▷ Revision Lecture 1: Nodal Analysis & Frequency Responses
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### Exam Format

Question 1 (40%): eight short parts covering the whole syllabus.

Questions 2 and 3: single topic questions (answer both)

### Syllabus

Does include: Everything in the notes.

Does not include: Two-port parameters (2008:1j), Gaussian elimination (2007:2c), Application areas (2008:3d), Nullators and Norators (2008:4c), Small-signal component models (2008:4e), Gain-bandwidth product (2006:4c), Zener Diodes (2008/9 syllabus), Non-ideal models of L, C and transformer (2011/12 syllabus), Transmission lines VSWR and crank diagram (2011/12 syllabus).

# Nodal Analysis

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(1) Pick reference node.

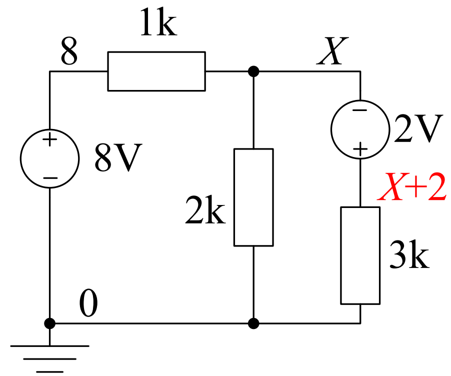
(2) Label nodes: $8$, $X$ and $X+2$ since it is joined to $X$ via a voltage source.

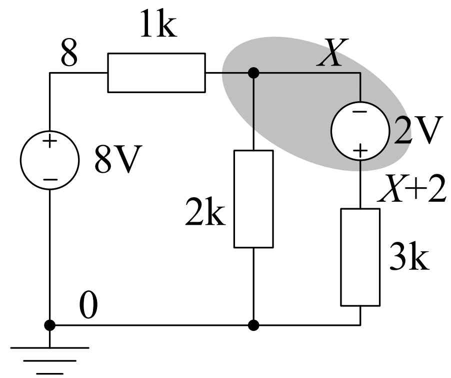
(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single "super-node" giving one equation

$$\frac{X-8}{1}+\frac{X}{2}+\frac{(X+2)-0}{3}=0$$

Ohm's law always involves the difference between the voltages at either end of a resistor. (Obvious but easily forgotten)

(4) Solve the equations: $X=4$



# Op Amps

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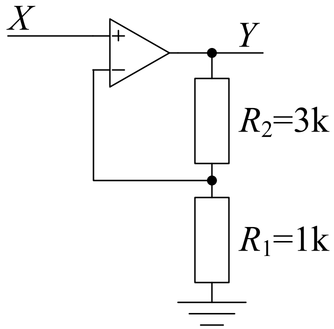
- Ideal Op Amp: (a) Zero input current, (b) Infinite gain
  (b) $\Rightarrow V_+ = V_-$ provided the circuit has negative feedback.
- Negative Feedback: An increase in $V_{out}$ makes $(V_+ - V_-)$ decrease.

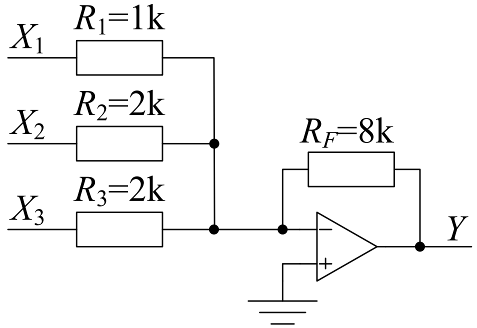
Non-inverting amplifier

$$Y = \left(1 + \tfrac{3}{1}\right) X$$




Inverting amplifier

$$Y = \tfrac{-8}{1} X_1 + \tfrac{-8}{2} X_2 + \tfrac{-8}{2} X_3$$




Nodal Analysis: Use two separate KCL equations at $V_+$ and $V_-$.
Do not do KCL at $V_{out}$ except to find the op-amp output current.

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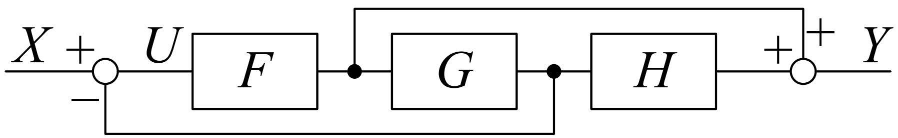
Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked $+$ for add or $-$ for subtract.




To analyse:

1. Label the inputs, the outputs and the output of each adder.
2. Write down an equation for each variable:
   - $U = X - FGU, \qquad Y = FU + FGHU$
   - Follow signals back though the blocks until you meet a labelled node.
3. Solve the equations (eliminate intermediate node variables):
   - $U(1 + FG) = X \quad \Rightarrow \quad U = \frac{X}{1+FG}$
   - $Y = (1 + GH)FU = \frac{(1+GH)F}{1+FG}X$

[Note: "Wires" carry information not current: KCL not valid]

# Diodes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Each diode in a circuit is in one of two modes; each has an equality condition and an inequality condition:

- ☐ Off: $I_D = 0$, $V_D < 0.7 \quad \Rightarrow \quad$ Diode = open circuit
- ☐ On: $V_D = 0.7$, $I_D > 0 \quad \Rightarrow \quad$ Diode = $0.7\,\text{V}$ voltage source

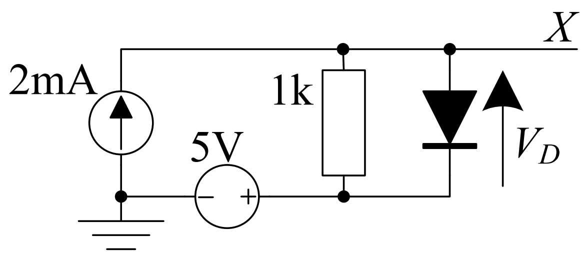
(a) Guess the mode
(b) Do nodal analysis assuming the equality condition
(c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- Assume Diode Off

$$X = 5 + 2 = 7$$
$$V_D = 2 \quad \text{Fail: } V_D > 0.7$$

- Assume Diode On

$$X = 5 + 0.7 = 5.7$$
$$I_D + \frac{0.7}{1\,\text{k}} = 2\,\text{mA} \quad \text{OK: } I_D > 0$$



# Reactive Components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- Impedances: $R$, $j\omega L$, $\frac{1}{j\omega C} = \frac{-j}{\omega C}$.
  - Admittances: $\frac{1}{R}$, $\frac{1}{j\omega L} = \frac{-j}{\omega L}$, $j\omega C$
- In a capacitor or inductor, the Current and Voltage are $90^\circ$ apart :
  - CIVIL: Capacitor - $I$ leads $V$; Inductor - $I$ lags $V$
- Average current (or DC current) through a capacitor is always zero
- Average voltage across an inductor is always zero
- Average power absorbed by a capacitor or inductor is always zero

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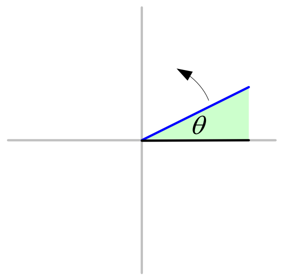
A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

| Waveform | Phasor | |
|---|---|---|
| $x(t) = F\cos\omega t - G\sin\omega t$ | $X = F + jG$ | [Note minus sign] |
| $x(t) = A\cos(\omega t + \theta)$ | $X = Ae^{j\theta} = A\angle\theta$ | |
| $\max(x(t)) = A$ | $\|X\| = A$ | |




$x(t)$ is the projection of a rotating rod onto the real (horizontal) axis.

$X = F + jG$ is its starting position at $t = 0$.

RMS Phasor: $\widetilde{V} = \frac{1}{\sqrt{2}}V \quad \Rightarrow \quad \left|\widetilde{V}\right|^2 = \left\langle x^2(t)\right\rangle$

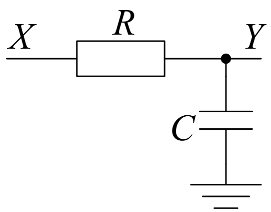
Complex Power: $\widetilde{V}\widetilde{I}^* = |\widetilde{I}|^2 Z = \frac{|\widetilde{V}|^2}{Z^*} = P + jQ$

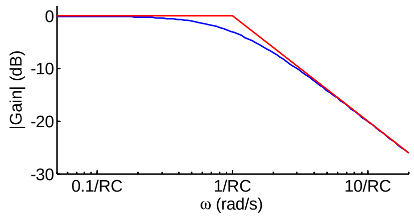
$P$ is average power (Watts), $Q$ is reactive power (VARs)

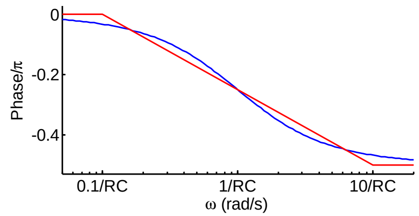
# Plotting Frequency Responses



- Plot the magnitude response and phase response as separate graphs. Use log scale for frequency and magnitude and linear scale for phase: this gives graphs that can be approximated by straight line segments.
- If $\frac{V_2}{V_1} = A\,(j\omega)^k = A \times j^k \times \omega^k$ (where $A$ is real)
  - magnitude is a straight line with gradient $k$:
    $$\log\left|\frac{V_2}{V_1}\right| = \log|A| + k\log\omega$$
  - phase is a constant $k \times \frac{\pi}{2}$ $(+\pi$ if $A<0)$:
    $$\angle\left(\frac{V_2}{V_1}\right) = \angle A + k\angle j = \angle A + k\frac{\pi}{2}$$
- Measure magnitude response using decibels $= 20\log_{10}\frac{|V_2|}{|V_1|}$.
  A gradient of $k$ on log axes is equivalent to $20k\,\text{dB}/\text{decade}$ $(\times 10$ in frequency) or $6k\,\text{dB}/\text{octave}$ ( $\times 2$ in frequency).




$$\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{j\omega RC+1} = \frac{1}{\frac{j\omega}{\omega_c}+1} \quad \text{where } \omega_c = \frac{1}{RC}$$

# LF and HF Asymptotes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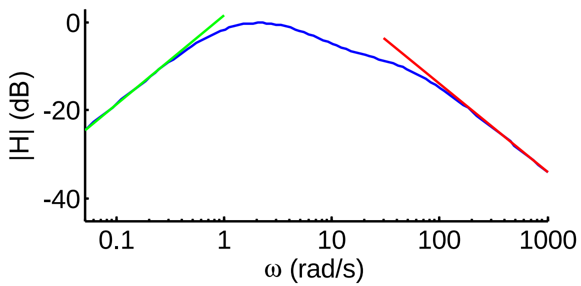
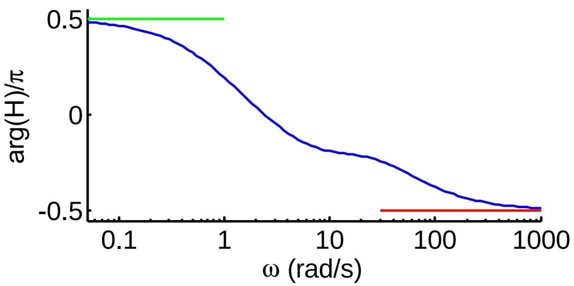
- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
  - The terms with the lowest power of $j\omega$ on top and bottom gives the low-frequency asymptote
  - The terms with the highest power of $j\omega$ on top and bottom gives the high-frequency asymptote.

Example: $H(j\omega) = \frac{60(j\omega)^2+720(j\omega)}{3(j\omega)^3+165(j\omega)^2+762(j\omega)+600}$




LF: $H(j\omega) \simeq 1.2j\omega$

HF: $H(j\omega) \simeq 20\,(j\omega)^{-1}$

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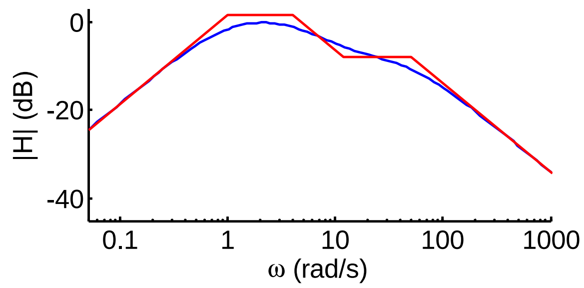
# Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form $(aj\omega + b) \simeq \begin{cases} b & \omega < \left|\frac{b}{a}\right| \\ aj\omega & \omega > \left|\frac{b}{a}\right| \end{cases}$.
- At the corner frequency, $\omega_c = \left|\frac{b}{a}\right|$, the slope of the magnitude response changes by $\pm 1$ ( $\pm 20$ dB/decade) because the linear term introduces another factor of $\omega$ into the numerator or denominator for $\omega > \omega_c$.
- The phase changes by $\pm\frac{\pi}{2}$ because the linear term introduces another factor of $j$ into the numerator or denominator for $\omega > \omega_c$.
  - The phase change is gradual and takes place over the range $0.1\omega_c$ to $10\omega_c$ ($\pm\frac{\pi}{2}$ spread over two decades in $\omega$).
- When $a$ and $b$ are real and positive, it is often convenient to write $(aj\omega + b) = b\left(\frac{j\omega}{\omega_c} + 1\right)$.
- The corner frequencies are the absolute values of the roots of the numerator and denominator polynomials (values of $j\omega$).

# Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots
2. Find LF and HF asymptotes. $A(j\omega)^k$ has a slope of $k$; substitute $\omega = \omega_c$ to get the response at first/last corner frequency.
3. At a corner frequency, the gradient of the magnitude response changes by $\pm 1$ ($\pm 20$ dB/decade). $+$ for numerator (top line) and $-$ for denominator (bottom line).
4. $|H(j\omega_2)| = \left(\frac{\omega_2}{\omega_1}\right)^k |H(j\omega_1)|$ if the gradient between them is $k$.

$$H(j\omega) = 1.2\frac{j\omega\left(\frac{j\omega}{12}+1\right)}{\left(\frac{j\omega}{1}+1\right)\left(\frac{j\omega}{4}+1\right)\left(\frac{j\omega}{50}+1\right)}$$




LF: $1.2j\omega \Rightarrow |H(j1)| = 1.2\,(1.6\,\text{dB})$

$|H(j4)| = \left(\frac{4}{1}\right)^0 \times 1.2 = 1.2$

$|H(j12)| = \left(\frac{12}{4}\right)^{-1} \times 1.2 = 0.4$

$|H(j50)| = \left(\frac{50}{12}\right)^0 \times 0.4 = 0.4\,(-8\,\text{dB})$. As a check: HF: $20\,(j\omega)^{-1}$

# [Sketching Responses (linear factors): Summary]

**LF and HF asymptotes**

The LF and HF asymptotes give you both the *magnitude* and *phase* at very low and very high frequencies. The LF asymptote is found by taking the terms with the lowest power of $\omega$ in numerator and denominator; the HF asymptote is found by taking the terms with the highest power of $\omega$.

**Magnitude response**

The corner frequency for a linear factor $(aj\omega + b)$ is at $\omega_c = \left|\frac{b}{a}\right|$. At each corner frequency, the slope of the magnitude response changes by $\pm 6\,\mathrm{dB/octave}$ $(= \pm 20\,\mathrm{dB/decade})$. The change is $+$ve for numerator corner frequencies and $-$ve for denominator corner frequencies. An octave is a factor of $2$ in frequency and a decade is a factor of $10$ in frequency. The number of decades between $\omega_1$ and $\omega_2$ is given by $\log_{10} \frac{\omega_2}{\omega_1}$.

**Phase Response**

For each corner frequency, $\omega_c$, the slope of the phase response changes *twice*: once at $0.1\omega_c$ and once, in the opposite direction, at $10\omega_c$. The change in slope is always $\pm 0.25\pi\,\mathrm{rad/decade}$. If $a$ and $b$ have the same sign (normal case), then the first slope change (at $0.1\omega_c$) is in the same direction as that of the magnitude response ($+$ve for numerator and $-$ve for denominator); if $a$ and $b$ have opposite signs (rare), then the sign of the slope change is reversed. The second slope change (at $10\omega_c$) always has the opposite sign from the first.

# Filters

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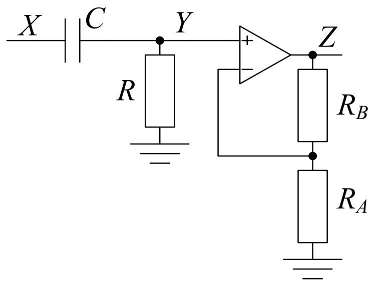
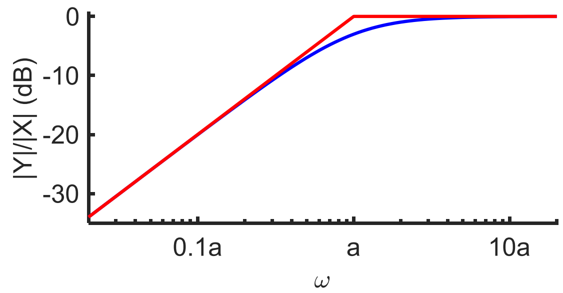
- Filter: a circuit designed to amplify some frequencies and/or attenuate others. Very widely used.
- The order of the filter is the highest power of $j\omega$ in the denominator of the frequency response.
- Often use op-amps to provide a low impedance output.




$$\frac{Y}{X} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{j\omega RC + 1} = \frac{j\omega RC}{\frac{j\omega}{a} + 1}$$

$$\frac{Z}{X} = \frac{Z}{Y} \times \frac{Y}{X} = \left(1 + \frac{R_B}{R_A}\right) \times \frac{j\omega RC}{\frac{j\omega}{a} + 1}$$

# Resonance

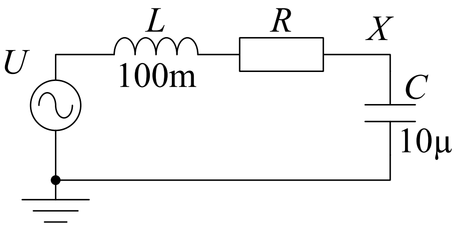
- Resonant circuits have quadratic factors that cannot be factorized
  - $H(j\omega) = a\,(j\omega)^2 + bj\omega + c = c\left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1\right)$
  - Corner frequency: $\omega_0 = \sqrt{\frac{c}{a}}$ determines the horizontal position
  - Damping Factor: $\zeta = \frac{b\omega_0}{2c} = \frac{b}{\sqrt{4ac}}$ determines the response shape
  - Equivalently Quality Factor: $Q \triangleq \frac{\omega \times \text{Average Stored Energy}}{\text{Average Power Dissipation}} \approx \frac{1}{2\zeta} = \frac{c}{b\omega_0}$
- At $\omega = \omega_0$, outer terms cancel $\left(a\,(j\omega)^2 = -c\right)$: $\Rightarrow H(j\omega) = jb\omega_0 = 2jc\zeta$
  - $|H(j\omega_0)| = 2\zeta$ times the straight line approximation at $\omega_0$.
  - 3 dB bandwidth of peak $\simeq 2\zeta\omega_0 \approx \frac{\omega_0}{Q}$. $\Delta\text{phase} = \pm\pi$ over $2\zeta$ decades

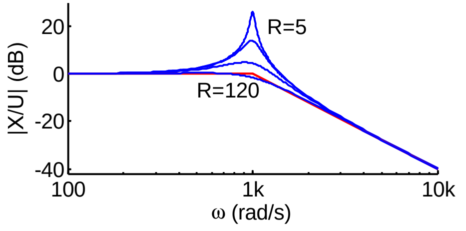
$R = 5, 20, 60, 120$

$\zeta = \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10}$

$Q = \frac{|Z_C(\omega_0) \text{ or } Z_L(\omega_0)|}{R} = 20, 5, \frac{5}{3}, \frac{5}{6}$

$\frac{\text{Gain@}\,\omega_0}{\text{CornerGain}} = \frac{1}{2\zeta} \approx Q$




$\frac{X}{U} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{(j\omega)^2 LC + j\omega RC + 1}$

$\omega_0 = \sqrt{\frac{1}{LC}}, \ \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}, \ Q = \frac{\omega_0 L}{R} = \frac{1}{2\zeta}$