E1.1 Circuit Analysis

Problem Sheet 5 (Lectures 11, 12 & 13)

Key: [A]= easy ... [E]=hard

Note: A "sketch" should show the values on the x and y axes corresonding to significant places on the corresponding graph.

1. [C] For each of the circuits in Fig. 1(i)-(vi),
   (a) Find the transfer function $\frac{Y}{X}(j\omega)$.
   (b) Find expressions for the low and high frequency asymptotes of $H(j\omega)$.
   (c) Sketch the straight line approximation to the magnitude response, $|H(j\omega)|$, indicating the frequency (in rad/s) and the gain of the approximation (in dB) at each of the corner frequencies.

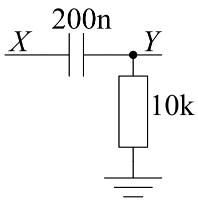

Fig. 1(i)

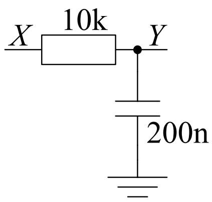

Fig. 1(ii)

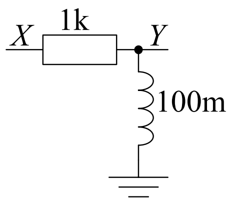

Fig. 1(iii)

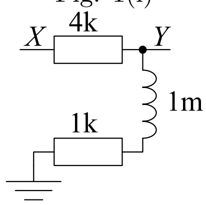

Fig. 1(iv)

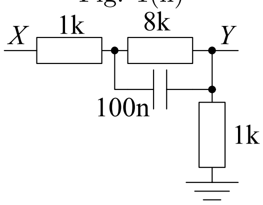

Fig. 1(v)

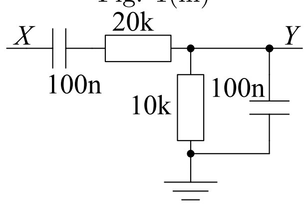

Fig. 1(vi)

2. [C] Sketch a straight line approximation for the phase response, $\angle H(j\omega)$, of the circuit in Fig. 1(v), indicating the frequency (in rad/s) and phase (in rad) at each of the corner frequencies.

3. [B] A "C-weighting filter" in audio engineering has the form $H(j\omega) = \frac{k(j\omega)^2}{(j\omega+a)^2(j\omega+b)^2}$ where $a = 129$ and $b = 76655$ rad/s. Calculate k exactly so that $|H(j\omega)| = 1$ at $\omega = 2000\pi$ rad/s.

4. [C] Design circuits with each of the magnitude responses given in Fig. 4(i)-(iii) using, in each case, a single capacitor and appropriate resistors.

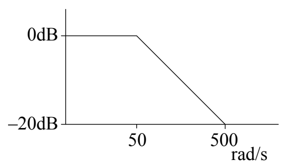

Fig. 4(i)

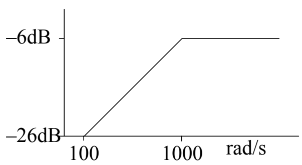

Fig. 4(ii)

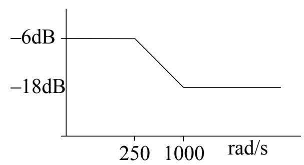

Fig. 4

5. [B] Express each of the following transfer functions in a standard form in which the numerator and denominator are factorized into linear and quadratic factors of the form $(j\omega+p)$ and $\left((j\omega)^2 + qj\omega + r^2\right)$ where p, q and r are real with an additional numerator factor of the form $A(j\omega)^k$ if required. Quadratic terms should be factorized if possible.
   (a) $\frac{-2\omega^2-2j\omega^3}{1-2\omega^2+\omega^4}$
   (b) $\frac{-2\left(1+\omega^2\right)}{(1-\omega^2)+2j\omega}$
   (c) $\frac{10(j\omega)^2+2j\omega+10}{(j\omega)^2+2j\omega+1}$
   (d) $\frac{1}{j\omega+6(j\omega)^{-1}+5}$

6. [B] Without doing any algebra, determine the low and high frequency asymptotes of the following transfer functions:
   (a) $\frac{-2\omega^2-2j\omega^3}{1+\omega^4}$
   (b) $\frac{2(j\omega)^3+3}{4(j\omega)^4+1}$
   (c) $\frac{j\omega\left(2(j\omega)^6+3\right)\left(5(j\omega)^3+4j\omega+3\right)}{2\left((j\omega)^5+1\right)\left((j\omega)^5+5\right)}$
   (d) $\frac{12}{j\omega+6(j\omega)^{-1}}$

7. [D] A circuit has a transfer function whose low and high frequency asymptotes are $A\,(j\omega)^{\alpha}$ and $B\,(j\omega)^{\beta}$ respectively. What constraints can you place on α and β if you know that the magnitude of the transfer function is less than G at all frequencies where G is a fixed real-valued constant.

8. [C] For each of the following transfer functions, sketch the straight line approximation to the magnitude response and determine the gain of this approximation at $\omega = 1000$ rad/s: (a) $\frac{5(1+j\omega/500)}{(1+j\omega/100)(1+j\omega/2000)}$, (b) $\frac{2(1+j\omega/5000)}{(1+j\omega/100)}$, (c) $\frac{3j\omega(1+j\omega/500)}{(1+j\omega/100)(1+j\omega/2000)(1+j\omega/5000)}$.

9. [C] The frequency response of the circuit in Fig. 9 is given by $\frac{\left(\frac{j\omega}{p}\right)^2}{\left(\frac{j\omega}{p}\right)^2+2\zeta\left(\frac{j\omega}{p}\right)+1}$ where the corner frquency, $p = \frac{1}{\zeta RC}$ rad/s. Using capacitors of value $C = 10\,\text{nF}$, design a filter with $\zeta = \sqrt{0.5}$ and a corner frequency of $\frac{p}{2\pi} = 1$ kHz. Determine the value of the transfer function at $\frac{\omega}{2\pi} = 100\,\text{Hz}$, 1 kHz and 10 kHz.

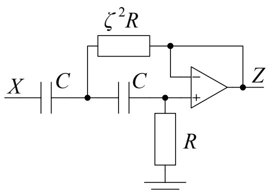

Fig. 9

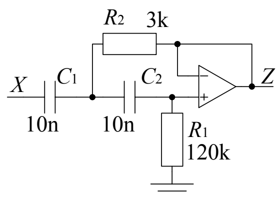

Fig. 11(a)

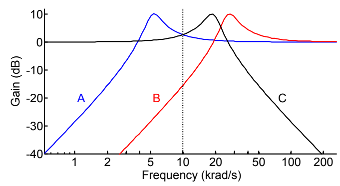

Fig. 11(b)

10. [D] A high-pass "Butterworth" filter of order $2N$ consists of N cascaded copies of Fig. 9 all having the same corner frequency, but with the k^{th} stage having $\zeta_k = \cos\left(\left(\frac{2k-1}{4N}\right)\pi\right)$ where $k = 1, \ldots, N$. ("cascaded" means you connect the output of one stage to the input of the next). Design a 4th order high-pass Butterworth filter with a corner frequency of 1 kHz. Write an expression for its transfer function and sketch its magnitude response using just the high and low asymptotes. Butterworth filters are widely used because they have a very smooth magnitude response without any peaks; the transfer function satisfies $|H(j\omega)|^2 = \frac{\left(\frac{\omega}{p}\right)^{4N}}{\left(\frac{\omega}{p}\right)^{4N}+1}$.

11. [C] The circuit of Fig. 11(a) is a high-pass filter whose magnitude response is marked "A" in Fig. 11(b) . Using the filter transformations described in lectures, design filters with the magnitude responses marked "B" and "C" on the graph. Relative to "A", these are respectively shifted up in frequency by a factor of 5 and reflected in the axis $\omega = 10,000$.

12. [C] (a) Find the resonant frequency, ω_r, at which the impedance of the network in Fig. 12(i) is real. (b) Determine the Q of the circuit at ω_r. (c) Find R_P and L_P in the circuit of Fig. 12(ii) so that the two networks have the same impedance at ω_r.

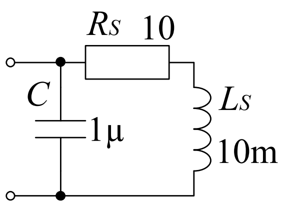

Fig. 12(i)

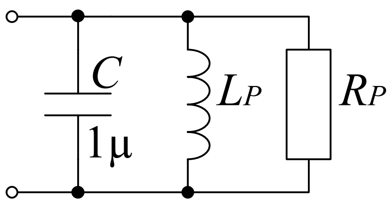

Fig. 12(ii)

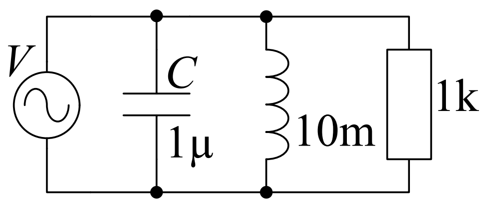

Fig. 13

13. [C] In the circuit of Fig. 13, $\omega = 10\,000$ and the phasor $V = 10$. Find (a) the peak power supplied by V and (b) the peak power absorbed by C.

14. [C] Determine the transfer function for each of the circuits Fig. 14(i)-(iv).

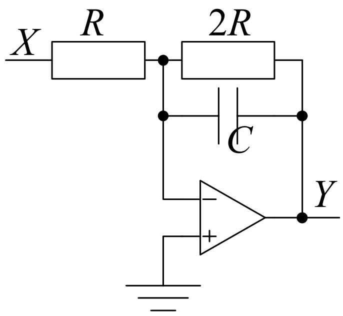

Fig. 14(i)

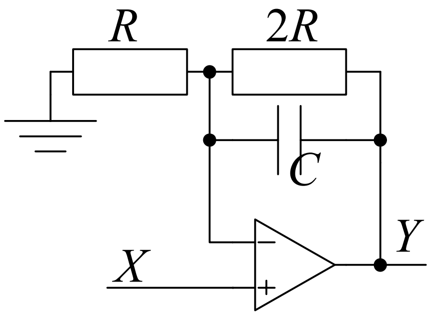

Fig. 14(ii)

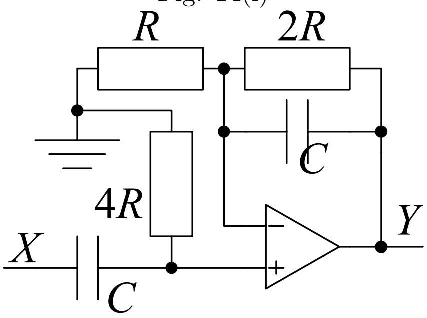

Fig. 14(iii)

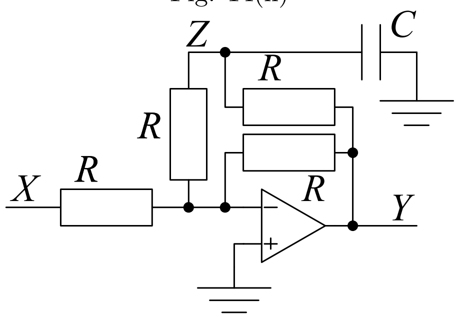

Fig. 14(iv)

15. [D] For the circuit in Fig. 15,
    (a) Find the transfer function $\frac{Z}{W}(j\omega)$ and explain why this is equal to $\frac{Y}{W}(j\omega)$
    (b) Hence, by applying KCL at node W and using part (a) to substitute for W, show that the transfer function $\frac{Y}{X}(j\omega) = \frac{1}{R_1R_2C_1C_2(j\omega)^2+(R_1+R_2)C_1j\omega+1}$.
    (c) From the transfer function expression we can express the corner frequency and damping factor as $p^2 = \frac{1}{R_1R_2C_1C_2}$ and $\zeta = \frac{p(R_1+R_2)C_1}{2}$. By eliminating p between these equations, show that $\left(1+\frac{R_2}{R_1}\right)\left(1+\sqrt{1-\frac{C_1}{\zeta^2C_2}}\right) = 2$. Explain why this means that we must choose $C_1 \le \zeta^2C_2$.
    (d) Assuming that you only have available capacitors of values 10 nF, 22 nF and 47 nF, design a filter with $p = 1000 \times 2\pi$ and $\zeta = 0.5$. Choose C_1 and C_2 first. then $\frac{R_2}{R_1}$ and lastly R_1.

16. [D] For the circuit of Fig. 16,
    (a) Find the transfer function $\frac{Y}{X}(j\omega)$.
    (b) Find the frequency, ω_0, at which $\left|\frac{Y}{X}(j\omega)\right|$ is maximum and its value at this maximum.
    (c) Find the 3dB bandwidth of the circuit and the value of $Q = \frac{1}{2\zeta}$.

    [Note: If you find yourself doing loads of algebra, you are using the wrong method.]

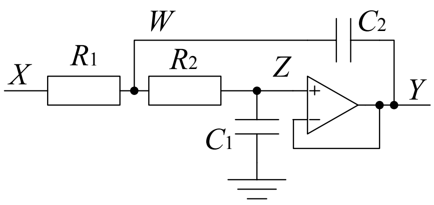

Fig. 15

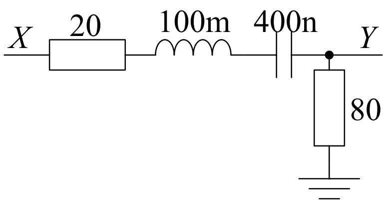

Fig. 16