E1.1 Circuit Analysis

Problem Sheet 6 (Lectures 14, 15 & 16)

Key: [A]= easy ... [E]=hard

Note: A tilde-superscript on a phasor denotes division by $\sqrt{2}$, i.e. $\widetilde{V} = \frac{1}{\sqrt{2}}V$. This means that $\left|\widetilde{V}\right|$ equals the RMS value of a phasor V.

1. [A] Say which of the following waveforms include negative exponentials and which include positive exponentials: (a) $2 - 4e^{-3t}$, (b) $2 + 4e^{3t}$, (c) $2 + 4e^{-3t}$, (d) $-2 - 4e^{3t}$, (e) $2 + 4e^{-t/-3}$.

2. [B] Suppose $v(t) = 5 + 2e^{-100t}$.
   (a) Determine the time constant, τ, of the negative exponential.
   (b) Determine the time at which $v(t) = 5.5\,\text{V}$.
   (c) Give an expression for the time taken for $v(t)$ to fall from A to B where $5 < B < A < 7$.

3. [B] If $V = -200j$ in Fig. 3, find the phasor value of I and the complex power absorbed by each of the components including the voltage source.

4. [B] If $v(t) = \begin{cases} 0 & t < 0 \\ 5 & t \geq 0 \end{cases}$ in Fig. 4 (below),
   (a) find an expression for $x(t)$ for $t \geq 0$.
   (b) Sketch a graph of $x(t)$ for $-RC \leq t \leq 3RC$.
   (c) Determine the time at which $x(t) = 4.5$.

5. [C] For each of the circuits shown in Fig. 5(i)-(vi) determine (a) the time constant (b) the DC gain $\left.\frac{Y}{X}\right|_{\omega=0}$ and (c) the high frequency gain $\left.\frac{Y}{X}\right|_{\omega=\infty}$. In each case, determine these in two ways: directly from the circuit and via the transfer function.

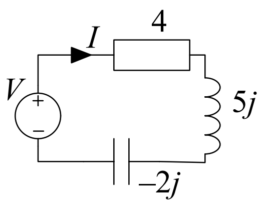

Fig. 3

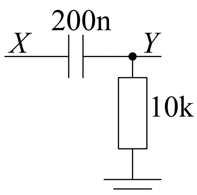

Fig. 5(i)

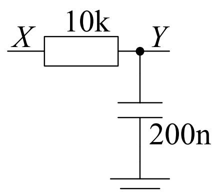

Fig. 5(ii)

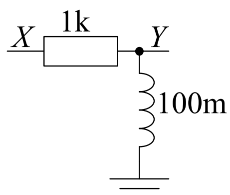

Fig. 5(iii)

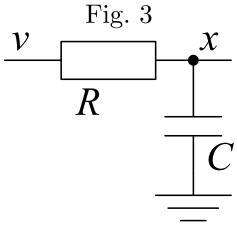

Fig. 4

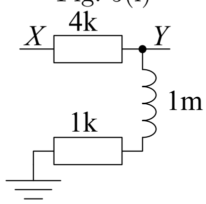

Fig. 5(iv)

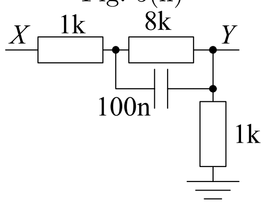

Fig. 5(v)

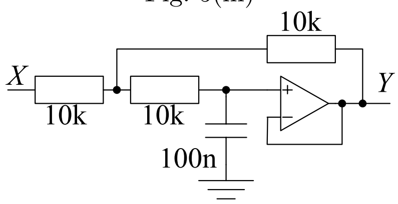

Fig. 5(vi)

6. [C] For each of the periodic waveforms shown in Fig. 6(i)-(iii) determine (a) the mean value and (b) the rms value.

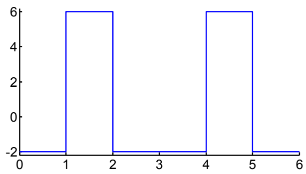
Fig. 6(i)

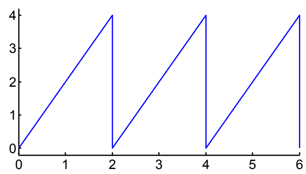
Fig. 6(ii)

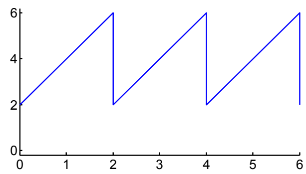
Fig. 6(iii)

7. [C] If $v(t) = \begin{cases} 0 & t < 0 \\ 5 & t \geq 0 \end{cases}$ in Fig. 7, determine an expression for $x(t)$ for $t \geq 0$ and sketch its graph.

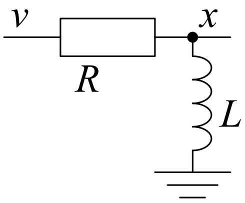

Fig. 7

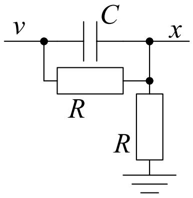

Fig. 8

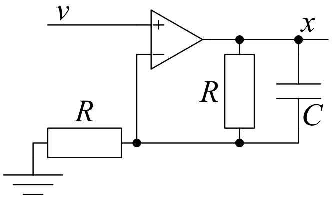

Fig. 9

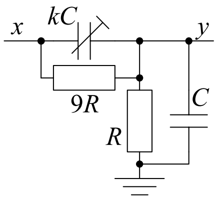

Fig. 10

8. [C] If $v(t) = \begin{cases} 2 & t < 0 \\ 6 & t \geq 0 \end{cases}$ in Fig. 8, determine an expression for $x(t)$ for $t \geq 0$ and sketch its graph.

9. [C] If $v(t) = \begin{cases} 4 & t < 0 \\ 1 & t \geq 0 \end{cases}$ in Fig. 9, find an expression for $x(t)$ for $t \geq 0$ and sketch a graph of $x(t)$ for the time interval $-RC \leq t \leq 3RC$.

10. [C] Fig. 10 shows a simplified circuit diagram for an oscilloscope probe which includes an adjustable capacitor of value kC.
    (a) Determine the transfer function, $\frac{Y}{X}(j\omega)$ and determine its value at $\omega = 0$ and $\omega = \infty$.
    (b) The variable capacitance, kC, is adjusted to the value that results in the same magnitude gain at $\omega = 0$ and $\omega = \infty$. Determine the value of k that achieves this.
    (c) Simplify the expression for $\frac{Y}{X}(j\omega)$ when k has the value calculated in the previous part.

11. [C] In the diagram of Fig. 11 power is being transmitted from a source to a load via two transformers having turns ratios of $1 : n$ and $n : 1$ respectively.
    (a) If $\widetilde{V}_L = 240\,\text{V}$ and the average power dissipated in R_L is $10\,\text{kW}$, calculate the value of R_L.
    (b) If $R_S = 0.5\,\Omega$, calculate the power dissipated in R_S when (i) $n = 1$ and (ii) $n = 5$.

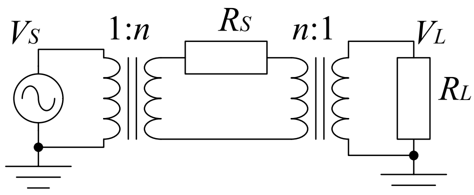

Fig. 11

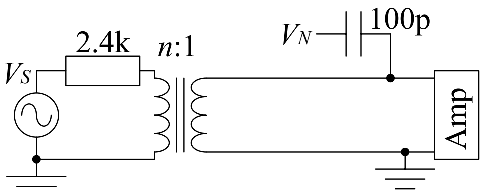

Fig. 12

12. [C] The circuit in Fig. 12 represents a microphone connected to an amplifier via a transformer and a long cable.
    (a) Determine the Thévenin output impedance of the microphone+transformer combination when $n = 4$.
    (b) The cable is subject to 50 Hz interference capacitively coupled from the mains, $\widetilde{V}_N = 230\,\text{V}$, via a capacitor of value 100 pF. If the RMS microphone signal amplitude is $\widetilde{V}_S = 1\,\text{V}$, calculate the ratio of the signal and the noise at the amplifier in dB if (i) $n = 1$ and (ii) $n = 4$.

13. [C] In the circuit of Fig. 13, the transformer may be assumed to be ideal.
    (a) Calculate the average power dissipated in each of R_1 and R_2 if $\widetilde{V}_s = 1$, $n_1 = 2$, $n_2 = 3$, $R_1 = 10$ and $R_2 = 20$.
    (b) Calculate, in terms of n_1, n_2, R_1 and R_2, the effective resistance seen by the voltage source.

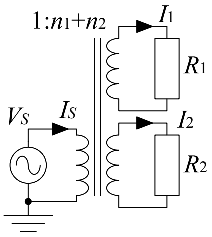

Fig. 13

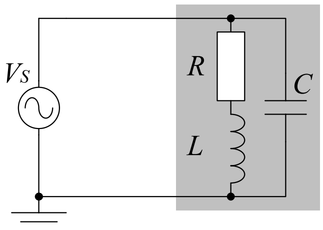

Fig. 14

14. [C] In the circuit of Fig. 14, $\widetilde{V}_S = 230$ at 50 Hz, $L = 8$ mH and $R = 1.6\,\Omega$.
    (a) If $C = 0$ (i.e. the capacitor is omitted), calculate the apparent power, average power and reactive power absorbed by the load (shaded region) and also its power factor.
    (b) Determine the value of C needed to increase the power factor to 0.9. Using this value, recalculate the quantities from part (a).

15. [D] Calculate the waveform $y(t)$ in Fig. 15(i) when,
    (a) $v(t) = \begin{cases} 0 & t \le 0 \\ 5\sin 2000\pi t & t > 0 \end{cases}$ as shown in Fig. 15(ii).
    (b) $v(t) = \begin{cases} 0 & t \le 0 \\ 5\sin 2000\pi t & 0 < t \le 1\,\text{ms} \\ 0 & t > 1\,\text{ms} \end{cases}$ as shown in Fig. 15(iii)

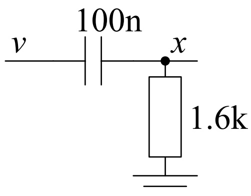

Fig. 15(i)

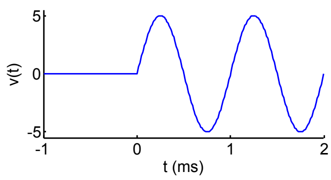

Fig. 15(ii)

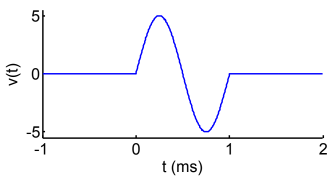

Fig. 15(iii)

16. [D] If the switch in Fig. 16 is $\begin{cases} \text{open} & t < 0 \\ \text{closed} & 0 \le t < 2\,\text{ms} \\ \text{open} & t \ge 2\,\text{ms} \end{cases}$, determine expressions for $i(t)$ for each of these periods and sketch graphs of $i(t)$ and $v(t)$ for $-1\,\text{ms} \le t \le 4\,\text{ms}$.

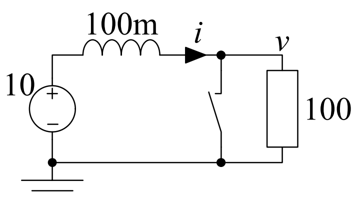

Fig. 16

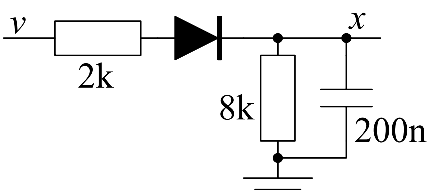

Fig. 17(i)

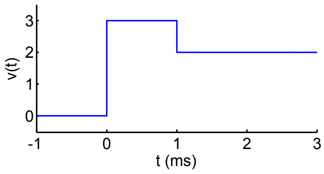

Fig. 17(ii)

17. [E] In Fig. 17(i), $v(t) = \begin{cases} 0 & t < 0 \\ 3 & 0 \le t < 1\,\text{ms} \\ 2 & t \ge 1\,\text{ms} \end{cases}$ as shown in Fig. 17(ii). If the diode has a forward voltage drop of 0.7 V, find expressions for $x(t)$ for $t \ge 0$.