

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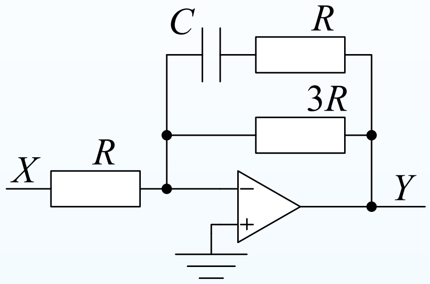

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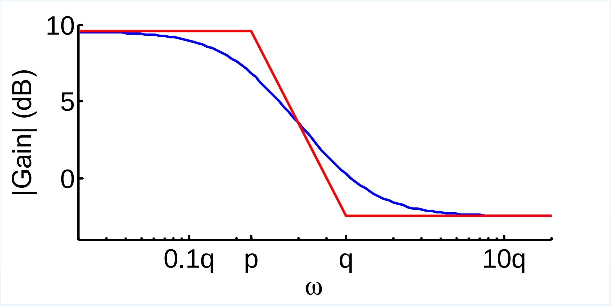

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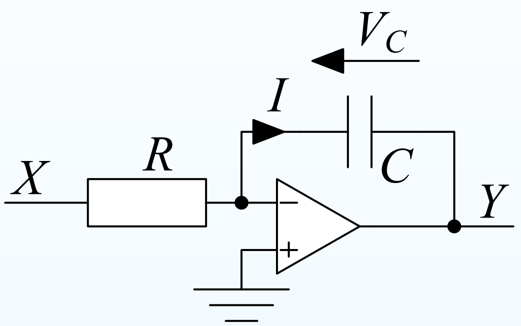
2. Low output impedance - loading does not affect filter
3. Resistive input impedance - does not vary with frequency

Integrator

$$\frac{Y}{X} = -\frac{1/j\omega C}{R}$$



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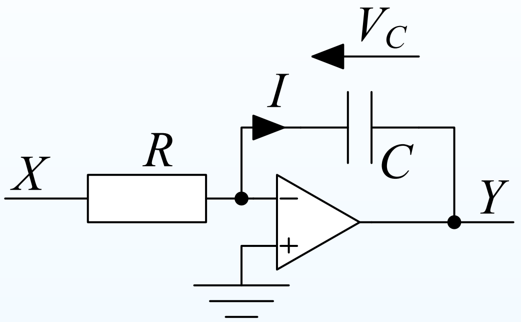

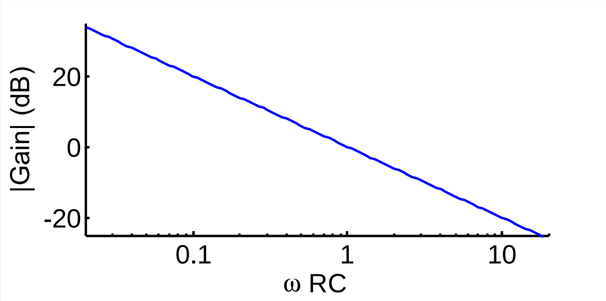

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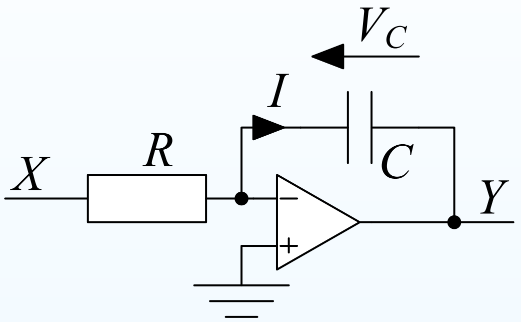

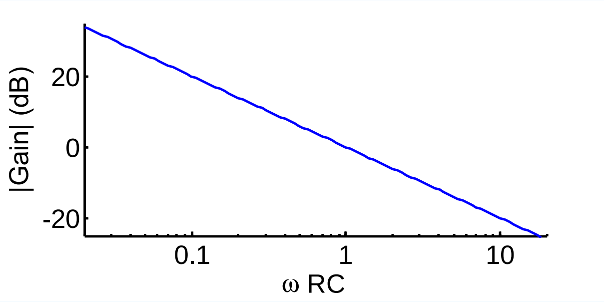

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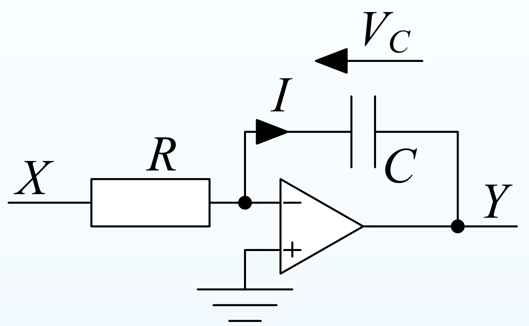
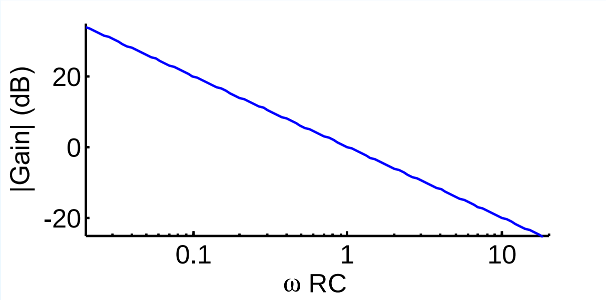
$$i = \frac{x}{R} = -C\frac{dy}{dt}$$





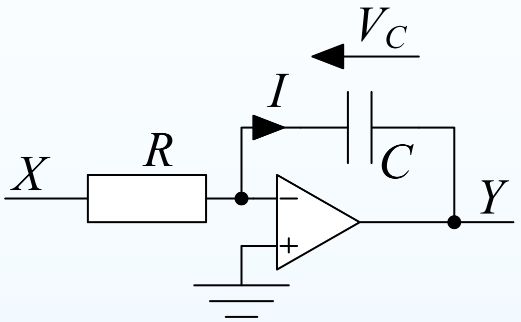
Integrator

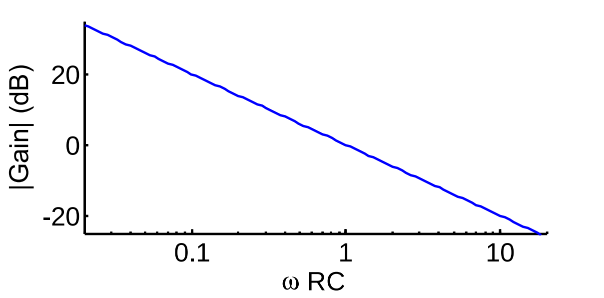
$$\frac{Y}{X} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC}$$

Capacitor: $i = C\frac{dv_C}{dt}$

$$i = \frac{x}{R} = -C\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{-1}{RC}x$$





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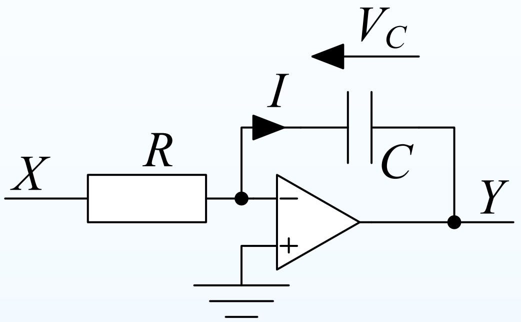

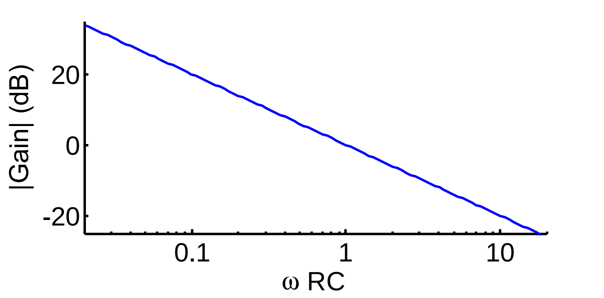

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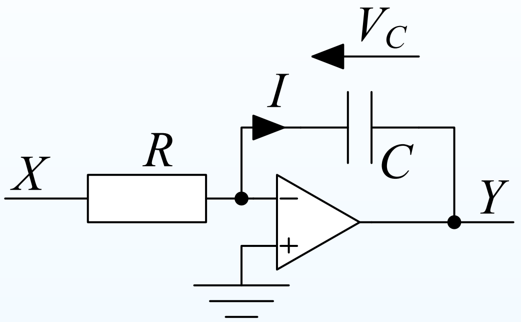

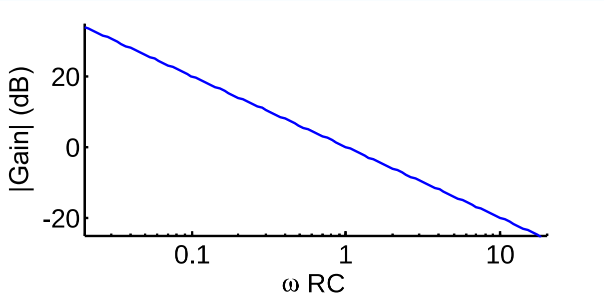

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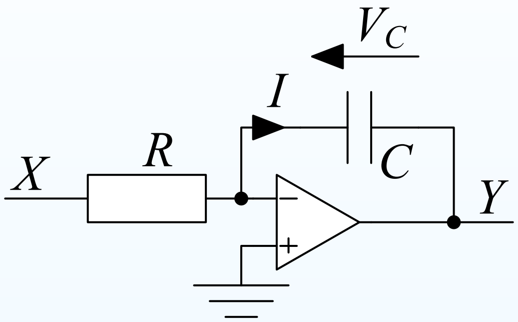

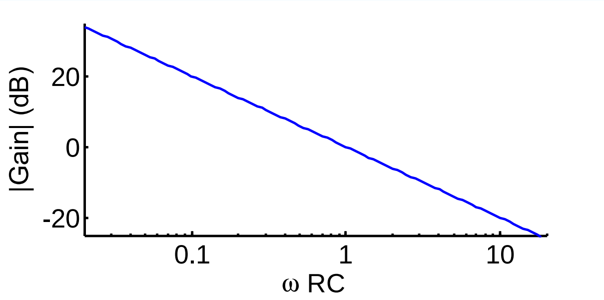

Note: if $x(t) = \cos\omega t$
$\int \cos(\omega t)dt = \frac{1}{\omega}\sin(\omega t) \Rightarrow$ gain $\propto \frac{1}{\omega}$.

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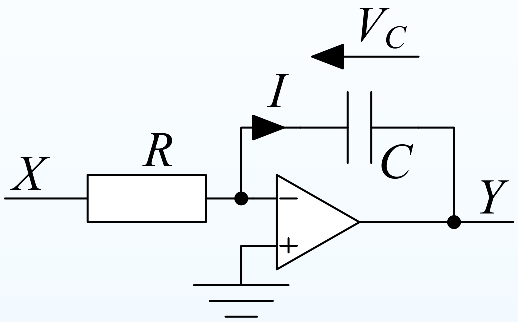

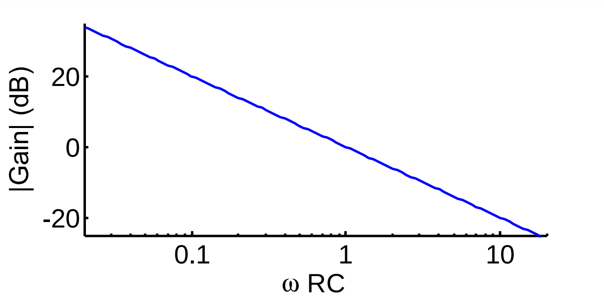

Note: if $x(t) = \cos\omega t$

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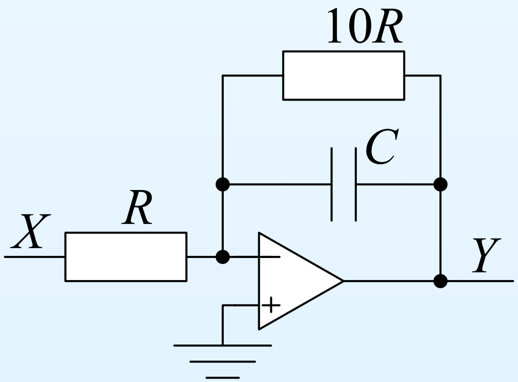
We can limit the LF gain to 20 dB:



Integrator

$$\frac{Y}{X} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC}$$

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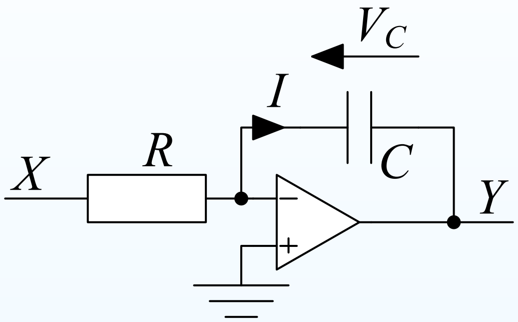
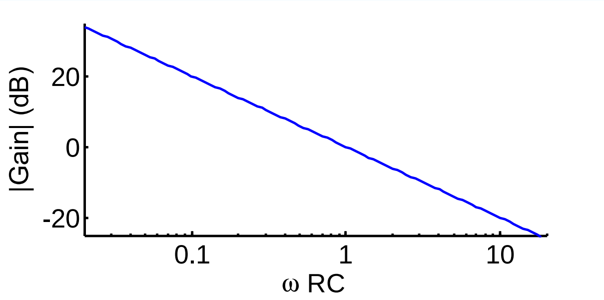


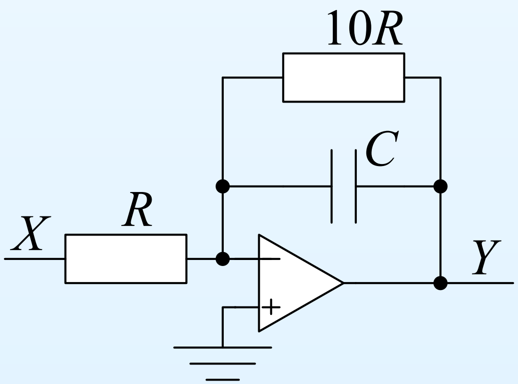


Note: if $x(t) = \cos\omega t$

$\int \cos(\omega t)dt = \frac{1}{\omega}\sin(\omega t) \Rightarrow$ gain $\propto \frac{1}{\omega}$.

We can limit the LF gain to 20 dB:

$$\frac{Y}{X} = -\frac{10R||1/j\omega C}{R}$$



Integrator

$$\frac{Y}{X} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC}$$

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$i = \frac{x}{R} = -C\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{-1}{RC}x$

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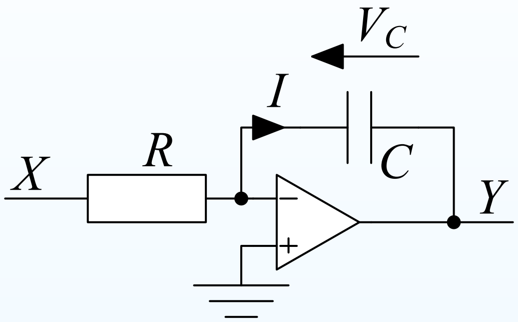

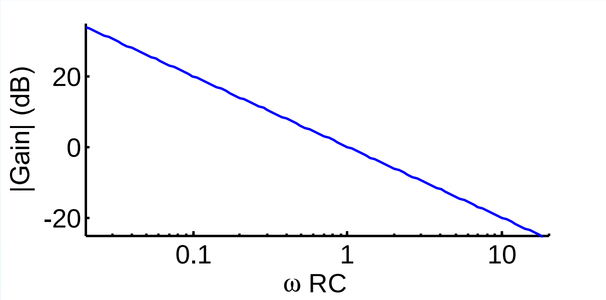

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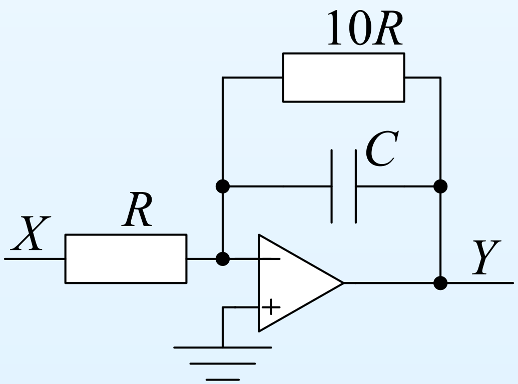
$$\frac{Y}{X} = -\frac{10R||1/j\omega C}{R} = -\frac{10R\times 1/j\omega C}{R(10R+1/j\omega C)}$$



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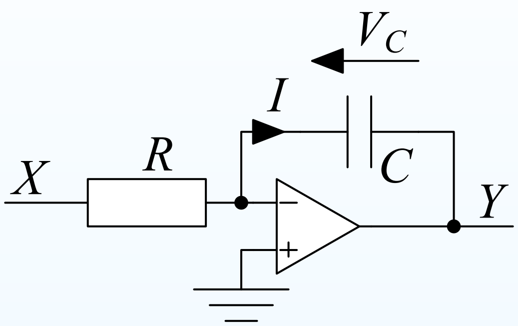

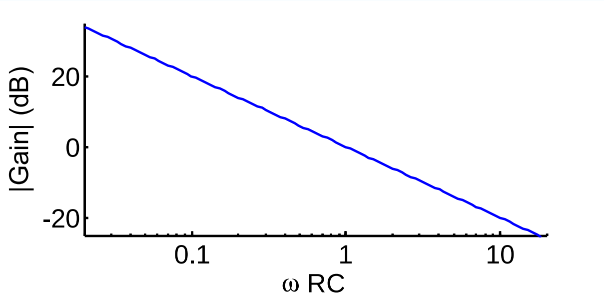

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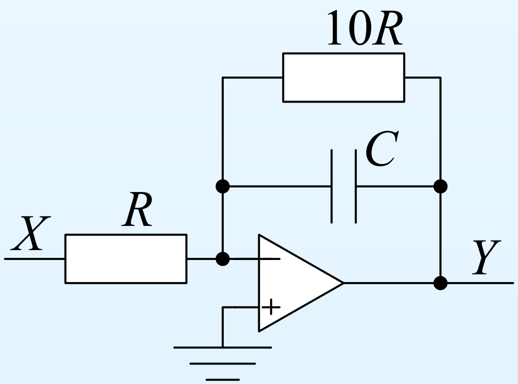
$$= -\frac{10}{j\omega 10RC+1}$$



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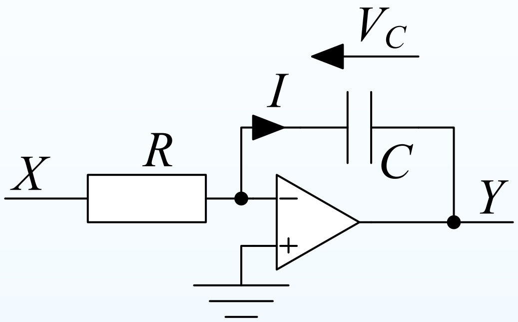

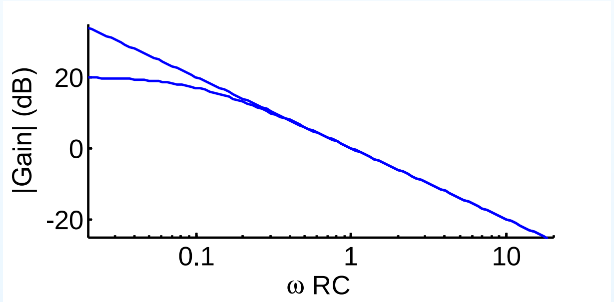

Note: if $x(t) = \cos\omega t$

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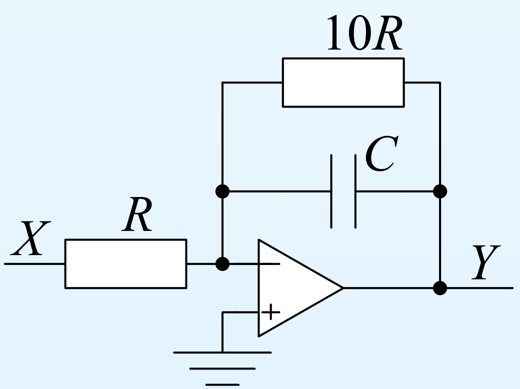
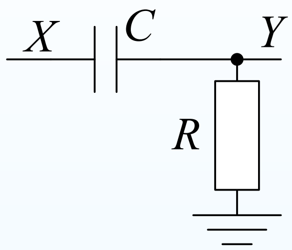
$$= -\frac{10}{j\omega 10RC+1} \qquad \left(\omega_c = \frac{0.1}{RC}\right)$$



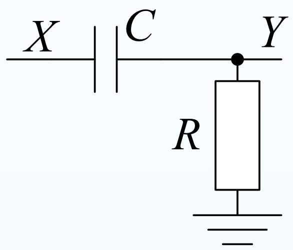
High Pass Filter

$$\frac{Y}{X} = \frac{R}{R + {}^{1}/_{j\omega C}}$$



High Pass Filter

$$\frac{Y}{X} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{j\omega RC + 1}$$



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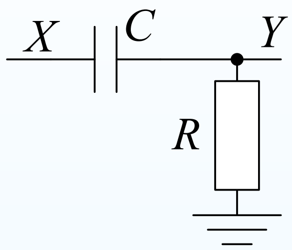
Corner Freq: $p = \frac{1}{RC}$



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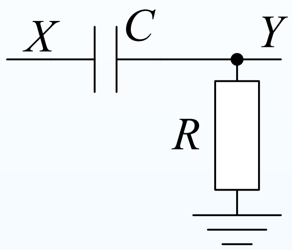

Corner Freq: $p = \frac{1}{RC}$

Asymptotes: $j\omega RC$ and 1

High Pass Filter

$$\frac{Y}{X} = \frac{R}{R + {}^{1}\!/\!_{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1}$$

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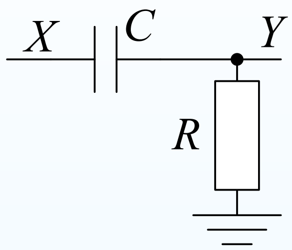

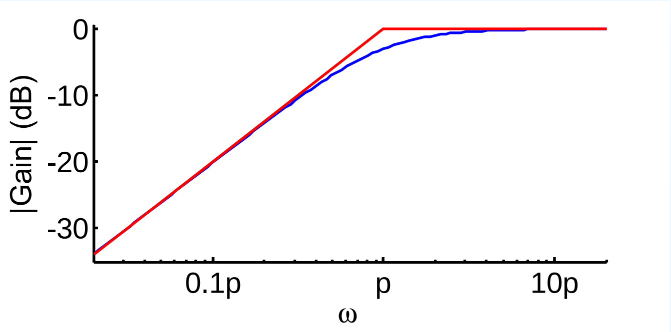

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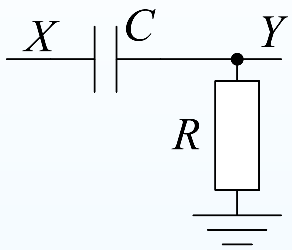
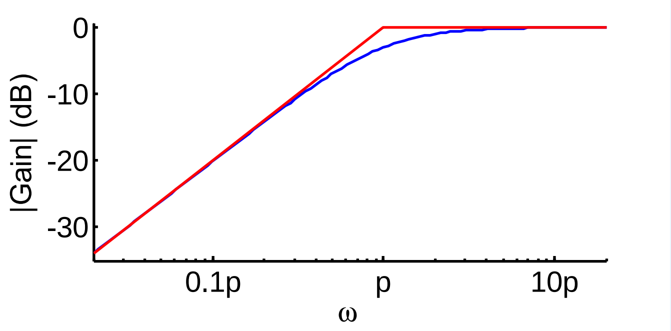


Corner Freq: $p = \frac{1}{RC}$

Asymptotes: $j\omega RC$ and 1

Very low ω: C open circuit: gain = 0

Very high ω: C short circuit: gain = 1



High Pass Filter

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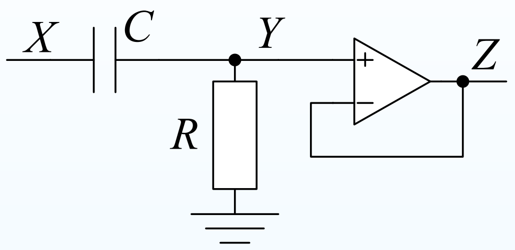

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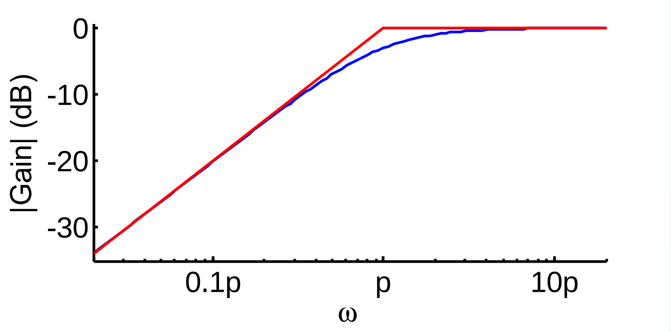
We can add an op-amp to give a low-impedance output.



High Pass Filter

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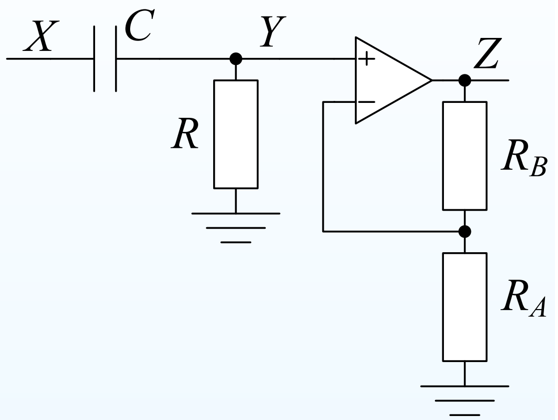


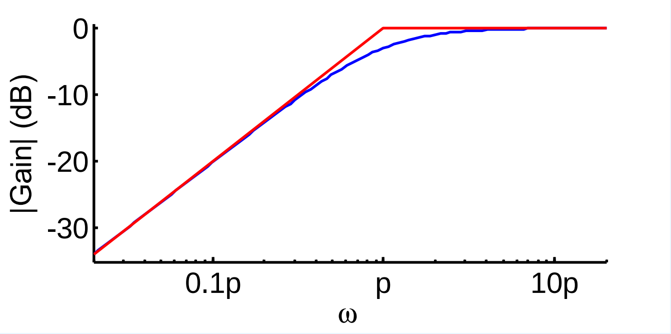
Corner Freq: $p = \frac{1}{RC}$

Asymptotes: $j\omega RC$ and 1

Very low ω: C open circuit: gain = 0
Very high ω: C short circuit: gain = 1

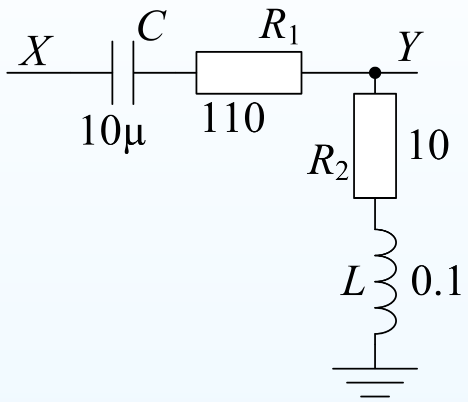
We can add an op-amp to give a low-impedance output. Or add gain:

$$\frac{Z}{X} = \left(1 + \frac{R_B}{R_A}\right) \times \frac{j\omega RC}{j\omega RC+1}$$



2nd order filter

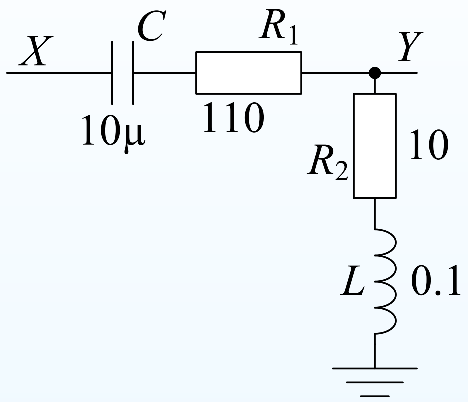
$$\frac{Y}{X} = \frac{R_2 + j\omega L}{{}^1/_{j\omega C} + R_1 + R_2 + j\omega L}$$



2nd order filter

$$\frac{Y}{X} = \frac{R_2 + j\omega L}{{}^1/_{j\omega C} + R_1 + R_2 + j\omega L}$$

$$= \frac{LC(j\omega)^2 + R_2 C j\omega}{LC(j\omega)^2 + (R_1 + R_2) C j\omega + 1}$$



2nd order filter

$$\frac{Y}{X} = \frac{R_2 + j\omega L}{1/j\omega C + R_1 + R_2 + j\omega L}$$

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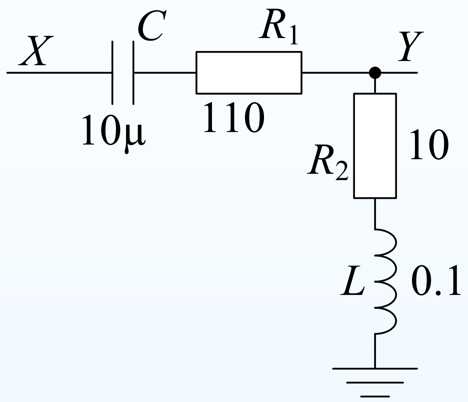
$$= \frac{j\omega C(j\omega L + R_2)}{LC(j\omega)^2 + (R_1 + R_2)Cj\omega + 1}$$



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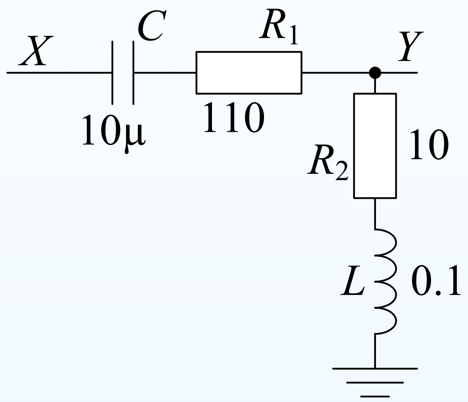

Asymptotes: $j\omega R_2 C$ and 1

2nd order filter

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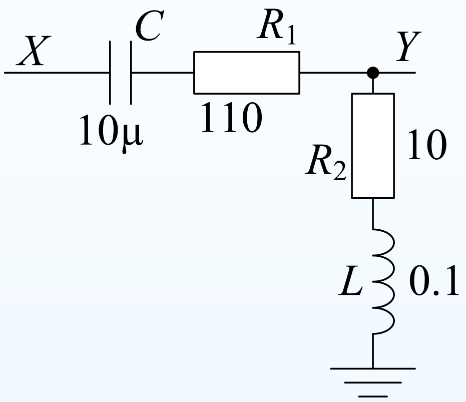


Asymptotes: $j\omega R_2 C$ and 1

Corner frequencies:

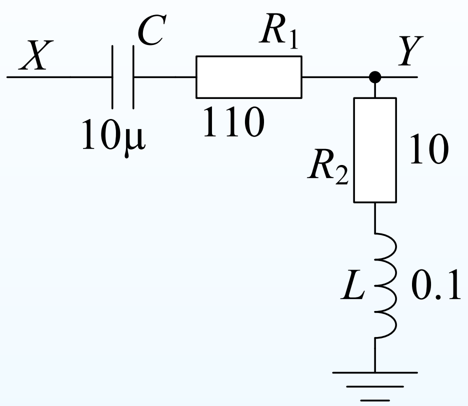
$+20$ dB/dec at $p = \frac{R_2}{L} = 100\,\text{rad/s}$

-40 dB/dec at

$$q = \sqrt{\frac{c}{a}} = \frac{1}{\sqrt{LC}} = 1000\,\text{rad/s}$$

2nd order filter

$$\frac{Y}{X} = \frac{R_2 + j\omega L}{1/j\omega C + R_1 + R_2 + j\omega L}$$
$$= \frac{LC(j\omega)^2 + R_2 C j\omega}{LC(j\omega)^2 + (R_1+R_2)Cj\omega + 1}$$
$$= \frac{j\omega C(j\omega L + R_2)}{LC(j\omega)^2 + (R_1+R_2)Cj\omega + 1}$$



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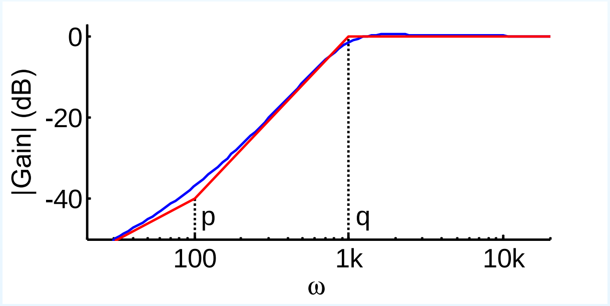
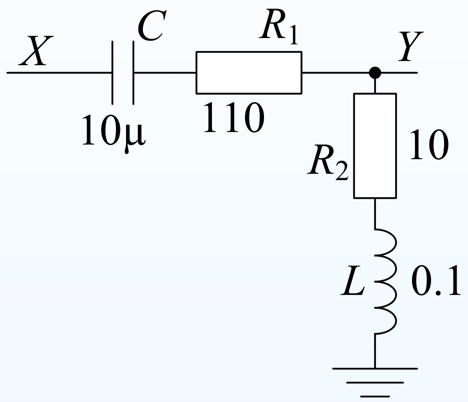


2nd order filter

$$\frac{Y}{X} = \frac{R_2 + j\omega L}{1/j\omega C + R_1 + R_2 + j\omega L}$$

$$= \frac{LC(j\omega)^2 + R_2 C j\omega}{LC(j\omega)^2 + (R_1 + R_2)Cj\omega + 1}$$

$$= \frac{j\omega C(j\omega L + R_2)}{LC(j\omega)^2 + (R_1 + R_2)Cj\omega + 1}$$



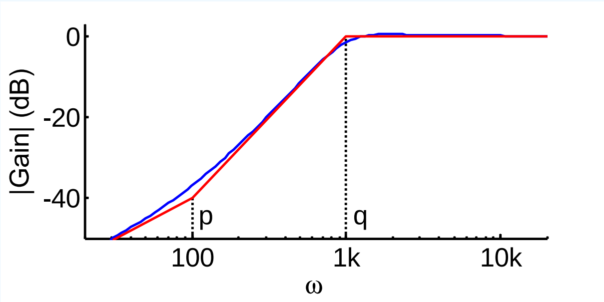
Asymptotes: $j\omega R_2 C$ and 1

Corner frequencies:

$+20$ dB/dec at $p = \frac{R_2}{L} = 100\,\mathrm{rad/s}$

-40 dB/dec at

$$q = \sqrt{\frac{c}{a}} = \frac{1}{\sqrt{LC}} = 1000\,\mathrm{rad/s}$$



Damping factor: $\zeta = \frac{b\,\mathrm{sgn}(a)}{\sqrt{4ac}} = \frac{qb}{2c} = \frac{q}{2}\left(R_1 + R_2\right)C = 0.6.$

Gain error at q is $\frac{1}{2\zeta} = Q = 0.83 = -1.6\,\mathrm{dB}$ ($+0.04\,\mathrm{dB}$ due to p)

2nd order filter

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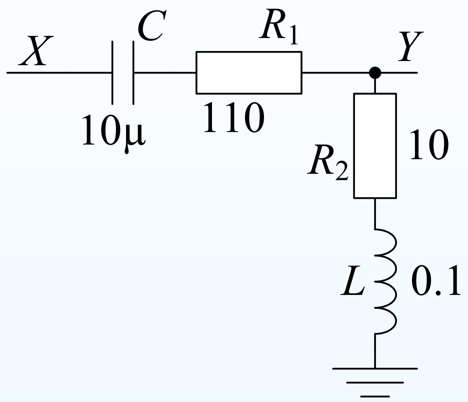

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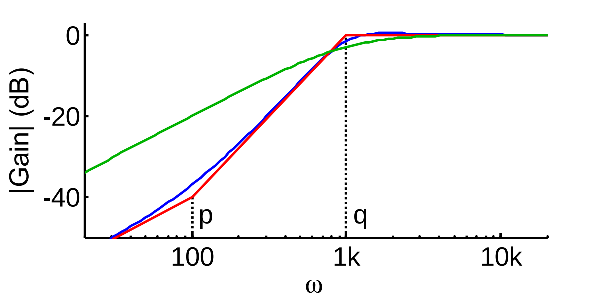

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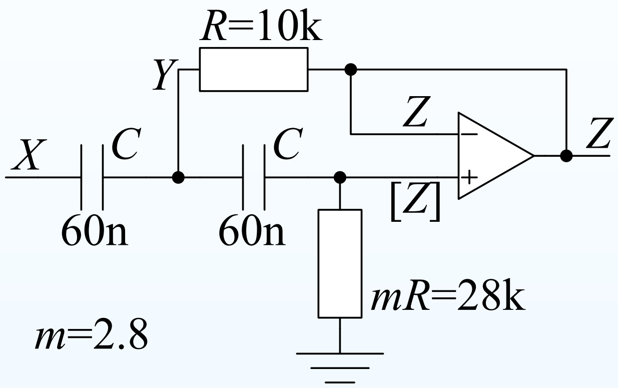
Compare with 1st order:

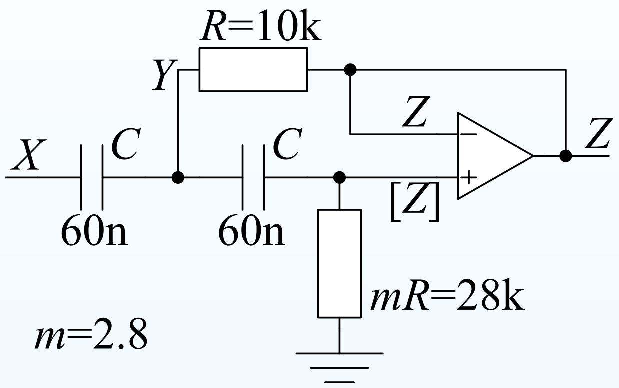
2nd order filter attenuates more rapidly than a 1st order filter.

Sallen-Key Filter



KCL @ Y: $\frac{Y-X}{1/j\omega C} + \frac{Y-Z}{1/j\omega C} + \frac{Y-Z}{R} = 0$ [assume $V_+ = V_- = Z$]

Sallen-Key Filter



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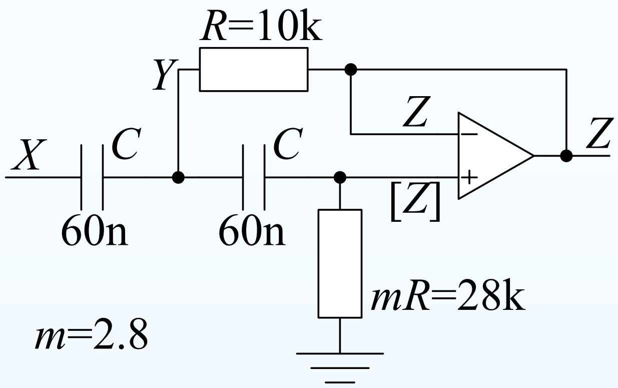
$$\Rightarrow Y\left(1+2j\omega RC\right)-Z\left(1+j\omega RC\right)=Xj\omega RC$$

Sallen-Key Filter



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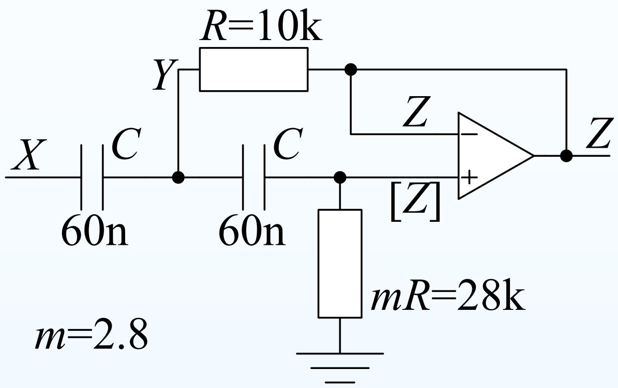

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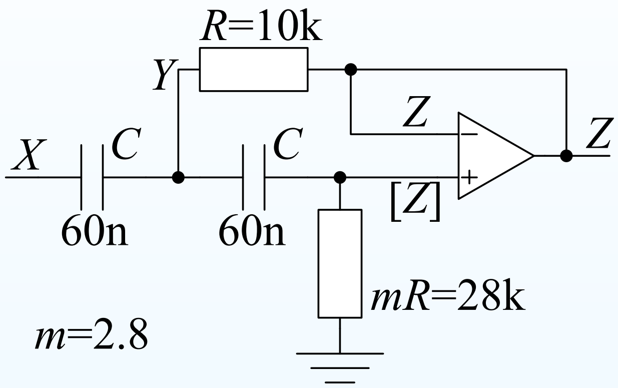

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Sub Y: $Z\frac{(1+j\omega mRC)}{j\omega mRC}\left(1 + 2j\omega RC\right) - Z\left(1 + j\omega RC\right) = Xj\omega RC$

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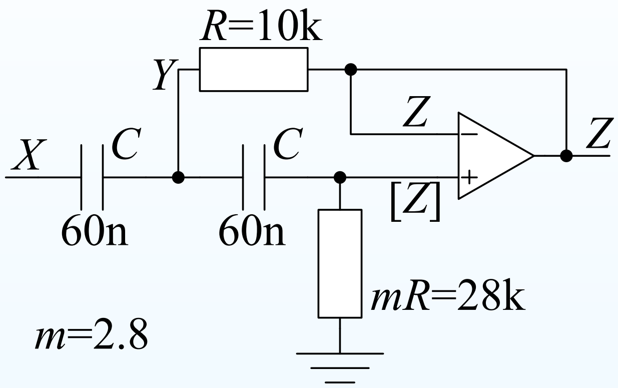

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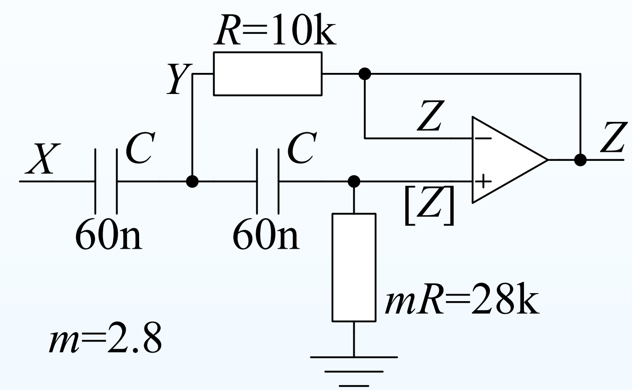
Sallen-Key Filter



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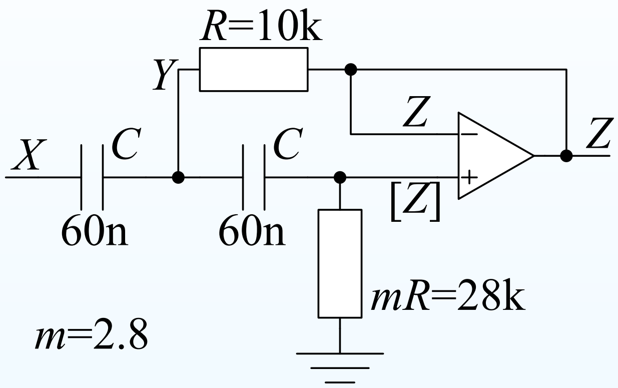
Corner freq: $p=\frac{1}{\sqrt{m}RC}=996$ rad/s

Sallen-Key Filter



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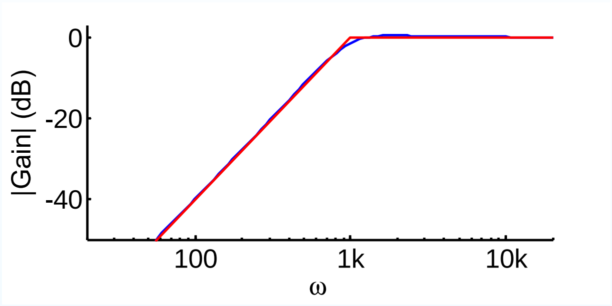

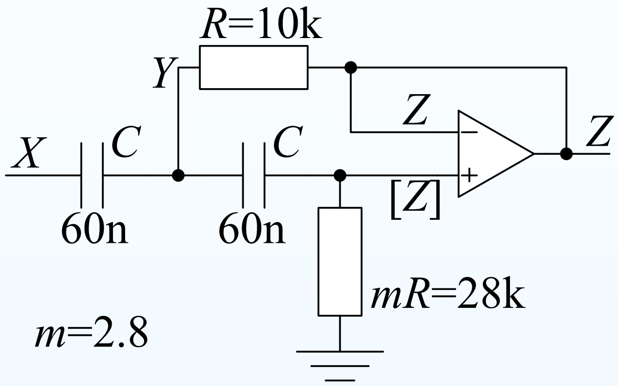

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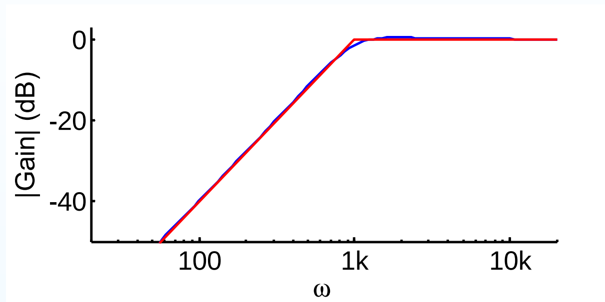
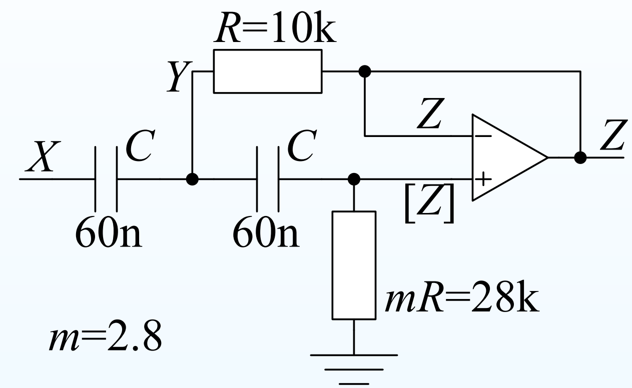
Corner freq: $p = \frac{1}{\sqrt{m}RC} = 996$ rad/s, $\zeta = \frac{1}{2Q} = pRC = \frac{1}{\sqrt{m}} = 0.6$

Sallen-Key Filter





Asymptotes: $\left(\frac{j\omega}{p}\right)^2$ and 1

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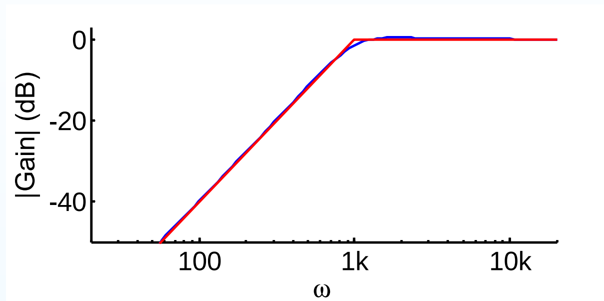

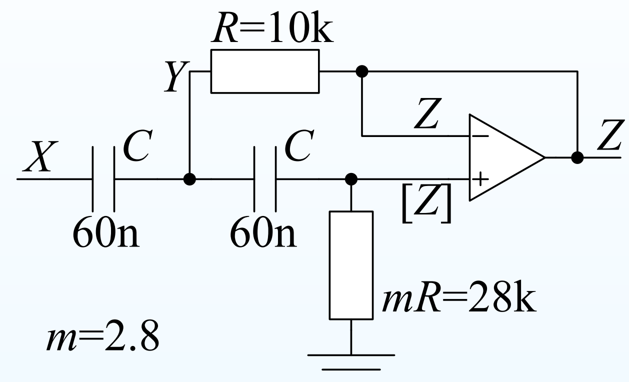

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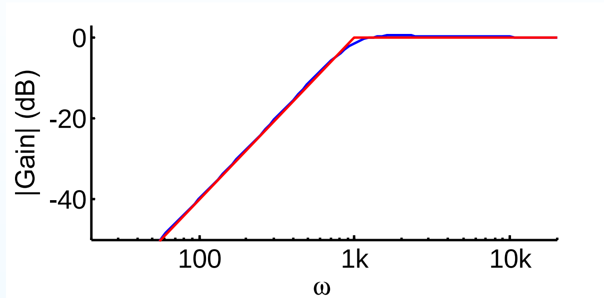
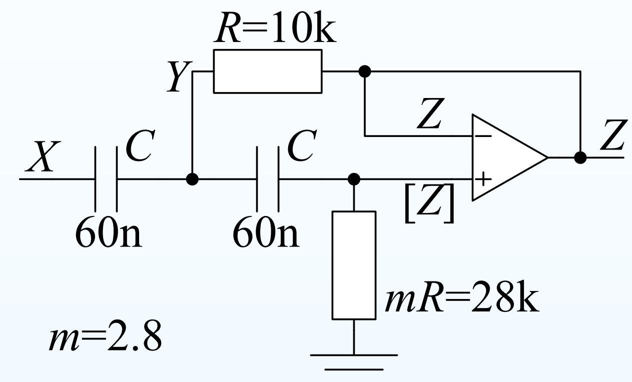
Sallen-Key: 2nd order filter without inductors. Can easily have gain >1.

Sallen-Key Filter





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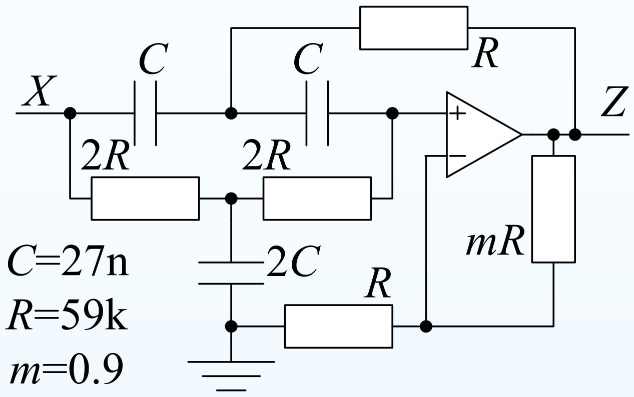
Sallen-Key: 2nd order filter without inductors. Can easily have gain >1.

Designing: Choose $m=\zeta^{-2}$; C any convenient value; $R=\frac{\zeta}{pC}$.

Twin-T Notch Filter

After much algebra:

$$\frac{Z}{X} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2+4(1-m)j\omega RC+1}$$



Do not try to memorize this circuit

Twin-T Notch Filter

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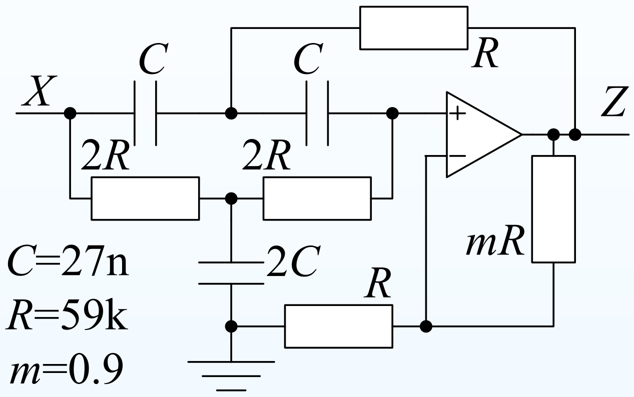

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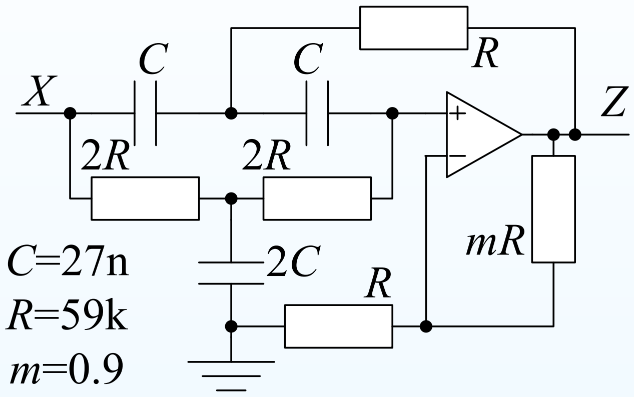
$$p = \frac{1}{2RC} = 314,\ \zeta = 1-m = 0.1$$



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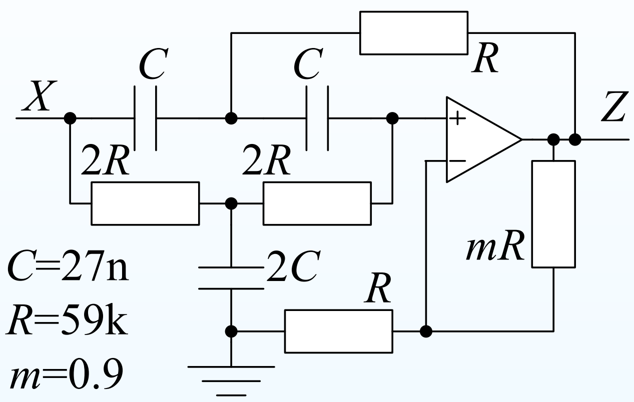

Very low ω: C open circuit

Do not try to memorize this circuit

Twin-T Notch Filter

After much algebra:

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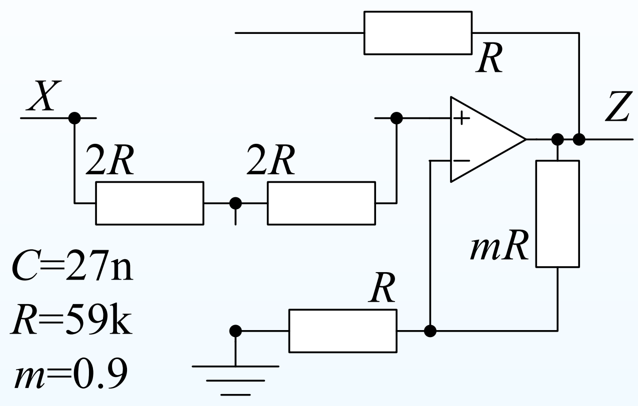

Very low ω: C open circuit

Non-inverting amp, $\frac{Z}{X} = 1 + m$

Do not try to memorize this circuit

Twin-T Notch Filter

After much algebra:

$$\frac{Z}{X} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2+4(1-m)j\omega RC+1}$$

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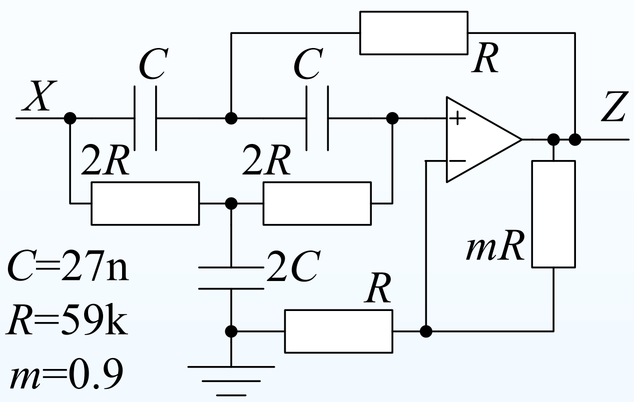

Very low ω: C open circuit

Non-inverting amp, $\frac{Z}{X} = 1 + m$

Very high ω: C short circuit

Do not try to memorize this circuit

Twin-T Notch Filter

After much algebra:

$$\frac{Z}{X} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2+4(1-m)j\omega RC+1}$$

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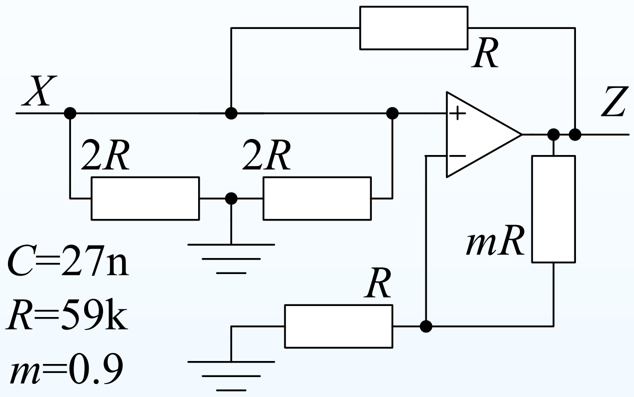

Very low ω: C open circuit

Non-inverting amp, $\frac{Z}{X} = 1+m$

Very high ω: C short circuit

Non-inverting amp, $\frac{Z}{X} = 1+m$

Do not try to memorize this circuit

Twin-T Notch Filter

After much algebra:

$$\frac{Z}{X} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2+4(1-m)j\omega RC+1}$$

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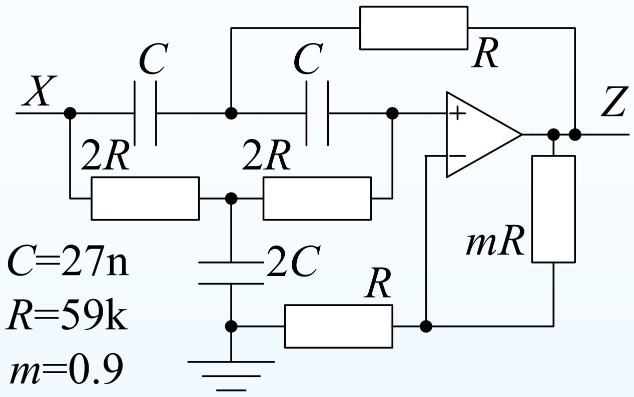


Very low ω: C open circuit

Non-inverting amp, $\frac{Z}{X} = 1+m$

Very high ω: C short circuit

Non-inverting amp, $\frac{Z}{X} = 1+m$

At $\omega = p$, $\left(\frac{j\omega}{p}\right)^2 = -1$: numerator = zero resulting in infinite attenuation.

Do not try to memorize this circuit

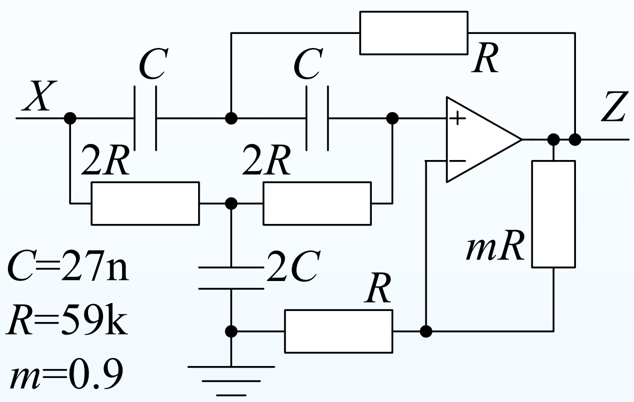
Twin-T Notch Filter

After much algebra:

$$\frac{Z}{X} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2+4(1-m)j\omega RC+1}$$

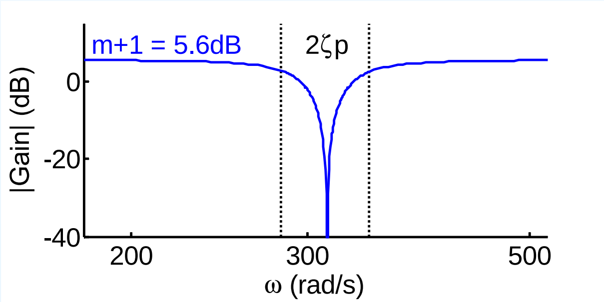
$$= \frac{(1+m)\left(({}^{j\omega}/_{p})^2+1\right)}{({}^{j\omega}/_{p})^2+2\zeta({}^{j\omega}/_{p})+1}$$

$$p = \frac{1}{2RC} = 314, \zeta = 1 - m = 0.1$$



Very low ω: C open circuit
Non-inverting amp, $\frac{Z}{X} = 1 + m$

Very high ω: C short circuit
Non-inverting amp, $\frac{Z}{X} = 1 + m$



At $\omega = p$, $\left(\frac{j\omega}{p}\right)^2 = -1$: numerator = zero resulting in infinite attenuation.

Do not try to memorize this circuit

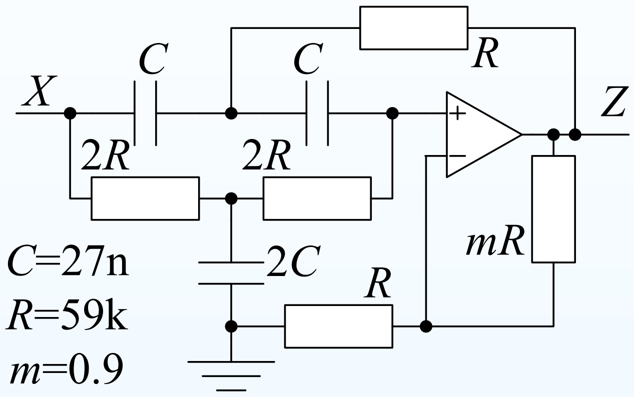
Twin-T Notch Filter

After much algebra:

$$\frac{Z}{X} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2+4(1-m)j\omega RC+1}$$

$$= \frac{(1+m)\left(\left(^{j\omega}/_{p}\right)^2+1\right)}{\left(^{j\omega}/_{p}\right)^2+2\zeta\left(^{j\omega}/_{p}\right)+1}$$

$$p = \frac{1}{2RC} = 314,\ \zeta = 1-m = 0.1$$



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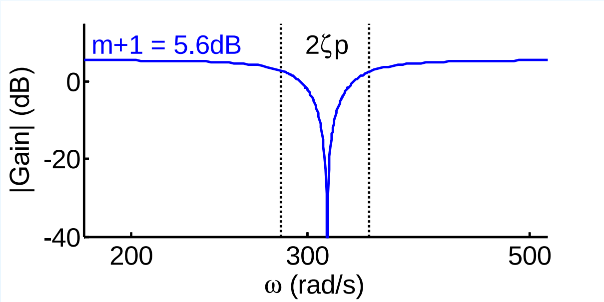

At $\omega = p$, $\left(\frac{j\omega}{p}\right)^2 = -1$: numerator = zero resulting in infinite attenuation.

The $3\,\text{dB}$ notch width is approximately $2\zeta p = 2(1-m)p$.

Do not try to memorize this circuit

Twin-T Notch Filter

After much algebra:

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$$= \frac{(1+m)\left((^{j\omega}/_{p})^2+1\right)}{(^{j\omega}/_{p})^2+2\zeta(^{j\omega}/_{p})+1}$$

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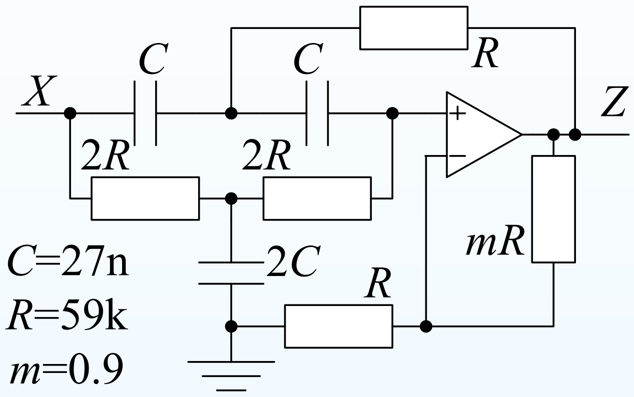

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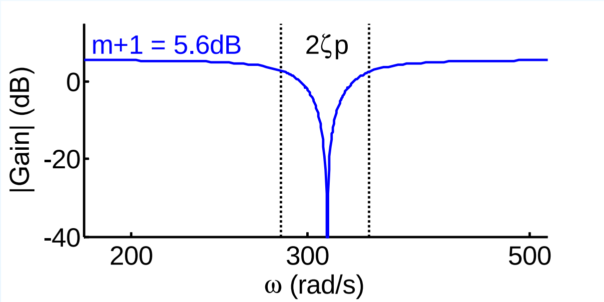

At $\omega = p$, $\left(\frac{j\omega}{p}\right)^2 = -1$: numerator = zero resulting in infinite attenuation.

The $3\,\mathrm{dB}$ notch width is approximately $2\zeta p = 2(1-m)p$.

Used to remove one specific frequency (e.g. mains hum @ 50 Hz)

Do not try to memorize this circuit

Conformal Filter Transformations (A)

A dimensionless gain, $\frac{V_Y}{V_X}$, can always be written using dimensionless impedance ratio terms: $\frac{Z_R}{Z_C} = j\omega RC$, $\frac{Z_L}{Z_R} = \frac{j\omega L}{R}$, $\frac{Z_L}{Z_C} = -\omega^2 LC$.

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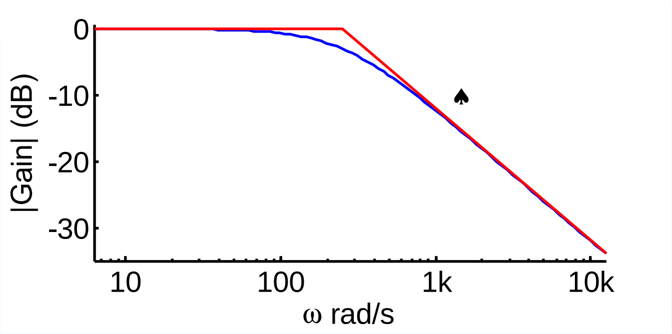

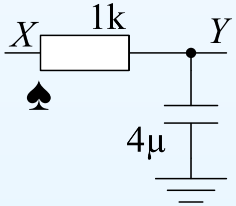

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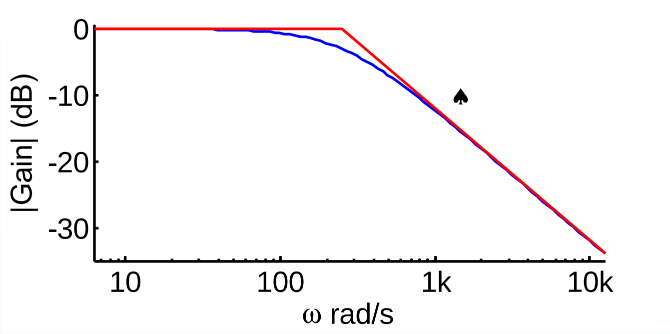
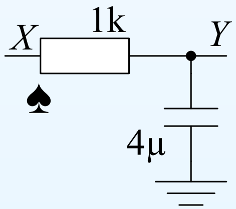
Impedance scaling:

Scale all impedances by k:





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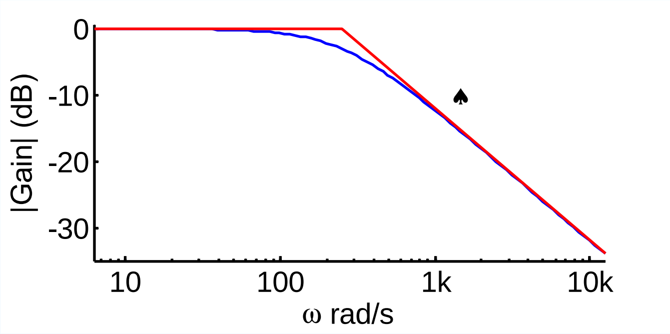

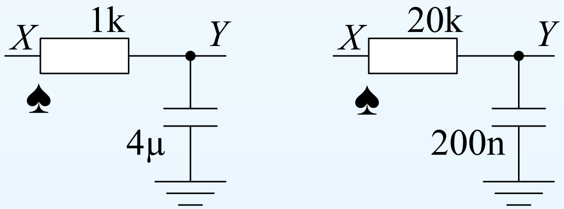

$k = 20$

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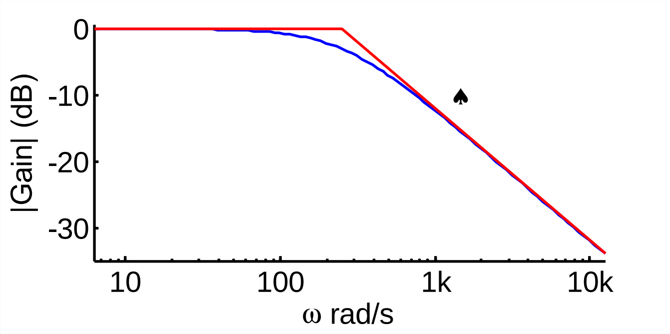

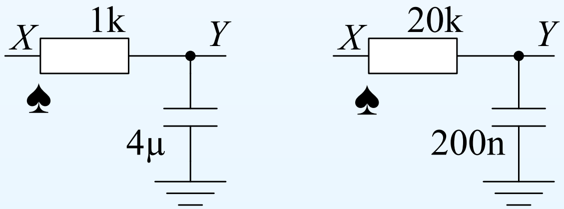

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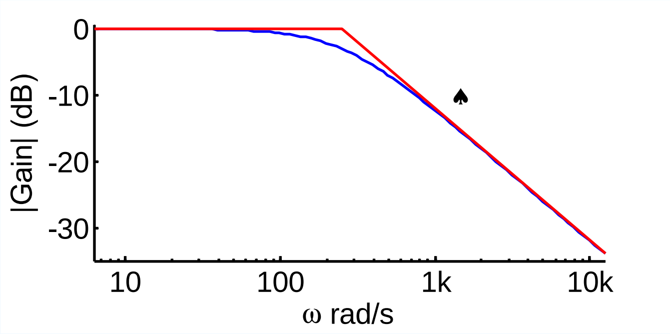

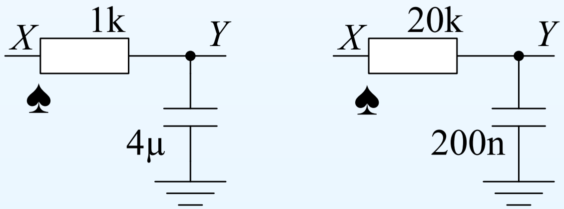

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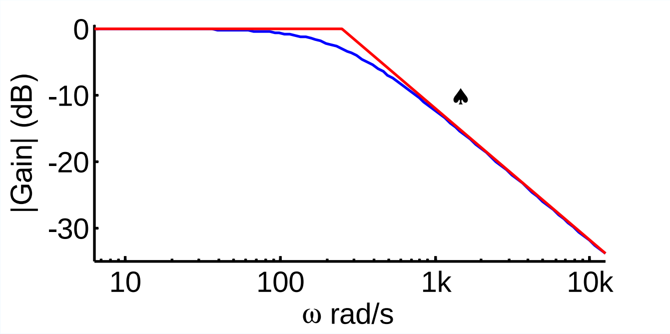
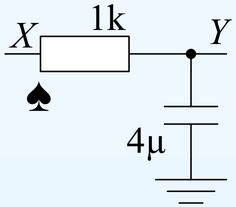
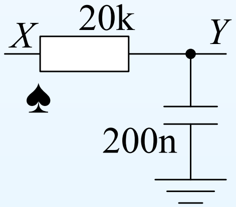


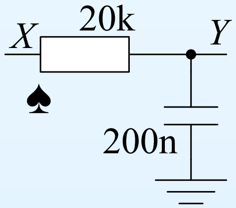




$k = 20$

Frequency Shift:

Scale reactive components by k:



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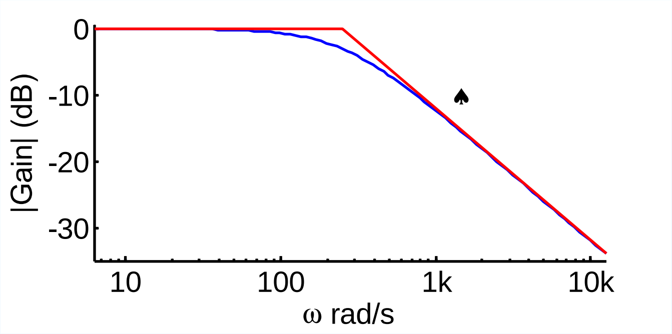

Impedance scaling:

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Impedance ratios are unchanged so graph stays the same.
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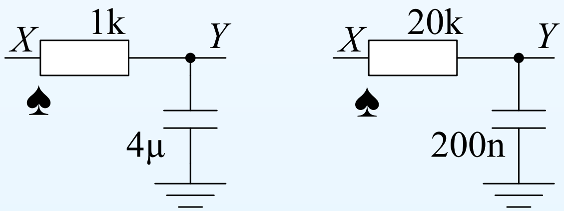
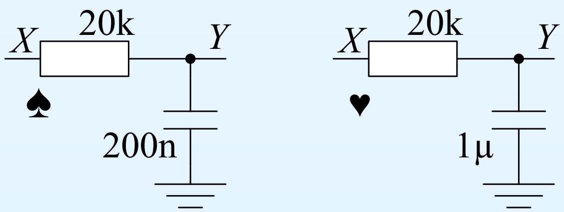


$k = 20$

Frequency Shift:

Scale reactive components by k:
$R' = R,\ C' = kC,\ L' = kL$



$k = 5$

Conformal Filter Transformations (A)

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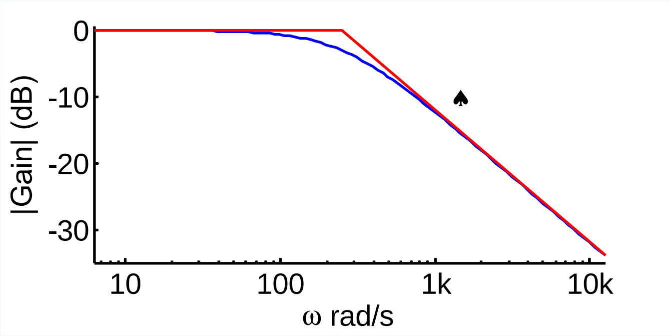

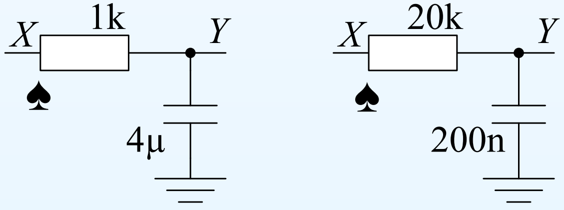

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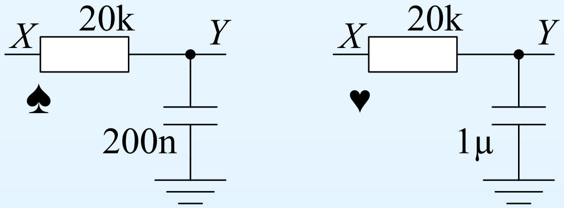
Frequency Shift:

Scale reactive components by k:
$R' = R,\ C' = kC,\ L' = kL$
$\Rightarrow Z'(k^{-1}\omega) \equiv Z(\omega)$



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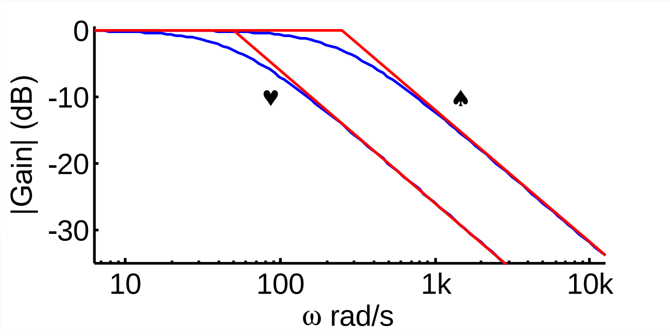

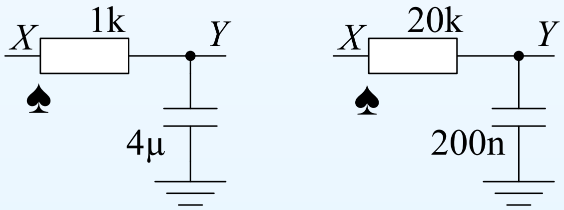

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Frequency Shift:

Scale reactive components by k:
$R' = R,\ C' = kC,\ L' = kL$
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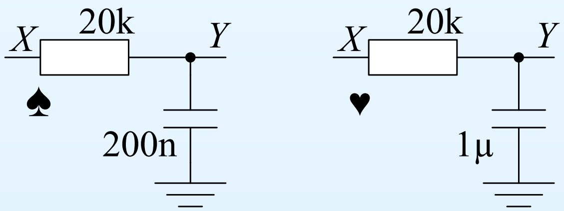
Graph shifts left by a factor of k.



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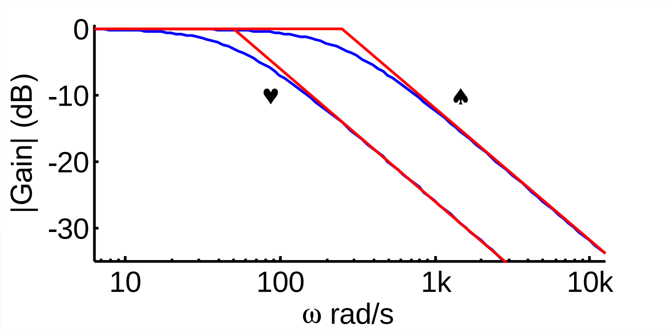

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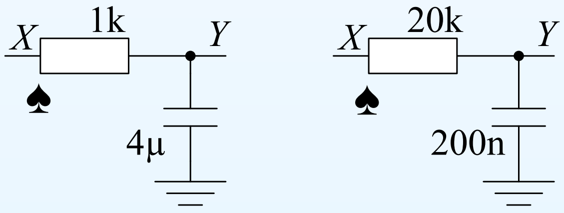

$k = 20$

Frequency Shift:

Scale reactive components by k:

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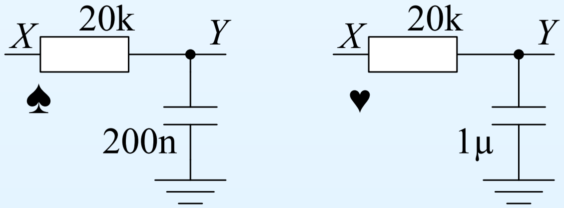
$\Rightarrow Z'(k^{-1}\omega) \equiv Z(\omega)$

Graph shifts left by a factor of k.

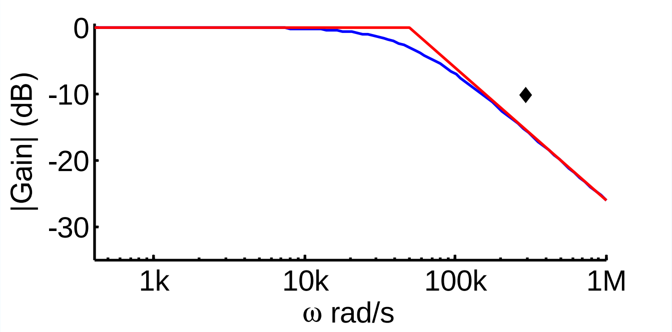


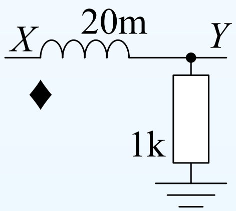
$k = 5$

Must scale all reactive components in the circuit by the same factor.

Conformal Filter Transformations (B)

Change LR circuit to RC:

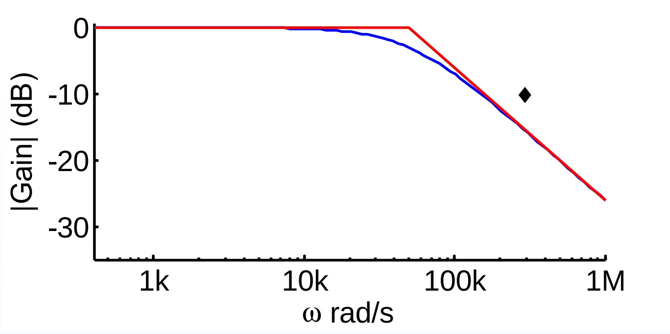


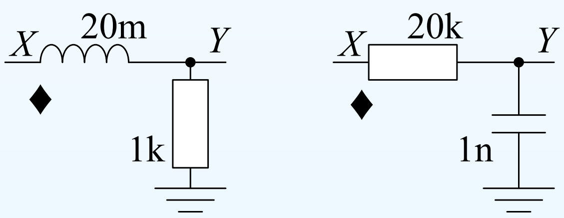


Conformal Filter Transformations (B)

Change LR circuit to RC:

Change $R' = kL,\ C' = \frac{1}{kR}$





$k = 10^6$

Conformal Filter Transformations (B)

Change LR circuit to RC:

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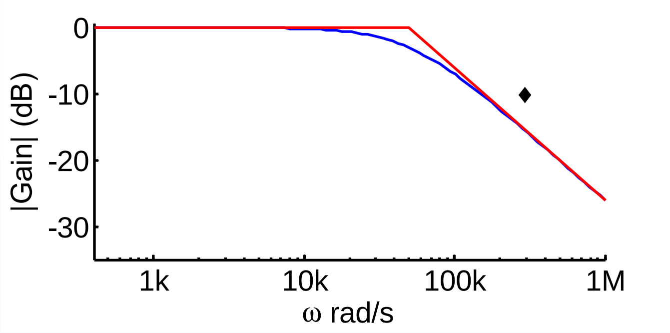
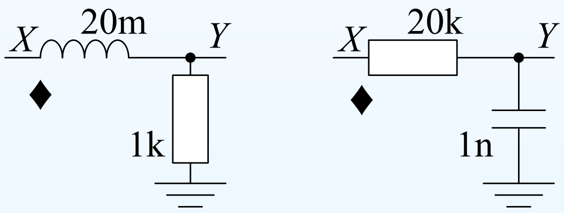
$$\Rightarrow \frac{Z_{R'}}{Z_{C'}} = j\omega R'C' = \frac{j\omega L}{R} = \frac{Z_L}{Z_R}$$





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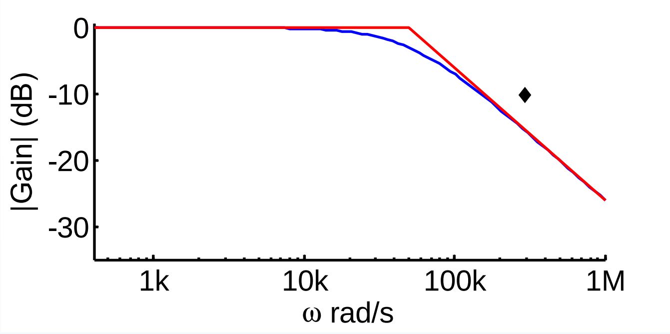
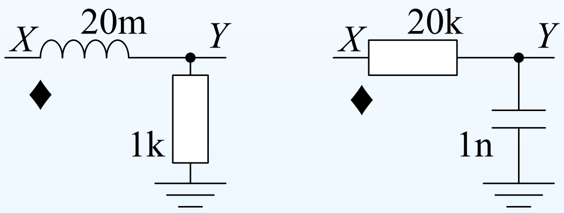
Impedance ratios are unchanged at all ω so graph stays the same.





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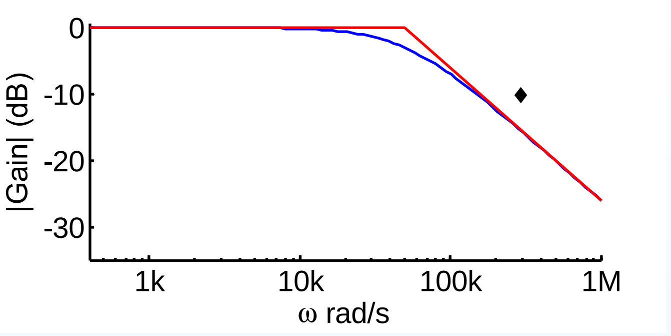

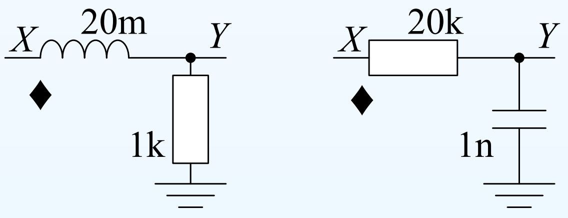

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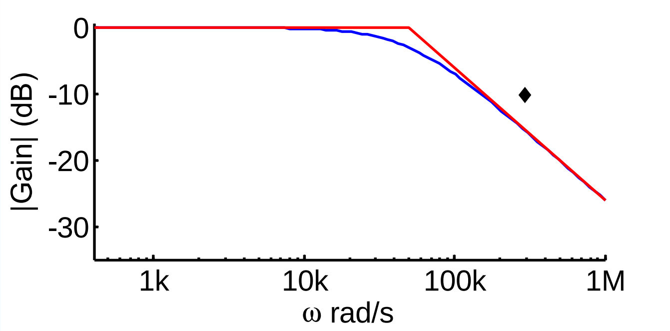
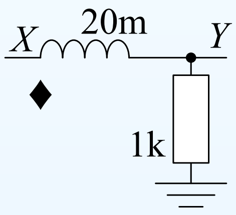
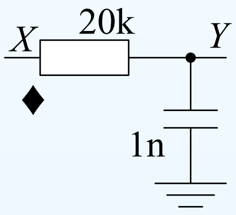


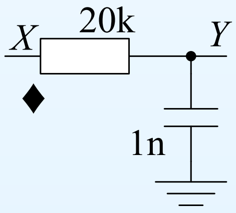




$k = 10^6$

Reflect frequency axis around ω_m:

Change $R' = \frac{k}{\omega_m C},\ C' = \frac{1}{\omega_m kR}$



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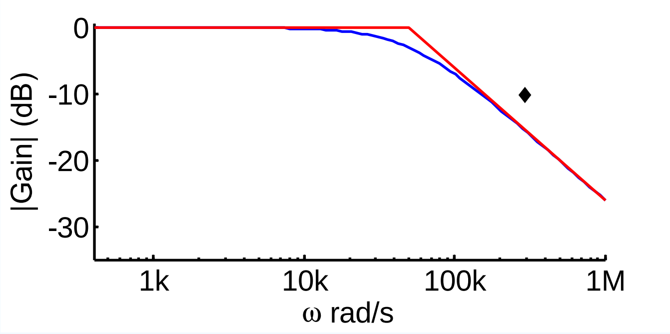

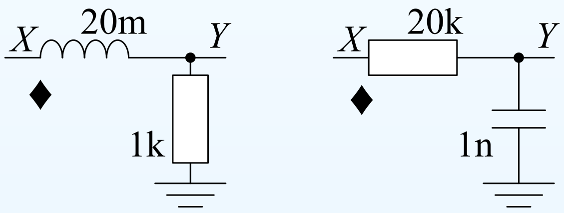

$k = 10^6$

Reflect frequency axis around ω_m:

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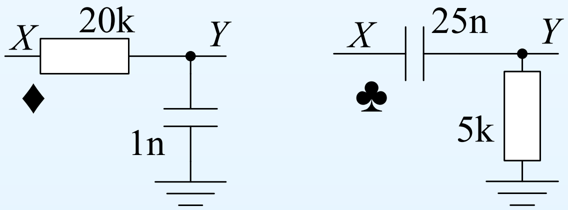
$\Rightarrow \frac{Z_{R'}}{Z_{C'}}\left(\frac{\omega_m^2}{\omega}\right) = \left(\frac{Z_C}{Z_R}(\omega)\right)^*$



$k = 0.1,\ \omega_m = 20\,\text{k}$

Conformal Filter Transformations (B)

Change LR circuit to RC:

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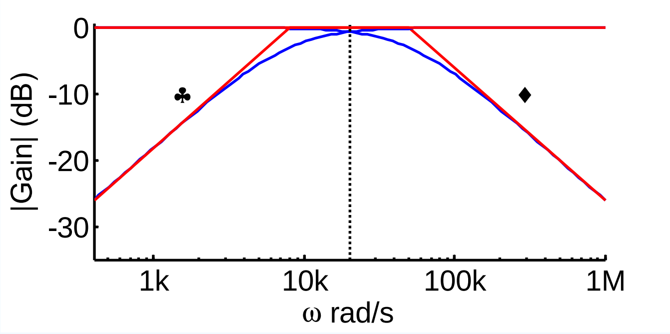

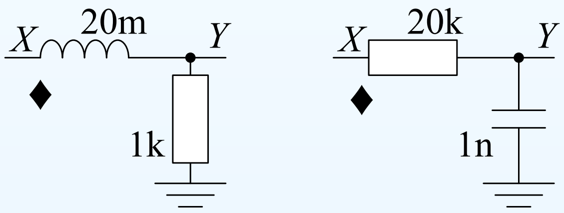

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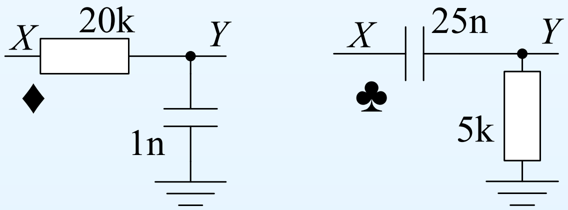

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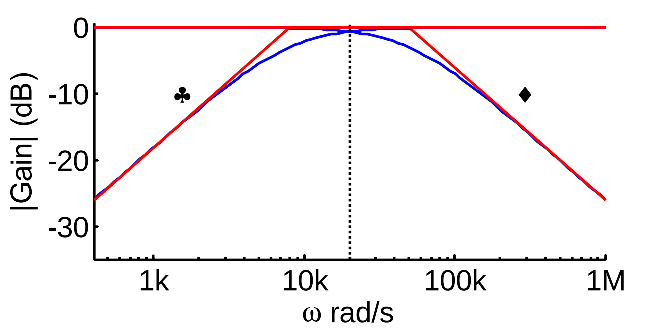

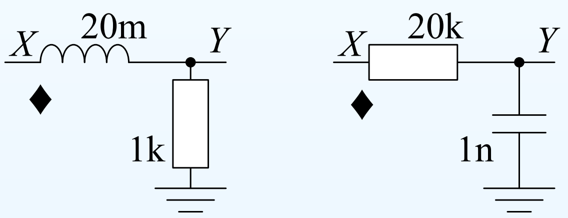

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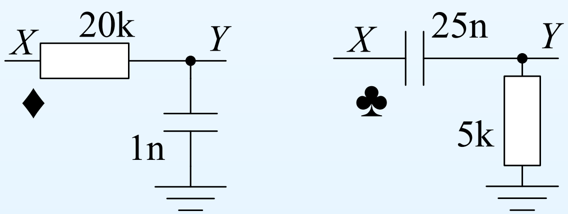

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(a) Magnitude graph flips

(b) Phase graph flips and negates since $\angle z^* = -\angle z$.
(k is arbitrary)

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For further details see Hayt Ch 16 or Irwin Ch 12.