ANALYSIS OF CIRCUITS

Wednesday, 6 June 10:00 am
Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Examiners responsible

First Marker(s): D.G. Haigh, D.G. Haigh
Second Marker(s): P.D. Mitcheson, P.D. Mitcheson
a) Use source transformations to derive a simplified equivalent circuit for the sub-circuit in Figure 1 (a) to the left of the terminals $x, x'$. Hence, determine the current $i$ when the sub-circuit is connected to the $2 \ Ohm$ resistor.

![Figure 1(a)](image)

b) Use the principle of superposition to find current $i$ in the circuit in Figure 1(b):

![Figure 1(b)](image)

c) Find the Norton equivalent circuit (current source with resistor in parallel with it) for the sub-circuit in Figure 1(c) to the left of the terminals $y, y'$. What is the voltage across the $3 \ Ohm$ resistor when it is connected to the sub-circuit?

![Figure 1(c)](image)

d) Use nodal analysis to determine the nodal voltages $v_1$ and $v_2$ in the circuit of Figure 1(d):

![Figure 1(d)](image)

e) In the circuit of Figure 1(e), the switch remains in position 1 for a long time before moving to position 2 at time $t = 0 \ s$. Find (i) capacitor voltage $v_c(t)$ at $t = 0 \ s$ before the switch moves, (ii) final value of $v_c(t)$ for $t \to \infty$, (iii) the time constant for $t \geq 0 \ s$ and (iv) an equation for $v_c(t)$ as a function of time.

![Figure 1(e)](image)
f) Draw the phasor equivalent circuit for the circuit in Figure 1(f). Hence determine the phasor for the current i(t) in the 4 Ω resistor. Hence, derive an expression for i(t).

Figure 1(f) 3cos2t

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\[ I \]

\[ 2 \text{ H} \]

\[ 4 \text{ Ω} \]

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\[ j(t) \]

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g) Figure 1(g) shows a parallel LC tuned-circuit with loss, driven by a current source (which could represent the output of a transistor). The transfer function \( V_o/I_i \) has the form of a bandpass filter. For this circuit, determine the centre frequency in rads/sec, the Q–factor and the bandwidth in rads/sec.

Figure 1(g) \[ 100 \text{ kΩ} \]

\[ 100 \text{ μF} \]

\[ 100 \text{ pF} \]

\[ V_o \]

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h) For the filter circuit shown in Figure 1(h), determine the frequency response function \( H(jω) = V_o/V_i \). By considering the behaviour of \( H(jω) \) at the resonant frequency, at zero frequency and for frequency \( ω \to \infty \), show that the filter is a low-pass filter.

Figure 1(h) \[ + \]

\[ V_i \]

\[ - \]

\[ + \]

\[ V_o \]

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i) For the filter circuit shown in Figure 1(i), where the op-amp may be assumed to be ideal, determine the voltage \( V_{out} \).

Figure 1(i) \[ + \]

\[ 13 \text{ V} \]

\[ - \]

\[ 7 \text{ mA} \]

\[ + \]

\[ 1 \text{ kΩ} \]

\[ 1 \text{ kΩ} \]

\[ 1 \text{ kΩ} \]

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j) A linear 2-port circuit with its 2-port impedance description is given in Figure 1(j). Determine, in terms of the z-parameters \( z_{11}, z_{12}, z_{21}, \text{ and } z_{22} \), expressions for (i) the voltage gain \( V_2/V_1 \) when port 2 is terminated in an open-circuit and (ii) the current gain \( I_2/I_1 \) when port 2 is terminated in a short-circuit.

Figure 1(j) \[ + \]

\[ + \]

\[ V_1 \]

\[ V_2 \]

\[ - \]

\[ + \]

\[ V_1 = z_{11}I_1 + z_{12}I_2 \]

\[ V_2 = z_{21}I_1 + z_{22}I_2 \]
a) Give definitions for the voltage between two nodes in a circuit and the current flowing through an element in a circuit in terms of electrical charge $Q$, work (or energy) $E$ and time $t$.

State Kirchoff's current law as it may be applied to any node in a circuit.

State Kirchoff's voltage law as it may be applied to any set of elements that form a loop in a circuit.

State how in general how the voltage across an element in a circuit is related to the voltages at its two nodes.

b) The circuit in Figure 2.1 consists of resistors and DC current sources only:

![Figure 2.1 Circuit for Question 2(b)](image)

Write the set of nodal equations that describe the circuit and may be used to solve for the node voltages, $v_1$, $v_2$, and $v_3$ (Do not solve the equations). You may use a by-inspection method if you wish.

c) State two methods that may be used to solve sets of linear simultaneous equations, such as those that are obtained in nodal analysis. List briefly their features, positive and/or negative.

d) The circuit in Figure 2.2 consists of a DC current source, two DC voltage sources and some resistors specified by their conductances (in Siemens):

![Figure 2.2 Circuit for Question 2(d)](image)

Write the set of nodal equations that describe the circuit and solve them in order to determine the node voltages, $v_1$, $v_2$, and $v_3$. 

EE1.1 Analysis of Circuits
3. a) Write down the phasors corresponding to the following current functions (for convenience, angles are shown in degrees):
   i) \( i_1(t) = 5\cos(2t - 90^\circ) \)
   ii) \( i_2(t) = 6\sin(t + 45^\circ) \)
   iii) \( i_3(t) = -2\cos(2t) \)

b) Give expressions for the impedance of an inductor of inductance value \( L \) and of a capacitor of capacitance value \( C \) as a function of frequency \( \omega \) in both rectangular and polar forms.

c) Two elements of impedance \( z_1 \) and \( z_2 \) are connected in series across a voltage source \( V \). Draw a sketch showing the circuit. Choose and indicate an orientation for the voltage \( v_2 \) across \( z_2 \), and state the voltage divider rule that determines \( v_2 \) in terms of \( V \) and the two impedances \( z_1 \) and \( z_2 \).

d) Consider the circuit in Figure 3.1:

\[ v(t) = 3\cos(2t) \]

![Figure 3.1 Circuit for Question 3(d)](image_url)

i) Draw the phasor equivalent circuit for this circuit.
ii) Carry out circuit analysis to determine the phasor form \( \vec{V} \) of voltage \( v(t) \).
iii) Convert the phasor \( \vec{V} \) into the corresponding time domain form \( v(t) \).

[12]

e) The circuit in Figure 3.2 has a periodic current excitation that consists of a fundamental sinusoidal component and its 2nd harmonic component, as shown:

\[ i(t) = \cos(2t) \]

![Figure 3.2 Circuit for Question 3(e)](image_url)

Show the two phasor equivalent circuits which can be used to solve for \( i(t) \) using the principle of superposition (Do not complete the solution for \( i(t) \)).

[6]
4. a) The dependent source, or controlled source, is a key element in circuit analysis because it can be used to model active elements. The dependent source is a 2-port circuit where the independent signal variable at the input port and the dependent signal variable at the output port may be a voltage or current, leading to four types of dependent source.
   
i) Draw symbols for the four types of dependent source, showing clearly the input and output signal variables.

ii) Give an equation for each of the four types of dependent source that expresses the dependent output variable as a function of the independent input variable, assuming that the sources may be treated as linear elements.

iii) State the units of the gain, defined as output variable divided by input variable for each of the four types of dependent source.

b) The circuit in Figure 4.1 contains a voltage-controlled current source, as well as an independent current source and two resistors. Node voltage $v_1$ is the controlling voltage for the voltage-controlled current source.

Determine the current $i$ in this circuit. It is recommended to use nodal analysis with the method of taping and then un-taping the dependent source.

![Figure 4.1 Circuit for Question 4(b)](image)

[c] Of the four types of dependent sources, identify one type most suitable to model the field-effect transistor and one type most suitable to model the voltage operational amplifier.

Show how the terminals of these dependent sources should be connected given that the dependent source has 4 terminals and the operational amplifier and transistor have only 3 terminals.

d) The sub-circuit in Figure 4.2 contains a resistor and a current-controlled voltage source. Suggest three values for the gain of the dependent source, $r_m$, such that the sub-circuit is equivalent to (i) a resistance of 10 $\Omega$, (ii) a short circuit and (iii) a resistance of $-10$ $\Omega$, respectively.

![Figure 4.2 Circuit for Question 4(d)](image)
Q1. a) 

\[ V = 12V \]
\[ I = 3A \leq 3A \]
\[ 2A \leq 6\Omega \]
\[ 5A \leq 2\Omega \]
\[ i = \frac{5}{2} = 2.5A \]

b) Replace V-source by 5V:
\[ i_1 = \frac{4}{2} = 2A \]
Replace I-source by 1A:
\[ i_2 = \frac{-6}{6} = -1A \]
\[ i = i_1 + i_2 = 1A \]

c) \[ i_{5/V} = \frac{24}{6} + \frac{12}{6} = 6A \]
\[ R_{eq} = \frac{6\Omega \parallel 6\Omega}{2} = 3\Omega \]
\[ V = 3A \times 3\Omega = 9V \]

d) Nodal equations:
\[ (3+2)V_1 - 2V_2 = 2 \]
\[ -2V_1 + (1+2)V_2 = 8 \]
\[ 5V_1 - 2V_2 = 2 \]
\[ -2V_1 + 3V_2 = 8 \]
\[ (5-\frac{4}{3})V_1 = 2 + \frac{16}{3} \]
\[ V_1 = 2V \]
\[ V_2 = \frac{1}{2}(5V_1 - 2) = 4V \]

e) \[ V_{cc} = 5V \]
\[ V_{cc0} = 0V \]
\[ C = 10^6 \times 10^{-6} = 1S \]
\[ V_c(t) = V_{cc} + (V_{cc0} - V_{cc})e^{-t/C} \]
1) \( 3 \theta^o \) circuit with 4 \( j \frac{\sqrt{2}}{2} \) impedance:

\[ \bar{I} = \frac{j^4}{4+4j} \times 3 \theta^o = \frac{4 \times 190^o}{4 \sqrt{2} \times 145^o} \times 3 \theta^o = \frac{3}{\sqrt{2}} \times 145^o \]

\( i(t) = \frac{3}{\sqrt{2}} \cos (2t + 45^o) \).

4) \( q \) For tuned circuit:

\[ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{10^{-4} \times 10^{-8}} = 10^7 \text{ rad/s} \]

\[ Q = \frac{R}{\omega_0 L} = \frac{10^5}{10^7 \times 10^{-4}} = 100 \]

\[ \omega_B = \frac{\omega_0}{Q} = 10^5 \text{ rad/sec} \]

4) \( h \) Using voltage divider rule:

\[ H(j\omega) = \frac{V_o}{V_i} = \frac{\frac{1}{Z_2}}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{1}{1 + j\omega L / (j\omega C + \frac{1}{R})} = \frac{1}{(j\omega)^2 LC + j\omega L + 1} \]

\[ \omega \to \infty \quad \frac{1}{\omega_0 R/\sqrt{LC}} \]

Filter is a \textit{lowpass} filter.

4) \( i \) Circuit diagram:

\[ V_{out} = 5V \]

4) \( j \)

\[ V_1 = \frac{211 I_1 + 212 I_2}{Z_{11}} \]

\[ V_2 = \frac{221 I_1 + 222 I_2}{Z_{21}} \]

\[ I_2 = 0, \quad V_2 = \frac{221 I_1}{Z_{21}} = \frac{221}{Z_{21}} \]

\[ V_2 = 0, \quad 0 = \frac{221 I_1 + 222 I_2}{Z_{21}} \]

\[ \frac{I_2}{I_1} = -\frac{221}{222} \]
2. a) Voltage between two nodes = \( \frac{E}{Q} \), where \( E \) is energy needed to move charge between the nodes. Current = \( \frac{dQ}{dt} \), i.e., rate of flow of charge.

Net sum of currents in elements incident at node is zero.

Net some of element voltage drops (or rises) is zero.

Element voltages are differences of nodal voltages.

b) \[
\begin{pmatrix}
3 & -2 & 0 \\
-2 & 7 & -2 \\
0 & -2 & 3
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix}
= \begin{pmatrix}
5 \\
6 \\
-1
\end{pmatrix}
\]

10. c) Substitution - Easy to understand - Very long Gaussian elimination - Algorithmic - Easy to program

d) \[
\begin{pmatrix}
1 & -1 & 0 \\
-1 & 3 & 0 \\
0 & 0 & 3
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
14
\end{pmatrix}
\]

\( V_2 - V_3 = -1 \quad V_1 = 1 \)

\[
\begin{pmatrix}
-1 & 3 & 3 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix}
= \begin{pmatrix}
14 \\
1 \\
-1
\end{pmatrix}
\]

9. \[
\begin{pmatrix}
6
\end{pmatrix}
\begin{pmatrix}
V_2
\end{pmatrix}
= 12 \quad V_2 = 2 V, \quad V_1 = 1 V, \quad V_3 = 3 V
\]

30.
3 a) $I_1 = 5 \angle -90^\circ = -j5$
$I_2 = 6 \angle -45^\circ = 3\sqrt{2} - j3\sqrt{2}$
$I_3 = 2 \angle 180^\circ = -2$

b) $\frac{Z_L}{j\omega L} = \frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90^\circ$

\[\begin{align*}
\text{c) } V_2 &= \frac{Z_L}{Z_1 + Z_2} V_s \\
\text{d) } \frac{V_s}{V} &= \frac{Z_L}{Z_1 + Z_2} \\
\text{Use voltage division:} \\
\frac{V}{V_s} &= \frac{j4}{2 - j2 + j4} = \frac{4190}{2\sqrt{2} \angle 45^\circ} = \frac{310^\circ}{3\sqrt{2} \angle 45^\circ} \\
\text{e) } V_{61} &= 12 \angle 60^\circ \\
V_{62} &= 15 \angle 145^\circ \\
\text{w} &= 2 \text{rad/s} \\
\text{w} &= 4 \text{rad/s}
\end{align*}\]
ii) \[ V_0 = \mu V_x \quad I_0 = g_m V_x \quad V_0 = R_m I \quad I_0 = \beta I_x \]

iii) \[ \mu \quad g_m \quad R_m \quad \beta \]

b) Tape the dependent source:

\[ \frac{1}{4} + V_i \rightarrow I_c \]

Nodal analysis:

\[ \frac{V_i}{3} + \frac{V_i}{6} = 9 + I_c \]

Untape, source:

\[ I_c = \frac{1}{4} V_i \]

\[ \frac{V_i}{2} - \frac{1}{4} V_i = 9 \]

\[ V_i = 36 \text{ V} \]

\[ i = \frac{V_i}{3} = 12 \text{ A} \]

c) For the FET - VCCS

For the op-amp - VCVS

FET

\[ \begin{array}{c}
\text{Op-amp} \\
\text{FET}
\end{array} \]

d) By analysis:

\[ Y = 10 I_x - R_m I_x \]

\[ = (10 - R_m) I_x \]

\[ = (10 - R_m) I \]

\[ \therefore \text{Equivalent} \Rightarrow \text{Thevenin resistance of } 10 - R_m \]

\[ \equiv \text{Res} \quad R_m (\Omega) \]

\[ \begin{array}{cc}
10 & 0 \\
0 & 10 \\
-10 & 20
\end{array} \]