Paper Number(s): **EE1-1**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

EEE/ISE PART I: MEng, BEng and ACGI

ANALYSIS OF CIRCUITS

Friday, 1 June 10:00 am

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Time allowed: 2:00 hours

Examiners responsible:

First Marker(s): D.M. Brookes

Second Marker(s): P. Georgiou

Information for Candidates:

The following notation is used in this paper:

1. The voltage waveform at node X in a circuit is denoted by $x(t)$, the phasor voltage by X and the root-mean-square (or RMS) phasor voltage by $\tilde{X} = \frac{X}{\sqrt{2}}$.
2. Component and source values in a circuit are normally given in Ohms, Farads, Henrys, Volts or Amps with the unit symbol omitted. Where an imaginary number is specified, it represents the complex impedance or phasor value.
3. Times are given in seconds unless otherwise stated.
4. Unless otherwise indicated, frequency response graphs should use a linear axis for phase and logarithmic axes for frequency and magnitude.

1. (a) Using nodal analysis calculate the voltages at nodes X and Y of *Figure 1.1*. [5]

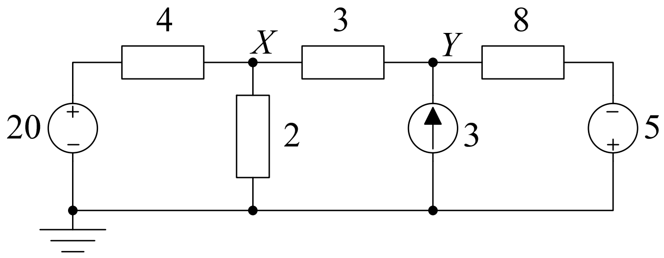


Figure 1.1

(b) Use the principle of superposition to find the voltage X in *Figure 1.2*. [5]

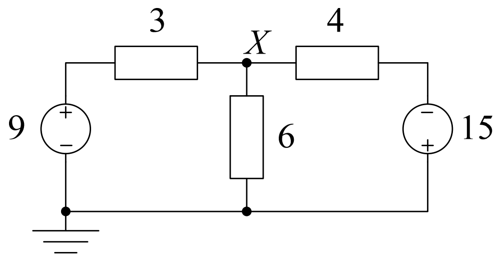


Figure 1.2

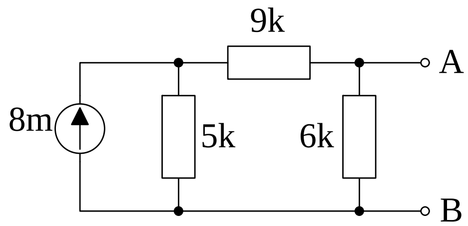


Figure 1.3

(c) Draw the Thévenin equivalent circuit of the network in *Figure 1.3* and find the values of its components. [5]

(d) Assuming the opamp in the circuit of *Figure 1.4* is ideal, give an expression for X in terms of U and V. [5]

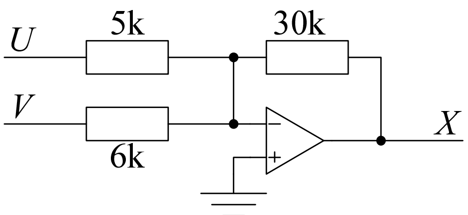


Figure 1.4

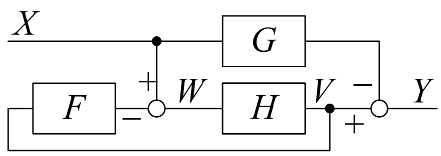


Figure 1.5

(e) Determine the gain $\frac{Y}{X}$ for the block diagram shown in *Figure 1.5*. The rectangular blocks are drawn with inputs at the left and outputs at the right and have gains of F, G and H respectively. The open circles represent adder/subtractors whose inputs have the signs indicated on the diagram and whose outputs are W and Y respectively. [5]

(f) The diode in *Figure 1.6* has a forward voltage drop of 0.7 V but is otherwise ideal. Determine Y as a function of X for the two cases:

(i) the diode is reverse biased

(ii) the diode is forward biased.

Hence sketch a dimensioned graph of Y versus X for $0 \leq X \leq 12$. [5]

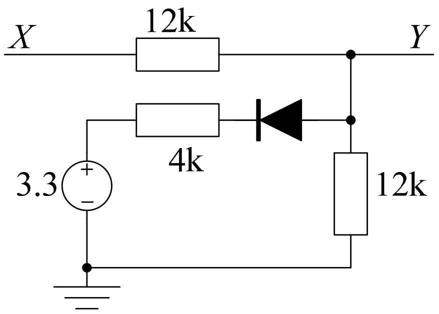


Figure 1.6

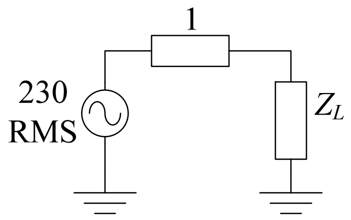


Figure 1.7

(g) The load, Z_L, in *Figure 1.7* is driven by a 230 V RMS, 50 Hz source through a line of resistance 1 Ω. The load impedance is $Z_L = 24 + 9j$.

(i) Determine the complex power absorbed by the load. [3]

(ii) If Z_L consists of a resistor, R, in parallel with an inductor, L, determine the values of R and L. [2]

(h) *Figure 1.9* shows a transmission line of length 100 m that is terminated in a resistive load, R, with reflection coefficient $\rho = -0.5$. The line has a characteristic impedance of $Z_0 = 100\ \Omega$ and a propagation velocity of $u = 2 \times 10^8$ m/s.

(i) At time $t = 0$, a forward-travelling pulse arrives at X with amplitude 5 V and duration 100 ns as shown in *Figure 1.8*. Draw a dimensioned sketch of the waveform at Y, the mid-point of the line, for $0 \leq t \leq 1\ \mu s$. [3]

(ii) Given that $\rho = \frac{R - Z_0}{R + Z_0}$, determine the value of R. [2]

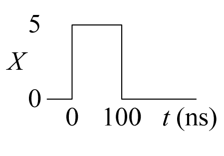


Figure 1.8

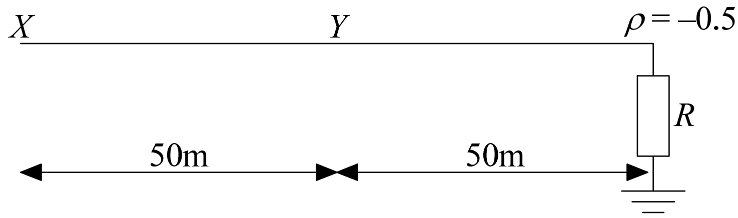


Figure 1.9

2. (a) Assuming that the op-amps in the circuit of *Figure 2.1* are ideal, explain why the nodes V_1, V_2 and V_3 have the same voltage stating any assumptions that you are making. [2]

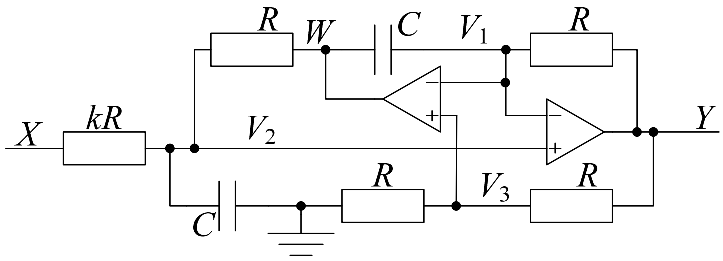


Figure 2.1

(b) Writing V for the common voltage at nodes V_1, V_2 and V_3, use nodal analysis to show that the transfer function, $\frac{Y}{X}(j\omega)$, is given by [10]

$$\frac{Y}{X}(j\omega) = \frac{2j\omega RC}{k(j\omega RC)^2 + j\omega RC + k}.$$

(c) For the transfer function $\frac{Y}{X}(j\omega)$, determine

(i) the value of the transfer function at $\omega = \frac{1}{RC}$, [2]

(ii) the low and high frequency asymptotes of the magnitude response, $\left|\frac{Y}{X}(j\omega)\right|$, [2]

(iii) the frequency at which the low and high frequency asymptotes cross and the gain magnitude of the asymptotes at this frequency. [2]

(d) On a single set of axes, draw dimensioned sketches of $\left|\frac{Y}{X}(j\omega)\right|$ for the cases $k = 4$ and $k = 10$ using a logarithmic frequency axis and a dB scale for gain. [5]

(e) If the input signal is $x(t) = A\cos\omega t$, determine the peak current that must be supplied by each of the op-amps at $\omega = \frac{1}{RC}$. [5]

(f) Using a value $k = 10$, determine value for R and C to give a corner frequency of 1 kHz while ensuring that neither opamp peak output current exceeds 4 mA at this frequency for an input voltage amplitude of $A = 10$ volts. [2]

3. In the circuit of *Figure 3.1* the waveform at node X is $x(t) = 10\sin \omega t$ where $\omega = 2000\ \text{rad/s}$.

(a) For each of the following cases, determine the steady state waveform at Y in the form $y(t) = A\cos \omega t + B\sin \omega t$:

(i) the switch in the circuit is open (as shown in the diagram), [4]

(ii) the switch in the circuit is closed. [4]

(b) Suppose that the switch has been open for a long time and is closed at time $t = 0$. Determine the waveform $y(t)$ for $t \geq 0$. [8]

(c) Suppose now that the switch has been open for a long time and is closed at time $t = t_0$. Determine the smallest positive value of t_0 for which the transient amplitude will equal zero. [6]

(d) Suppose that the switch has been open for a long time and is closed at time $t = 0$ and then re-opened at time $t = \frac{\pi}{\omega}$. Determine the waveform $y(t)$ for $t \geq \frac{\pi}{\omega}$. [8]

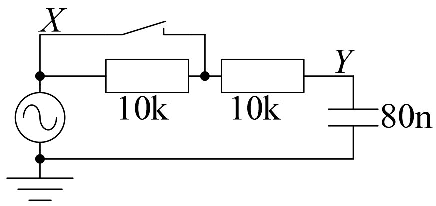


Figure 3.1

2012 Paper E1.1: Analysis of Circuits - Solutions

Key to letters on mark scheme: B=Bookwork, C=New computed example, A=Analysis of new example, D=design of new example

1. (a) KCL equations are

$$\frac{X-20}{4}+\frac{X}{2}+\frac{X-Y}{3}=0 \Rightarrow 3X-60+6X+4X-4Y=0 \;\Rightarrow 13X-4Y=60$$

$$\frac{Y-X}{3}-3+\frac{Y+5}{8}=0 \Rightarrow 8Y-8X-72+3Y+15=0 \;\Rightarrow\; -8X+11Y=57$$

Solving these equations gives $X=8$ and $Y=11$. [5A]

Almost everyone wrote down the correct equations although occasionally the current source had the wrong sign. Quite often though, there were algebra errors (usually sign errors) in solving the equations; the provided calculator will solve them directly (in EQN mode) or else you can multiply the two equations above by 11 and 4 respectively and add them together (quite a bit easier than substituting in for X or Y which seems the most popular approach). A few people used the calculator to solve the equations but got the answer wrong either because they reversed the order of the X and Y coefficients in one of the equations (presumably to avoid starting with a negative coefficient) or because they negated the right hand sides of the equations.

(b) Setting the 15 V source to zero gives a potential divider:

$$X_9 = 9\times\frac{4||6}{4||6+3} = 9\times\frac{2.4}{2.4+3} = 4$$

Now setting the 9 V source to zero also gives a potential divider:

$$X_{15} = -15\times\frac{3||6}{3||6+4} = -15\times\frac{2}{2+4} = -5$$

Hence $X=4-5=-1$. [5A]

Mostly correct. Some people open-circuited the unused voltage source rather than short-circuiting it. Some just used nodal analysis to find X – even if done correctly, this gets 0 marks since the question is about whether you know how to use superposition.

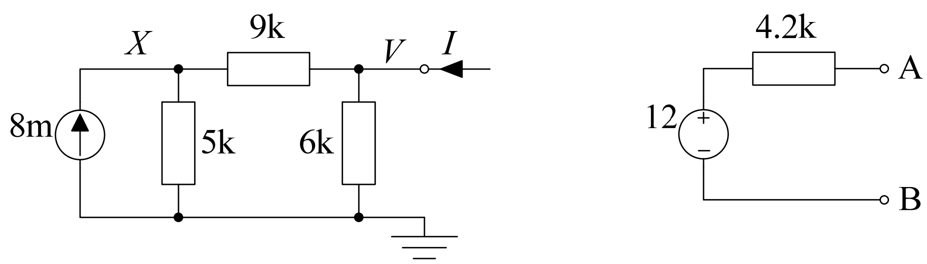
(c) The Thévenin resistance is found by open-circuiting the current source. This leaves 6 k in parallel with $9+5=14$ k. Thus $R_{Th}=\frac{6\times14}{6+14}=4.2$ k.

For the open-circuit voltage, we can think of the circuit as a current divider: the 8 mA current divides between the 5 k and 15 k branches with 2 mA through the 15 k branch. Thus $V_{Th}=2\times6=12$ V.

So the Thévenin equivalent is as shown at the right below. [5A]

An alternative, more laborious, method is to apply an input current I and use nodal analysis to calculate V in the left diagram below. This gives two KCL equations:

$-8+\frac{X}{5}+\frac{X-V}{9}=0$ and $\frac{V-X}{9}+\frac{V}{6}-I=0 \Rightarrow V=12+4.2I=V_{Th}+R_{Th}\times I$



Several took the Thévenin voltage to be 30 V which is the no-load voltage across the current source but not the voltage across A-B. Others calculated the Thévenin voltage of the current source and 5k resistor as 40V, but then applied this to a potential divider of 9k and 6k without taking the 5k into account. When calculating the Thévenin resistance, a few people made the current source a short circuit rather than an open circuit. Quite a good method used by a few people was to convert the current source and 5k resistor into its Thévenin equivalent (40V + 5k) which makes the circuit much simpler.

(d) This is an inverting amplifier so $X=-\frac{30}{5}U-\frac{30}{6}V=-6U-5V$. [5A]

Alternatively, KCL at the –ve input gives

$$\frac{0-U}{5}+\frac{0-V}{6}+\frac{0-X}{30}=0 \Rightarrow X=-6U-5V$$

Correctly done by almost everyone. A few people didn't make the assumption that $V_-=0$ without which it is impossible to solve.

(e) From the diagram we get: $Y = V - GX$, $V = HW$, $W = X - FV$. We need to eliminate V and W from these three equations so we can write Y in terms of X.

Eliminate W from the last two: $V = HW = HX - FHV \Rightarrow V(1 + FH) = HX$

From which $Y = V - GX = \frac{H}{1+FH}X - GX = \frac{H-G-FGH}{1+FH}X$ [5A]

Several people did not appreciate that blocks F, G and H have inputs on the left and outputs on the right; this was not stated explicitly in the original exam question although it is implied by the statement about the adders. Quite a few people got the correct equations but either did not realize that they needed to eliminate V and W or else were unable to do so; this is just conventional algebra. Many people did not even attempt this question. Others treated the blocks as resistors and used nodal analysis.

(f) (i) Diode off gives a potential divider with $Y = 0.5X$.

(ii) Diode on means the left terminal of the diode is at Y-0.7. We do KCL with a supernode enclosing the diode to give:

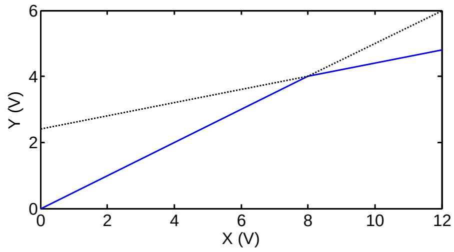
$$\frac{Y-0.7-3.3}{4} + \frac{Y}{12} + \frac{Y-X}{12} = 0 \;\Rightarrow 3Y - 12 + Y + Y - X = 0\,,$$

$$\Rightarrow Y = \frac{X+12}{5} = 0.2X + 2.4.$$

Many were unable to get the correct equation for part (ii). A reverse biased diode acts like an open circuit, whereas a forward biased diode acts like a 0.7 V voltage source. So for part (ii), you replace the diode by a 0.7 V voltage source and then use nodal analysis.

These two expressions are equal when $0.5X = 0.2X + 2.4 \;\Rightarrow X = 8$. This is therefore when the switchover occurs: the diode is off when $X < 8$.

The graph is therefore the solid curve shown below: [5A]



A surprising number of people took the voltage source to be 3 V instead of 3.3 V. Quite a lot of people answered parts (i) and (ii) but lost marks because they did not or could not draw the graph. Many people had the switching point at the wrong input voltage giving a discontinuity in the graph: at the diode switching point, both expressions (i) and (ii) must be true and it is therefore at the intersection of the two graphs. Quite a lot had curved plots on the graph even though they got the correct linear expressions for Y(X).

(g) (i) The RMS current is $\tilde{I} = \frac{230}{1+Z_L} = \frac{230}{25+9j} = 8.14 - 2.93j = 8.66\angle - 19.8°$.

So the complex power is $\left|\tilde{I}\right|^2 Z_L = 1.8 + 0.674j$ kVA $= 1921\angle 20.6°$. [3A]

Although, not required, $I = \tilde{I}\sqrt{2} = 11.52 - 4.14j = 12.23\angle - 19.8°$

Also not required is $\tilde{V_L} = 230 \times \frac{24+9j}{25+9j} = 221.9 + 2.93j = 221.9\angle 0.76°$

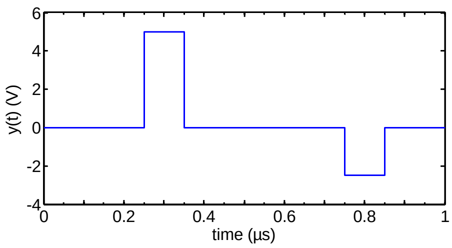
Most people got this correct although often with lots of working. People who did complex arithmetic without using the calculator COMP mode usually made mistakes. A few thought that "complex power" meant just the imaginary part of the answer given above (that would be "reactive power"). There were several wrong expressions for complex power; the correct ones are $S = \tilde{V} \times \tilde{I}^* = \frac{|\tilde{V}|^2}{Z^*} = \left|\tilde{I}\right|^2 Z$ where $\tilde{V}$ and $\tilde{I}$ must of course be the voltage across or current through Z. Several people omitted modulus signs or used the wrong voltage (e.g. 230V which is not the voltage across Z_L). Several people divided the voltage by $\sqrt{2}$ unnecessarily: "230 V RMS" means that it has already been divided by $\sqrt{2}$. Quite a few weirdly used 240 V instead of 230 V.

(ii) $Y_L = \frac{1}{Z_L} = \frac{1}{24+9j} = 36.5 - 13.7$ mS. So $R = \frac{1}{36.5\text{ m}} = \frac{219}{8} = 27.375\ \Omega$ and

$L = \frac{1}{\omega \times 13.7\text{ m}} = \frac{73}{100\pi} = 0.232\text{ H}.$ [2A]

This question is easy if you work in terms of admittances because parallel admittances add. It is quite messy if you use the parallel impedance formula (product over sum) and this is then made much worse if you multiply numerator and denominator by the complex conjugate of the denominator; this is almost always a very bad thing to do in algebra since it doubles the order of the equation you have to solve. Unfortunately the overwhelming majority tried to solve it this way and few of them got the right answer. Some thought $\omega = 50$ rather than $50 \times 2\pi$. Many people just took R=24. Another way of solving it is to work out the load voltage and then to say $P = \frac{|\widetilde{V}_L|^2}{R}$ and a similar expression for Q. This is easier than the parallel impedance approach but more effort than the method given above.

(h) (i) The time taken to travel 50 m is $\frac{50}{u} = 250\text{ ns}$. Therefore the forward-travelling pulse will arrive at Y at $t = 250$ ns and the reflection will arrive at $t = 750$ ns. Therefore the waveform at Y will be: [3A]



Several made the delayed forward pulse start at 200 ns instead of 250 ns. Quite a few people solved the more difficult problem of determining Y when the forward wave at X is a step function rather than a pulse (perhaps because I solved this problem in a study group).

(ii) Rearranging the given equation gives $\rho R + \rho Z_0 = R - Z_0$ from which we get $R = \frac{1+\rho}{1-\rho} Z_0 = \frac{0.5}{1.5} \times 100 = 33\ \Omega.$ [2A]

Almost everyone got this right except for those who did not attempt it.

2. (a) We assume that the opamp gains are infinite and that the overall feedback around each opamp is negative. The latter is not obvious since there are multiple feedback loops. Under these assumptions, the + and – terminals of the opamps will be at the same voltage implying that $V_1 = V_3$ for the left opamp and $V_1 = V_2$ for the right opamp. [2B]

Many did not mention the requirement that the feedback is negative; one person even said it had to be positive. Many mentioned that the inputs take no current; this is true but nothing to do with $V_1 = V_2 = V_3$.

(b) We apply Kirchoff's current law at each of V_1, V_2 and V_3 to get:

At V_1: $(V-W)j\omega C + \frac{V-Y}{R} = 0 \Rightarrow (V-W)j\omega RC = Y - V$

At V_2: $\frac{V-X}{kR} + \frac{V-W}{R} + Vj\omega C = 0 \Rightarrow (V-X) + (V-W)k + Vjk\omega RC = 0$

At V_3: $\frac{V}{R} + \frac{V-Y}{R} = 0 \Rightarrow Y = 2V$ [6A]

A few people took the capacitor impedance to be C rather than $\frac{1}{j\omega C}$. Several people wrote down invalid KCL equations at nodes W or Y that ignored the opamp output current; you can <u>never</u> use KCL at the output node of an opamp in nodal analysis since you don't know what the current in or out of the opamp is. It follows that you can only do KCL at V_1, V_2 and V_3. A few wrote down a single enormous equation which incorporated all the currents leaving all three nodes; this is valid but only gives one of the three needed equations. Even though the nodes have the same voltage, you still get one equation per node.

We have three equations in three unknowns and we need to eliminate V and W from them. Using the third equation to substitute for Y in the first gives:

$$(V-W)j\omega RC = Y - V = V$$

Now use this to substitute for $(V-W)$ in the second equation:

$$(V-X) + \frac{kV}{j\omega RC} + Vjk\omega RC = 0$$

$$\Rightarrow (V-X)j\omega RC + kV + Vk(j\omega RC)^2 = 0$$

$$\Rightarrow V(j\omega RC + k + k(j\omega RC)^2) = j\omega RCX$$

Hence [4A]

$$\frac{Y}{X} = \frac{2V}{X} = \frac{2j\omega RC}{k(j\omega RC)^2 + j\omega RC + k}$$

There was often quite a lot of rather aimless algebra; for nodal analysis you need to (a) write down the KCL equations and then (b) systematically eliminate the unwanted variables (V and W in this case). People who did it methodically like this were usually successful. Substituting one complicated equation into another often gives much messier algebra than multiplying two equations by appropriate constants and adding or subtracting them (as in the elimination of W above).

Quite a few people ignored the instruction in the question to use nodal analysis and tried to solve it by writing down the gains of the two opamp. Even if you do this correctly (which was very rare), it only gives two of the three equations you need. Others treated the circuit as a combination of potential dividers falsely assuming the opamp output currents to be zero.

A few people wrote down the KCL equations and then said "solving these gives the answer". This does not count as a proof.

(c) (i) $\frac{Y}{X}\left(\frac{j}{RC}\right) = \frac{2j}{-k+j+k} = 2.$ [2A]

Most got this right although a few made mistakes with the complex arithmetic: $\frac{2j}{j} = 2j$ *was quite common as was* $kj^2 = k$.

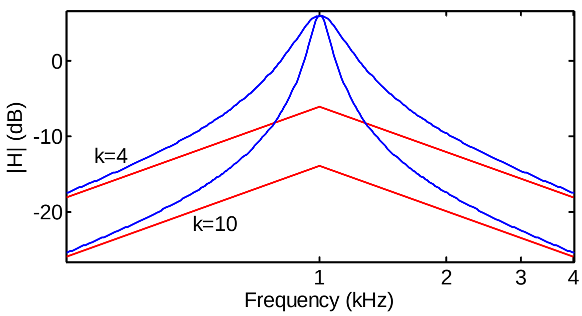
(ii) LF magnitude response asymptote is $\frac{2\omega RC}{k}$ and HF asymptote is $\frac{2}{k\omega RC}$. [2A]

Many people included "j" in the asymptote expressions although the question asked for the magnitude response asymptotes. Several people missed out the ω factor in one or both of the expressions. All the asymptotes are always of the form $A(j\omega)^r$ (omitting the "j" if you want only the magnitude).

(iii) The asymptotes cross when $\left|\frac{2j\omega RC}{k}\right| = \left|\frac{2}{kj\omega RC}\right| \Rightarrow \omega = \frac{1}{RC}$. At this frequency the magnitude asymptote value is $\frac{2}{k}$. [2A]

Many assumed that the gain at the crossing point of the asymptotes was 2 as in part (i). For a quadratic resonance the asymptote crossing point and the transfer function gain differ by a factor of 2ζ. If the modulus signs are omitted then ω becomes complex. Surprisingly many obtained $\omega^2 = \frac{1}{(RC)^2}$ and then wrongly deduced that $\omega = \sqrt{\frac{1}{RC}}$.

(d) The peak gain is $20\log_{10} 2 = 6$ dB for all values of k. The asymptotes cross at gains of $20\log_{10}\frac{2}{k} = -6$ dB and -14 dB. Although the two curves have the same peak value of 6 dB, the asymptotes for the two cases are parallel but separated by 8 dB. The gradient of the asymptotes changes by -2 at the corner frequency. [5A]



Many found this hard, often because of an assumption in part (c)(iii) that the asymptotes cross at a gain of 2. Often the LF asymptote was drawn with a slope of 0 even though the correct asymptote was calculated in part (c)(ii). Quite a few people had two corner frequencies, but, since the denominator cannot be factorized, there is only one, at $\omega = \frac{1}{RC}$. *Many people had non-integer slopes for the asymptotes:* $0.2\omega RC$ *has a slope of +1 not 0.2 as was sometimes claimed.*

Some found the corner frequency as $\omega = \sqrt{\frac{c}{a}}$ *where c and a are coefficients from the denominator polynomial. This is fine so long as you take* $a = kR^2C^2$ *and not just k as was sometimes assumed.*

(e) From part (b), we know that $V = \frac{Y}{2} = X$ and $W = \frac{j\omega RC-1}{2j\omega RC}Y = \frac{j-1}{j}X = \frac{1+j}{1}X$.

The current from the rightmost opamp is $\frac{Y-V_3}{R} + \frac{Y-V_1}{R} = \frac{2(Y-V)}{R} = \frac{2X}{R}$ with a peak value of $\frac{2A}{R}$.

The current from the leftmost opamp is $\frac{W-V_2}{R} + \frac{W-V_1}{1/j\omega C} = (W-V)\left(\frac{1}{R} + j\omega C\right)$. Substituting for W and V gives $W - V = (1 + j - 1)X = jX$.

Hence the current is $\frac{j}{R}(1+j)X = \frac{j-1}{R}X$ with a peak value of $\frac{\sqrt{2}A}{R}$. [5A]

Working out the current supplied by an opamp just amounts to applying Kirchoff's current law at the opamp output node. However few people were able to do this correctly.

(f) From the previous part we require $\frac{2A}{R} = \frac{20}{R} \leq 4$ mA $\Rightarrow R \geq 5$ kΩ. Using this value, $C = \frac{1}{2\pi\times 1000\times R} = 31.8$ nF. [2D]

Many people missed out the factor of 2π.

3. (a) (i) The transfer function $\frac{Y}{X}(j\omega) = \frac{1}{j\omega RC+1}$ where $R = 20\text{ k}$ may be determined using nodal analysis or a potential divider.

Using the values given, $\omega RC = 3.2$ and, as a phasor, $X = -10j$. From this, $Y = -10j \times \frac{1}{1+3.2j} = -10j \times (0.089 - 0.285j) = -2.85 - 0.89j$ giving $y(t) = -\frac{800}{281}\cos\omega t + \frac{250}{281}\sin\omega t = -2.85\cos\omega t + 0.89\sin\omega t$. [4A]

Mostly done well. A few people forgot to multiply by $-10j$. Those who did their own complex algebra (rather than using the COMP calculator mode) often made errors.

(ii) With the switch closed, we now have $R = 10\text{ k} \Rightarrow \omega RC = 1.6$. Hence:

$Y = -10j \times \frac{1}{1+1.6j} = -10j \times (0.281 - 0.449j) = -4.49 - 2.81j$ giving $y(t) = -\frac{400}{89}\cos\omega t + \frac{250}{89}\sin\omega t = -4.49\cos\omega t + 2.81\sin\omega t$. [4A]

Also correct from most people.

(b) Substituting $t = 0$ into the answer to (a)(i) gives $y(0-) = -2.85$. For $t \geq 0$, $y(t) = -4.49\cos\omega t + 2.81\sin\omega t + Ae^{-\frac{t}{\tau_C}}$ where $\tau_C = 0.8\text{ ms}$ since the switch is now closed.

Because the voltage across the capacitor cannot change instantly, we must have $y(0+) = y(0-) = -2.85$. So, substituting $t = 0$ into the expression above gives $-2.85 = y(0+) = -4.49 + A$ from which $A = \frac{41200}{25009} = 1.64$.

So, for $t \geq 0$,

$$y(t) = -4.49\cos\omega t + 2.81\sin\omega t + 1.64e^{-\frac{t}{\tau_C}}$$

where $\tau_C = \frac{1}{1250} = 0.8\text{ ms}$. [8A]

Quite a few people thought that $\tau = \frac{1}{RC}$ instead of RC; several others used the formula $\tau = \frac{L}{R}$ and took $L = 80\text{n}$. Several gave the transient amplitude the wrong sign: The transient + the new steady state must equal the correct value (which in this case equals the old steady state). A small number had transient amplitudes that were either complex or else time-varying; both of these are always wrong: the transient amplitude is always a real-valued constant. Several people calculated the correct transient but had the steady state a DC value (either -4.49 or even -2.85 or occasionally $-4.49 - 2.81j$). Some used a formula for the transient amplitude of $A = y(0) - y(\infty)$: this formula is only true for DC input signals and is not valid for sine waves (for a sine wave $y(\infty)$ doesn't even make sense). Several converted the steady states into the form $A\cos(\omega t + \phi)$ which makes the calculation messier and so is a bad idea.

(c) The transient amplitude is equal to the difference between the old and the new steady state waveforms. It will therefore equal zero iff the old and new steady state waveforms have the same value at $t = t_0$. That is:

$$-2.85\cos\omega t + 0.89\sin\omega t = -4.49\cos\omega t + 2.81\sin\omega t$$

$$\Leftrightarrow 1.64\cos\omega t - 1.92\sin\omega t = 0 \Leftrightarrow 103\cos\omega t - 120\sin\omega t = 0$$

$$\Leftrightarrow \tan\omega t = \frac{103}{120} = 0.858 \Leftrightarrow \omega t = \operatorname{atan} 0.858 + k\pi = 0.709 + k\pi$$

The first positive t for which this is true is when $t = \frac{0.709}{\omega} = 0.355$ ms. [6A]

Several people missed the point of this part and thought it was asking when the transient from part (b) would decay to zero (which is of course "never") or else the first time the waveform from part (b) equalled zero. Some took the arctangent in degrees rather than radians (personally, I always leave my calculator set to radians to avoid this sort of error).

(d) From part (b), at $t_1 = \frac{\pi}{\omega} = 1.57$ ms, $\omega t_1 = \pi$ and so

$$y(t_1 -) = -4.49\cos\omega t + 2.81\sin\omega t + 1.64e^{-\frac{t}{\tau_C}} = 4.49 + 0 + 1.64e^{-0.625\pi}$$
$$= 4.49 + 1.64e^{-1.963} = 4.49 + 1.64 * 0.14 = 4.49 + 0.23 = 4.72.$$

From part (a), the steady state output for $t > t_1$ is $y_{SS}(t) = -2.85\cos\omega t + 0.89\sin\omega t$, so $y_{SS}(t_1) = +2.85$.

The full output expression for $t > t_1$ is $y(t) = y_{SS}(t) + Be^{\frac{t-t_1}{\tau_O}}$

Because of the capacitor, y must be continuous, so

$$y(t_1 +) = y_{SS}(t_1) + B = 2.85 + B = y(t_1 -) = 4.72$$

Hence the transient amplitude is $B = 4.72 - 2.85 = 1.87$ and the complete waveform is

$$y(t) = -2.85\cos\omega t + 0.89\sin\omega t + 1.87e^{-\frac{t-t_1}{\tau_O}}$$
$$= -2.85\cos\omega t + 0.89\sin\omega t + 4.99e^{-\frac{t}{\tau_O}}$$

where now the time constant is $\tau_O = \frac{1}{625} = 1.6$ ms since the switch is open. [8A]

The commonest mistake was to neglect the exponential transient term when calculating $y(t_1-)$. A few people gave a final answer that included two transients: the original one plus a new one starting at t_1. In circuits that do not include switches, this approach can be correct mathematically, but it is incorrect for this circuit and, in any case, it is always better to lump everything together as a single new transient. As in the previous part, some people took ωt to be in degrees rather than radians which gives the wrong answer. Several people wrongly used $\omega(t - t_1)$ as the argument for the cos and sin terms.