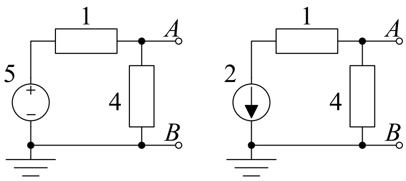
E1.1 Circuit Analysis

Problem Sheet 3 (Lectures 5, 6, 7 & 8)

Key: [A]= easy ... [E]=hard

1. [B] Calculate the Thévenin and Norton equivalent networks at the terminals A and B for each of the following.



(a) (b)

2. [B] Use nodal analysis to calculate an expression for A in Fig. 2 in terms of I and then rearrange this to give I in terms of A. Show how these expressions are related to the Thévenin and Norton equivalent networks at the terminals A and B.

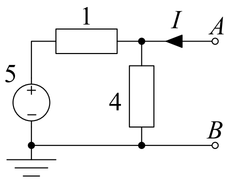


Fig. 2

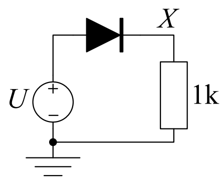


Fig. 3

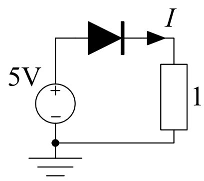
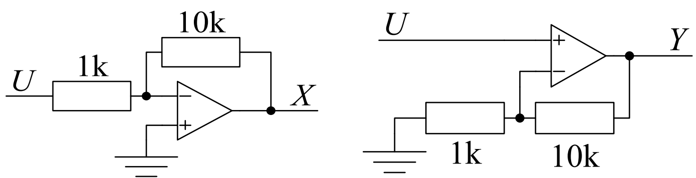


Fig. 4

3. [B] Determine X in Fig. 3 when (a) $U = +5\,\text{V}$ and (b) $U = -5\,\text{V}$. Assume that the diode has a forward voltage drop of 0.7 V.

4. [B] In Fig. 4, calculate I and the power dissipation in the resistor and in the diode. Assume that the diode has a forward voltage drop of 0.7 V.

5. [B] Find the gains $\frac{X}{U}$ and $\frac{Y}{U}$ in the following circuits:



(a) (b)

6. [C] Calculate the Thévenin and Norton equivalent networks at the terminals A and B in Fig. 6 in two ways (a) by combining resistors to simplify the circuit and (b) by using nodal analysis to express A in terms of I.

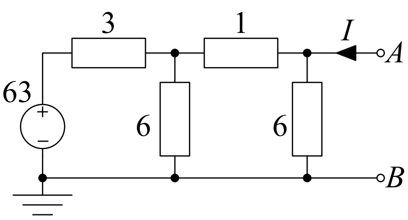


Fig. 6

7. [C] Find the current I in Fig. 7 in two ways: (a) by nodal analysis and (b) by combining the leftmost three components into their Thévenin equivalent.

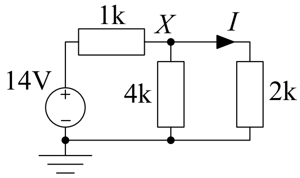


Fig. 7

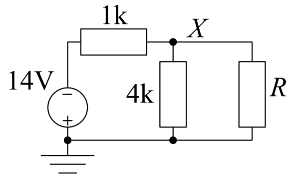
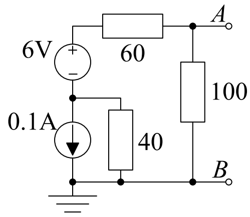


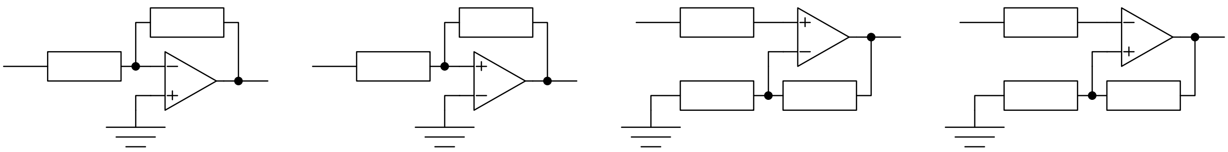
Fig. 8

8. [C] For what value of R in Fig. 8 will the power dissipation in R be maximized. Find the power dissipation in R in this case.

9. [C] Find the Thévenin equivalent of the circuit between nodes A and B in two ways: (a) by performing a sequence of Norton↔Thévenin transformations and (b) using superposition to find the open-circuit voltage and combining resistors to find the Thévenin resistance.

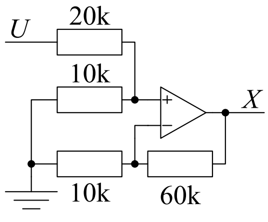


10. [C] State whether the feedback in the following circuits is positive or negative:

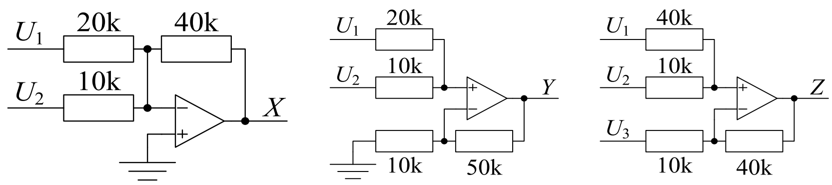


(a) (b) (c) (d)

11. [C] Find the gain $\frac{X}{U}$ in the following circuit:



12. [C] Find expressions for X, Y and Z in terms of U_1, U_2 and U_3.



(a) (b) (c)

13. [C] Choose values of R_1 and R_2 in Fig. 13 so that $X = 2U_2 - 3U_1$.

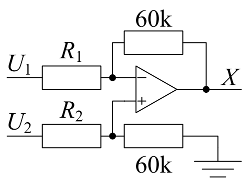


Fig. 13

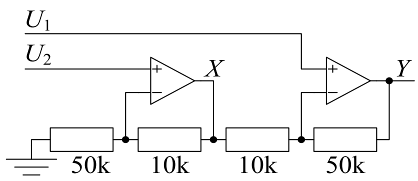
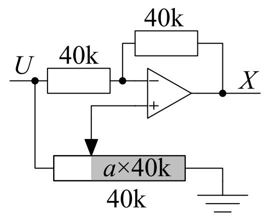


Fig. 14

14. [C] Find an expression for Y in Fig. 14 in terms of U_1 and U_2.

15. [C] In the circuit diagram, the potentiometer resistance between the slider and ground is $a \times 40\,\text{k}\Omega$ where $0 \le a \le 1$. Find the gain of the circuit, $\frac{X}{U}$ as a function of a. What is the range of gains that the circuit can generate as a is varied.



16. [C] By replacing the rightmost three components in Fig. 16 by their Thévenin equivalent, find X when (a) $U = 0\,\text{V}$ and (b) $U = 5\,\text{V}$. Assume that the diode has a forward voltage drop of 0.7 V. Determine the value of U at which the diode switches between operating regions.

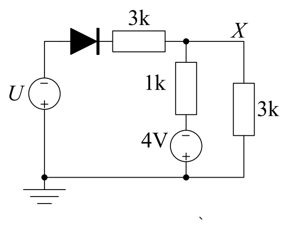


Fig. 16

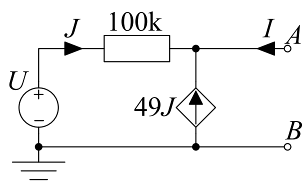


Fig. 18

17. [C] In the block diagram of Fig. 17, F, G, H represent the gains of the blocks. Find the overall gain $\frac{Y}{X}$.

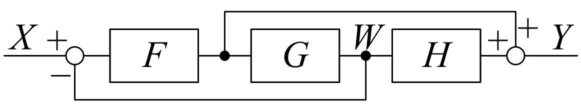


Fig. 17

18. [D] The circuit of Fig. 18 includes a dependent current source whose value is proportional to the current J. Find the Thévenin equivalent of the circuit by two methods: (a) use nodal analysis including the current I and express the voltage V_{AB} as a function of I and (b) assume $I = 0$ and find (i) the open circuit voltage, V_{AB} as a function of U and (ii) the short-circuit current (with A joined to B) as a function of U. Hence find the Thévenin equivalent of the circuit.

19. [D] Choose resistor values in Fig. 19 so that (a) $X = \frac{1}{2}U_3 + \frac{1}{3}U_2 + \frac{1}{6}U_1$ and (b) the Thévenin resistance between X and ground is $50\,\Omega$. Why would the question be impossible if it had asked for $X = \frac{1}{2}U_3 + \frac{1}{3}U_2 + \frac{1}{3}U_1$?

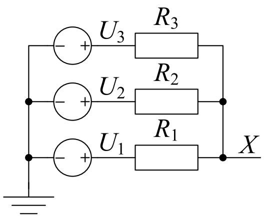


Fig. 19

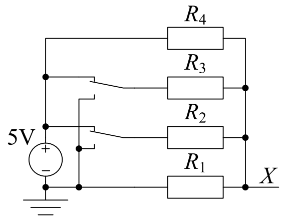
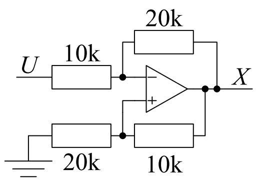


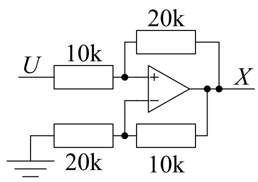
Fig. 20

20. [D] Choose resistor values in Fig. 20 so that (a) the Thévenin between X and ground is $50\,\Omega$ and (b) it is possible to set X to 1, 2, 3 or 4 V by setting the switches appropriately.

21. [D] State whether the feedback in the following circuits is positive or negative:



(a)



(b)

22. [D] Find an expression for Z in Fig. 22 in terms of U_1 and U_2.

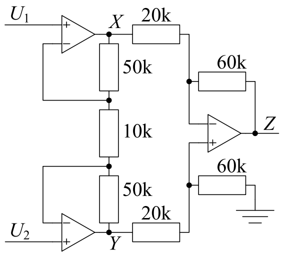


Fig. 22

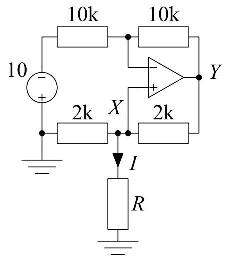


Fig. 23

23. [D] The circuit of Fig. 23 is called a Howland current source. Show that the circuit has negative feedback. Use nodal analysis to determine the current I and show that it does not depend on R.

24. [D] A non-linear device has a characteristic $Y = \sqrt{X}$ for inputs in the range $0 \le X \le 1$. To improve its linearity, the device is placed in a feedback loop using an op-amp with a gain of A as shown in Fig. 24. Determine an expression for Y in terms of U and A. Simplify the expression by using the Taylor series approximation: $\sqrt{v+w} \approx \sqrt{v}\left(1 + \frac{w}{2v} - \frac{w^2}{8v^2}\right)$ valid for $v \gg w$. Estimate how large A must be to ensure that $Y|_{U=0.5} = 0.5 \times Y|_{U=1}$ to within 1% of $Y|_{U=1}$.

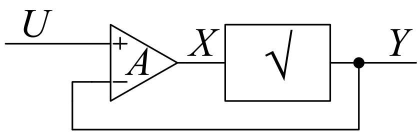


Fig. 24

25. [D] The diodes in Fig. 25 have a characteristic $I = k \exp \frac{V}{V_T}$ where $V_T = 25\,\text{mV}$.

    (a) For the circuit of in Fig. 25(a), find an expression for U in terms of X assuming that $X > 0$.

    (b) For the circuit of in Fig. 25(b), find and expression for Y in terms of X assuming that $X > 0$.

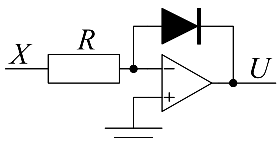


Fig. 25(a)

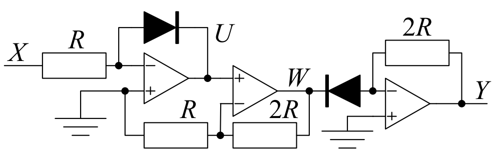
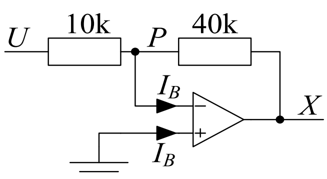
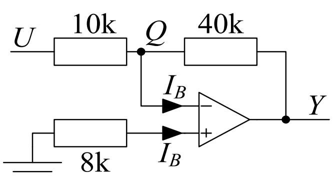


Fig. 25(b)

26. [C] We normally assume that the current at the inputs to an opamp are negligible. However this is not always true and this question investigates the effect of non-zero input bias currents. If $I_B = 100\,\text{nA}$, find expressions for $\frac{X}{U}$ and $\frac{Y}{U}$ in the circuits below. Explain the advantage of circuit (b) and derive a general principal that should be followed when using opamps having non-negligible input bias currents, I_B.



(a)



(b)