E1.1 Circuit Analysis

Problem Sheet 7 (Lectures 17 & 18)

Key: [A]= easy ... [E]=hard

Note: In this problem sheet u and Z_0 are the propagation velocity and characteristic impedance of a transmission line and the forward and backward waves at the point x are $f_x(t) = f(t - \frac{x}{u})$ and $g_x(t) = g(t + \frac{x}{u})$ with, in the case of sinusoidal waves, the corresponding phasors being F_x and G_x.

1. [A] Find the propagation velocity, u, and characteristic impedance, Z_0, of a transmission line whose capacitance and inductance are 50 pf/m and 500 nH/m respectively. Express the propagation velocity also as a fraction of the speed of light.

2. [B] The line in Fig. 2 has $Z_0 = 100\,\Omega$. For each of the cases below, calculate the reflection coefficients at both ends of the line and describe the waves that would arise from a short positive pulse at V_S.
   (a) $R_S = 10$ and $R_L = 100$.
   (b) $R_S = 10$ and $R_L = 1000$.
   (c) $R_S = 100$ and $R_L = 1000$.

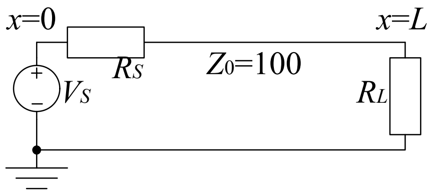

Fig. 2

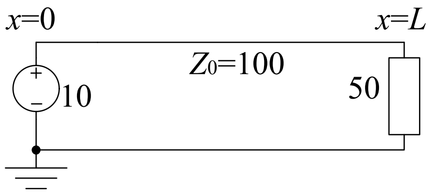

Fig. 3

3. [C] The line in Fig. 3 is driven by a 10 V DC voltage source. Determine the voltage and current in the line and hence the forward and backward waves $f(t - \frac{x}{u})$ and $g(t + \frac{x}{u})$. Determine also the power carried by the two waves and verify that their differnce equals the total power delivered to the load.

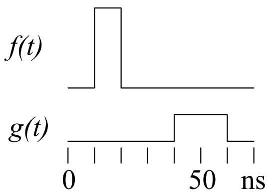

Fig. 4

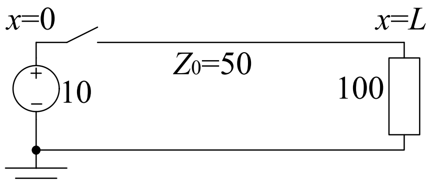

Fig. 5

4. [C] A transmission line has a propagation velocity of 15 cm/ns and a characteristic impedance of $100\,\Omega$. The forward and backward waves are shown in Fig. 4 and have amplitudes of 9 V and 3 V respectively. Draw dimensioned sketches of the voltage and current waveforms at (a) $x = 0$ and (b) $x = 300$ cm. In each case, give the value of the peak voltage and peak current.

5. [C] The transmission line shown in Fig. 5 has a propagation velocity of 15 cm/ns and a characteristic impedance of $50\,\Omega$. The length of the line is $L = 300$ cm.
   (a) Determine the reflection coefficients at both ends of the line when the switch is held closed.
   (b) Calculate the steady state DC forward and backward waves when the switch has been closed for a long time.
   (c) If the switch is closed at time $t = 0$, determine the forward and backward waves at $x = 0$. Hence determine the voltage waveforms at $x = 0$ and $x = L$.

6. [C] A length of transmission line with $Z_0 = 100$ and $u = 20$ cm/ns is terminated in a short circuit at $x = L$. Find the shortest lengths of line, L, for which the impedance at 20 MHz at $x = 0$ will equal (a) 50 pF and (b) 1 μH.

7. [C] In Fig. 7, $L = 5$ m, $u = 20$ cm/ns, $Z_0 = 100$, $R_L = 50$ and the frequency of operation is 50 MHz.
   (a) If the forward wave phasor at $x = 0$ is $F_0 = 6j$, determine the forward wave phasors, F_x, at $x = 1$, 2, 3, 4 and 5 metres.
   (b) Calculate the reflection coefficient at $x = L$.
   (c) Determine the backward wave phasors, G_x, at $x = 0$, 1, 2, 3, 4 and 5.
   (d) Determine the line voltage phasors, V_x, at $x = 0$, 1, 2, 3, 4 and 5.
   (e) Determine the Voltage Standing Wave Ratio: $VSWR = \frac{\max(|V_x|)}{\min(|V_x|)}$.
   (f) Determine the line impedance, $\frac{V_0}{I_0}$, at $x = 0$.

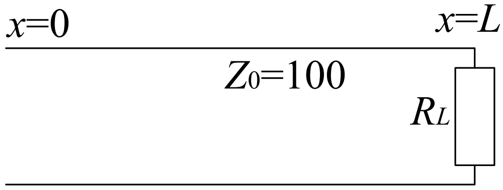

Fig. 7

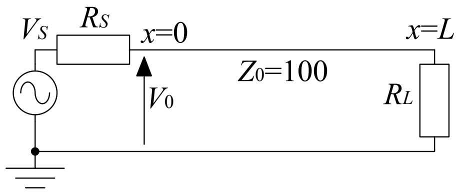

Fig. 9

8. [C] Repeat question 7 for $R_L = 100$.

9. [C] In Fig. 9, $L = 1$ m, $u = 15$ cm/ns, $Z_0 = 100$, $R_S = 10$, $R_L = 150$ and the frequency of operation is 20 MHz.
   (a) Calculate the reflection coefficient, ρ_L at $x = L$. Hence calculate the phasor ratio $\frac{G_0}{F_0}$ at $x = 0$.
   (b) Calculate the line impedance $\frac{V_0}{I_0}$ at $x = 0$.
   (c) By treating the circuit at the source end as a potential divider, calculate V_0 if $V_S = 10$.
   (d) Calculate F_0 and hence calculate F_L, G_L and the load voltage, V_L.
   (e) Calculate the complex power supplied by the source.