Revision Lecture 1: Nodal Analysis & Frequency Responses

- Exam
- Nodal Analysis
- Op Amps
- Block Diagrams
- Diodes
- Reactive Components
- Phasors
- Plotting Frequency Responses
- LF and HF Asymptotes
- Corner frequencies (linear factors)
- Sketching Magnitude Responses (linear factors)
- Filters
- Resonance

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Exam

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Exam Format

Question 1 (40%): eight short parts covering the whole syllabus.

Questions 2 and 3: single topic questions (answer both)

Syllabus

Does include: Everything in the notes.

Does not include: Two-port parameters (2008:1j), Gaussian elimination (2007:2c), Application areas (2008:3d), Nullators and Norators (2008:4c), Small-signal component models (2008:4e), Gain-bandwidth product (2006:4c), Zener Diodes (2008/9 syllabus), Non-ideal models of L, C and transformer (2011/12 syllabus), Transmission lines VSWR and crank diagram (2011/12 syllabus).

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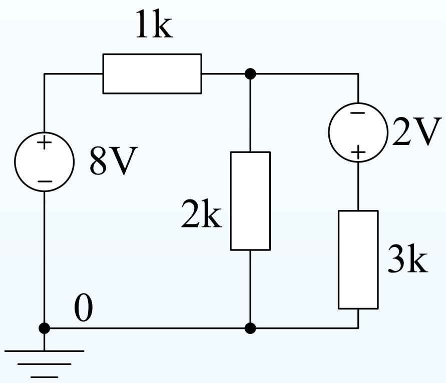

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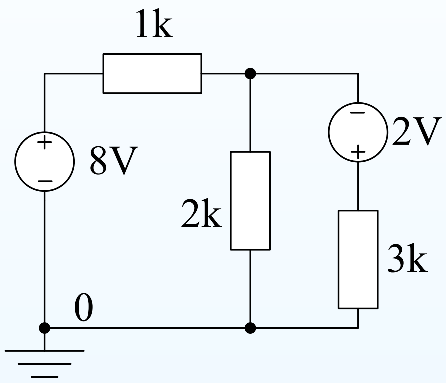
(1) Pick reference node.



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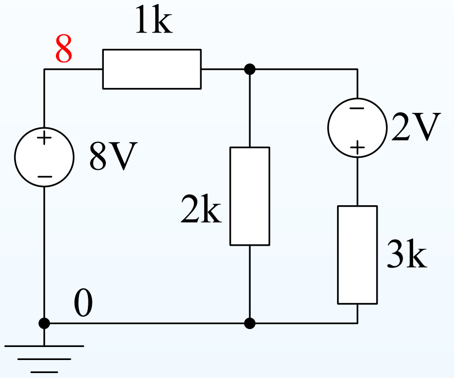
(1) Pick reference node.

(2) Label nodes: 8



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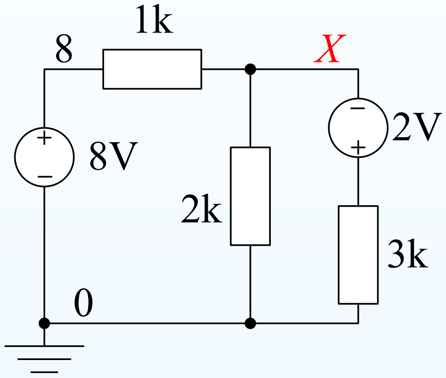
(1) Pick reference node.

(2) Label nodes: 8, X



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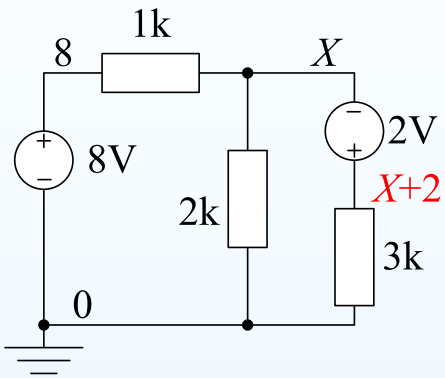
(1) Pick reference node.

(2) Label nodes: 8, X and $X+2$ since it is joined to X via a voltage source.



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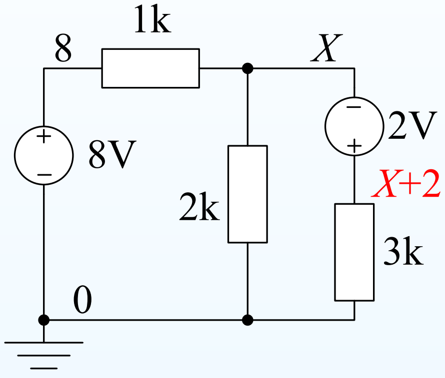
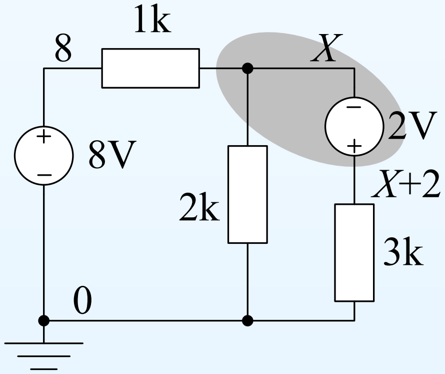
(1) Pick reference node.

(2) Label nodes: 8, X and $X+2$ since it is joined to X via a voltage source.

(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single "super-node" giving one equation

$$\frac{X-8}{1}+\frac{X}{2}+\frac{(X+2)-0}{3}=0$$





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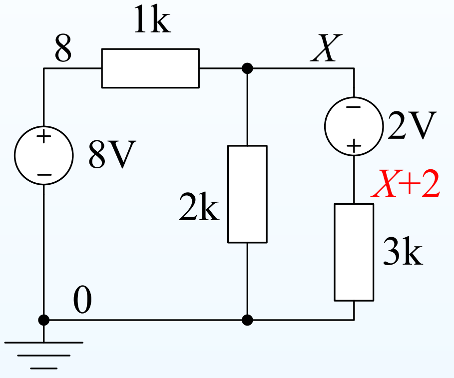
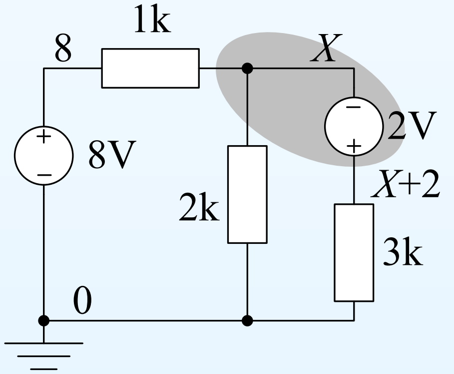
Ohm's law always involves the difference between the voltages at either end of a resistor. (Obvious but easily forgotten)





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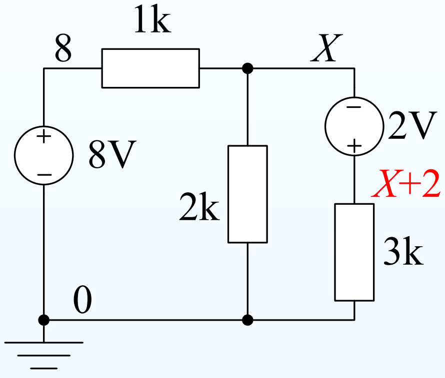
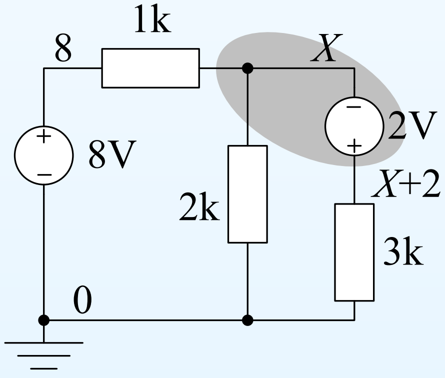
$$\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0$$

Ohm's law always involves the difference between the voltages at either end of a resistor. (Obvious but easily forgotten)

(4) Solve the equations: $X = 4$





Op Amps

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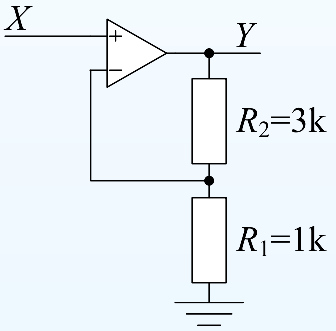
- Ideal Op Amp: (a) Zero input current, (b) Infinite gain
  (b) $\Rightarrow V_+ = V_-$ provided the circuit has negative feedback.
- Negative Feedback: An increase in V_{out} makes $(V_+ - V_-)$ decrease.



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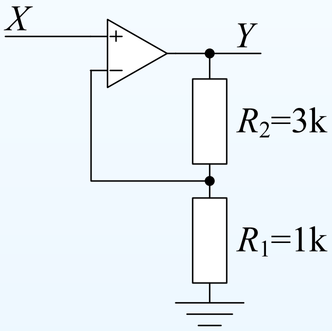
Non-inverting amplifier

$$Y = \left(1 + \frac{3}{1}\right) X$$



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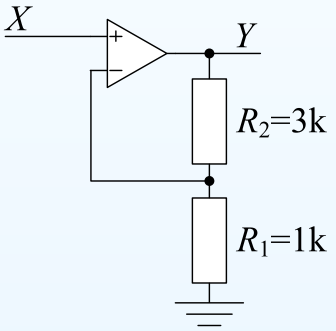
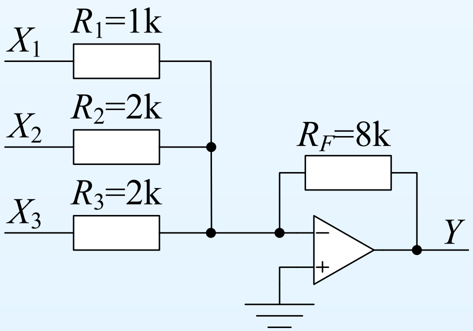


Inverting amplifier

$$Y = \frac{-8}{1} X_1 + \frac{-8}{2} X_2 + \frac{-8}{2} X_3$$



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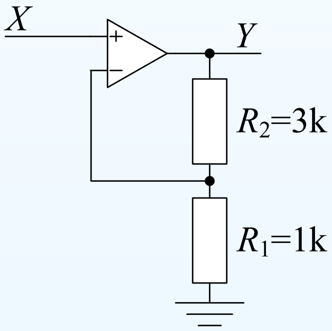

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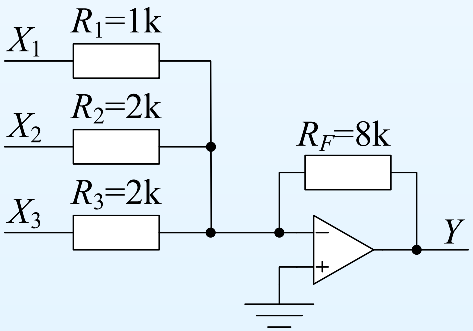

Nodal Analysis: Use two separate KCL equations at V_+ and V_-.

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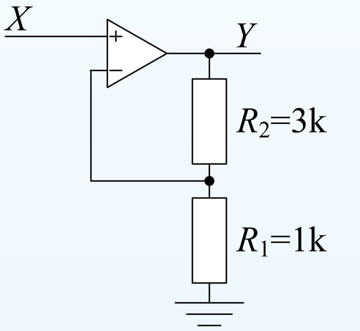

Inverting amplifier

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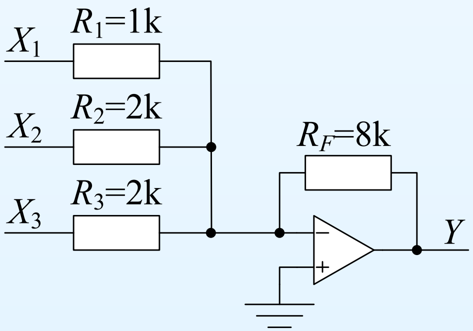

Nodal Analysis: Use two separate KCL equations at V_+ and V_-.
Do not do KCL at V_{out} except to find the op-amp output current.

Block Diagrams

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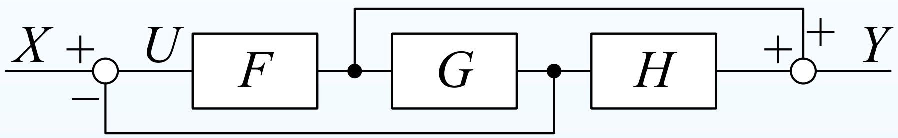
Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked $+$ for add or $-$ for subtract.



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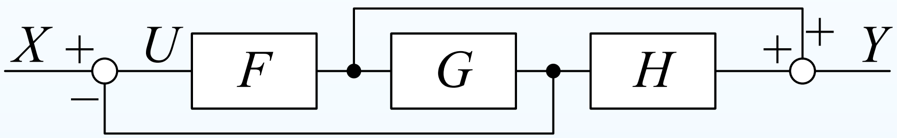

To analyse:

1. Label the inputs, the outputs and the output of each adder.

Block Diagrams

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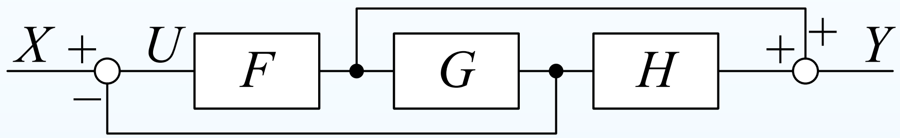

To analyse:

1. Label the inputs, the outputs and the output of each adder.
2. Write down an equation for each variable:
   - $U = X - FGU$
   - Follow signals back though the blocks until you meet a labelled node.

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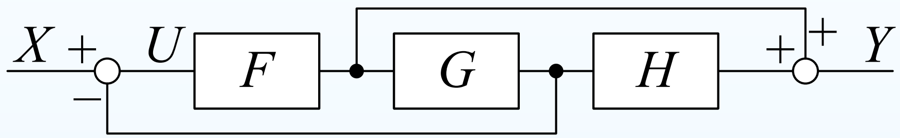

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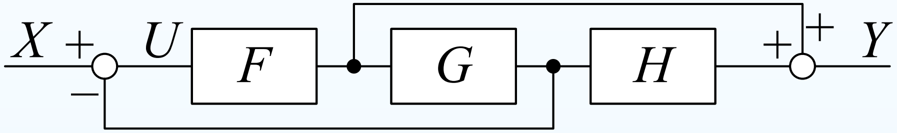

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3. Solve the equations (eliminate intermediate node variables):
   - $U(1 + FG) = X$

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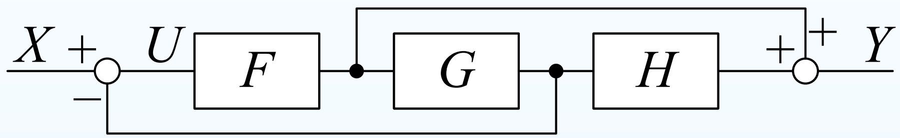

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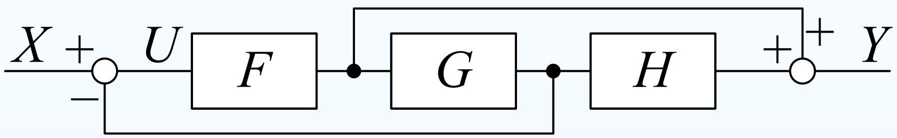

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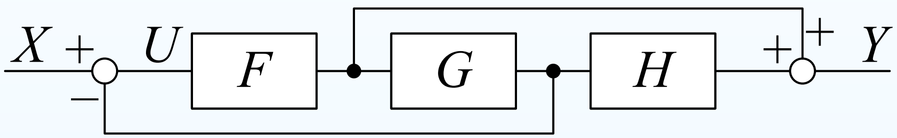

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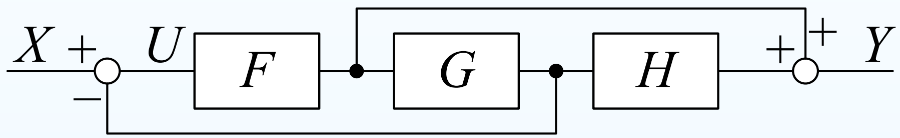

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[Note: "Wires" carry information not current: KCL not valid]

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Diodes

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- Off: $I_D = 0$, $V_D < 0.7$
- On: $V_D = 0.7$, $I_D > 0$

(a) Guess the mode

(b) Do nodal analysis assuming the equality condition

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Diodes

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- Sketching Magnitude Responses (linear factors)
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- Resonance

Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- Off: $I_D = 0, V_D < 0.7 \quad \Rightarrow \quad$ Diode = open circuit
- On: $V_D = 0.7, I_D > 0$

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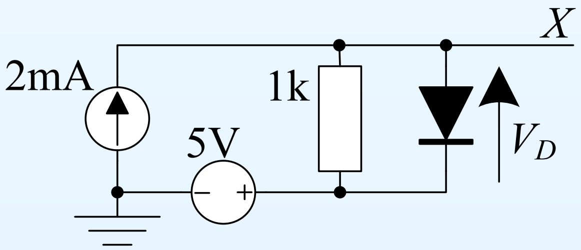
- Assume Diode Off

$$X = 5 + 2 = 7$$



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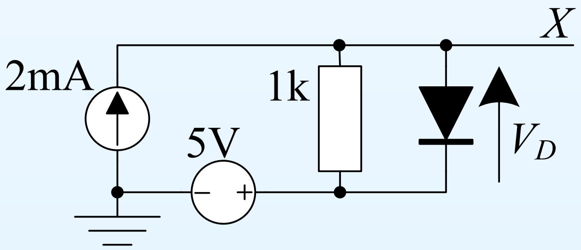
$$V_D = 2$$



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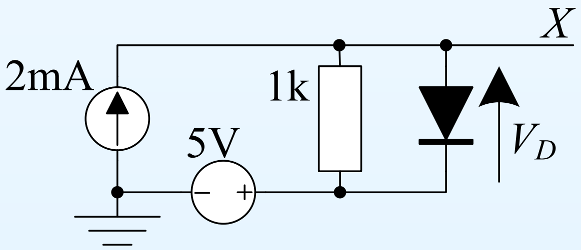
$$X = 5 + 2 = 7$$
$$V_D = 2 \quad \text{Fail: } V_D > 0.7$$



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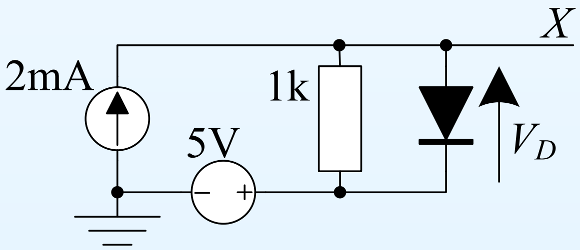

- Assume Diode On

$$X = 5 + 0.7 = 5.7$$

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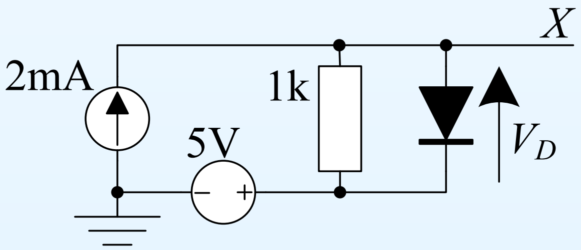

- Assume Diode On

$$X = 5 + 0.7 = 5.7$$
$$I_D + \frac{0.7}{1\,\mathrm{k}} = 2\,\mathrm{mA}$$

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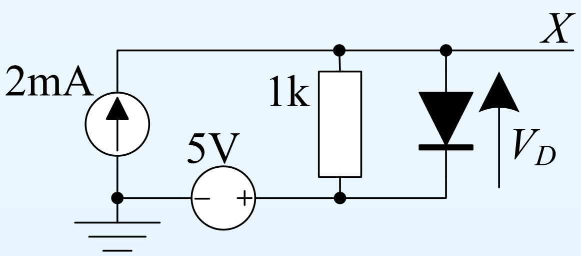

- Assume Diode On

$$X = 5 + 0.7 = 5.7$$
$$I_D + \frac{0.7}{1\,\text{k}} = 2\,\text{mA} \quad \text{OK: } I_D > 0$$

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- Impedances: $R,\ j\omega L,\ \frac{1}{j\omega C} = \frac{-j}{\omega C}$.
  - Admittances: $\frac{1}{R},\ \frac{1}{j\omega L} = \frac{-j}{\omega L},\ j\omega C$

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  - CIVIL: Capacitor - I leads V; Inductor - I lags V

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- Average current (or DC current) through a capacitor is always zero
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  - CIVIL: Capacitor - I leads V; Inductor - I lags V
- Average current (or DC current) through a capacitor is always zero
- Average voltage across an inductor is always zero
- Average power absorbed by a capacitor or inductor is always zero

Phasors

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A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

Waveform	Phasor
$x(t) = F\cos\omega t - G\sin\omega t$	$X = F + jG$

Phasors

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$x(t) = A\cos(\omega t + \theta)$	$X = Ae^{j\theta} = A\angle\theta$	

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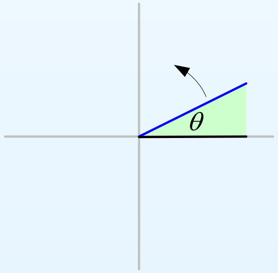

$x(t)$ is the projection of a rotating rod onto the real (horizontal) axis.

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Phasors

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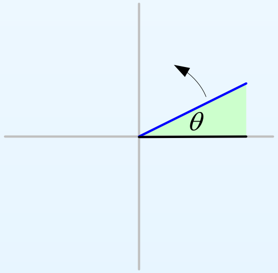

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RMS Phasor: $\widetilde{V} = \frac{1}{\sqrt{2}}V$

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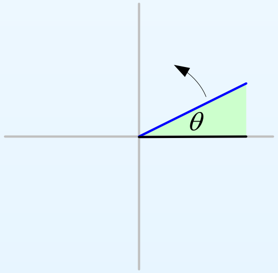

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RMS Phasor: $\widetilde{V} = \frac{1}{\sqrt{2}}V \quad \Rightarrow \quad \left|\widetilde{V}\right|^2 = \left\langle x^2(t)\right\rangle$

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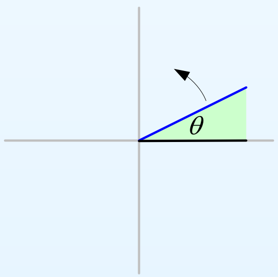

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Complex Power: $\widetilde{V}\tilde{I}^* = |\tilde{I}|^2 Z = \frac{|\widetilde{V}|^2}{Z^*} = P + jQ$

P is average power (Watts), Q is reactive power (VARs)

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Plotting Frequency Responses

- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.

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- If $\frac{V_2}{V_1} = A\,(j\omega)^k = A \times j^k \times \omega^k$ (where A is real)
  - magnitude is a straight line with gradient k:

$$\log\left|\frac{V_2}{V_1}\right| = \log|A| + k\log\omega$$

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  - phase is a constant $k \times \frac{\pi}{2}$ ($+\pi$ if $A < 0$):
    $$\angle\left(\frac{V_2}{V_1}\right) = \angle A + k\angle j = \angle A + k\frac{\pi}{2}$$

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- Measure magnitude response using decibels $= 20\log_{10}\frac{|V_2|}{|V_1|}$. A gradient of k on log axes is equivalent to $20k\,\mathrm{dB}/\mathrm{decade}$ ($\times 10$ in frequency) or $6k\,\mathrm{dB}/\mathrm{octave}$ ($\times 2$ in frequency).

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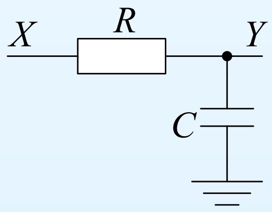

$$\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{j\omega RC+1}$$

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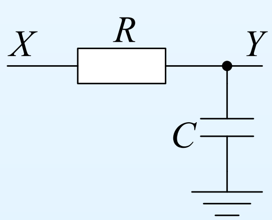
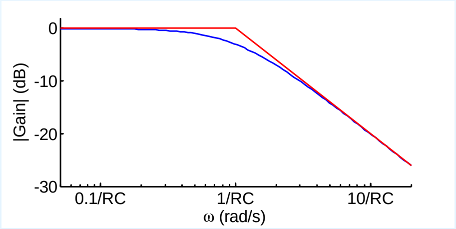
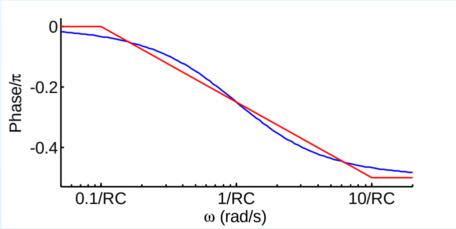
  - magnitude is a straight line with gradient k:
  $$\log\left|\frac{V_2}{V_1}\right| = \log|A| + k\log\omega$$
  - phase is a constant $k \times \frac{\pi}{2}$ ($+\pi$ if $A < 0$):
  $$\angle\left(\frac{V_2}{V_1}\right) = \angle A + k\angle j = \angle A + k\frac{\pi}{2}$$
- Measure magnitude response using decibels $= 20\log_{10}\frac{|V_2|}{|V_1|}$. A gradient of k on log axes is equivalent to $20k\,\mathrm{dB/decade}$ ($\times 10$ in frequency) or $6k\,\mathrm{dB/octave}$ ($\times 2$ in frequency).







$\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{j\omega RC+1} = \frac{1}{\frac{j\omega}{\omega_c}+1}$ where $\omega_c = \frac{1}{RC}$

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LF and HF Asymptotes

- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
  - The terms with the lowest power of $j\omega$ on top and bottom gives the low-frequency asymptote
  - The terms with the highest power of $j\omega$ on top and bottom gives the high-frequency asymptote.

LF and HF Asymptotes

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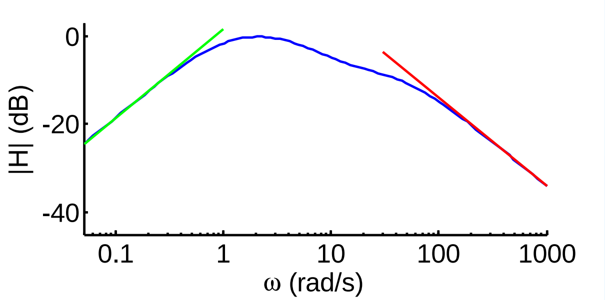
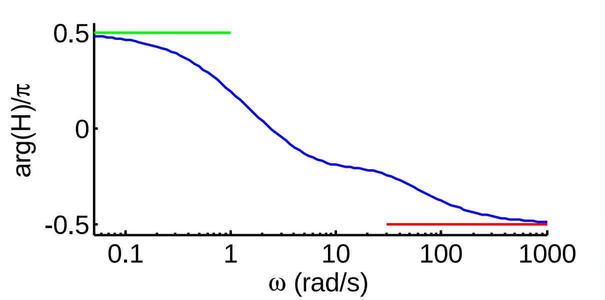
Example: $H(j\omega) = \frac{60(j\omega)^2 + 720(j\omega)}{3(j\omega)^3 + 165(j\omega)^2 + 762(j\omega) + 600}$





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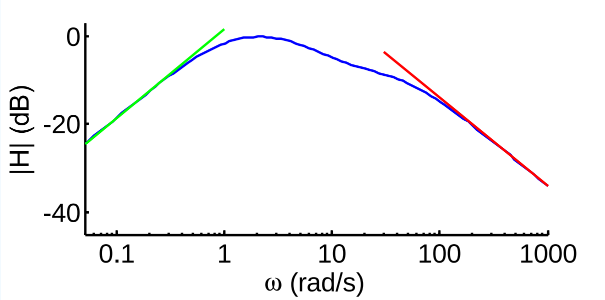

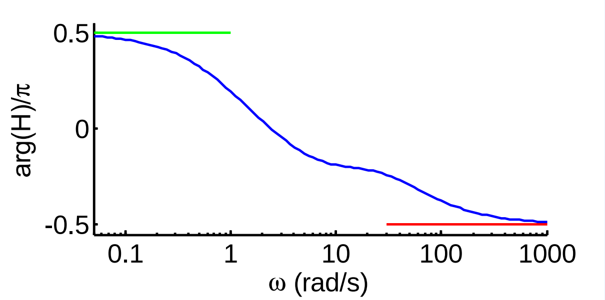

LF: $H(j\omega) \simeq 1.2j\omega$

LF and HF Asymptotes

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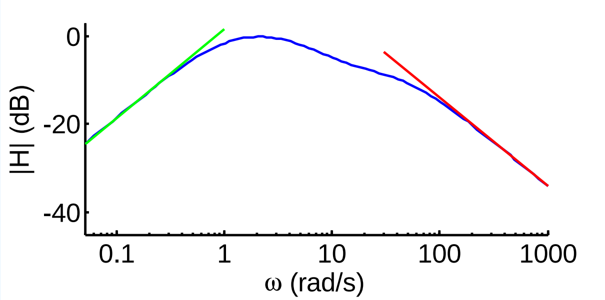

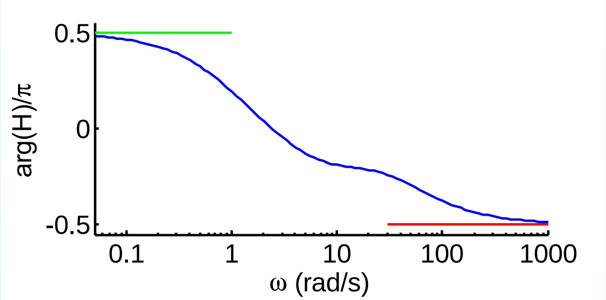

LF: $H(j\omega) \simeq 1.2j\omega$

HF: $H(j\omega) \simeq 20\,(j\omega)^{-1}$

Corner frequencies (linear factors)

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- We can factorize the numerator and denominator into linear terms of the form $(aj\omega + b) \simeq \begin{cases} b & \omega < \left|\frac{b}{a}\right| \\ aj\omega & \omega > \left|\frac{b}{a}\right| \end{cases}$.

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- The phase changes by $\pm\frac{\pi}{2}$ because the linear term introduces another factor of j into the numerator or denominator for $\omega > \omega_c$.
  - The phase change is gradual and takes place over the range $0.1\omega_c$ to $10\omega_c$ ($\pm\frac{\pi}{2}$ spread over two decades in ω).

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- When a and b are real and positive, it is often convenient to write $(aj\omega + b) = b\left(\frac{j\omega}{\omega_c} + 1\right)$.

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- When a and b are real and positive, it is often convenient to write $(aj\omega + b) = b\left(\frac{j\omega}{\omega_c} + 1\right)$.
- The corner frequencies are the absolute values of the roots of the numerator and denominator polynomials (values of $j\omega$).

Sketching Magnitude Responses (linear factors)

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3. At a corner frequency, the gradient of the magnitude response changes by ± 1 (± 20 dB/decade). $+$ for numerator (top line) and $-$ for denominator (bottom line).
4. $|H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)|$ if the gradient between them is k.

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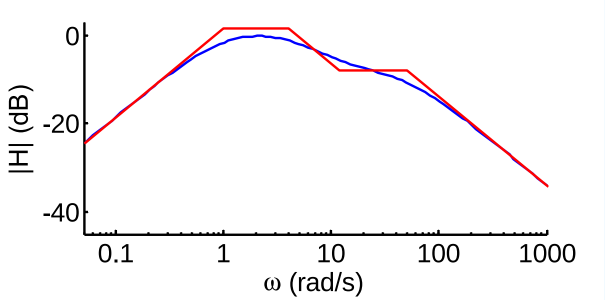

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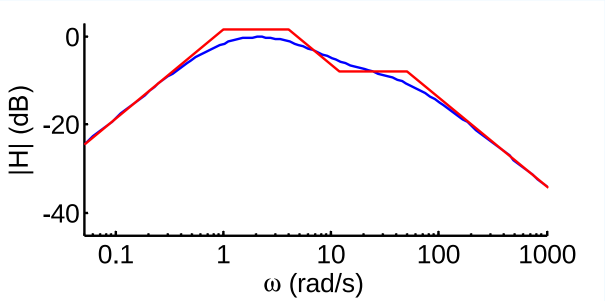
LF: $1.2j\omega \Rightarrow |H(j1)| = 1.2\,(1.6\,\text{dB})$



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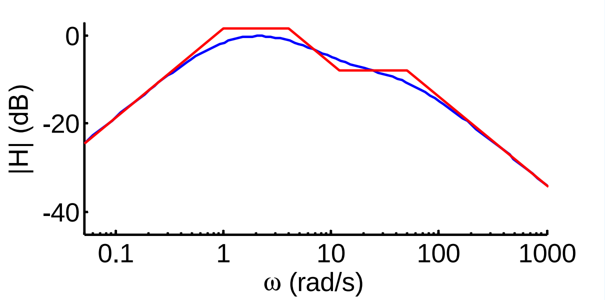
$|H(j4)| = \left(\frac{4}{1}\right)^0 \times 1.2 = 1.2$



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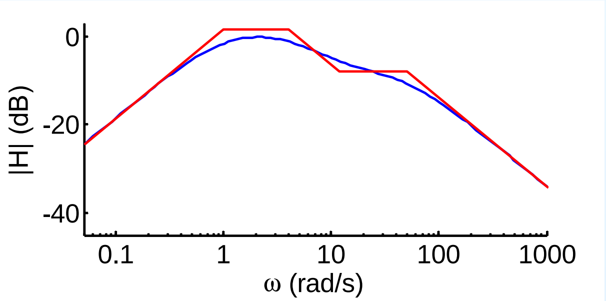
$|H(j12)| = \left(\frac{12}{4}\right)^{-1} \times 1.2 = 0.4$



Sketching Magnitude Responses (linear factors)

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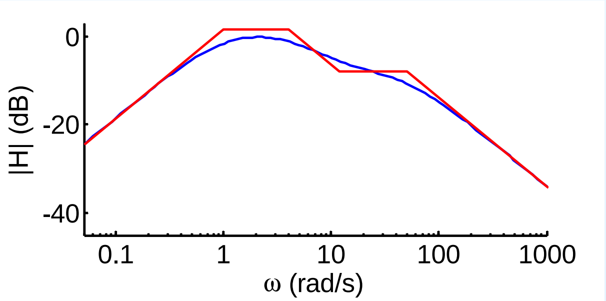
$|H(j50)| = \left(\frac{50}{12}\right)^0 \times 0.4 = 0.4\,(-8\text{ dB})$.



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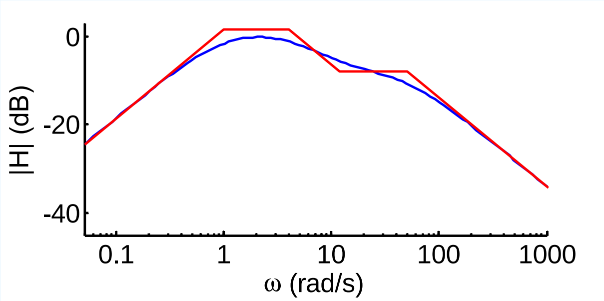

Filters

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- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.

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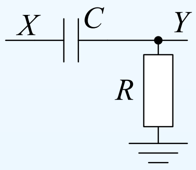

$$\frac{Y}{X} = \frac{R}{R + {}^{1}/_{j\omega C}}$$

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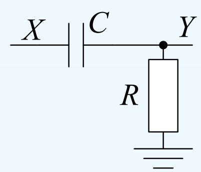

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Revision Lecture 1: Nodal Analysis & Frequency Responses

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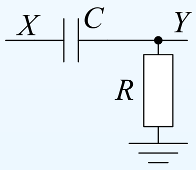
- Op Amps
- Block Diagrams
- Diodes
- Reactive Components
- Phasors
- Plotting Frequency Responses
- LF and HF Asymptotes
- Corner frequencies (linear factors)
- Sketching Magnitude Responses (linear factors)
- Filters
- Resonance

Filters

- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.



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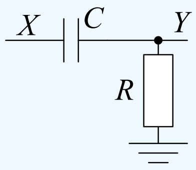

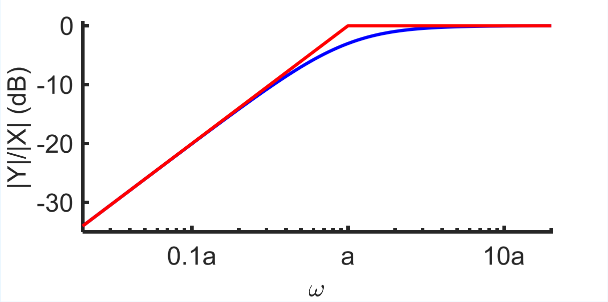

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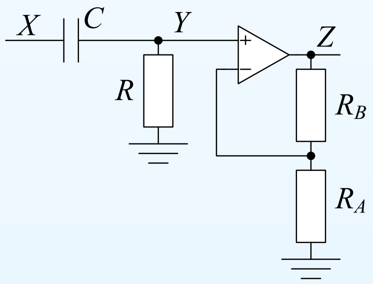

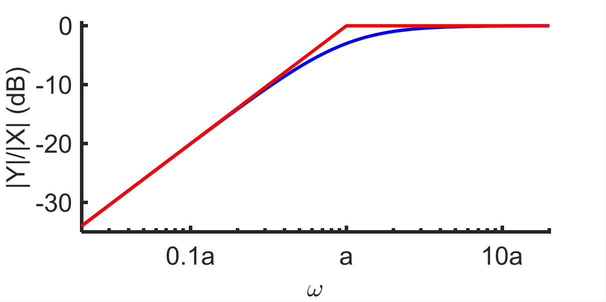

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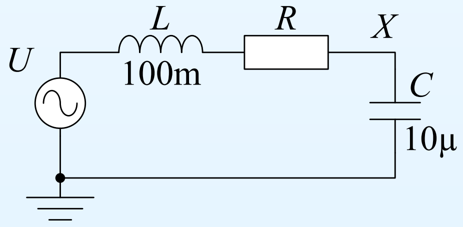

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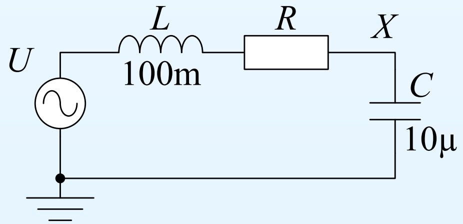

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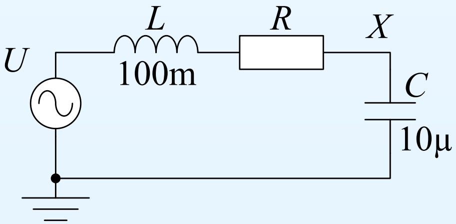

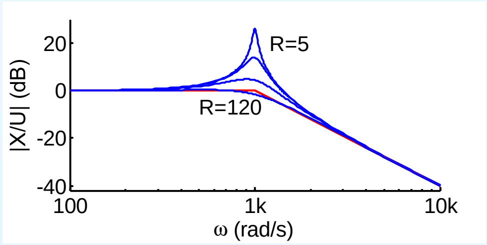

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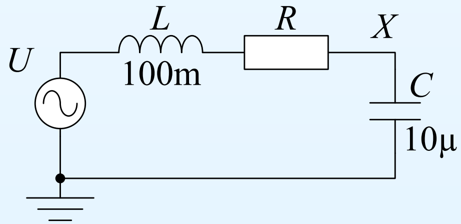

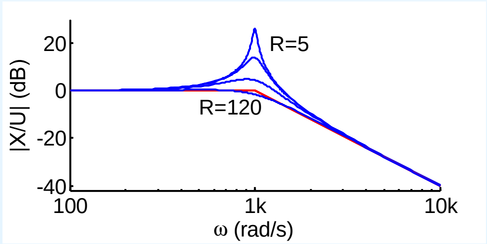

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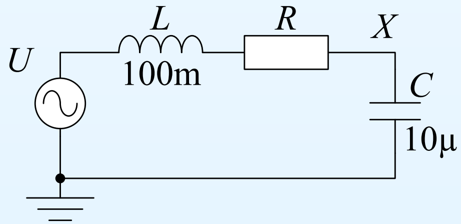

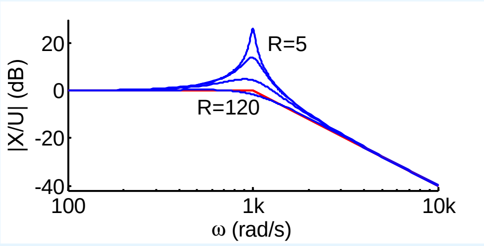

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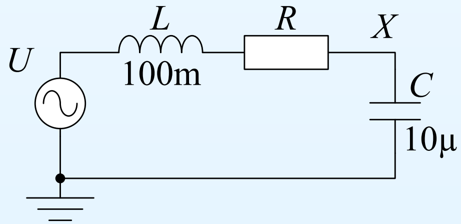

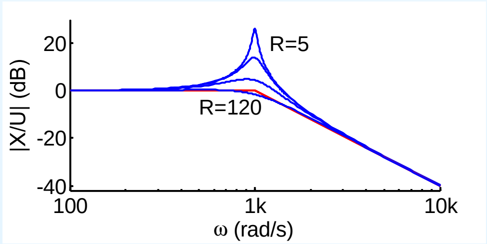

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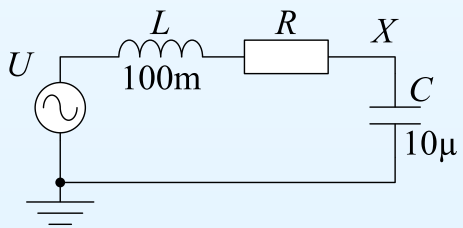

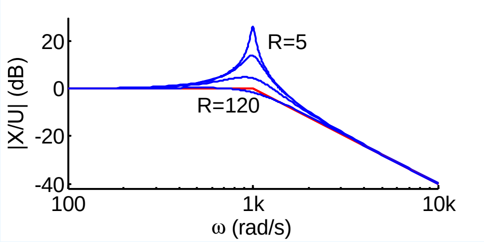

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