E1.1 Circuit Analysis

Problem Sheet 4 (lectures 9 & 10)

Key: [A]= easy ... [E]=hard

Note: A "dimensioned sketch" should show the values on the x and y axes corresonding to significant places on the corresponding graph.

1. [B] For each of the following waveforms, determine the corresponding phasor in both the form $a+jb$ and $r\angle\theta$.
   (a) $8\cos\omega t$.
   (b) $3\cos\omega t + 4\sin\omega t$.
   (c) $2\cos\left(\omega t + \frac{\pi}{4}\right)$.
   (d) $8\sin\omega t$.
   (e) $-2\cos\omega t$.
   (f) $-4\sin\left(\omega t - \frac{\pi}{2}\right)$.
   (g) $8\cos\left(\omega t + \frac{\pi}{4}\right) + 5\sin\left(\omega t - \frac{\pi}{4}\right)$.

2. [B] For each of the following phasors, determine the corresponding waveform in both the form $a\cos\omega t + b\sin\omega t$ and $a\cos(\omega t+\theta)$. (a) 1, (b) -2, (c) $3j$, (d) $-4j$, (e) $j-1$, (f) $3-4j$, (g) $2e^{j\frac{\pi}{2}}$, (h) $4e^{-j\frac{\pi}{6}}$.

3. [B] For each of the following cases say which of the two waveforms or phasors leads the other:
   (a) $\sin\omega t$ and $\cos\omega t$.
   (b) $\sin(\omega t+\pi)$ and $\cos\omega t$.
   (c) $\sin(\omega t-\pi)$ and $\cos\omega t$.
   (d) $(1+j)$ and $(2+j)$.
   (e) $(1+j)$ and $(1-j)$.
   (f) $(-1+j)$ and $(-1-j)$.
   (g) 1 and $1\angle 350°$.

4. [B] Draw a dimensioned sketch of the waveform of i in the circuit of Fig. 4(a) when v has the waveform shown in Fig. 4(b).

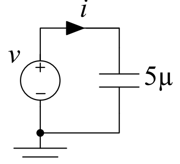

Fig. 4(a)

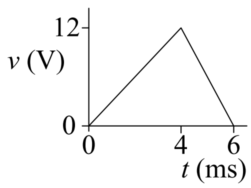

Fig. 4(b)

5. [B] For each of the circuits shown in Fig. 5(a)-(d) determine the average value of $y(t)$ when $x(t) = 4 + 2\cos\omega t$ for some non-zero frequency ω.

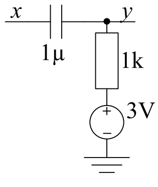

Fig. 5(a)

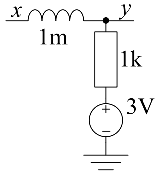

Fig. 5(b)

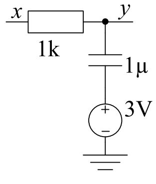

Fig. 5(c)

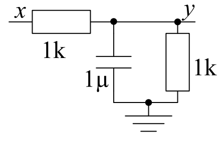

Fig. 5(d)

6. [B] Find the value of a single inductor equivalent to the circuit shown in Fig. 6.

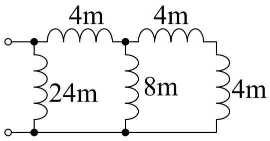

Fig. 6

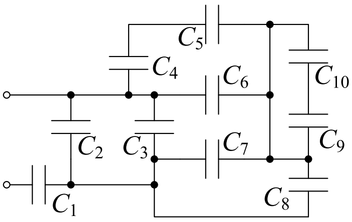

Fig. 7

7. [B] Find the value of a single capacitor equivalent to the circuit shown in Fig. 7 given that each of the capacitors has a value of $1\,\mu$F.

8. [B] Find the average value of v in the circuit of Fig. 8 if $u(t) = 2 + 3\cos\omega t$.

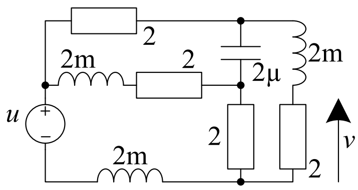

Fig. 8

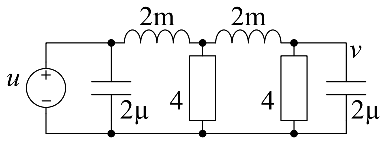

Fig. 9

9. [B] Find the average value of v in the circuit of Fig. 9 if $u(t) = 8 - 2\cos\omega t$.

10. [B] Find the complex impedance of the circuit shown in Fig. 10 for (a) $\omega = 0$, (b) $\omega = 1000$, (c) $\omega = 2000$ and (d) $\omega = \infty$.

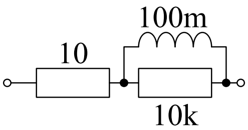

Fig. 10

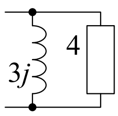

Fig. 11(a)

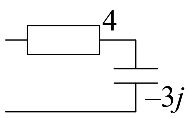

Fig. 11(b)

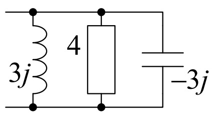

Fig. 11(c)

11. [B] The components in Fig. 11 are labelled with their impedances. Calculate both the complex impedance and the complex admittance for each of the three networks.

12. [B] The components in Fig. 12 are labelled with their impedances. Determine the values of a parallel inductor and resistor that will have the same overall impedance at (a) 1 kHz and (b) 10 kHz. Hint: first calculate the admittance of the original network.

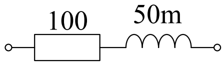

Fig. 12

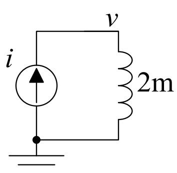

Fig. 13(a)

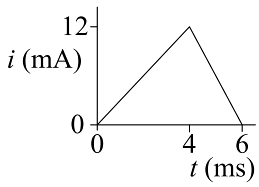

Fig. 13(b)

13. [C] Draw a dimensioned sketch of the waveform of $v(t)$ in the circuit of Fig. 13(a) when i has the waveform shown in Fig. 13(b).

14. [C] The three current i_1, i_2, i_3 in Fig. 14 are equal to $5\cos\left(\omega t + \frac{3\pi}{4}\right)$, $2\cos\left(\omega t + \frac{\pi}{4}\right)$ and $\sqrt{8}\cos\omega t$ but not necessarily in that order. Determine which current is which and find both the phasor I and time-waveform $i(t)$.

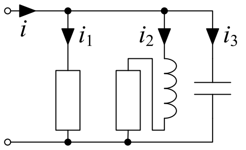

Fig. 14

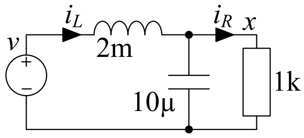

Fig. 17(a)

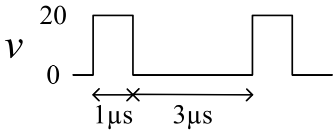

Fig. 17(b)

15. [D] Draw a dimensioned sketch of the waveform of i in the circuit of Fig. 15 when v has the waveform shown in Fig. 4(b) given that at time $t = 0$, (a) $i(0) = 0$ and (b) $i(0) = 2\,\text{A}$.

16. [D] Draw a dimensioned sketch of the waveform of v in the circuit of Fig. 16(a) when i has the waveform shown in Fig. 16(b) given that at time $t = 0$, (a) $v(0) = 0$ and (b) $v(0) = -5\,\text{V}$.

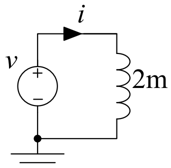

Fig. 15

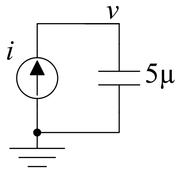

Fig. 16(a)

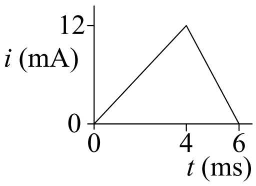

Fig. 16(b)

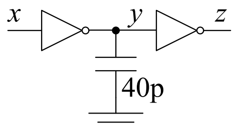

Fig. 18

17. [D] In the circuit of Fig. 17(a), the voltage v has the periodic waveform shown in Fig. 17(b) with a period of $4\,\mu$s and an amplitude of 20 V.
   (a) State the duty cycle of v.
   (b) Determine the average value of x.
   (c) Determine the average value of i_R.
   (d) Determine the average value of i_L.
   (e) Assuming that x is constant (at its average value), draw a dimensioned sketch of the waveform of $i_L(t)$ and determine its maximum and minimum values.
   (f) Assuming that x is constant (at its average value), determine the average, positive peak and negative peak of the powers absorbed by each of R, L and C.

18. [D] In the circuit of Fig. 18, the output logic levels from the inverter are 0 V and 5 V and the inverter has a maximum output current of $\pm 2\,\text{mA}$. The inverter senses a low voltage when its input is $< 1.5\,\text{V}$. If x changes from logic 0 to logic 1, determine the delay until z changes. Ignore the inverter input currents and any delays inside the inverters themselves.