IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

EEE/ISE PART I: MEng, BEng and ACGI

ANALYSIS OF CIRCUITS

Wednesday, 6 June 10:00 am

Corrected Copy

Time allowed: 2:00 hours

There are FOUR questions on this paper.

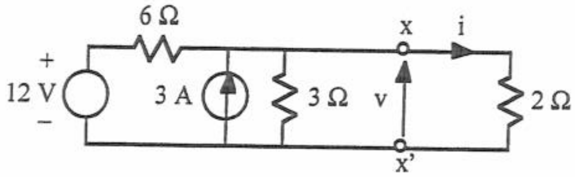
Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	D.G. Haigh, D.G. Haigh
	Second Marker(s) :	P.D. Mitcheson, P.D. Mitcheson

1 a) Use source transformations to derive a simplified equivalent circuit for the sub-circuit in Figure 1 (a) to the left of the terminals x, x'. Hence, determine the current i when the sub-circuit is connected to the 2 Ω resistor.

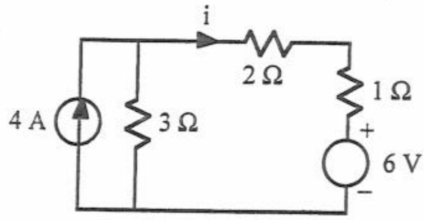
Figure 1(a)



[4]

b) Use the principle of superposition to find current i in the circuit in Figure 1(b):

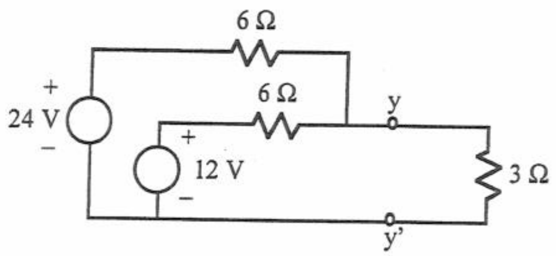
Figure 1(b)



[4]

c) Find the Norton equivalent circuit (current source with resistor in parallel with it) for the sub-circuit in Figure 1(c) to the left of the terminals y, y'. What is the voltage across the 3 Ω resistor when it is connected to the sub-circuit?

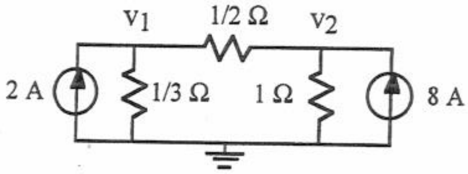
Figure 1(c)



[4]

d) Use nodal analysis to determine the nodal voltages v_1 and v_2 in the circuit of Figure 1(d):

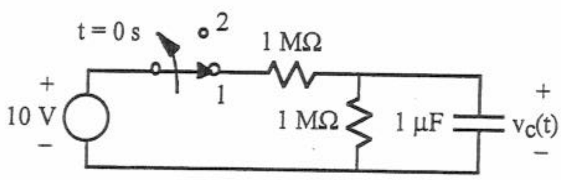
Figure 1(d)



[4]

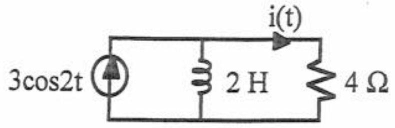
e) In the circuit of Figure 1(e), the switch remains in position 1 for a long time before moving to position 2 at time $t = 0$ s. Find (i) capacitor voltage $v_c(t)$ at $t = 0$ s before the switch moves, (ii) final value of $v_c(t)$ for $t \rightarrow \infty$, (iii) the time constant for $t \geq 0$ s and (iv) an equation for $v_c(t)$ as a function of time.

Figure 1(e)



[4]

f) Draw the phasor equivalent circuit for the circuit in Figure 1(f). Hence determine the phasor for the current i(t) in the 4 Ω resistor. Hence, derive an expression for i(t).

Figure 1(f)


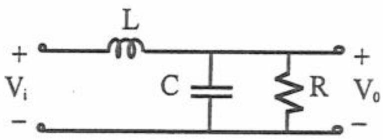
[4]

g) Figure 1(g) shows a parallel LC tuned-circuit with loss, driven by a current source (which could represent the output of a transistor). The transfer function V_o/I_i has the form of a bandpass filter. For this circuit, determine the centre frequency in rads/sec, the Q–factor and the bandwidth in rads/sec.

Figure 1(g)

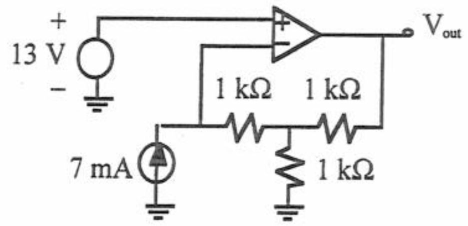

[4]

h) For the filter circuit shown in Figure 1(h), determine the frequency response function $H(j\omega) = V_o/V_i$. By considering the behaviour of $H(j\omega)$ at the resonant frequency, at zero frequency and for frequency $\omega \to \infty$, show that the filter is a low-pass filter.

Figure 1(h)


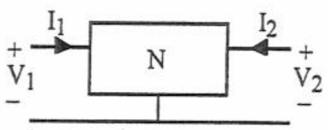
[4]

i) For the filter circuit shown in Figure 1(i), where the op-amp may be assumed to be ideal, determine the voltage V_{out}.

Figure 1(i)


[4]

j) A linear 2-port circuit with its 2-port impedance description is given in Figure 1(j). Determine, in terms of the z-parameters z_{11}, z_{12}, z_{21} and z_{22}, expressions for (i) the voltage gain V_2/V_1 when port 2 is terminated in an open-circuit and (ii) the current gain I_2/I_1 when port 2 is terminated in a short-circuit.

Figure 1(j)


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

[4]

2 a) Give definitions for the voltage between two nodes in a circuit and the current flowing through an element in a circuit in terms of electrical charge Q, work (or energy) E and time t.

State Kirchoff's current law as it may be applied to any node in a circuit.

State Kirchoff's voltage law as it may be applied to any set of elements that form a loop in a circuit.

State how in general how the voltage across an element in a circuit is related to the voltages at its two nodes.

[6]

b) The circuit in Figure 2.1 consists of resistors and DC current sources only:

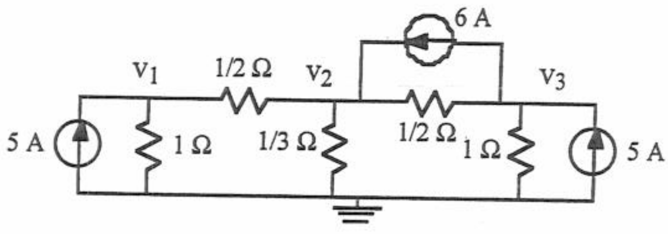


Figure 2.1 Circuit for Question 2(b)

Write the set of nodal equations that describe the circuit and may be used to solve for the node voltages, v_1, v_2, and v_3 (Do not solve the equations). You may use a by-inspection method if you wish.

[10]

c) State two methods that may be used to solve sets of linear simultaneous equations, such as those that are obtained in nodal analysis. List briefly their features, positive and/or negative.

[5]

d) The circuit in Figure 2.2 consists of a DC current source, two DC voltage sources and some resistors specified by their conductances (in Siemens):

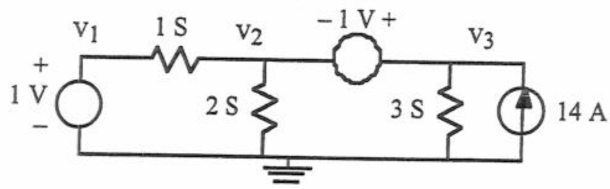


Figure 2.2 Circuit for Question 2(d)

Write the set of nodal equations that describe the circuit and solve them in order to determine the node voltages, v_1, v_2, and v_3.

[9]

3. a) Write down the phasors corresponding to the following current functions (for convenience, angles are shown in degrees):

i) $i_1(t) = 5\cos(2t - 90°)$

ii) $i_2(t) = 6\sin(t + 45°)$

iii) $i_3(t) = -2\cos(2t)$

[6]

b) Give expressions for the impedance of an <u>inductor</u> of inductance value L and of a <u>capacitor</u> of capacitance value C as a function of frequency ω in both <u>rectangular</u> and <u>polar</u> forms.

[4]

c) Two elements of impedance z_1 and z_2 are connected in series across a voltage source V_s. Draw a sketch showing the circuit. Choose and indicate an orientation for the voltage v_2 across z_2, and state the *voltage divider rule* that determines v_2 in terms of V_s and the two impedances z_1 and z_2.

[2]

d) Consider the circuit in Figure 3.1:

Figure 3.1

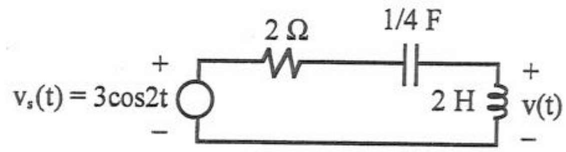


Figure 3.1 Circuit for Question 3(d)

i) Draw the phasor equivalent circuit for this circuit.

ii) Carry out circuit analysis to determine the phasor form $\overline{V}$ of voltage $v(t)$.

iii) Convert the phasor $\overline{V}$ into the corresponding time domain form $v(t)$.

[12]

e) The circuit in Figure 3.2 has a periodic current excitation that consists of a fundamental sinusoidal component and its 2nd harmonic component, as shown:

Figure 3.2

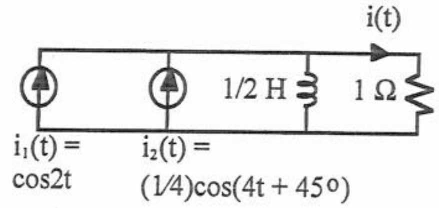


Figure 3.2 Circuit for Question 3(e)

Show the two phasor equivalent circuits which can be used to solve for $i(t)$ using the principle of superposition (Do not complete the solution for $i(t)$).

[6]

4. a) The *dependent source*, or controlled source, is a key element in circuit analysis because it can be used to model active elements. The dependent source is a 2-port circuit where the independent signal variable at the input port and the dependent signal variable at the output port may be a voltage or current, leading to four types of dependent source.

   i) Draw symbols for the four types of dependent source, showing clearly the input and output signal variables.

   ii) Give an equation for each of the four types of dependent source that expresses the dependent output variable as a function of the independent input variable, assuming that the sources may be treated as linear elements.

   iii) State the units of the gain, defined as output variable divided by input variable for each of the four types of dependent source.

[12]

b) The circuit in Figure 4.1 contains a voltage-controlled current source, as well as an independent current source and two resistors. Node voltage v_1 is the controlling voltage for the voltage-controlled current source.

Determine the current i in this circuit. It is recommended to use nodal analysis with the method of taping and then un-taping the dependent source.

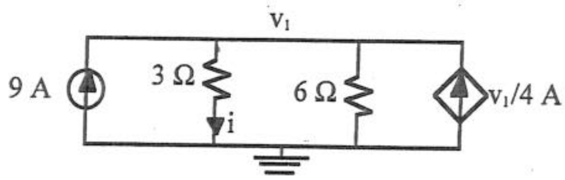


Figure 4.1 Circuit for Question 4(b)

[8]

c) Of the four types of dependent sources, identify one type most suitable to model the field-effect transistor and one type most suitable to model the voltage operational amplifier.

Show how the terminals of these dependent sources should be connected given that the dependent source has 4 terminals and the operational amplifier and transistor have only 3 terminals.

[4]

d) The sub-circuit in Figure 4.2 contains a resistor and a current-controlled voltage source. Suggest three values for the gain of the dependent source, r_m, such that the sub-circuit is equivalent to (i) a resistance of 10 Ω, (ii) a short circuit and (iii) a resistance of –10 Ω, respectively.

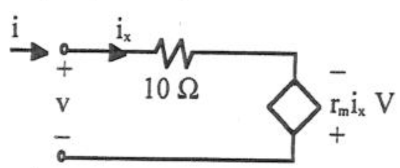


Figure 4.2 Circuit for Question 4(d)

[6]

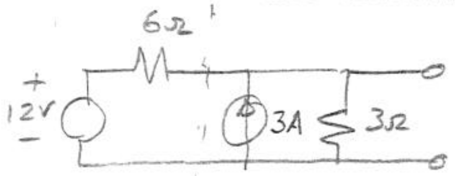
SOLUTIONS

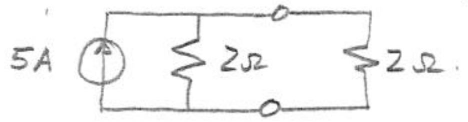
CIRCUIT ANALYSIS E1.1 2007

KEY	
A	Application
B	Bookwork
T	Theory

Q1. a)

A





$i = \frac{5}{2} = 2.5A$

4

b). Replace V-source by s/c:

A

$i_1 = 4/2 = 2A$

Replace I-source by o/c:

$i_2 = -6/6 = -1A$

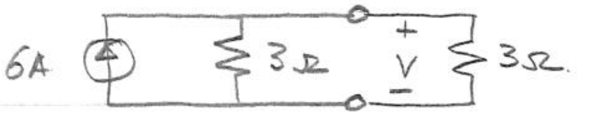
$i = i_1 + i_2 = 1A$

4

c). $i_{s/c} = \frac{24}{6} + \frac{12}{6} = 6A$

A

$R_{eq} = 6\Omega // 6\Omega = 3\Omega$



$V = 3A \times 3\Omega = 9V$

4

d). Nodal equations

A

$(3+2)V_1 - 2V_2 = 2$

$-2V_1 + (1+2)V_2 = 8$

$5V_1 - 2V_2 = 2$

$-2V_1 + 3V_2 = 8$

$\left(5 - \frac{4}{3}\right)V_1 = 2 + \frac{16}{3}$

$V_1 = 2V \qquad V_2 = \frac{1}{2}(5V_1 - 2) = 4V$

4

e).

A

$V_{C0} = 5V$

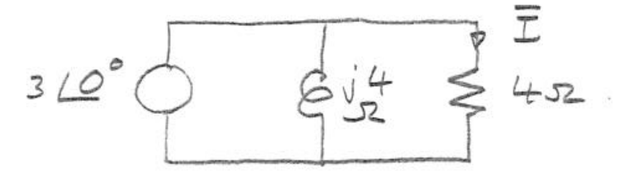
$V_{C\infty} = 0V$

$\tau = 10^6 \times 10^{-6} = 1s$

$V_C(t) = V_{C0} + (V_{C\infty} - V_{C0})(1 - e^{-t/\tau})$

4

1 f)



$\omega = 2\ r/s$

$\boxed{A}$

$$\bar{I} = \frac{j4}{4+j4} \times 3\angle 0^\circ = \frac{4\angle 90^\circ}{4\sqrt{2}\angle 45^\circ} \times 3\angle 0^\circ = \frac{3}{\sqrt{2}}\angle 45^\circ$$

$$i(t) = \frac{3}{\sqrt{2}} \cos(2t + 45^\circ).$$

4

g) For tuned-circuit,

$\boxed{A}$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-4}\cdot 10^{-10})}} = 10^7\ \text{rad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{10^5}{10^7\cdot 10^{-4}} = 100$$

$$\omega_B = \frac{\omega_0}{Q} = 10^5\ \text{rad/sec.}$$

4

h) Using voltage divider rule:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{Z_2}{Z_1+Z_2} = \frac{1}{1+Z_1Y_2}$$

$\boxed{A}$

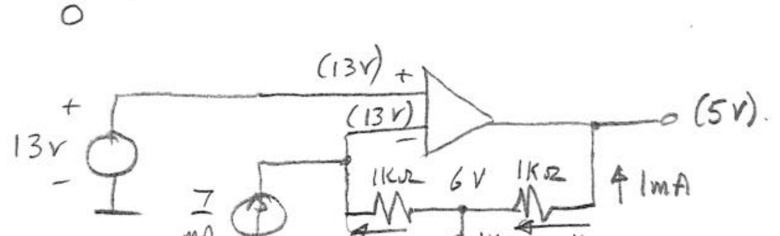
$$= \frac{1}{1+j\omega L\left(j\omega C + \frac{1}{R}\right)} = \frac{1}{(j\omega)^2 LC + j\omega \frac{L}{R} + 1}$$

ω	$\lvert H\rvert$
0	1
ω_0	$\frac{1}{R\sqrt{L/C}}$
$\to\infty$	0

∴ Filter is a <u>Lowpass</u> filter

4

i)



$\boxed{A}$

$V_{out} = 5V$

4

j)

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$\boxed{A}$

$I_2 = 0$, $\frac{V_2}{V_1} = \frac{Z_{21}I_1}{Z_{11}I_1} = \frac{Z_{21}}{Z_{11}}$; $V_2 = 0$, $0 = Z_{21}I_1 + Z_{12}I_2$

$$\frac{I_2}{I_1} = -\frac{Z_{21}}{Z_{22}}$$

$\boxed{40}$ 4

Solutions
2007
EE1.1

2

a) Voltage between two nodes = E/Q, where E is energy needed to move charge between the nodes.
Current = dQ/dt, i.e. rate of flow of charge.

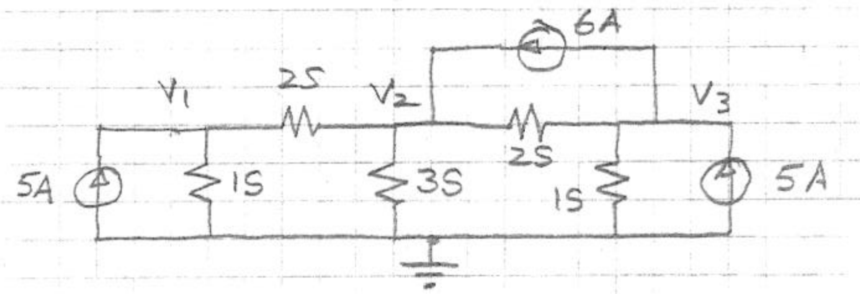
B

Net sum of currents in elements incident at node is zero.

Net some of element voltage drops (or rises) is zero.

6 Element voltages are differences of nodal voltages.

b)



A

10

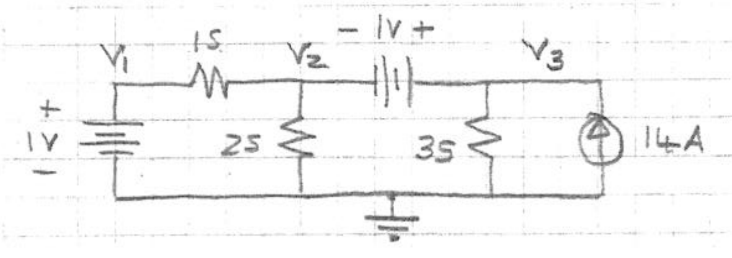
$$\begin{bmatrix} 3 & -2 & 0 \\ -2 & 7 & -2 \\ 0 & -2 & 3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix}$$

c) Substitution – Easy to understand – Very long
Gaussian elimination – Algorithmic – Easy to program

5

B

d)



$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 14 \end{bmatrix}$$

$V_2 - V_3 = -1$ $\qquad V_1 = 1$

A

$$\begin{bmatrix} -1 & 3 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & \boxed{-1} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 1 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} \boxed{-1} & 6 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 1 \end{bmatrix}$$

9

30

$[6][V_2] = 12$ $\qquad V_2 = 2V,\ V_1 = 1V,\ V_3 = 3V$

3 a) $\bar{I}_1 = 5\angle{-90^\circ} = -j5$

$\bar{I}_2 = 6\angle{-45^\circ} = 3\sqrt{2} - j\,3\sqrt{2}$

(6) $\bar{I}_3 = 2\angle{180^\circ} = -2$

$\boxed{A}$

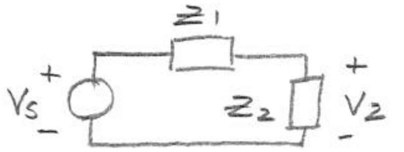
b) $Z_L = j\omega L = \omega L \angle 90^\circ$

$\boxed{B}$ $Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{\omega C}\angle{-90^\circ}$

(4)

c)

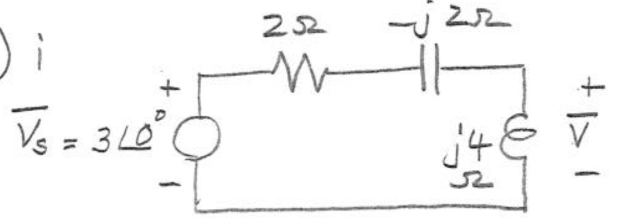
$\boxed{B}$



$V_2 = \dfrac{Z_2}{Z_1 + Z_2} V_s$

(2)

d) i



$\omega = 2\ \text{rad/s}$

ii Use voltage division:

$$\bar{V} = \frac{Z_2}{Z_1 + Z_2}\bar{V}_s = \frac{j4}{2 - j2 + j4}\, 3\angle 0^\circ$$

$$= \frac{4\angle 90^\circ}{2\sqrt{2}\angle 45^\circ}\, 3\angle 0^\circ$$

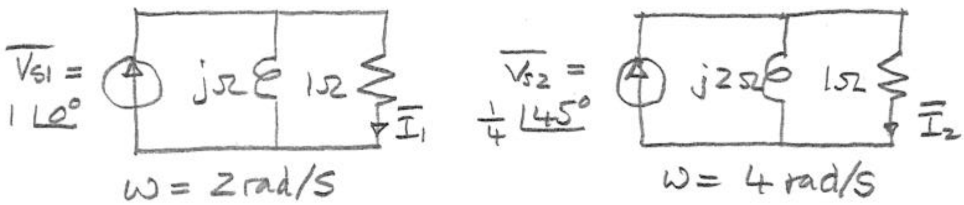
$\boxed{A}$

$$= 3\sqrt{2}\angle 45^\circ$$

(12) iii $v(t) = 3\sqrt{2}\cos(2t + 45^\circ)$

e)

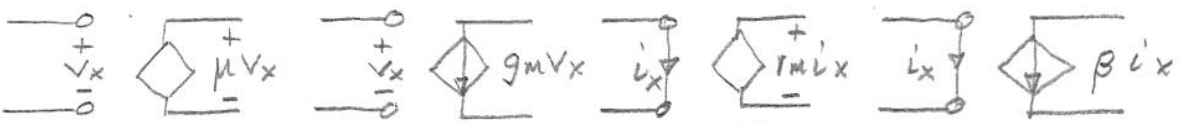
$\boxed{A}$



(6)

$\boxed{30}$

4 a) i)



VCVS VCCS CCVS CCCS

B

ii) $V_o = \mu V_x$ $i_o = g_m V_x$ $V_o = r_m i_x$ $i_o = \beta i_x$

iii)

μ	g_m	r_m	β
Dimensionless	A/V	V/A	Dimensionless

(12)

b) Tape the dependent source:

$\frac{1}{4}V_1 \rightarrow i_C$

Nodal analysis:

$$\frac{V_1}{3} + \frac{V_1}{6} = 9 + i_C$$

Untape source:

$$i_C = \frac{1}{4}V_1$$

$$\frac{V_1}{2} - \frac{1}{4}V_1 = 9$$

A

$$V_1 = 36\ V$$

$$i = \frac{V_1}{3} = 12\ A$$

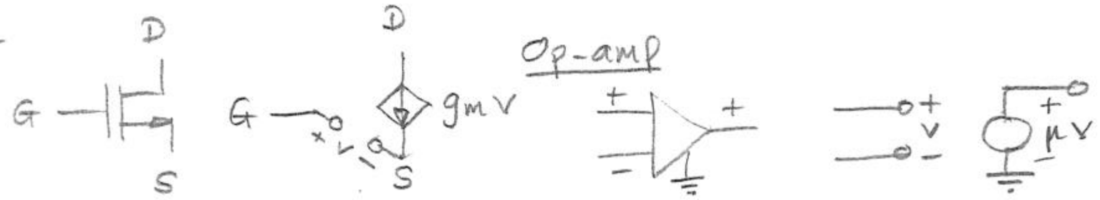
(8)

c) For the FET − VCCS

For the op-amp − VCVS

FET

B



(4)

d) By analysis:

$$\begin{aligned} V &= 10 i_x - r_m i_x \\ &= (10 - r_m) i_x \\ &= (10 - r_m) i \end{aligned}$$

A

∴ Equivalent to Thevenin resistance of $10 - r_m\ \Omega$

≡ Res	r_m (Ω)
10	0
0	10
−10	20

(6)

30