Revision Lecture 1: Nodal Analysis & Frequency Responses

- Exam
- Nodal Analysis
- Op Amps
- Block Diagrams
- Diodes
- Reactive Components
- Phasors
- Plotting Frequency Responses
- LF and HF Asymptotes
- Corner frequencies (linear factors)
- Sketching Magnitude Responses (linear factors)
- Filters
- Resonance

Revision Lecture 1: Nodal Analysis & Frequency Responses
Exam

Exam Format

Question 1 (40%): eight short parts covering the whole syllabus.

Questions 2 and 3: single topic questions (answer both)

Syllabus

Does include: Everything in the notes.

Nodal Analysis

(1) Pick reference node.
Nodal Analysis

(1) Pick reference node.

(2) Label nodes: 8
Nodal Analysis

(1) Pick reference node.

(2) Label nodes: 8, X
Nodal Analysis

(1) Pick reference node.

(2) Label nodes: $8$, $X$ and $X + 2$ since it is joined to $X$ via a voltage source.
Nodal Analysis

(1) Pick reference node.

(2) Label nodes: 8, \(X\) and \(X + 2\) since it is joined to \(X\) via a voltage source.

(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single “super-node” giving one equation

\[
\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0
\]
(1) Pick reference node.

(2) Label nodes: 8, X and X + 2 since it is joined to X via a voltage source.

(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single “super-node” giving one equation

\[
\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0
\]

Ohm’s law always involves the difference between the voltages at either end of a resistor. (Obvious but easily forgotten)
Nodal Analysis

(1) Pick reference node.

(2) Label nodes: 8, X and X + 2 since it is joined to X via a voltage source.

(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single “super-node” giving one equation

\[
\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0
\]

Ohm’s law always involves the difference between the voltages at either end of a resistor. (Obvious but easily forgotten)

(4) Solve the equations: \( X = 4 \)
Op Amps

• **Ideal Op Amp:** (a) Zero input current, (b) Infinite gain
  (b) ⇒ \( V_+ = V_- \) provided the circuit has negative feedback.

• **Negative Feedback:** An increase in \( V_{out} \) makes \( (V_+ - V_-) \) decrease.

\[ \begin{align*}
&\text{X} \\
&\downarrow \\
&\text{R}_2=3k \\
&\uparrow \\
&\text{Y} \\
&\downarrow \\
&\text{R}_1=1k
\end{align*} \]
Op Amps

- **Ideal Op Amp:** (a) Zero input current, (b) Infinite gain
  (b) ⇒ $V_+ = V_-$ provided the circuit has **negative feedback**.

- **Negative Feedback:** An **increase** in $V_{out}$ makes $(V_+ - V_-)$ **decrease**.

### Non-inverting amplifier

\[
Y = (1 + \frac{3}{1}) \times X
\]
Op Amps

- **Ideal Op Amp:** (a) Zero input current, (b) Infinite gain
  (b) ⇒ $V_+ = V_-$ provided the circuit has **negative feedback**.

- **Negative Feedback:** An increase in $V_{out}$ makes $(V_+ - V_-)$ decrease.

Non-inverting amplifier

$$Y = \left(1 + \frac{3}{1}\right)X$$

Inverting amplifier

$$Y = -\frac{8}{1}X_1 + \frac{8}{2}X_2 + \frac{8}{2}X_3$$

![Non-inverting amplifier circuit diagram](image1)

![Inverting amplifier circuit diagram](image2)
Op Amps

- **Ideal Op Amp:** (a) Zero input current, (b) Infinite gain
  \( (b) \Rightarrow V_+ = V_- \) provided the circuit has negative feedback.

- **Negative Feedback:** An increase in \( V_{out} \) makes \( (V_+ - V_-) \) decrease.

### Non-inverting amplifier

\[
Y = \left( 1 + \frac{3}{1} \right) X
\]

### Inverting amplifier

\[
Y = \frac{-8}{1} X_1 + \frac{-8}{2} X_2 + \frac{-8}{2} X_3
\]

**Nodal Analysis:** Use two separate KCL equations at \( V_+ \) and \( V_- \).
Op Amps

- **Ideal Op Amp**: (a) Zero input current, (b) Infinite gain
  
  \[ (b) \Rightarrow V_+ = V_- \] provided the circuit has negative feedback.

- **Negative Feedback**: An increase in \( V_{out} \) makes \( (V_+ - V_-) \) decrease.

  
  **Non-inverting amplifier**
  
  \[ Y = \left(1 + \frac{3}{1}\right) X \]

  
  **Inverting amplifier**
  
  \[ Y = -\frac{8}{1} X_1 + -\frac{8}{2} X_2 + -\frac{8}{2} X_3 \]

  
  **Nodal Analysis**: Use two separate KCL equations at \( V_+ \) and \( V_- \).
  
  Do not do KCL at \( V_{out} \) except to find the op-amp output current.
Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked + for add or − for subtract.

![Block Diagram](image-url)
Block Diagrams

Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked $+$ for add or $-$ for subtract.

To analyse:

1. Label the inputs, the outputs and the output of each adder.
Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked $+$ for add or $-$ for subtract.

To analyse:

1. Label the inputs, the outputs and the output of each adder.

2. Write down an equation for each variable:

   - $U = X - FGU$
   - Follow signals back though the blocks until you meet a labelled node.
Block Diagrams

Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked $+$ for add or $-$ for subtract.

To analyse:

1. Label the inputs, the outputs and the output of each adder.

2. Write down an equation for each variable:
   - $U = X - FGU$, $Y = FU + FGHU$
   - Follow signals back though the blocks until you meet a labelled node.
Block Diagrams

Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked + for add or − for subtract.

To analyse:

1. Label the inputs, the outputs and the output of each adder.

2. Write down an equation for each variable:
   - \( U = X - FGU \), \( Y = FU + FGHU \)
   - Follow signals back though the blocks until you meet a labelled node.

3. Solve the equations (eliminate intermediate node variables):
   - \( U(1 + FG) = X \)
Block Diagrams

Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked + for add or − for subtract.

To analyse:

1. Label the inputs, the outputs and the output of each adder.
2. Write down an equation for each variable:
   - \( U = X - FGU \), \( Y = FU + FGHU \)
   - Follow signals back though the blocks until you meet a labelled node.
3. Solve the equations (eliminate intermediate node variables):
   - \( U(1 + FG) = X \) \( \Rightarrow \) \( U = \frac{X}{1+FG} \)
Block Diagrams

Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked + for add or – for subtract.

To analyse:

1. Label the inputs, the outputs and the output of each adder.
2. Write down an equation for each variable:
   - \[ U = X - FGU, \quad Y = FU + FGHU \]
   - Follow signals back though the blocks until you meet a labelled node.
3. Solve the equations (eliminate intermediate node variables):
   - \[ U(1 + FG) = X \quad \Rightarrow \quad U = \frac{X}{1+FG} \]
   - \[ Y = (1 + GH)FU \]
Block Diagrams

Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked + for add or − for subtract.

\[
\begin{align*}
X & \quad U & \quad F & \quad G & \quad H & \quad Y \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
\end{align*}
\]

To analyse:

1. Label the inputs, the outputs and the output of each adder.
2. Write down an equation for each variable:
   - \( U = X - FGU \), \( Y = FU + FGHU \)
   - Follow signals back through the blocks until you meet a labelled node.
3. Solve the equations (eliminate intermediate node variables):
   - \( U(1 + FG) = X \quad \Rightarrow \quad U = \frac{X}{1+FG} \)
   - \( Y = (1 + GH)FU = \frac{(1+GH)F}{1+FG}X \)
Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked $+$ for add or $-$ for subtract.

![Block Diagram](image)

To analyse:

1. Label the inputs, the outputs and the output of each adder.
2. Write down an equation for each variable:
   - $U = X - FGU$, $Y = FU + FGHU$
   - Follow signals back though the blocks until you meet a labelled node.
3. Solve the equations (eliminate intermediate node variables):
   - $U(1 + FG) = X \implies U = \frac{X}{1+FG}$
   - $Y = (1 + GH)FU = \frac{(1+GH)F}{1+FG}X$

[Note: “Wires” carry information not current: KCL not valid]
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- **Off**: $I_D = 0$, $V_D < 0.7$
- **On**: $V_D = 0.7$, $I_D > 0$

(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- **Off**: \( I_D = 0, \ V_D < 0.7 \ \Rightarrow \ \text{Diode = open circuit} \\
- **On**: \( V_D = 0.7, \ I_D > 0 \)

(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- **Assume Diode Off**
  \[ X = 5 + 2 = 7 \]
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- Off: $I_D = 0, \ V_D < 0.7 \ \Rightarrow \ \text{Diode} = \text{open circuit}$
- On: $V_D = 0.7, \ I_D > 0$

(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- Assume Diode Off

$$X = 5 + 2 = 7$$
$$V_D = 2$$
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- Off: \( I_D = 0, \ V_D < 0.7 \Rightarrow \text{Diode = open circuit} \)
- On: \( V_D = 0.7, \ I_D > 0 \)

(a) Guess the mode  
(b) Do nodal analysis assuming the equality condition  
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- Assume Diode Off

\[
X = 5 + 2 = 7 \\
V_D = 2 \quad \text{Fail:} \ V_D > 0.7
\]
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- Off: \( I_D = 0, V_D < 0.7 \) \( \Rightarrow \) Diode = open circuit
- On: \( V_D = 0.7, I_D > 0 \) \( \Rightarrow \) Diode = 0.7 V voltage source

(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- Assume Diode Off
  \[
  X = 5 + 2 = 7 \\
  V_D = 2 \quad \text{Fail: } V_D > 0.7
  \]
- Assume Diode On
  \[
  X = 5 + 0.7 = 5.7
  \]
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- **Off:** \( I_D = 0, \ V_D < 0.7 \ \Rightarrow \ \text{Diode} = \text{open circuit} \\
- **On:** \( V_D = 0.7, \ I_D > 0 \ \Rightarrow \ \text{Diode} = 0.7 \ \text{V} \ \text{voltage source} \\

(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- **Assume Diode Off**
  \[ X = 5 + 2 = 7 \]
  \[ V_D = 2 \ \text{Fail:} \ V_D > 0.7 \]

- **Assume Diode On**
  \[ X = 5 + 0.7 = 5.7 \]
  \[ I_D + \frac{0.7}{1k} = 2 \ \text{mA} \]
Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- **Off**: $I_D = 0$, $V_D < 0.7$  $\Rightarrow$  Diode = open circuit
- **On**: $V_D = 0.7$, $I_D > 0$  $\Rightarrow$  Diode = $0.7$ V voltage source

(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- **Assume Diode Off**
  \[ X = 5 + 2 = 7 \]
  \[ V_D = 2 \quad \text{Fail: } V_D > 0.7 \]

- **Assume Diode On**
  \[ X = 5 + 0.7 = 5.7 \]
  \[ I_D + \frac{0.7}{1\,k} = 2 \, mA \quad \text{OK: } I_D > 0 \]
Reactive Components

- **Impedances:** $R, j\omega L, \frac{1}{j\omega C} = -\frac{j}{\omega C}$.

  - **Admittances:** $\frac{1}{R}, \frac{1}{j\omega L} = -\frac{j}{\omega L}, j\omega C$. 
Reactive Components

- **Impedances:** \( R, j\omega L, \frac{1}{j\omega C} = \frac{-j}{\omega C} \).
  - **Admittances:** \( \frac{1}{R}, \frac{1}{j\omega L} = \frac{-j}{\omega L}, j\omega C \)

- In a capacitor or inductor, the Current and Voltage are 90° apart:
  - **CIVIL:** Capacitor - \( I \) leads \( V \); Inductor - \( I \) lags \( V \)
Reactive Components

- **Impedances:** $R, j\omega L, \frac{1}{j\omega C} = -\frac{j}{\omega C}$
  - **Admittances:** $\frac{1}{R}, \frac{1}{j\omega L} = -\frac{j}{\omega L}, j\omega C$

- In a capacitor or inductor, the Current and Voltage are $90^\circ$ apart:
  - **CIVIL:** Capacitor - $I$ leads $V$; Inductor - $I$ lags $V$

- **Average current** (or DC current) through a **capacitor** is always zero
- **Average voltage** across an **inductor** is always zero
Reactive Components

- **Impedances:** \( R, j\omega L, \frac{1}{j\omega C} = \frac{-j}{\omega C} \).
  - **Admittances:** \( \frac{1}{R}, \frac{1}{j\omega L} = \frac{-j}{\omega L}, j\omega C \)

- In a capacitor or inductor, the Current and Voltage are 90° apart:
  - **CIVIL:** Capacitor - \( I \) leads \( V \); Inductor - \( I \) lags \( V \)

- Average current (or DC current) through a capacitor is always zero
- Average voltage across an inductor is always zero
- Average power absorbed by a capacitor or inductor is always zero
Phasors

A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

Waveform

\[ x(t) = F \cos \omega t - G \sin \omega t \]

Phasor

\[ X = F + jG \]
Phasors

A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

Waveform

\[ x(t) = F \cos \omega t - G \sin \omega t \]

Phasor

\[ X = F + jG \]

[Note minus sign]
A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

\[
x(t) = F \cos \omega t - G \sin \omega t \\
x(t) = A \cos (\omega t + \theta)
\]

\[
X = F + jG \\
X = Ae^{j\theta} = A\angle\theta
\]

[Note minus sign]
A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

**Waveform**

\[ x(t) = F \cos \omega t - G \sin \omega t \]

\[ x(t) = A \cos (\omega t + \theta) \]

\[ \max (x(t)) = A \]

**Phasor**

\[ X = F + jG \]

\[ X = Ae^{j\theta} = A \angle \theta \]

\[ |X| = A \]
A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

**Waveform**

\[ x(t) = F \cos \omega t - G \sin \omega t \]

\[ x(t) = A \cos (\omega t + \theta) \]

\[ \text{max} (x(t)) = A \]

**Phasor**

\[ X = F + jG \]

\[ X = Ae^{j\theta} = A \angle \theta \]

\[ |X| = A \]

\( x(t) \) is the projection of a rotating rod onto the real (horizontal) axis.

\( X = F + jG \) is its starting position at \( t = 0 \).
A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

Waveform

\[ x(t) = F \cos \omega t - G \sin \omega t \]

\[ x(t) = A \cos (\omega t + \theta) \]

\[ \max (x(t)) = A \]

Phasor

\[ X = F + jG \]

\[ X = Ae^{j\theta} = A \angle \theta \]

\[ |X| = A \]

\( x(t) \) is the projection of a rotating rod onto the real (horizontal) axis.

\( X = F + jG \) is its starting position at \( t = 0 \).

RMS Phasor:

\[ \tilde{V} = \frac{1}{\sqrt{2}} V \]
Phasors

A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

Waveform
\[ x(t) = F \cos \omega t - G \sin \omega t \]

\[ x(t) = A \cos (\omega t + \theta) \]

\[ \max(x(t)) = A \]

RMS Phasor: \( \tilde{V} = \frac{1}{\sqrt{2}} V \)  \( \Rightarrow \)  \( |\tilde{V}|^2 = \langle x^2(t) \rangle \)

Phasor
\[ X = F + jG \]

[Note minus sign]

\[ X = Ae^{j\theta} = A \angle \theta \]

\[ |X| = A \]

\( x(t) \) is the projection of a rotating rod onto the real (horizontal) axis.

\( X = F + jG \) is its starting position at \( t = 0 \).
A phasor represents a **time-varying sinusoidal waveform** by a **fixed complex number**.

**Waveform**

\[ x(t) = F \cos \omega t - G \sin \omega t \]

**Phasor**

\[ X = F + jG \quad \text{[Note minus sign]} \]

\[ x(t) = A \cos (\omega t + \theta) \]

\[ X = Ae^{j\theta} = A \angle \theta \]

\[ \max (x(t)) = A \]

\[ |X| = A \]

\( x(t) \) is the projection of a rotating rod onto the real (horizontal) axis.

\( X = F + jG \) is its starting position at \( t = 0 \).

**RMS Phasor**

\[ \tilde{V} = \frac{1}{\sqrt{2}} V \quad \Rightarrow \quad \left| \tilde{V} \right|^2 = \langle x^2(t) \rangle \]

**Complex Power**

\[ \tilde{V} \tilde{I}^* = |\tilde{I}|^2 Z = \frac{\left| \tilde{V} \right|^2}{Z^*} = P + jQ \]

\( P \) is average power (Watts), \( Q \) is reactive power (VARs)
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.

- If \( \frac{V_2}{V_1} = A (j\omega)^k = A \times j^k \times \omega^k \) (where \( A \) is real)
  - magnitude is a straight line with gradient \( k \):
    \[
    \log \left| \frac{V_2}{V_1} \right| = \log |A| + k \log \omega
    \]
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.

- If \( \frac{V_2}{V_1} = A (j\omega)^k = A \times j^k \times \omega^k \) (where \( A \) is real)
  - magnitude is a straight line with gradient \( k \):
    \[
    \log \left| \frac{V_2}{V_1} \right| = \log |A| + k \log \omega
    \]
  - phase is a constant \( k \times \frac{\pi}{2} \) (+\( \pi \) if \( A < 0 \)):
    \[
    \angle \left( \frac{V_2}{V_1} \right) = \angle A + k \angle j = \angle A + k \frac{\pi}{2}
    \]
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.

- If $\frac{V_2}{V_1} = A (j\omega)^k = A \times j^k \times \omega^k$ (where $A$ is real)
  - magnitude is a straight line with gradient $k$:
    $$\log \left| \frac{V_2}{V_1} \right| = \log |A| + k \log \omega$$
  - phase is a constant $k \times \frac{\pi}{2}$ ($+\pi$ if $A < 0$):
    $$\angle \left( \frac{V_2}{V_1} \right) = \angle A + k \angle j = \angle A + k \frac{\pi}{2}$$

- Measure magnitude response using decibels $= 20 \log_{10} \left| \frac{V_2}{V_1} \right|$. 

Exam
Nodal Analysis
Op Amps
Block Diagrams
Diodes
Reactive Components
Phasors
Plotting Frequency Responses
LF and HF Asymptotes
Corner frequencies (linear factors)
Sketching Magnitude Responses (linear factors)
Filters
Resonance
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use **log scale** for frequency and magnitude and **linear scale** for phase: this gives graphs that can be approximated by straight line segments.

- If \( \frac{V_2}{V_1} = A \left(j \omega \right)^k = A \times j^k \times \omega^k \) (where \( A \) is real)
  - magnitude is a straight line with gradient \( k \):
    \[
    \log \left| \frac{V_2}{V_1} \right| = \log |A| + k \log \omega
    \]
  - phase is a constant \( k \times \frac{\pi}{2} \) (+\( \pi \) if \( A < 0 \)):
    \[
    \angle \left( \frac{V_2}{V_1} \right) = \angle A + k \angle j = \angle A + k \frac{\pi}{2}
    \]

- Measure magnitude response using **decibels** = \( 20 \log_{10} \left| \frac{V_2}{V_1} \right| \).
  A gradient of \( k \) on log axes is equivalent to \( 20k \) dB/decade (\( \times 10 \) in frequency) or \( 6k \) dB/octave (\( \times 2 \) in frequency).
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.

- If \( \frac{V_2}{V_1} = A (j\omega)^k = A \times j^k \times \omega^k \) (where \( A \) is real)
  
  - magnitude is a straight line with gradient \( k \):
    \[
    \log \left| \frac{V_2}{V_1} \right| = \log |A| + k \log \omega
    \]
  
  - phase is a constant \( k \times \frac{\pi}{2} \) (+\( \pi \) if \( A < 0 \)):
    \[
    \angle \left( \frac{V_2}{V_1} \right) = \angle A + k \angle j = \angle A + k \frac{\pi}{2}
    \]

- Measure magnitude response using decibels \( = 20 \log_{10} \left| \frac{V_2}{V_1} \right| \).
  A gradient of \( k \) on log axes is equivalent to \( 20k \) dB/decade (\( \times 10 \) in frequency) or \( 6k \) dB/octave (\( \times 2 \) in frequency).

\[
\frac{Y}{X} = \frac{1}{R} + \frac{j\omega C}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1}
\]
Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.

- If \( \frac{V_2}{V_1} = A \left( j\omega \right)^k = A \times j^k \times \omega^k \) (where \( A \) is real)
  - magnitude is a straight line with gradient \( k \):
    \[
    \log \left| \frac{V_2}{V_1} \right| = \log |A| + k \log \omega
    \]
  - phase is a constant \( k \times \frac{\pi}{2} (+\pi \text{ if } A < 0) \):
    \[
    \angle \left( \frac{V_2}{V_1} \right) = \angle A + k \angle j = \angle A + k \frac{\pi}{2}
    \]

- Measure magnitude response using decibels \( = 20 \log_{10} \left| \frac{V_2}{V_1} \right| \).
  A gradient of \( k \) on log axes is equivalent to \( 20k \text{ dB/decade} \) (\( \times 10 \) in frequency) or \( 6k \text{ dB/octave} \) (\( \times 2 \) in frequency).

\[
\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega^2 C + 1} \quad \text{where} \quad \omega_c = \frac{1}{RC}
\]
LF and HF Asymptotes

- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
  - The terms with the lowest power of $j\omega$ on top and bottom gives the low-frequency asymptote
  - The terms with the highest power of $j\omega$ on top and bottom gives the high-frequency asymptote.
LF and HF Asymptotes

- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
  - The terms with the **lowest** power of $j\omega$ on top and bottom gives the **low-frequency** asymptote.
  - The terms with the **highest** power of $j\omega$ on top and bottom gives the **high-frequency** asymptote.

Example: $H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$
LF and HF Asymptotes

- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
  - The terms with the lowest power of $j\omega$ on top and bottom gives the low-frequency asymptote.
  - The terms with the highest power of $j\omega$ on top and bottom gives the high-frequency asymptote.

Example: $H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$

LF: $H(j\omega) \approx 1.2j\omega$
LF and HF Asymptotes

- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
  - The terms with the lowest power of $j\omega$ on top and bottom gives the low-frequency asymptote.
  - The terms with the highest power of $j\omega$ on top and bottom gives the high-frequency asymptote.

Example: $H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$

LF: $H(j\omega) \simeq 1.2j\omega$

HF: $H(j\omega) \simeq 20(j\omega)^{-1}$
Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form \((a j \omega + b) \simeq \begin{cases} b & \omega < \left| \frac{b}{a} \right| \\ a j \omega & \omega > \left| \frac{b}{a} \right| \end{cases}\).
Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form 
  \[(a j \omega + b) \simeq \begin{cases} 
  b & \omega < \left| \frac{b}{a} \right| \\
  a j \omega & \omega > \left| \frac{b}{a} \right| 
  \end{cases} \]

- At the corner frequency, \( \omega_c = \left| \frac{b}{a} \right| \), the slope of the magnitude response changes by \( \pm 1 \) (\( \pm 20 \text{ dB/decade} \)) because the linear term introduces another factor of \( \omega \) into the numerator or denominator for \( \omega > \omega_c \).
Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form \((a_j\omega + b) \simeq \begin{cases} b & \omega < |b/a| \\ a_j\omega & \omega > |b/a| \end{cases}\).

- At the corner frequency, \(\omega_c = |b/a|\), the slope of the magnitude response changes by \(\pm 1\) (\(\pm 20\) dB/decade) because the linear term introduces another factor of \(\omega\) into the numerator or denominator for \(\omega > \omega_c\).

- The phase changes by \(\pm \frac{\pi}{2}\) because the linear term introduces another factor of \(j\) into the numerator or denominator for \(\omega > \omega_c\).

  - The phase change is gradual and takes place over the range \(0.1\omega_c\) to \(10\omega_c\) (\(\pm \frac{\pi}{2}\) spread over two decades in \(\omega\)).
Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form \((a j\omega + b) \approx \begin{cases} b & \omega < \left|\frac{b}{a}\right| \\ a j\omega & \omega > \left|\frac{b}{a}\right| \end{cases}\).

- At the corner frequency, \(\omega_c = \left|\frac{b}{a}\right|\), the slope of the magnitude response changes by \(\pm 1\) (\(\pm 20\) dB/decade) because the linear term introduces another factor of \(\omega\) into the numerator or denominator for \(\omega > \omega_c\).

- The phase changes by \(\pm \frac{\pi}{2}\) because the linear term introduces another factor of \(j\) into the numerator or denominator for \(\omega > \omega_c\).
  - The phase change is gradual and takes place over the range \(0.1\omega_c\) to \(10\omega_c\) (\(\pm \frac{\pi}{2}\) spread over two decades in \(\omega\)).

- When \(a\) and \(b\) are real and positive, it is often convenient to write \((a j\omega + b) = b \left(\frac{j\omega}{\omega_c} + 1\right)\).
Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form \((a j\omega + b) \simeq \begin{cases} \frac{b}{a} & \omega < \left|\frac{b}{a}\right| \\ a j\omega & \omega > \left|\frac{b}{a}\right| \end{cases}\).

- At the corner frequency, \(\omega_c = \left|\frac{b}{a}\right|\), the slope of the magnitude response changes by \(\pm 1\) (\(\pm 20\, \text{dB/decade}\)) because the linear term introduces another factor of \(\omega\) into the numerator or denominator for \(\omega > \omega_c\).

- The phase changes by \(\pm \frac{\pi}{2}\) because the linear term introduces another factor of \(j\) into the numerator or denominator for \(\omega > \omega_c\).
  - The phase change is **gradual** and takes place over the range \(0.1\omega_c\) to \(10\omega_c\) (\(\pm \frac{\pi}{2}\) spread over two decades in \(\omega\)).

- When \(a\) and \(b\) are real and positive, it is often convenient to write \((a j\omega + b) = b \left(\frac{j\omega}{\omega_c} + 1\right)\).

- The **corner frequencies** are the absolute values of the roots of the numerator and denominator polynomials (values of \(j\omega\)).
1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots

2. Find LF and HF asymptotes. $A (j\omega)^k$ has a slope of $k$; substitute $\omega = \omega_c$ to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by $\pm 1$ ($\pm 20$ dB/decade). $+$ for numerator (top line) and $-$ for denominator (bottom line).

4. $|H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)|$ if the gradient between them is $k$. 
Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots

2. Find LF and HF asymptotes. $A (j\omega)^k$ has a slope of $k$; substitute $\omega = \omega_c$ to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by $\pm 1$ ($\pm 20$ dB/decade). $+$ for numerator (top line) and $-$ for denominator (bottom line).

4. $|H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)|$ if the gradient between them is $k$.

$$H(j\omega) = 1.2 \frac{j\omega (\frac{j\omega}{12}+1)}{(\frac{j\omega}{1}+1)(\frac{j\omega}{4}+1)(\frac{j\omega}{50}+1)}$$
Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots

2. Find LF and HF asymptotes. \( A (j\omega)^k \) has a slope of \( k \); substitute \( \omega = \omega_c \) to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by \( \pm 1 \) \( (\pm 20 \text{ dB/decade}) \). \( + \) for numerator (top line) and \( - \) for denominator (bottom line).

4. \[
|H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)| \text{ if the gradient between them is } k.
\]

\[
H(j\omega) = 1.2 \frac{j\omega(\frac{j\omega}{12}+1)}{(\frac{j\omega}{1}+1)(\frac{j\omega}{4}+1)(\frac{j\omega}{50}+1)}
\]

LF: \( 1.2j\omega \Rightarrow |H(j1)| = 1.2 \ (1.6 \text{ dB}) \)
Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots

2. Find LF and HF asymptotes. \( A (j\omega)^k \) has a slope of \( k \); substitute \( \omega = \omega_c \) to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by \( \pm 1 \) \( (\pm 20 \text{ dB/decade}) \). + for numerator (top line) and − for denominator (bottom line).

4. \[ |H(j\omega_2)| = \left( \frac{\omega_2}{\omega_1} \right)^k |H(j\omega_1)| \] if the gradient between them is \( k \).

\[ H(j\omega) = 1.2 \frac{j\omega}{(j\omega + 1)(\frac{j\omega}{12} + 1)(\frac{j\omega}{4} + 1)(\frac{j\omega}{50} + 1)} \]

LF: \( 1.2j\omega \Rightarrow |H(j1)| = 1.2 \ (1.6 \text{ dB}) \)

\[ |H(j4)| = \left( \frac{4}{1} \right)^0 \times 1.2 = 1.2 \]
Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots

2. Find LF and HF asymptotes. \( A (j\omega)^k \) has a slope of \( k \); substitute \( \omega = \omega_c \) to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by \( \pm 1 \) (\( \pm 20 \) dB/decade). + for numerator (top line) and − for denominator (bottom line).

4. \(|H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)|\) if the gradient between them is \( k \).

\[
H(j\omega) = 1.2 \frac{j\omega \left(\frac{j\omega}{12} + 1\right)}{(\frac{j\omega}{1} + 1)(\frac{j\omega}{4} + 1)(\frac{j\omega}{50} + 1)}
\]

LF: \(1.2j\omega \Rightarrow |H(j1)| = 1.2 (1.6 \text{ dB})\)

\[|H(j4)| = \left(\frac{4}{1}\right)^0 \times 1.2 = 1.2\]

\[|H(j12)| = \left(\frac{12}{4}\right)^{-1} \times 1.2 = 0.4\]
Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots

2. Find LF and HF asymptotes. $A (j\omega)^k$ has a slope of $k$; substitute $\omega = \omega_c$ to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by $\pm 1$ ($\pm 20$ dB/decade). $+$ for numerator (top line) and $-$ for denominator (bottom line).

4. $|H(j\omega_2)| = \left( \frac{\omega_2}{\omega_1} \right)^k |H(j\omega_1)|$ if the gradient between them is $k$.

\[
H(j\omega) = 1.2 \frac{j\omega\left(\frac{j\omega}{12} + 1\right)}{(\frac{j\omega}{12} + 1)(\frac{j\omega}{4} + 1)(\frac{j\omega}{50} + 1)}
\]

**LF:** $1.2j\omega \Rightarrow |H(j1)| = 1.2 (1.6$ dB$)$

$|H(j4)| = (\frac{4}{1})^0 \times 1.2 = 1.2$

$|H(j12)| = (\frac{12}{4})^{-1} \times 1.2 = 0.4$

$|H(j50)| = (\frac{50}{12})^0 \times 0.4 = 0.4 (-8$ dB$)$.
Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots.

2. Find LF and HF asymptotes. \( A (j\omega)^k \) has a slope of \( k \); substitute \( \omega = \omega_c \) to get the response at first/last corner frequency.

3. At a corner frequency, the gradient of the magnitude response changes by \( \pm 1 \) (\( \pm 20 \) dB/decade). + for numerator (top line) and − for denominator (bottom line).

4. \[ |H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)| \] if the gradient between them is \( k \).

\[
H(j\omega) = 1.2 \frac{j\omega\left(\frac{j\omega}{12}+1\right)}{\left(\frac{j\omega}{1}+1\right)\left(\frac{j\omega}{4}+1\right)\left(\frac{j\omega}{50}+1\right)}
\]

LF: \( 1.2j\omega \Rightarrow |H(j1)| = 1.2 \ (1.6 \) dB

\[ |H(j4)| = \left(\frac{4}{1}\right)^0 \times 1.2 = 1.2 \]

\[ |H(j12)| = \left(\frac{12}{4}\right)^{-1} \times 1.2 = 0.4 \]

\[ |H(j50)| = \left(\frac{50}{12}\right)^0 \times 0.4 = 0.4 \ (\ -8 \) dB). As a check: HF: \( 20 \ (j\omega)^{-1} \)
Filters

- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.
Filters

- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.

\[
\frac{Y}{X} = \frac{R}{R + \frac{1}{j\omega C}}
\]
Filters

- **Filter**: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- **The order** of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.

\[
\frac{Y}{X} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{j\omega RC + 1}
\]
Filters

- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.

\[
\frac{Y}{X} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1} = \frac{j\omega RC}{\frac{j\omega RC}{\alpha} + 1}
\]
Filters

- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.

\[
\frac{Y}{X} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{j\omega RC + 1} = \frac{j\omega RC}{j\omega + 1}
\]
Filters

- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.

\[
\frac{Y}{X} = \frac{R}{R+1/j\omega C} = \frac{j\omega RC}{j\omega RC+1} = \frac{j\omega RC}{\frac{j\omega}{a}+1}
\]

\[
\frac{Z}{X} = \frac{Z}{Y} \times \frac{Y}{X} = \left(1 + \frac{R_B}{R_A}\right) \times \frac{j\omega RC}{\frac{j\omega}{a}+1}
\]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized

\[ H(j\omega) = a (j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  - \[ H(j\omega) = a(j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]
  - **Corner frequency**: \[ \omega_0 = \sqrt{\frac{c}{a}} \] determines the horizontal position
  - **Damping Factor**: \[ \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \] determines the response shape
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  - \( H(j\omega) = a(j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \)
  - Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
  - Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape
  - Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  \[ H(j\omega) = a(j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]
  - Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
  - Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape
  - Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel \( a(j\omega)^2 = -c \): \( \Rightarrow H(j\omega) = jb\omega_0 = 2jc\zeta \)
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  
  \[ H(j\omega) = a\,(j\omega)^2 + bj\omega + c = c\left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta \left(\frac{j\omega}{\omega_0}\right) + 1\right) \]

- Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position

- Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape

- Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel \( (a\,(j\omega)^2 = -c) \): \( \Rightarrow H(j\omega) = jb\omega_0 = 2j\,c\zeta \)

  - \( |H(j\omega_0)| = 2\zeta \) times the straight line approximation at \( \omega_0 \).
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  \[ H(j\omega) = a(j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]

- Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
- Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape

- Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel \( (a(j\omega)^2 = -c) \): \( \Rightarrow H(j\omega) = jb\omega_0 = 2jc\zeta \)
  \( |H(j\omega_0)| = 2\zeta \) times the straight line approximation at \( \omega_0 \).

\[
\frac{X}{U} = \frac{1}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{(j\omega)^2 LC + j\omega RC + 1}
\]
**Resonance**

- Resonant circuits have quadratic factors that cannot be factorized
  
  \[ H(j\omega) = a (j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]
  
  - Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
  - Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape
  - Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel \( (a (j\omega)^2 = -c) \): \( \Rightarrow H(j\omega) = jb\omega_0 = 2jc\zeta \)
  
  - \( |H(j\omega_0)| = 2\zeta \) times the straight line approximation at \( \omega_0 \).

![Resonance Circuit Diagram](image)

\[ \frac{X}{U} = \frac{1}{R+j\omega L+\frac{1}{j\omega C}} = \frac{1}{(j\omega)^2LC+j\omega RC+1} \]

\[ \omega_0 = \sqrt{\frac{1}{LC}}, \zeta = \frac{R}{2} \sqrt{\frac{C}{L}}, Q = \frac{\omega_0 L}{R} = \frac{1}{2\zeta} \]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  \[ H(j\omega) = a(j\omega)^2 + bj\omega + c = c\left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1\right) \]

- Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
- Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape

- Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel (\( a(j\omega)^2 = -c \)): \( H(j\omega) = jb\omega_0 = 2j\sqrt{c} \zeta \)
  \[ |H(j\omega_0)| = 2\zeta \text{ times the straight line approximation at } \omega_0. \]

\[ R = 5, 20, 60, 120 \]
\[ \zeta = \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10} \]

\[ \frac{X}{U} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{(j\omega)^2LC + j\omega RC + 1} \]
\[ \omega_0 = \sqrt{\frac{1}{LC}}, \ \zeta = \frac{R}{2} \sqrt{\frac{C}{L}}, \ \frac{Q}{R} = \frac{\omega_0L}{R} = \frac{1}{2\zeta} \]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  
  \[ H(j\omega) = a \left(j\omega\right)^2 + bj\omega + c = c \left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta \left(\frac{j\omega}{\omega_0}\right) + 1\right) \]

- Corner frequency: \(\omega_0 = \sqrt{\frac{c}{a}}\) determines the horizontal position
- Damping Factor: \(\zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}}\) determines the response shape

- Equivalently Quality Factor: \(Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0}\)

- At \(\omega = \omega_0\), outer terms cancel \((a \left(j\omega\right)^2 = -c)\): \(\Rightarrow H(j\omega) = jb\omega_0 = 2jc\zeta\)
  
  \(|H(j\omega_0)| = 2\zeta\) times the straight line approximation at \(\omega_0\).

\[
\begin{align*}
R &= 5, 20, 60, 120 \\
\zeta &= \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10} \\
Q &= \frac{|Z_C(\omega_0)|}{R} \text{ or } \frac{|Z_L(\omega_0)|}{R} = 20, 5, \frac{5}{3}, \frac{5}{6}
\end{align*}
\]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  \[ H(j\omega) = a(j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]
  - Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
  - Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape
  - Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel (\( a(j\omega)^2 = -c \)): \( H(j\omega) = jb\omega_0 = 2jc\zeta \)
  \[ |H(j\omega_0)| = 2\zeta \text{ times the straight line approximation at } \omega_0. \]

\[
\begin{align*}
R &= 5, 20, 60, 120 \\
\zeta &= \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10} \\
Q &= \frac{|Z_C(\omega_0) \text{ or } Z_L(\omega_0)|}{R} = 20, 5, \frac{5}{3}, \frac{5}{6} \\
\frac{\text{Gain@}\omega_0}{\text{CornerGain}} &= \frac{1}{2\zeta} \approx Q
\end{align*}
\]

\[
\begin{align*}
X &= \frac{1}{1+j\omega C} \\
\frac{X}{U} &= \frac{1}{R+j\omega L + \frac{1}{j\omega C}} = \frac{1}{\frac{(j\omega)^2 LC + j\omega RC + 1}{R+j\omega L + \frac{1}{j\omega C}}} \\
\omega_0 &= \sqrt{\frac{1}{LC}}, \quad \zeta = \frac{R}{2\sqrt{\frac{C}{L}}}, \quad Q = \frac{\omega_0 L}{R} = \frac{1}{2\zeta}
\end{align*}
\]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  \[ H(j\omega) = a(j\omega)^2 + bj\omega + c = c \left( \left( \frac{j\omega}{\omega_0} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_0} \right) + 1 \right) \]

- Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
- Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape

- Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel \( (a(j\omega)^2 = -c) \): \( H(j\omega) = jb\omega_0 = 2jc\zeta \)
  \[ |H(j\omega_0)| = 2\zeta \text{ times the straight line approximation at } \omega_0. \]
  \( 3 \text{ dB bandwidth of peak} \approx 2\zeta\omega_0 \approx \frac{\omega_0}{Q} \).

\[ R = 5, 20, 60, 120 \]
\[ \zeta = \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10} \]
\[ Q = \frac{|Z_C(\omega_0) \text{ or } Z_L(\omega_0)|}{R} = 20, 5, \frac{5}{3}, \frac{5}{6} \]
\[ \text{Gain@}\omega_0 \quad \text{CornerGain} = \frac{1}{2\zeta} \approx Q \]
Resonance

- Resonant circuits have quadratic factors that cannot be factorized
  - \( H(j\omega) = a(j\omega)^2 + bj\omega + c = c\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1 \)
  - Corner frequency: \( \omega_0 = \sqrt{\frac{c}{a}} \) determines the horizontal position
  - Damping Factor: \( \zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}} \) determines the response shape
  - Equivalently Quality Factor: \( Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0} \)

- At \( \omega = \omega_0 \), outer terms cancel \( (a(j\omega)^2 = -c) \): \( H(j\omega) = jb\omega_0 = 2jc\zeta \)
  - \( |H(j\omega_0)| = 2\zeta \) times the straight line approximation at \( \omega_0 \).
  - 3 dB bandwidth of peak \( \approx 2\zeta\omega_0 \approx \frac{\omega_0}{Q} \). \( \Delta \text{phase} = \pm \pi \) over \( 2\zeta \) decades

\[ R = 5, 20, 60, 120 \]
\[ \zeta = \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10} \]
\[ Q = \frac{|Z_C(\omega_0)| \text{ or } Z_L(\omega_0)|}{R} = 20, 5, \frac{5}{3}, \frac{5}{6} \]
\[ \frac{\text{Gain@} \omega_0}{\text{CornerGain}} = \frac{1}{2\zeta} \approx Q \]

\[ \omega_0 = \sqrt{\frac{1}{LC}}, \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}, Q = \frac{\omega_0 L}{R} = \frac{1}{2\zeta} \]