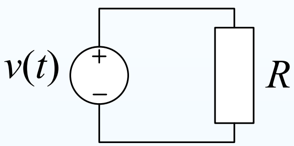
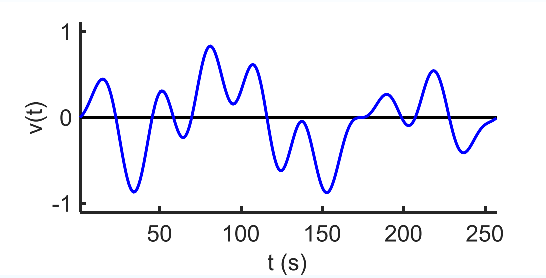
14: Power in AC Circuits

Average Power





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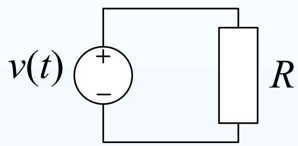
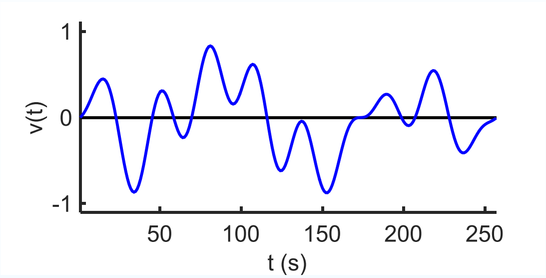
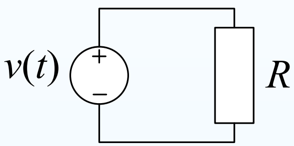
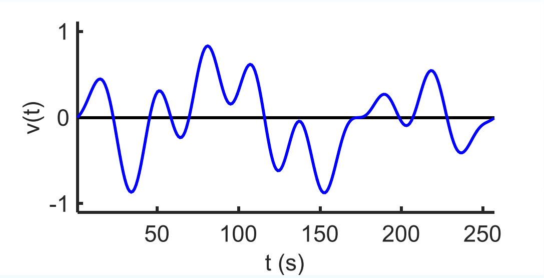


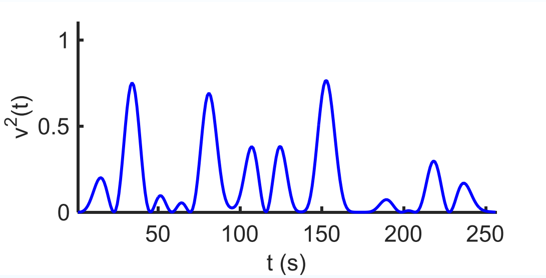


Intantaneous Power dissipated in R: $p(t) = \frac{v^2(t)}{R}$

Average Power







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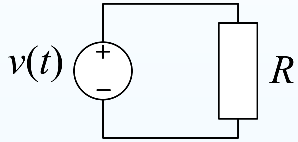

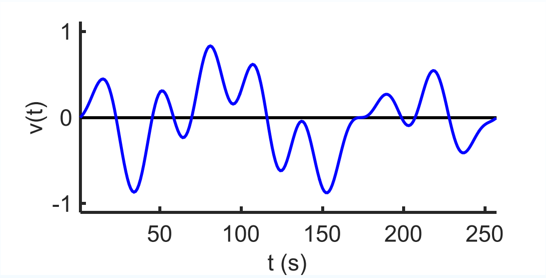

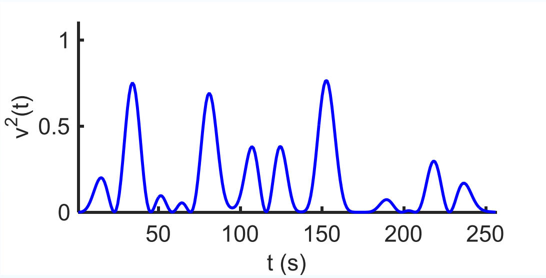

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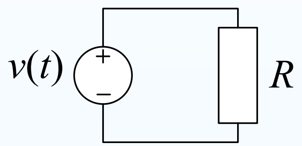

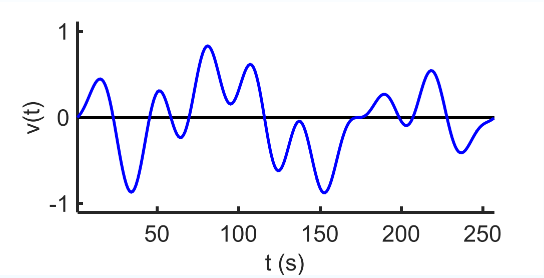

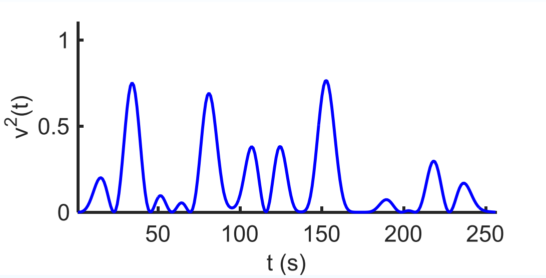

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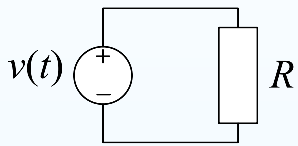

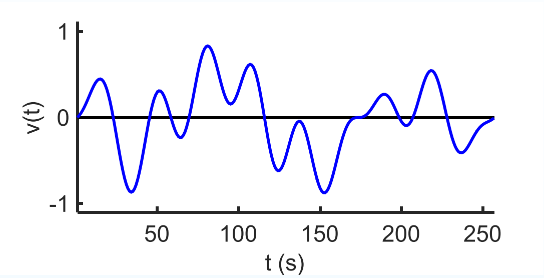

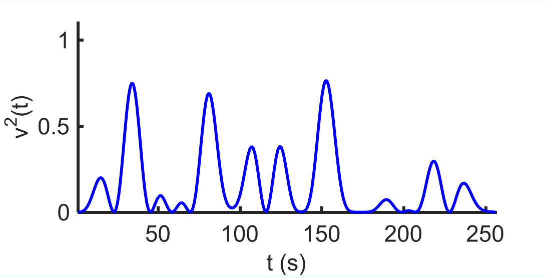

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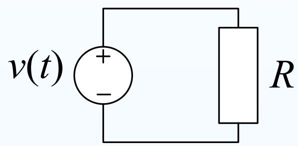

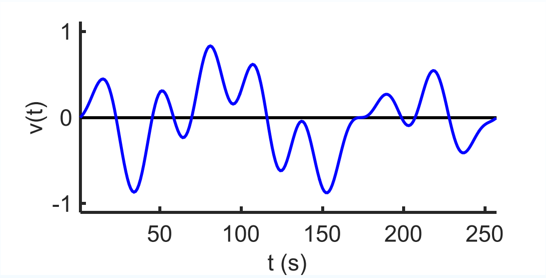

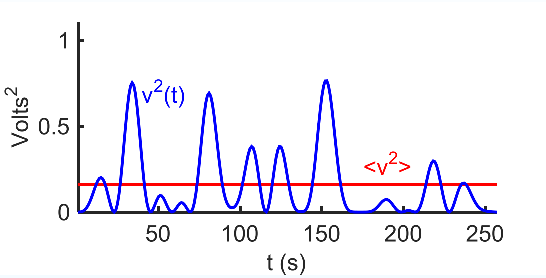

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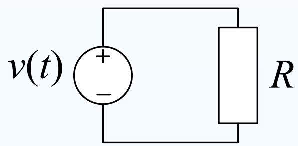

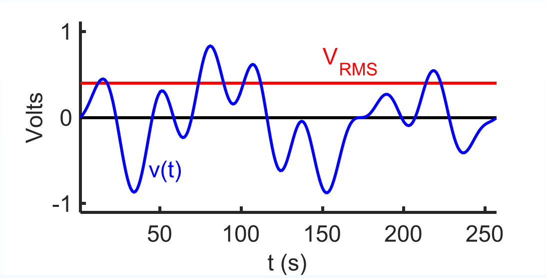

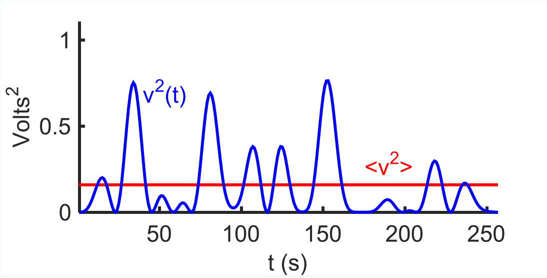

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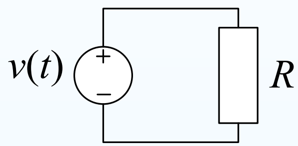

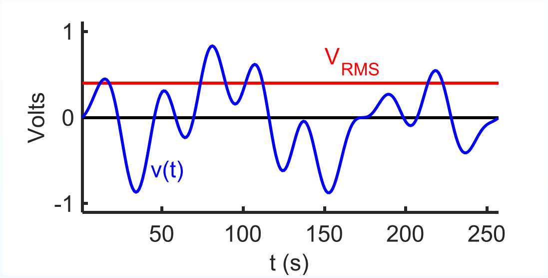

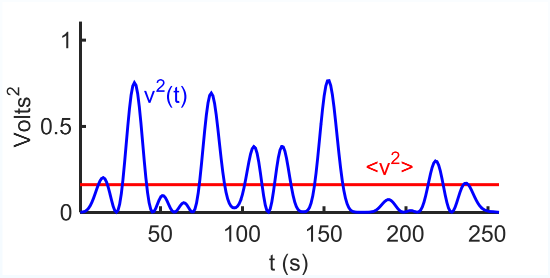

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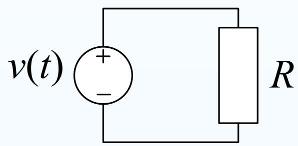
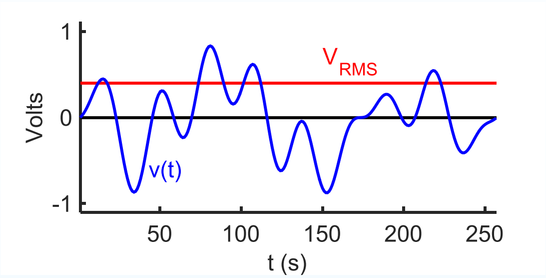
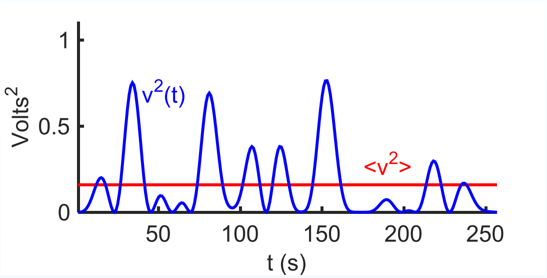
V_{rms} is the DC voltage that would cause R to dissipate the same power.

Average Power







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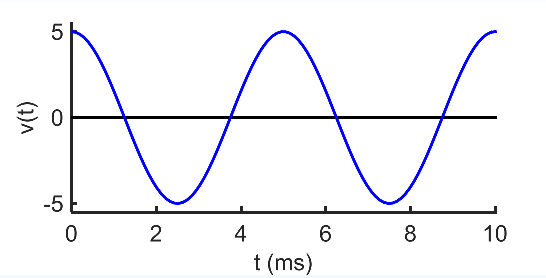
We use *small letters* for time-varying voltages and *capital letters* for time-invariant values.

Cosine Wave RMS



Cosine Wave: $v(t) = 5\cos\omega t$. Amplitude is $V = 5$ V.

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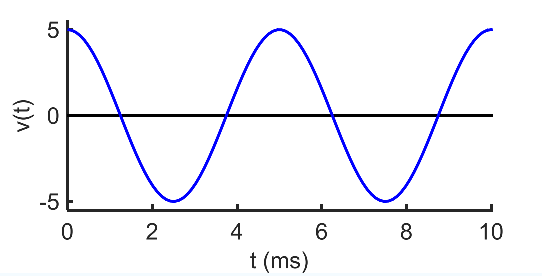

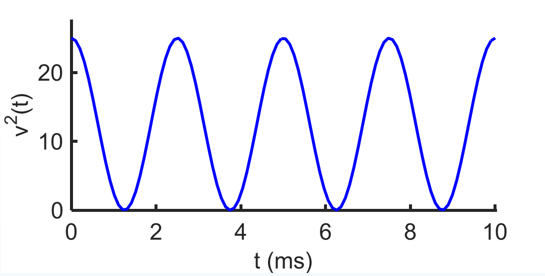

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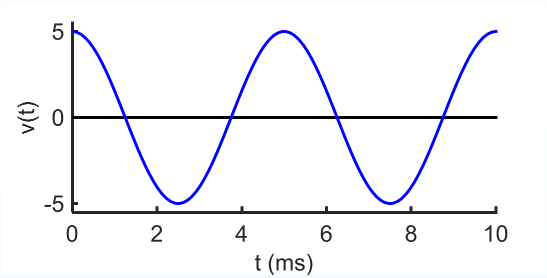

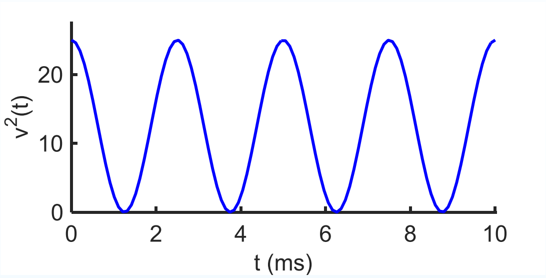

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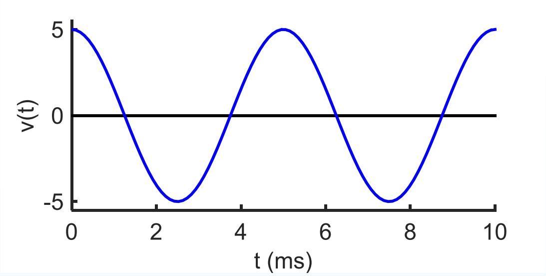

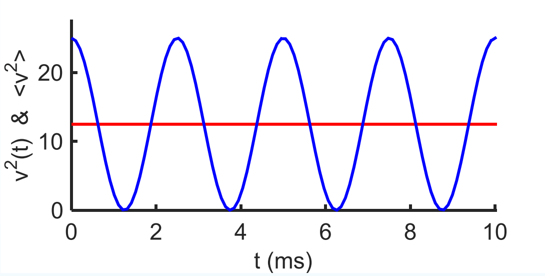

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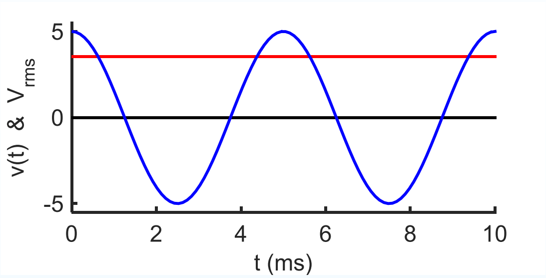

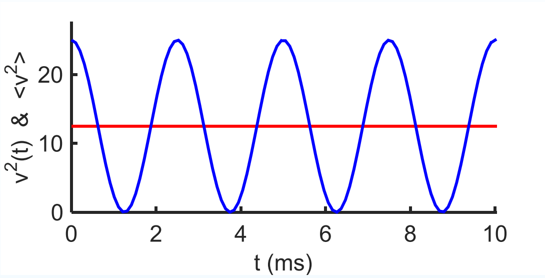

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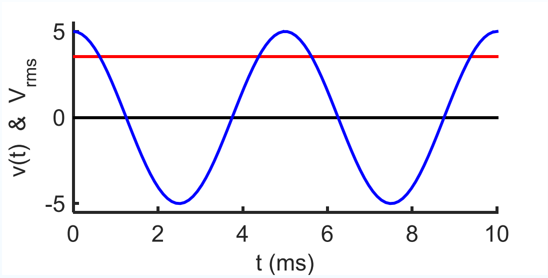

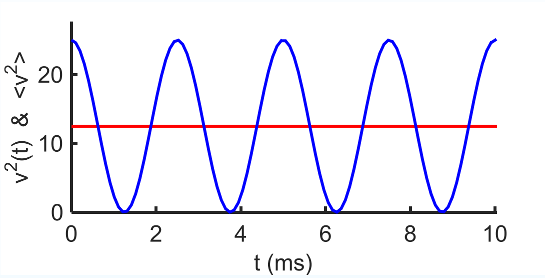

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Note: Power engineers *always* use RMS voltages and currents exclusively and omit the "rms" subscript.
For example UK Mains voltage = 230 V rms = 325 V peak.

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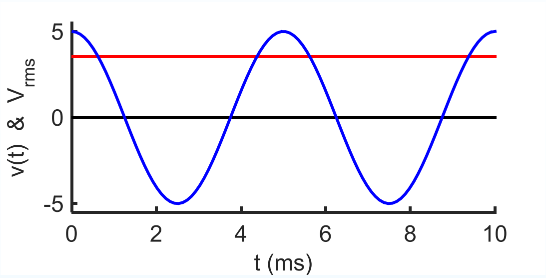

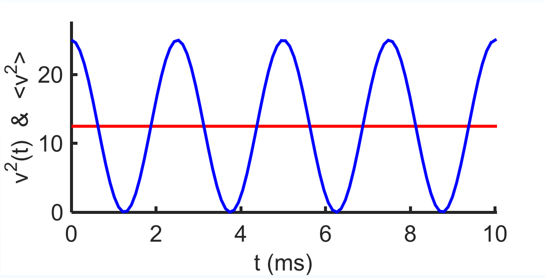

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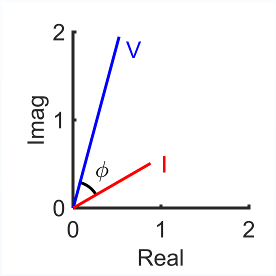
In this lecture course only, a ~ overbar means $\div\sqrt{2}$: thus $\widetilde{V} = \frac{1}{\sqrt{2}}V$.

Power Factor



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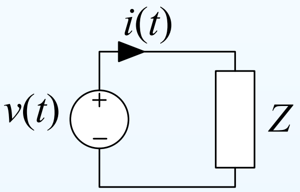
$$V = |V| \, e^{j\theta_V}$$
$$I = |I| \, e^{j\theta_I}$$



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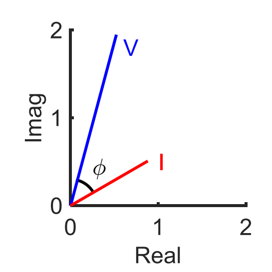

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$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos(\omega t + \theta_V)$$
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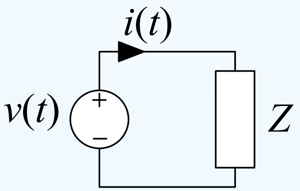

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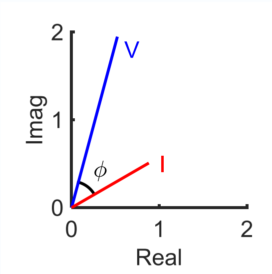

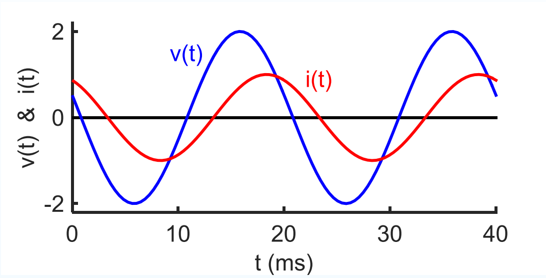

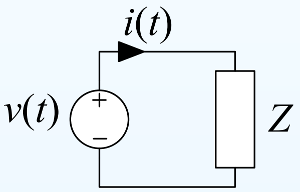

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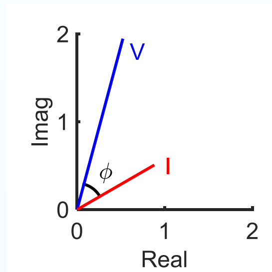
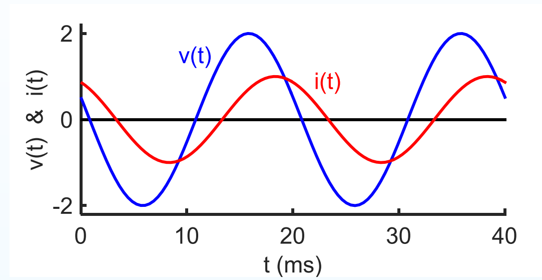
Power Factor

+





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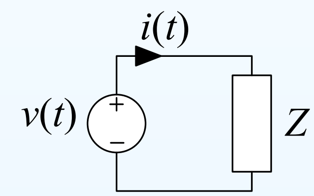
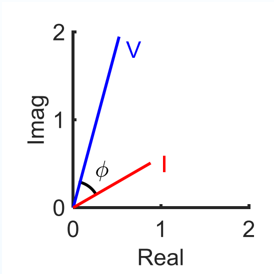


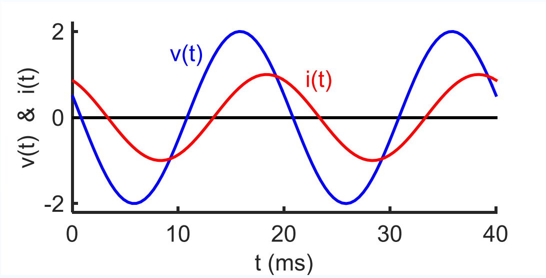
Power dissipated in load Z is

$$p(t) = v(t)i(t) = |V|\,|I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I)$$

Power Factor

+





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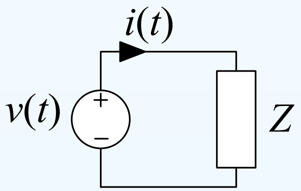
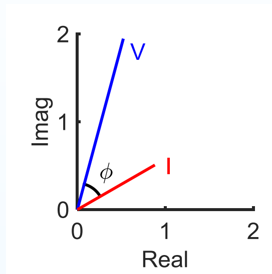
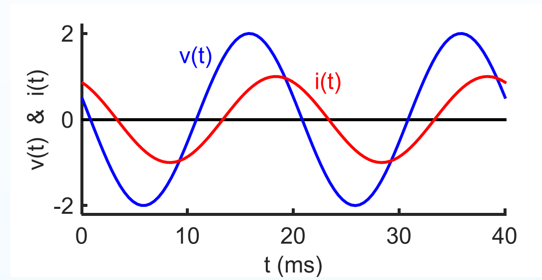


Power dissipated in load Z is

$$p(t) = v(t)i(t) = |V||I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I)$$
$$= |V||I| \left(\tfrac{1}{2}\cos(2\omega t + \theta_V + \theta_I) + \tfrac{1}{2}\cos(\theta_V - \theta_I)\right)$$

Power Factor

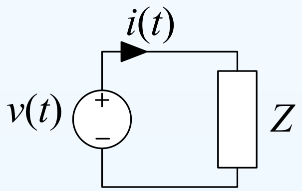




Suppose voltage and current phasors are:

$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos\left(\omega t + \theta_V\right)$$

$$I = |I|\, e^{j\theta_I} \quad \Leftrightarrow \quad i(t) = |I| \cos\left(\omega t + \theta_I\right)$$



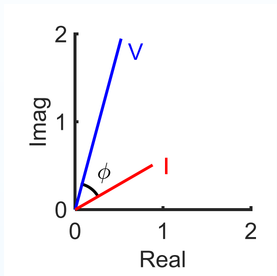
Power dissipated in load Z is

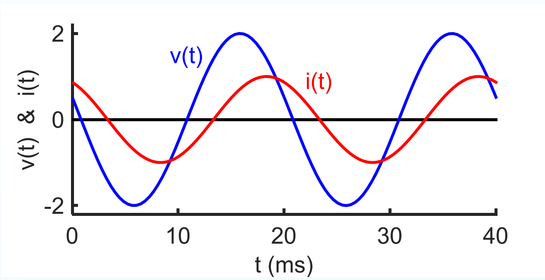
$$p(t) = v(t)i(t) = |V|\,|I| \cos\left(\omega t + \theta_V\right) \cos\left(\omega t + \theta_I\right)$$

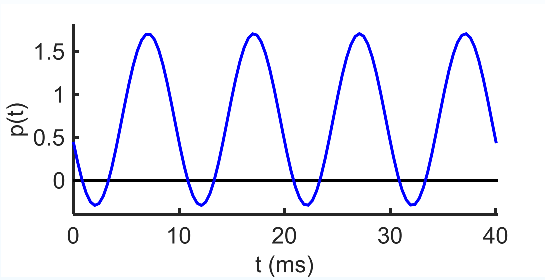
$$= |V|\,|I| \left(\tfrac{1}{2} \cos\left(2\omega t + \theta_V + \theta_I\right) + \tfrac{1}{2} \cos\left(\theta_V - \theta_I\right)\right)$$

$$= \tfrac{1}{2}\,|V|\,|I| \cos\left(\theta_V - \theta_I\right) + \tfrac{1}{2}\,|V|\,|I| \cos\left(2\omega t + \theta_V + \theta_I\right)$$

Power Factor



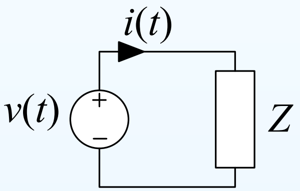




Suppose voltage and current phasors are:

$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos(\omega t + \theta_V)$$

$$I = |I|\, e^{j\theta_I} \quad \Leftrightarrow \quad i(t) = |I| \cos(\omega t + \theta_I)$$

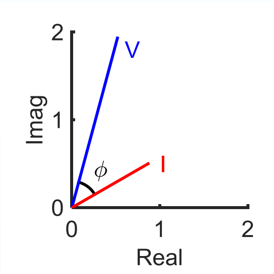


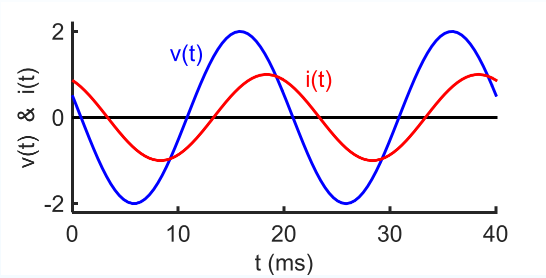
Power dissipated in load Z is

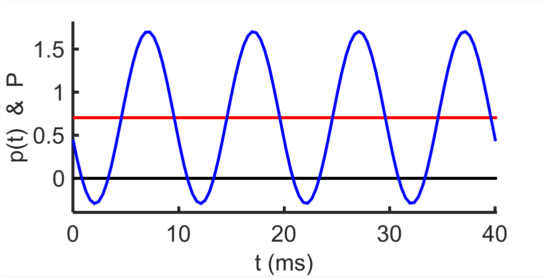
$$\begin{aligned} p(t) = v(t)i(t) &= |V|\,|I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I) \\ &= |V|\,|I| \left(\tfrac{1}{2}\cos(2\omega t + \theta_V + \theta_I) + \tfrac{1}{2}\cos(\theta_V - \theta_I)\right) \\ &= \tfrac{1}{2}|V|\,|I| \cos(\theta_V - \theta_I) + \tfrac{1}{2}|V|\,|I| \cos(2\omega t + \theta_V + \theta_I) \end{aligned}$$

Power Factor

+



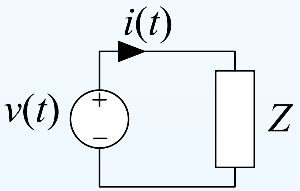




Suppose voltage and current phasors are:

$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos(\omega t + \theta_V)$$

$$I = |I|\, e^{j\theta_I} \quad \Leftrightarrow \quad i(t) = |I| \cos(\omega t + \theta_I)$$

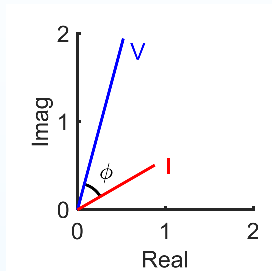


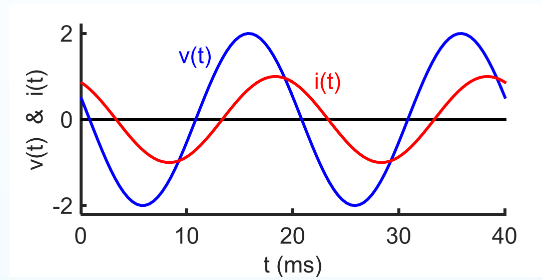
Power dissipated in load Z is

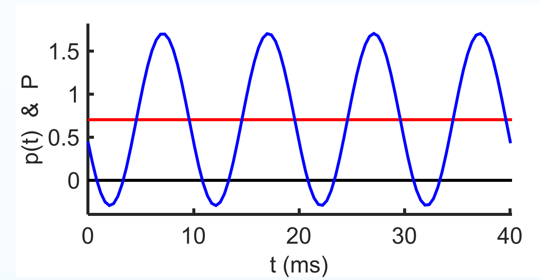
$$\begin{aligned} p(t) = v(t)i(t) &= |V|\,|I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I) \\ &= |V|\,|I| \left(\tfrac{1}{2}\cos(2\omega t + \theta_V + \theta_I) + \tfrac{1}{2}\cos(\theta_V - \theta_I)\right) \\ &= \tfrac{1}{2}|V|\,|I| \cos(\theta_V - \theta_I) + \tfrac{1}{2}|V|\,|I| \cos(2\omega t + \theta_V + \theta_I) \end{aligned}$$

Average power: $P = \frac{1}{2}|V|\,|I| \cos(\phi)$ where $\phi = \theta_V - \theta_I$

Power Factor



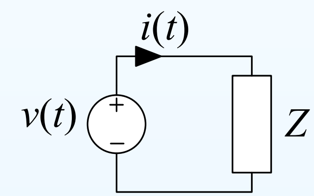




Suppose voltage and current phasors are:

$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos(\omega t + \theta_V)$$

$$I = |I|\, e^{j\theta_I} \quad \Leftrightarrow \quad i(t) = |I| \cos(\omega t + \theta_I)$$



Power dissipated in load Z is

$$p(t) = v(t)i(t) = |V|\,|I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I)$$

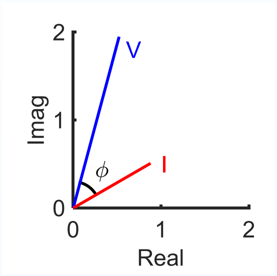
$$= |V|\,|I| \left(\tfrac{1}{2}\cos(2\omega t + \theta_V + \theta_I) + \tfrac{1}{2}\cos(\theta_V - \theta_I)\right)$$

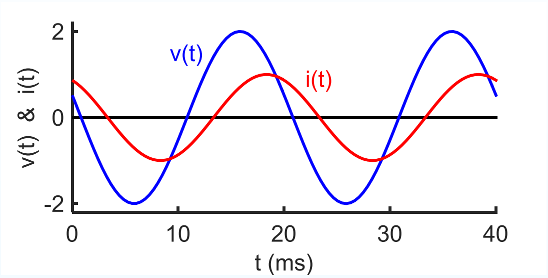
$$= \tfrac{1}{2}|V|\,|I| \cos(\theta_V - \theta_I) + \tfrac{1}{2}|V|\,|I| \cos(2\omega t + \theta_V + \theta_I)$$

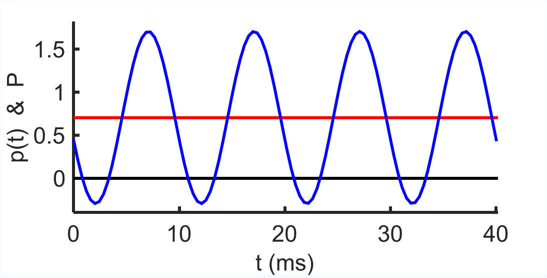
Average power: $P = \frac{1}{2}|V|\,|I| \cos(\phi)$ where $\phi = \theta_V - \theta_I$

$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos(\phi)$$

Power Factor



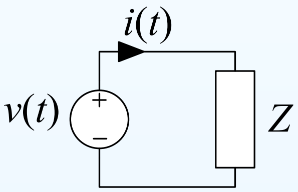




Suppose voltage and current phasors are:

$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos(\omega t + \theta_V)$$

$$I = |I|\, e^{j\theta_I} \quad \Leftrightarrow \quad i(t) = |I| \cos(\omega t + \theta_I)$$



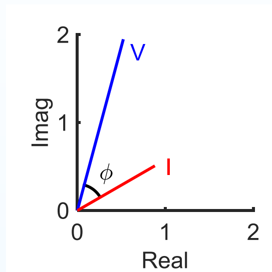
Power dissipated in load Z is

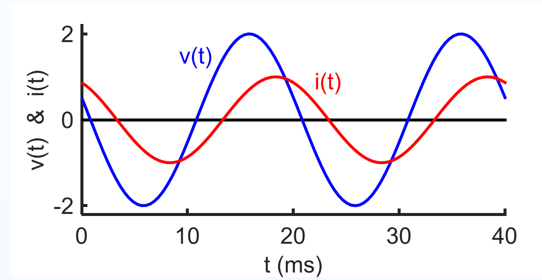
$$\begin{aligned} p(t) = v(t)i(t) &= |V|\,|I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I) \\ &= |V|\,|I| \left(\tfrac{1}{2}\cos(2\omega t + \theta_V + \theta_I) + \tfrac{1}{2}\cos(\theta_V - \theta_I)\right) \\ &= \tfrac{1}{2}|V|\,|I| \cos(\theta_V - \theta_I) + \tfrac{1}{2}|V|\,|I| \cos(2\omega t + \theta_V + \theta_I) \end{aligned}$$

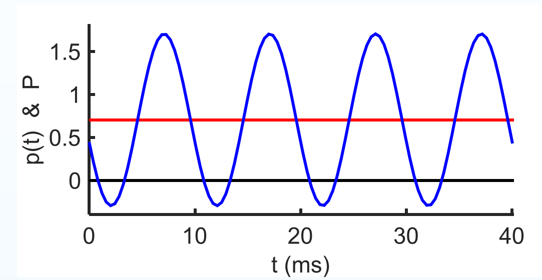
Average power: $P = \frac{1}{2}|V|\,|I|\cos(\phi)$ where $\phi = \theta_V - \theta_I$

$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos(\phi)$ $\quad$ $\cos\phi$ is the *power factor*

Power Factor







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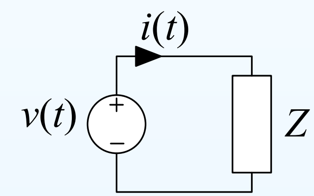
$$V = |V|\, e^{j\theta_V} \quad \Leftrightarrow \quad v(t) = |V| \cos(\omega t + \theta_V)$$

$$I = |I|\, e^{j\theta_I} \quad \Leftrightarrow \quad i(t) = |I| \cos(\omega t + \theta_I)$$



Power dissipated in load Z is

$$\begin{aligned} p(t) = v(t)i(t) &= |V|\,|I| \cos(\omega t + \theta_V) \cos(\omega t + \theta_I) \\ &= |V|\,|I| \left(\tfrac{1}{2}\cos(2\omega t + \theta_V + \theta_I) + \tfrac{1}{2}\cos(\theta_V - \theta_I)\right) \\ &= \tfrac{1}{2}|V|\,|I| \cos(\theta_V - \theta_I) + \tfrac{1}{2}|V|\,|I| \cos(2\omega t + \theta_V + \theta_I) \end{aligned}$$

Average power: $P = \frac{1}{2}|V|\,|I| \cos(\phi)$ where $\phi = \theta_V - \theta_I$

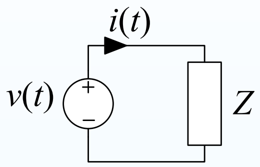
$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos(\phi)$ $\quad$ $\cos\phi$ is the *power factor*

$\phi > 0 \Leftrightarrow$ a *lagging power factor* (normal case: Current lags Voltage)

$\phi < 0 \Leftrightarrow$ a *leading power factor* (rare case: Current leads Voltage)

Complex Power

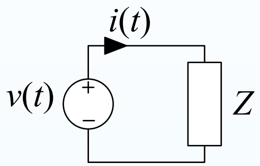
If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$



The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$

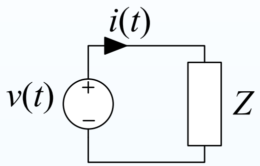


The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I}$$

Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$

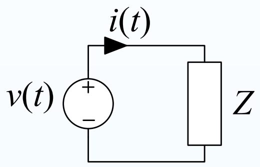


The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$

Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$



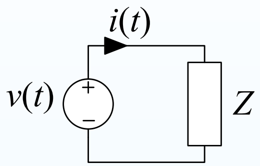
The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$

$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi}$$

Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$



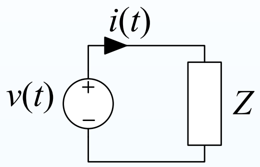
The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$

$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos\phi + j \left|\widetilde{V}\right| \left|\widetilde{I}\right| \sin\phi$$

Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$



The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

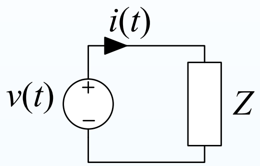
$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$

$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos\phi + j \left|\widetilde{V}\right| \left|\widetilde{I}\right| \sin\phi$$

$$= P + jQ$$

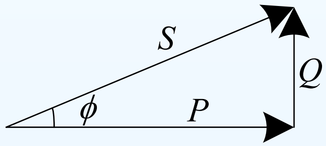
Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$



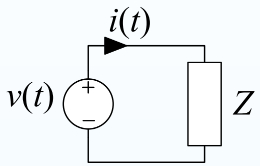
The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$
where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$
$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos\phi + j \left|\widetilde{V}\right| \left|\widetilde{I}\right| \sin\phi$$
$$= P + jQ$$



Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$

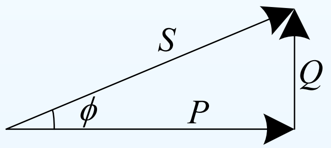


The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$

$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos\phi + j \left|\widetilde{V}\right| \left|\widetilde{I}\right| \sin\phi$$

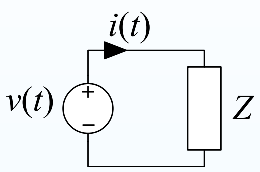
$$= P + jQ$$



Complex Power: $S \triangleq \widetilde{V}\widetilde{I}^* = P + jQ$ measured in Volt-Amps (VA)

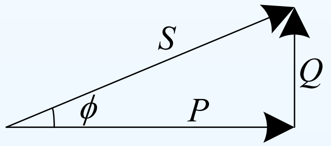
Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$



The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$
$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos\phi + j \left|\widetilde{V}\right| \left|\widetilde{I}\right| \sin\phi$$
$$= P + jQ$$

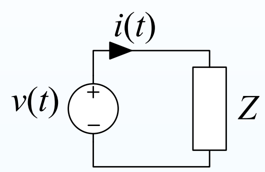


Complex Power: $S \triangleq \widetilde{V}\widetilde{I}^* = P + jQ$ measured in Volt-Amps (VA)

Apparent Power: $|S| \triangleq \left|\widetilde{V}\right| \left|\widetilde{I}\right|$ measured in Volt-Amps (VA)

Complex Power

If $\widetilde{V} = \frac{1}{\sqrt{2}} |V| e^{j\theta_V}$ and $\tilde{I} = \frac{1}{\sqrt{2}} |I| e^{j\theta_I}$

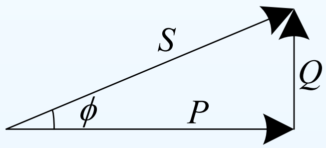


The *complex power* absorbed by Z is $S \triangleq \widetilde{V} \times \widetilde{I}^*$ where * means complex conjugate.

$$\widetilde{V} \times \widetilde{I}^* = \left|\widetilde{V}\right| e^{j\theta_V} \times \left|\widetilde{I}\right| e^{-j\theta_I} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j(\theta_V - \theta_I)}$$

$$= \left|\widetilde{V}\right| \left|\widetilde{I}\right| e^{j\phi} = \left|\widetilde{V}\right| \left|\widetilde{I}\right| \cos\phi + j \left|\widetilde{V}\right| \left|\widetilde{I}\right| \sin\phi$$

$$= P + jQ$$



Complex Power: $S \triangleq \widetilde{V}\widetilde{I}^* = P + jQ$ measured in Volt-Amps (VA)

Apparent Power: $|S| \triangleq \left|\widetilde{V}\right| \left|\widetilde{I}\right|$ measured in Volt-Amps (VA)

Average Power: $P \triangleq \Re(S)$ measured in Watts (W)

Complex Power

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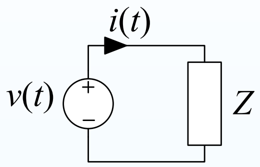

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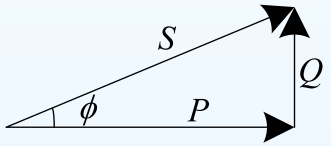

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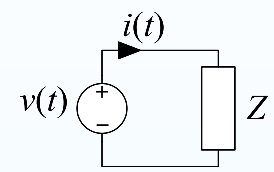

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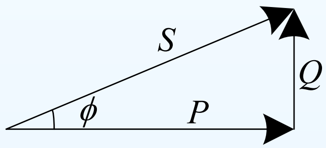

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Power Factor: $\cos\phi \triangleq \cos\left(\angle\widetilde{V} - \angle\widetilde{I}\right) = \frac{P}{|S|}$

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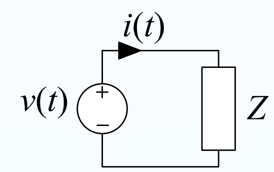

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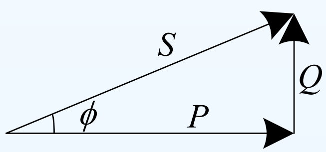

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Machines and transformers have capacity limits and power losses that are independent of $\cos\phi$; their ratings are always given in apparent power.

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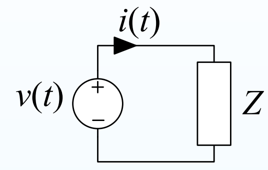

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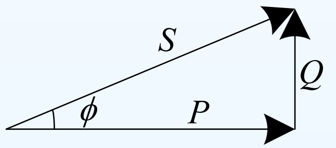

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Power Company: Costs $\propto$ apparent power, Revenue $\propto$ average power.

Power in R, L, C

For any impedance, Z, complex power absorbed: $S = \widetilde{V}\widetilde{I}^* = P + jQ$

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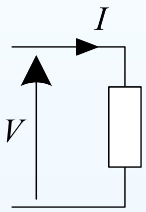

Absorbs average power, no VARs ($Q = 0$)

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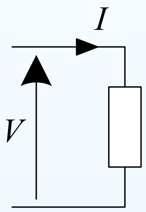

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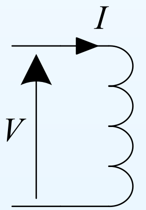

No average power, Absorbs VARs ($Q > 0$)

Power in R, L, C

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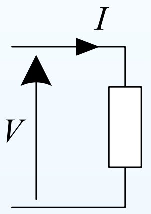

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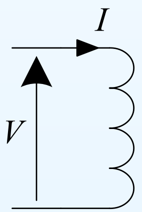
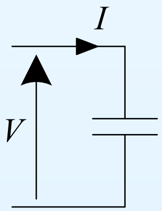


Capacitor: $S = -j\frac{\left|\widetilde{I}\right|^2}{\omega C} = -j\left|\widetilde{V}\right|^2 \omega C \qquad \phi = -90°$

No average power, Generates VARs ($Q < 0$)



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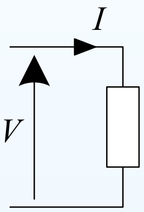

Absorbs average power, no VARs ($Q = 0$)

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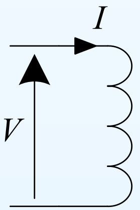

No average power, Absorbs VARs ($Q > 0$)

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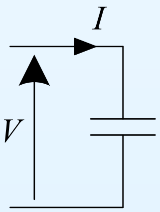

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VARs are generated by capacitors and absorbed by inductors

Power in R, L, C

For any impedance, Z, complex power absorbed: $S = \widetilde{V}\widetilde{I}^* = P + jQ$

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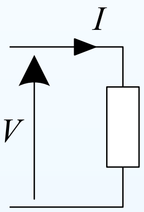

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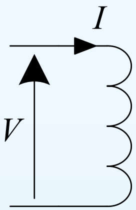

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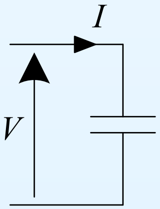


No average power, Generates VARs ($Q < 0$)

VARs are generated by capacitors and absorbed by inductors

The phase, ϕ, of the absorbed power, S, equals the phase of Z

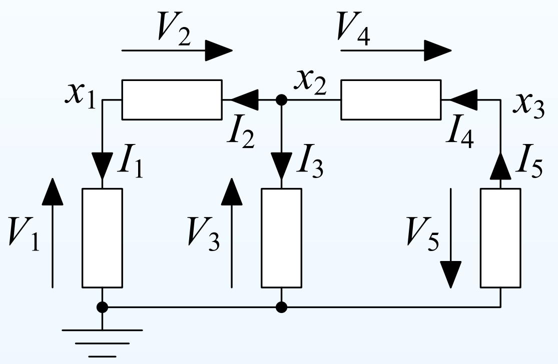
Tellegen's Theorem

Tellegen's Theorem: The complex power, S, dissipated in any circuit's components sums to zero.

$x_n =$ voltage at node n

$V_b, I_b =$ voltage/current in branch b

(obeying passive sign convention)



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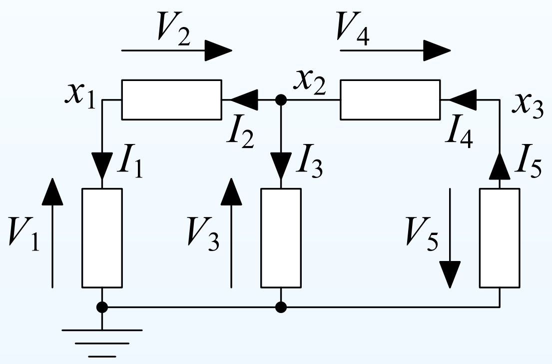
$$a_{bn} \triangleq \begin{cases} -1 & \text{if } V_b \text{ starts from node } n \\ +1 & \text{if } V_b \text{ ends at node } n \\ 0 & \text{else} \end{cases}$$



	a_{bn}	nodes (n) 1	2	3
branches (b)	1	1	0	0
	2	–1	1	0
	3	0	1	0
	4	0	–1	1
	5	0	0	–1

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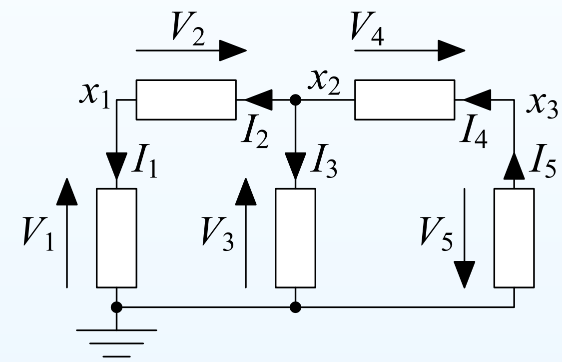

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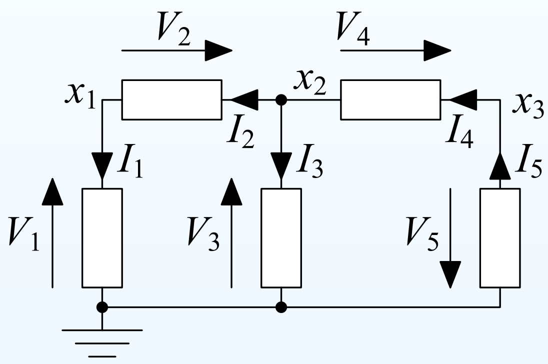

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branches (b) \ nodes (n): a_{bn}	1	2	3
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Branch voltages: $V_b = \sum_n a_{bn} x_n$ (e.g. $V_4 = x_3 - x_2$)

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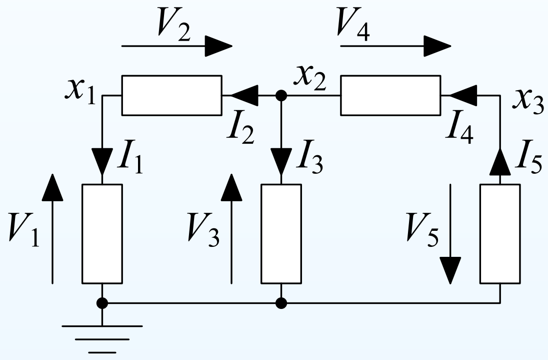

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nodes (n), branches (b)

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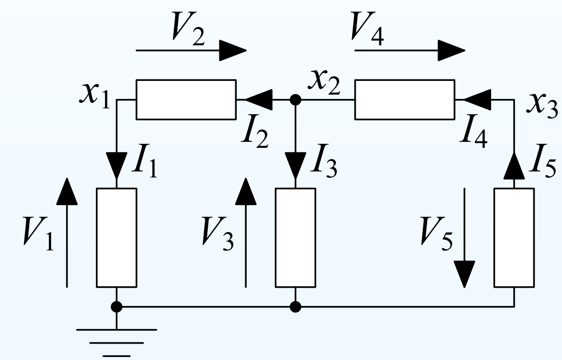

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e.g. branch 4 goes from 2 to 3 $\Rightarrow a_{4*} = [0,\ -1,\ 1]$

a_{bn} (branches (b) \ nodes (n))	1	2	3
1	1	0	0
2	–1	1	0
3	0	1	0
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Tellegen: $\sum_b V_b I_b^* = \sum_b \sum_n a_{bn} x_n I_b^*$

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$V_b,\ I_b$ = voltage/current in branch b
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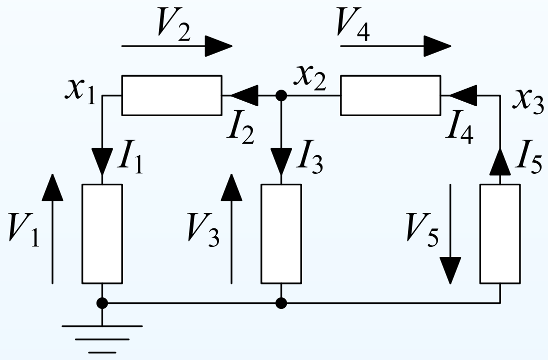

$$a_{bn} \triangleq \begin{cases} -1 & \text{if } V_b \text{ starts from node } n \\ +1 & \text{if } V_b \text{ ends at node } n \\ 0 & \text{else} \end{cases}$$

e.g. branch 4 goes from 2 to 3 $\Rightarrow a_{4*} = [0,\ -1,\ 1]$

a_{bn} (branches (b) \ nodes (n))	1	2	3
1	1	0	0
2	–1	1	0
3	0	1	0
4	0	–1	1
5	0	0	–1

Branch voltages: $V_b = \sum_n a_{bn} x_n$ (e.g. $V_4 = x_3 - x_2$)

KCL @ node n: $\sum_b a_{bn} I_b = 0 \quad \Rightarrow \quad \sum_b a_{bn} I_b^* = 0$

Tellegen: $\sum_b V_b I_b^* = \sum_b \sum_n a_{bn} x_n I_b^*$

$$= \sum_n \sum_b a_{bn} I_b^* x_n$$

Tellegen's Theorem

Tellegen's Theorem: The complex power, S, dissipated in any circuit's components sums to zero.

$$x_n = \text{voltage at node } n$$
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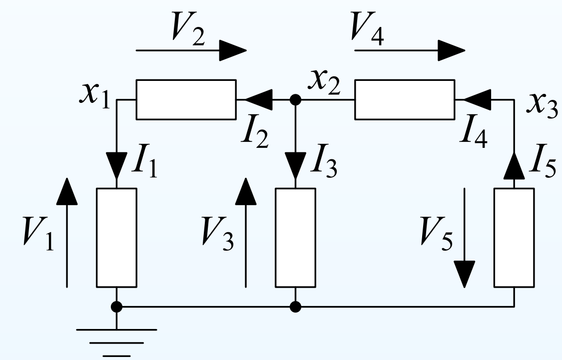

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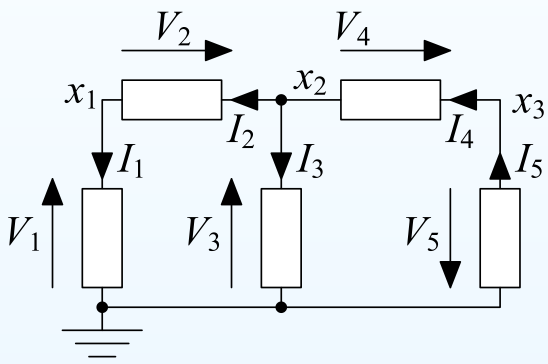

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$$= \sum_n \sum_b a_{bn} I_b^* x_n = \sum_n x_n \sum_b a_{bn} I_b^* = \sum_n x_n \times 0$$

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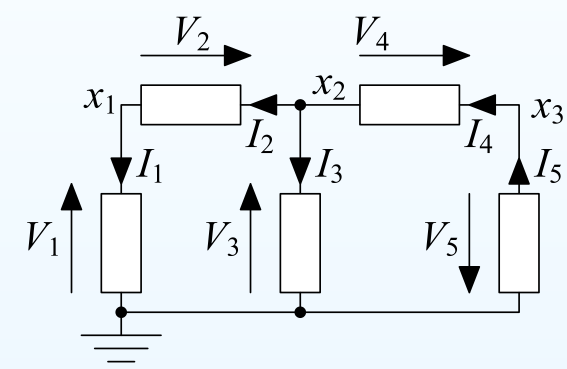

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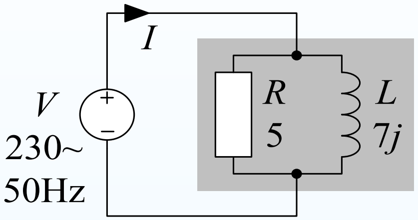
Note: $\sum_b S_b = 0 \quad \Rightarrow \quad \sum_b P_b = 0$ and also $\sum_b Q_b = 0$.

Power Factor Correction

$\widetilde{V} = 230$. Motor modelled as $5||7j\,\Omega$.

$\widetilde{I} = \frac{\widetilde{V}}{R} + \frac{\widetilde{V}}{Z_L}$



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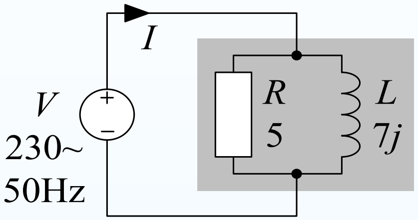
$\widetilde{I} = \frac{\widetilde{V}}{R} + \frac{\widetilde{V}}{Z_L} = 46 - j32.9$ A



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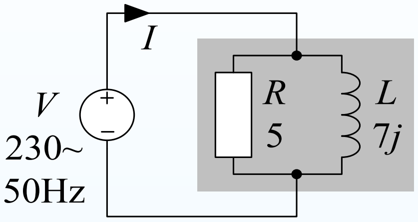
$\widetilde{I} = \frac{\widetilde{V}}{R} + \frac{\widetilde{V}}{Z_L} = 46 - j32.9$ A $= 56.5\angle - 36°$



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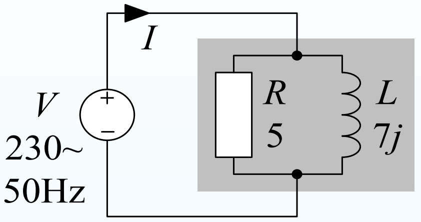

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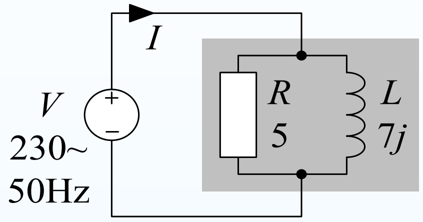

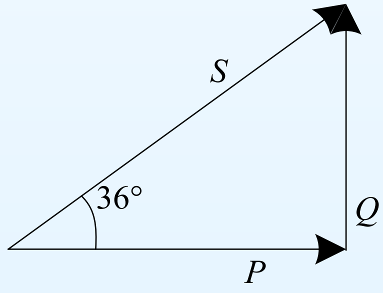

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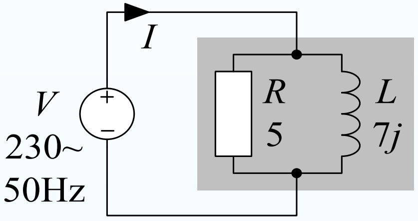

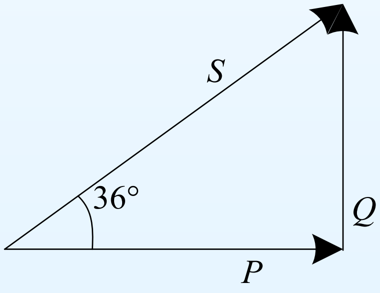

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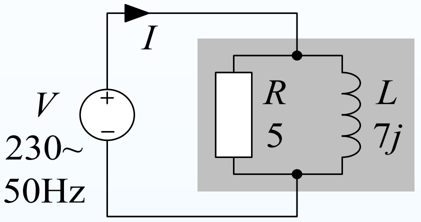
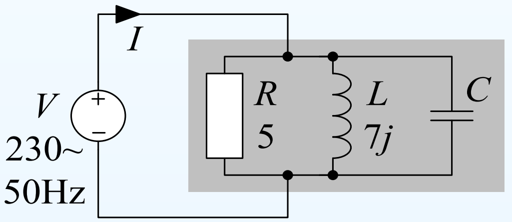
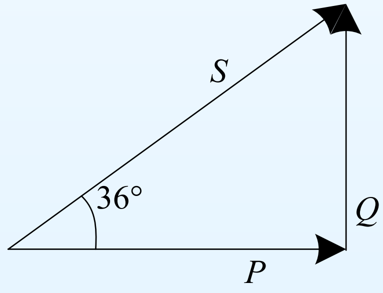


Add parallel capacitor of $300\,\mu$F:





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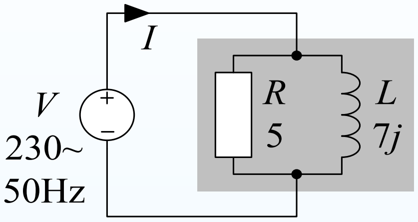

Add parallel capacitor of $300\,\mu$F:

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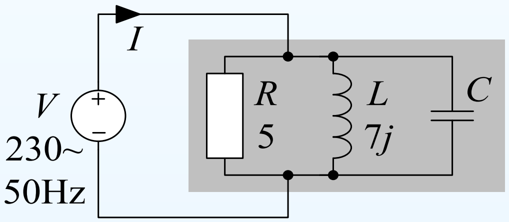

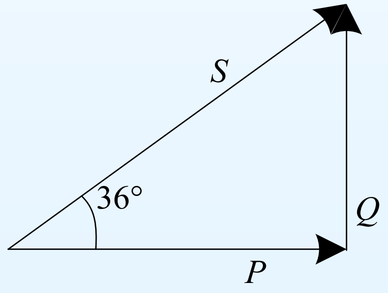

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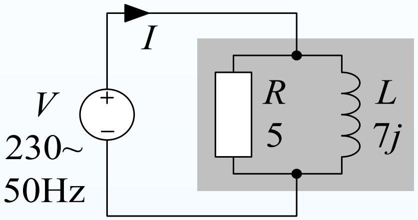

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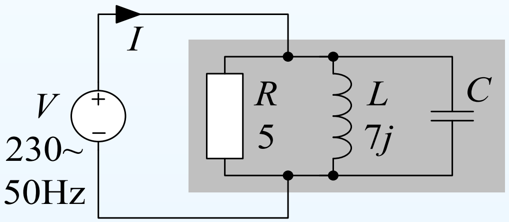

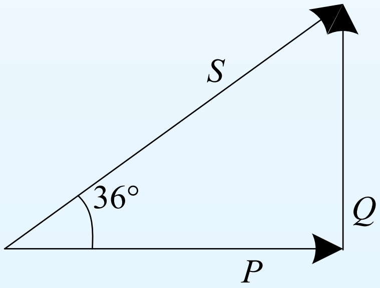

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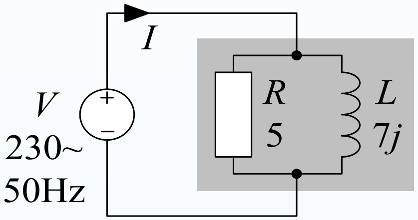
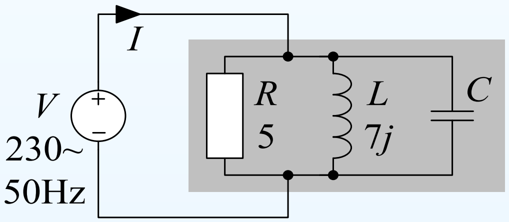
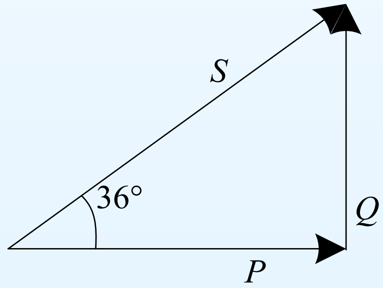


Add parallel capacitor of $300\,\mu$F:

$Z_C = \frac{1}{j\omega C} = -10.6j\,\Omega \Rightarrow \widetilde{I}_C = 21.7j$ A

$\widetilde{I} = 46 - j11.2\,\text{A} = 47\angle - 14°$ A





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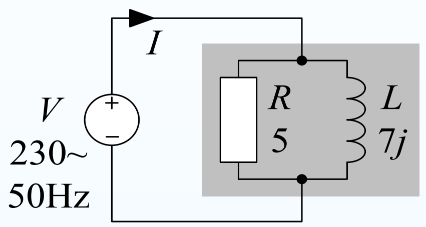

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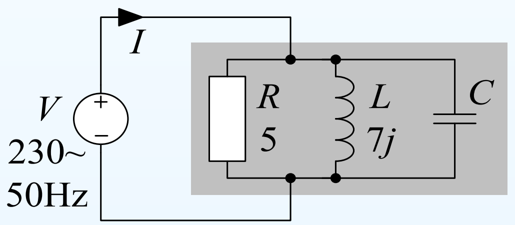
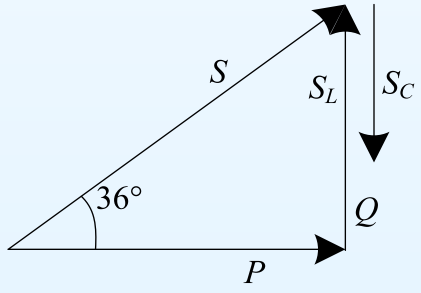


$S_C = \widetilde{V}\widetilde{I}_C^* = -j5\,\text{kVA}$



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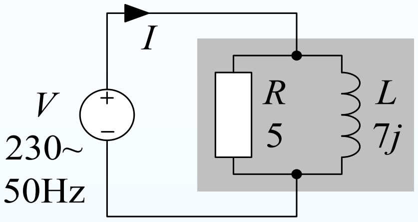

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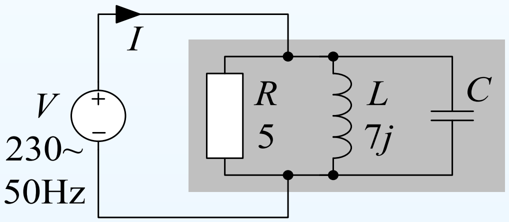

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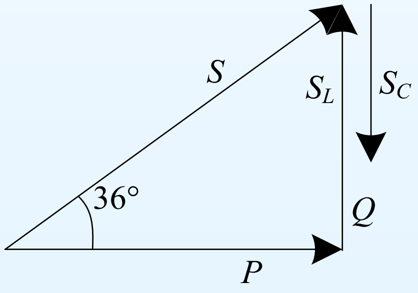

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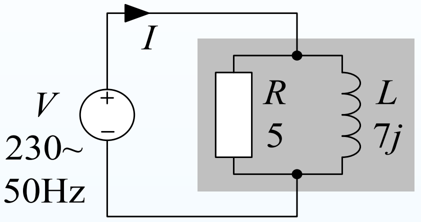

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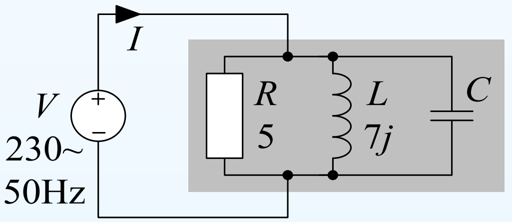

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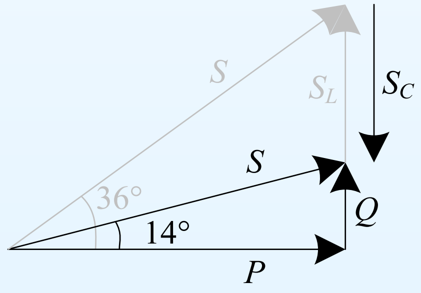

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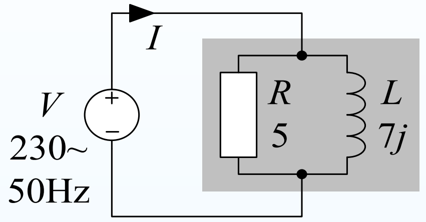

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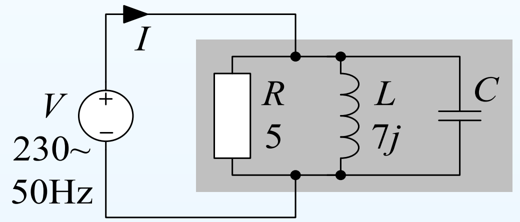

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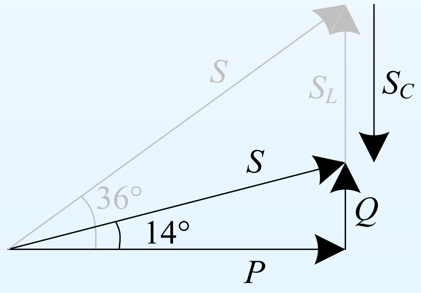
$\cos\phi = \frac{P}{|S|} = \cos 14° = 0.97$



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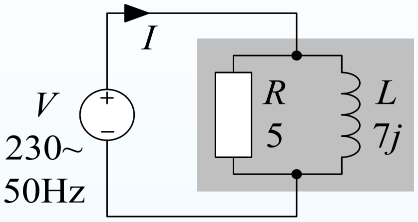

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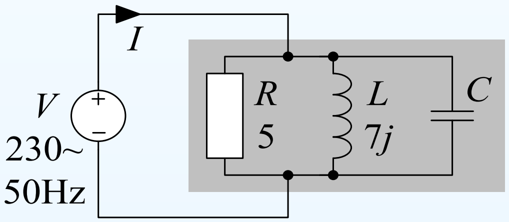

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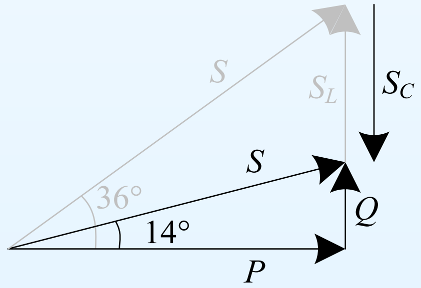

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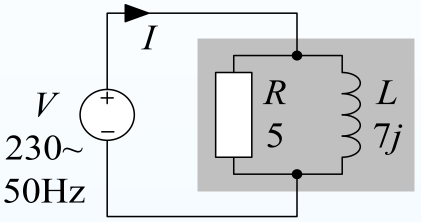

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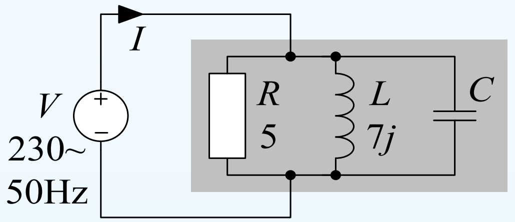

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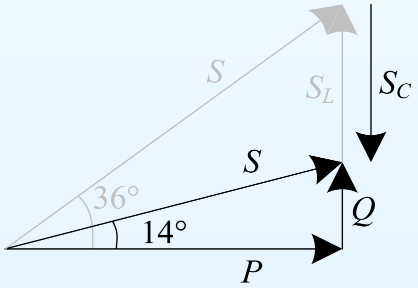

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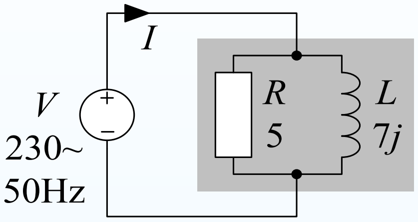

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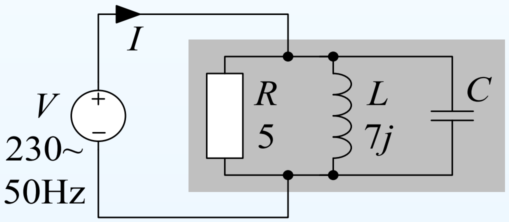

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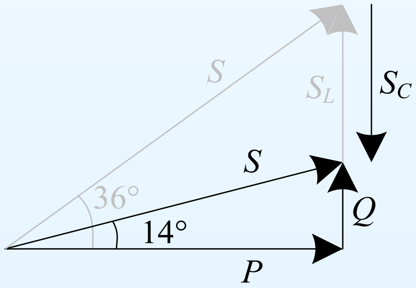

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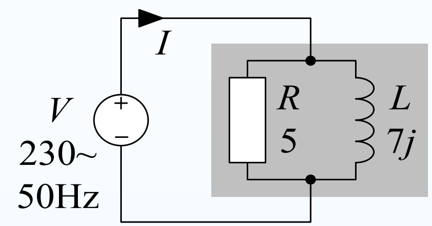

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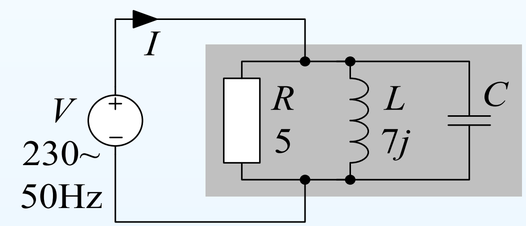

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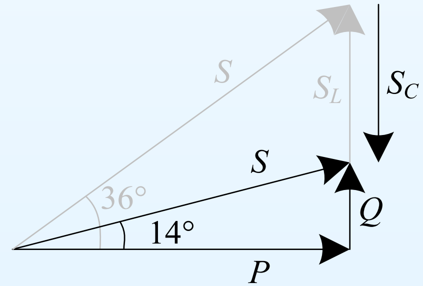

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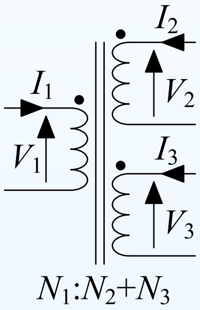

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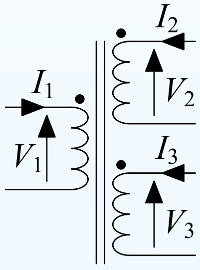

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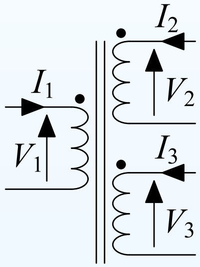

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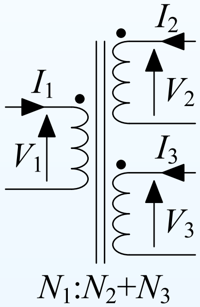

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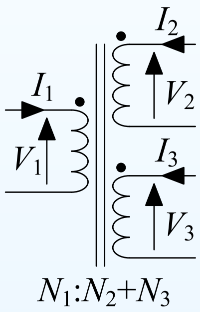

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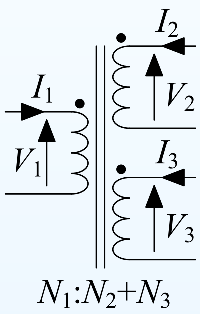

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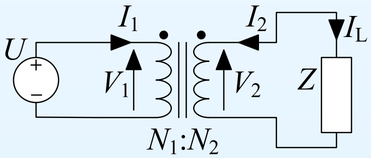

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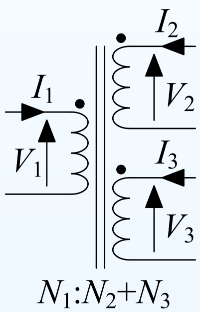

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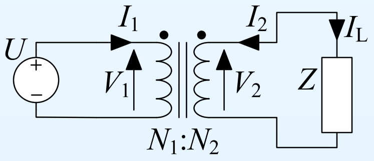

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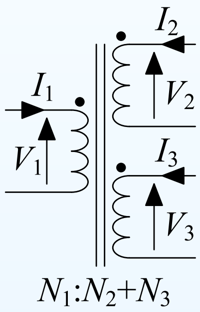

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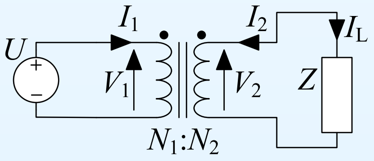

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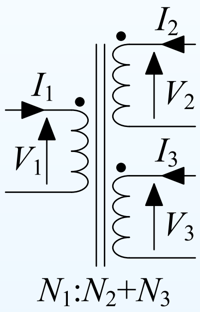

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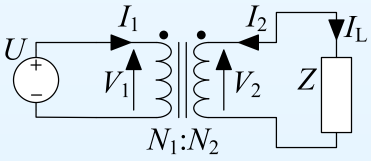

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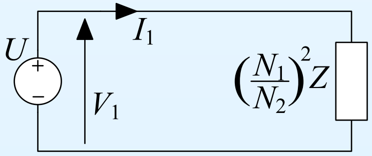

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Equivalent to a *reflected impedance* of $\left(\frac{N_1}{N_2}\right)^2 Z$

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There is no electrical connection between the windings of a transformer so circuitry (or people) on one side will not be endangered by a failure that results in high voltages on the other side.

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Summary

- Complex Power: $S = \widetilde{V}\widetilde{I}^* = P + jQ$ where $\widetilde{V} = V_{rms} = \frac{1}{\sqrt{2}}V$.
  - For an impedance Z: $S = \left|\widetilde{I}\right|^2 Z = \frac{\left|\widetilde{V}\right|^2}{Z^*}$
  - Apparent Power: $|S| = \left|\widetilde{V}\right|\left|\widetilde{I}\right|$ used for machine ratings.
  - Average Power: $P = \Re(S) = \left|\widetilde{V}\right|\left|\widetilde{I}\right|\cos\phi$ (in Watts)
  - Reactive Power: $Q = \Im(S) = \left|\widetilde{V}\right|\left|\widetilde{I}\right|\sin\phi$ (in VARs)
  - Power engineers *always* use $\widetilde{V}$ and $\widetilde{I}$ and omit the ~.
- Tellegen: In any circuit $\sum_b S_b = 0 \Rightarrow \sum_b P_b = \sum_b Q_b = 0$
- Power Factor Correction: add parallel C to generate extra VARs
- Ideal Transformer: $V_i \propto N_i$ and $\sum N_i I_i = 0$ (implies $\sum S_i = 0$)

For further details see Hayt Ch 11 or Irwin Ch 9.