Revision Lecture 2: Transients & Lines

Transients: Basic Ideas

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    - τ and A can also be found from the transfer function.

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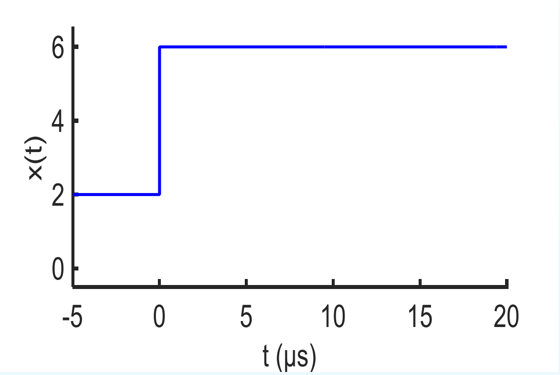

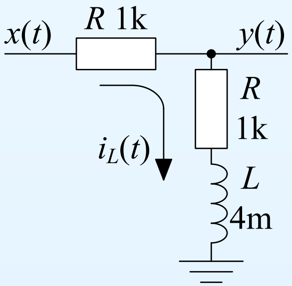

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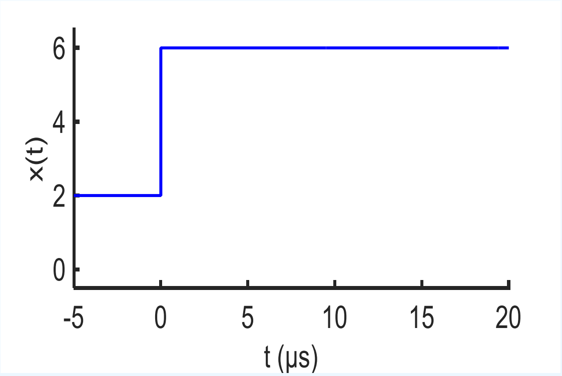

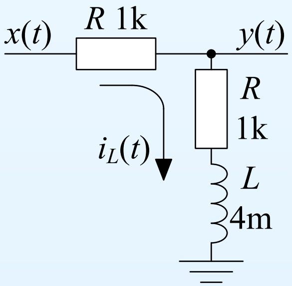

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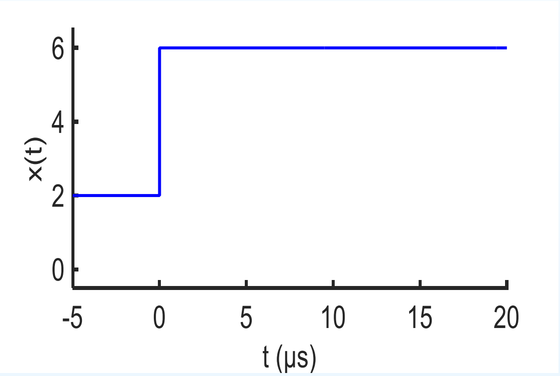

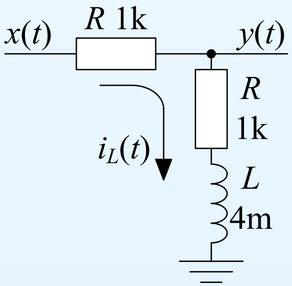

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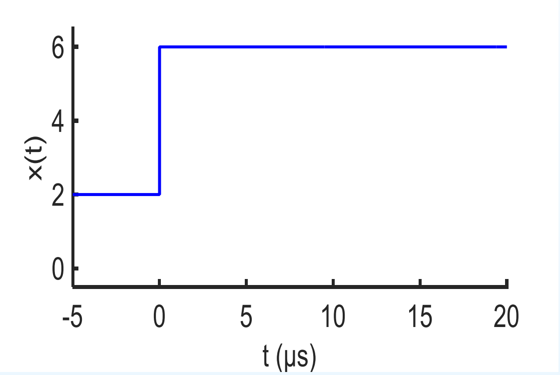

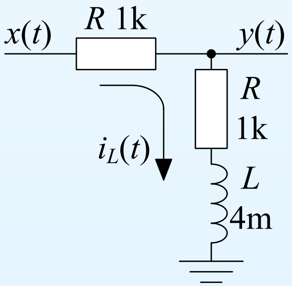

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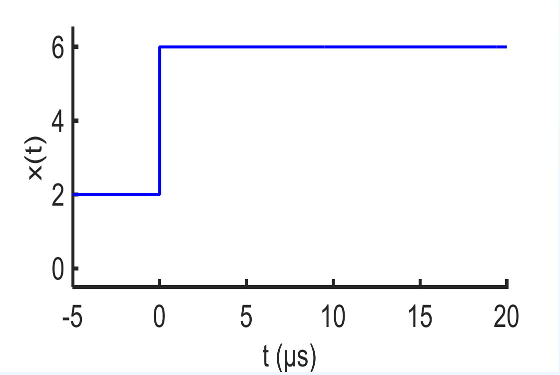

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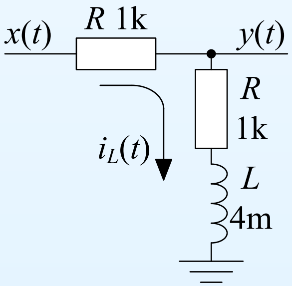

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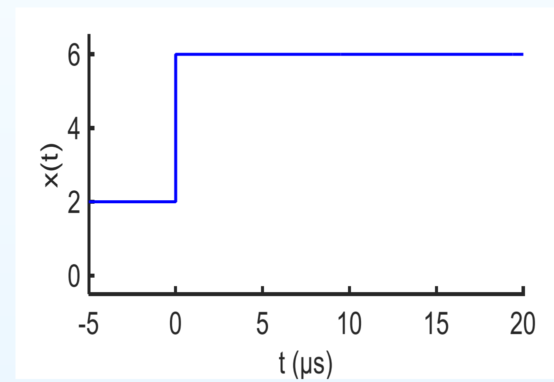

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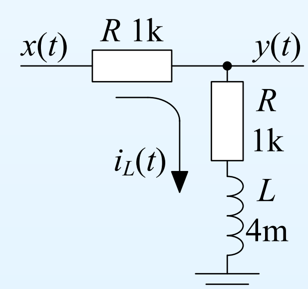

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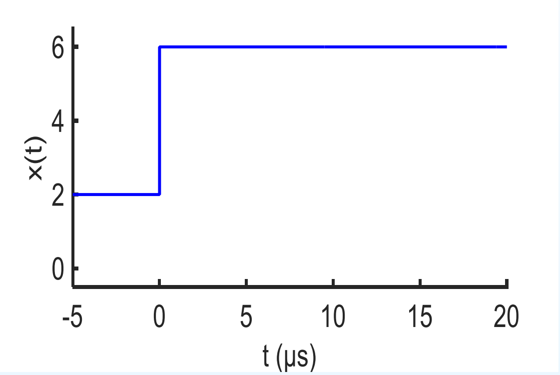

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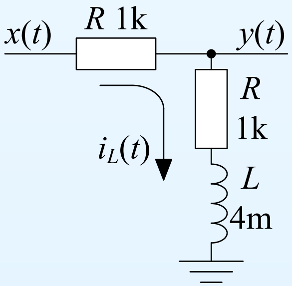

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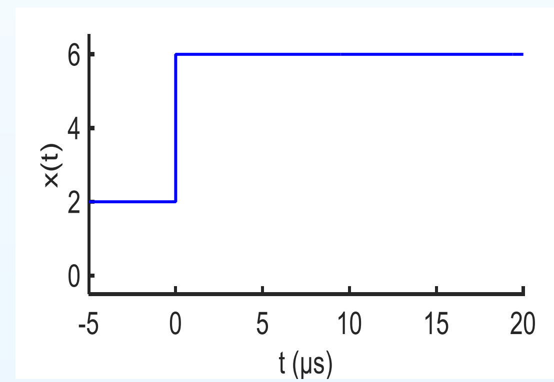

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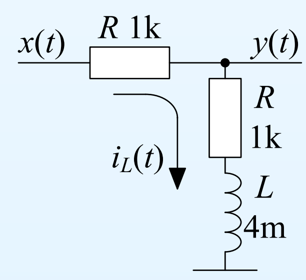

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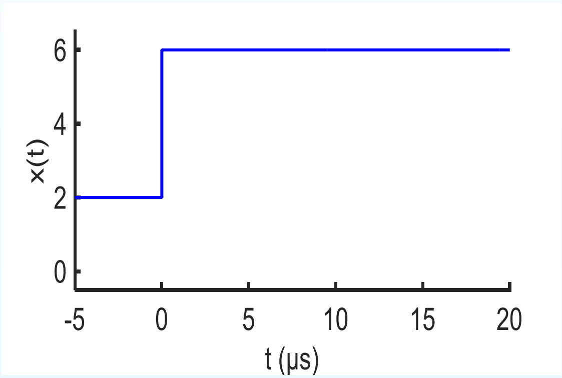

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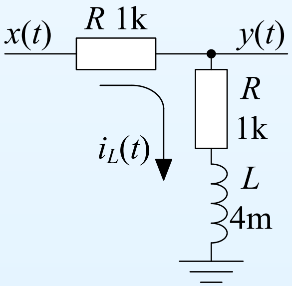

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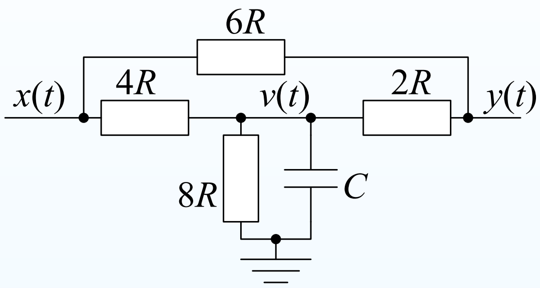
Sinusoidal input $\Rightarrow$ Sinusoidal steady state $\Rightarrow$ use phasors.
Then convert phasors to time waveforms to calculate the actual output voltages $y_{SS}(0-)$ and $y_{SS}(0+)$ at $t = 0$.

Determining Time Constant

Method 1: Thévenin



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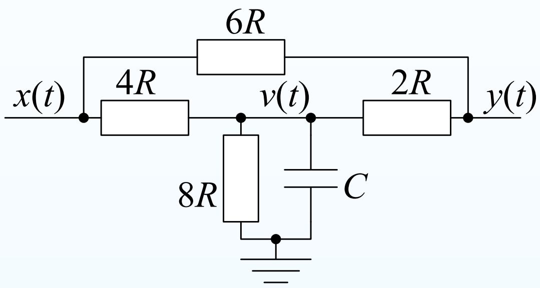

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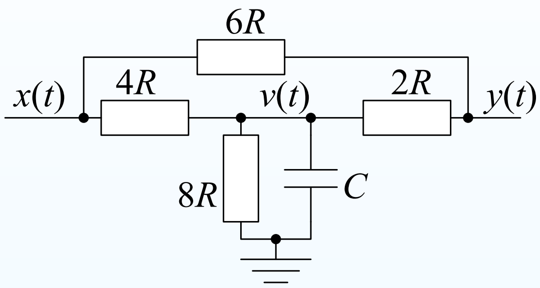

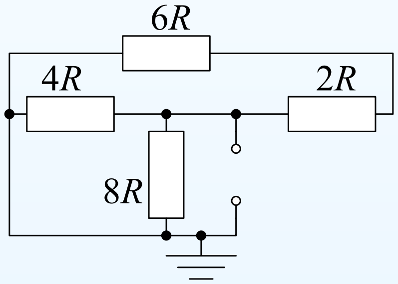

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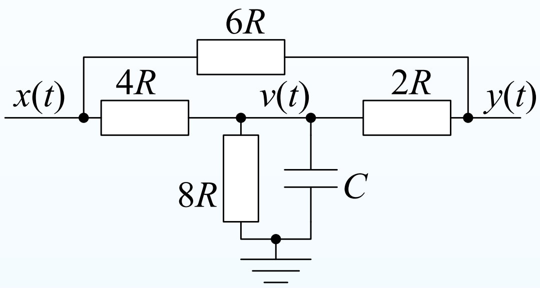

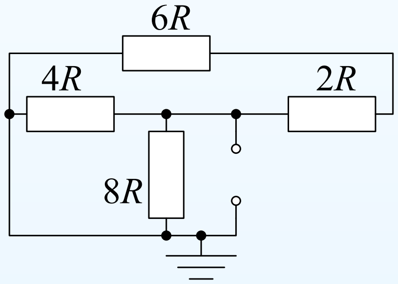

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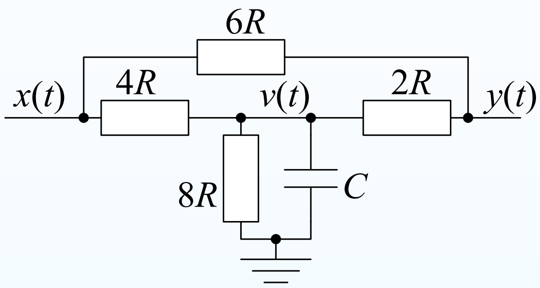

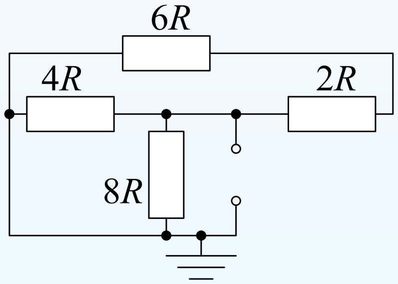

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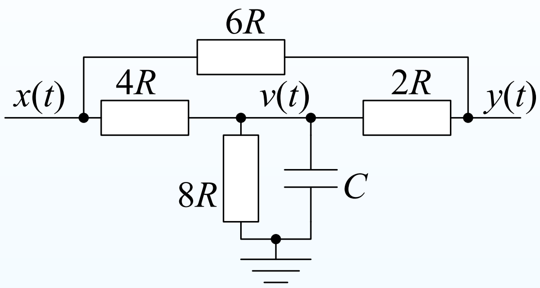

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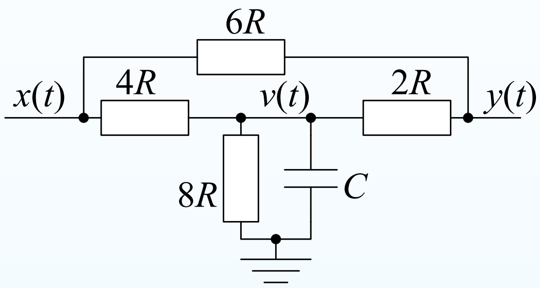

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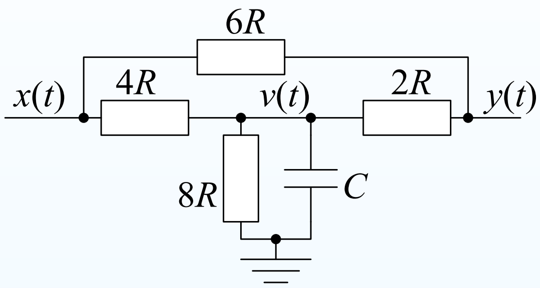

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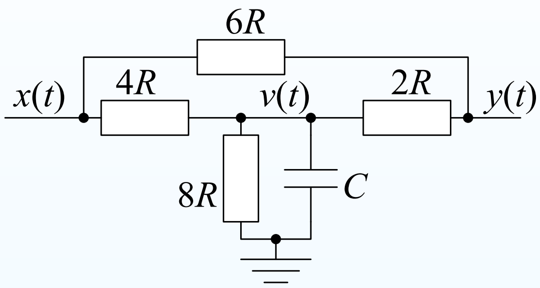

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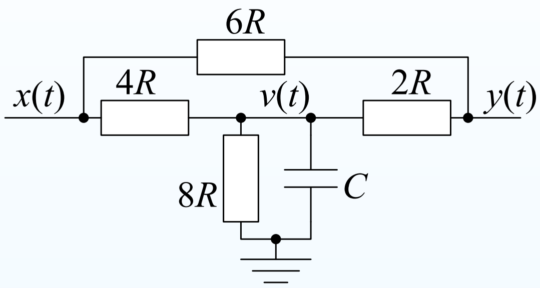

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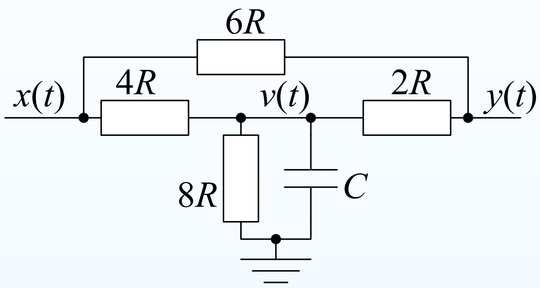

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$$\omega_d = \frac{16}{32RC} \Rightarrow \tau = \frac{1}{\omega_d} = 2RC$$

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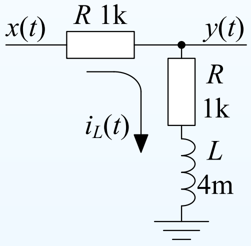

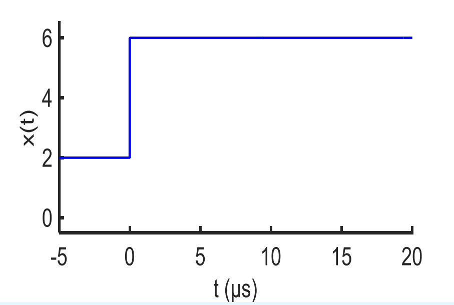

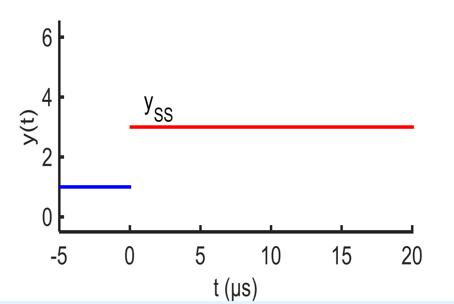

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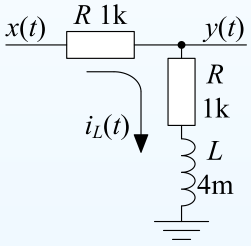
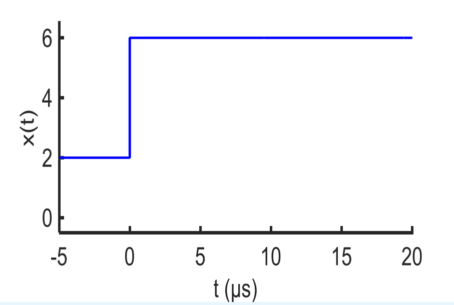
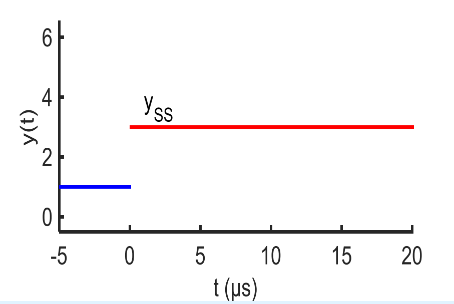
$\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$







Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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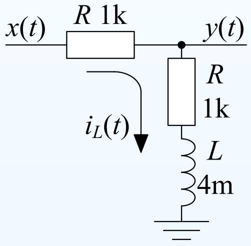
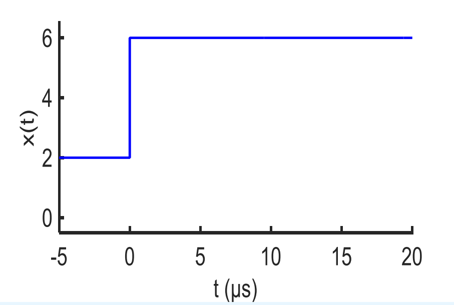
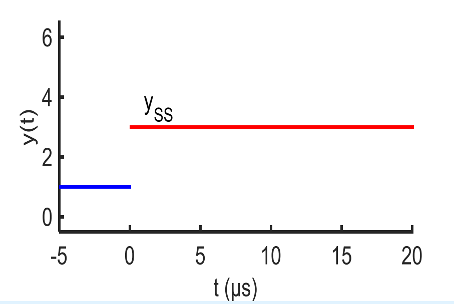
Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A







Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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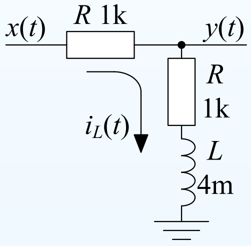
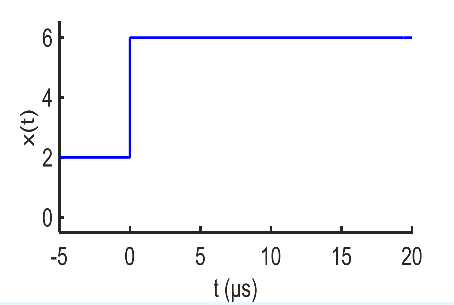
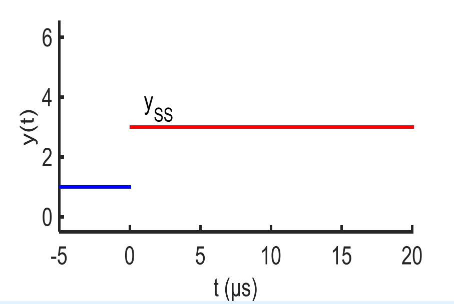
Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A

(i) Version 1: v_C or i_L continuity







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After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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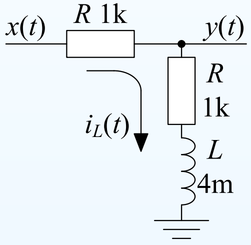
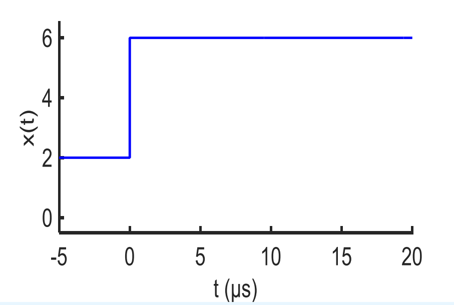
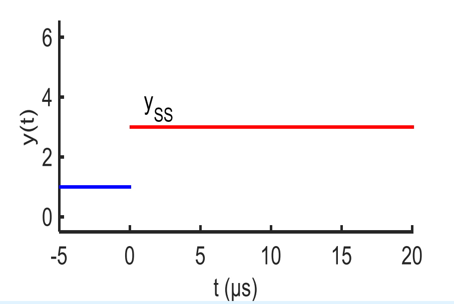
Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A

(i) Version 1: v_C or i_L continuity

$x(0-) = 2$





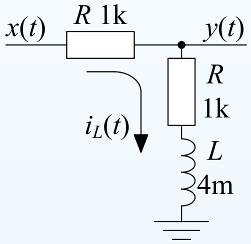


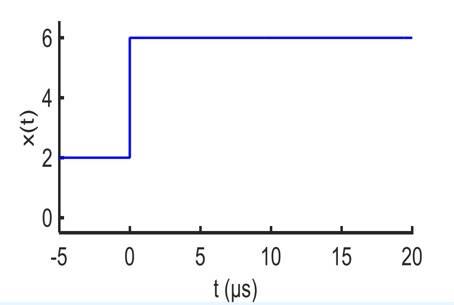
Determining Transient Amplitude

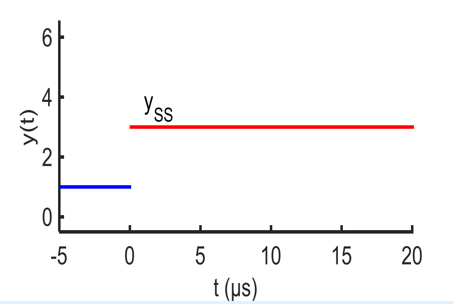
After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
$\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$
Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A

(i) Version 1: v_C or i_L continuity

$$x(0-) = 2 \Rightarrow i_L(0-) = 1\,\text{mA}$$







Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

$\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$

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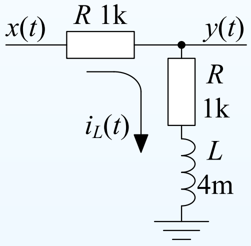
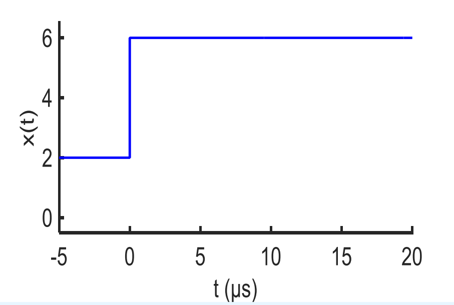
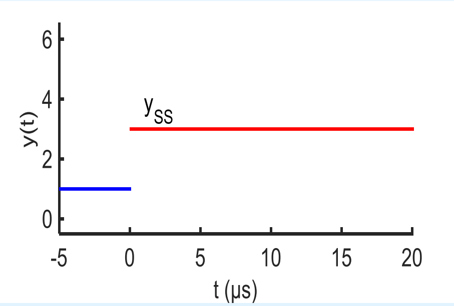
(i) Version 1: v_C or i_L continuity

$x(0-) = 2 \Rightarrow i_L(0-) = 1\,\text{mA}$

Continuity $\Rightarrow i_L(0+) = i_L(0-)$







Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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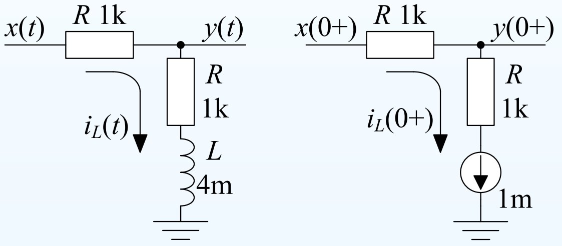
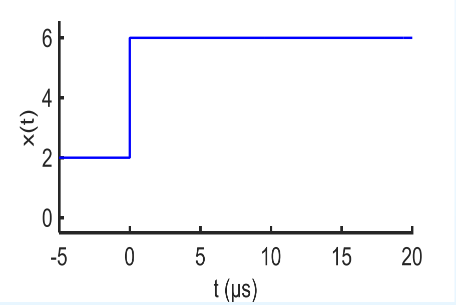
(i) Version 1: v_C or i_L continuity

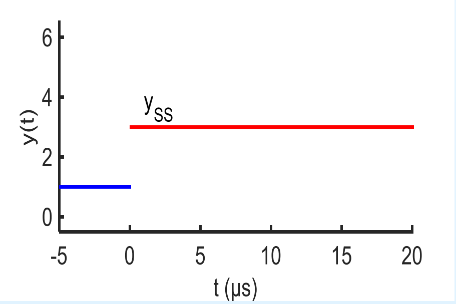
$x(0-) = 2 \Rightarrow i_L(0-) = 1\,\mathrm{mA}$

Continuity $\Rightarrow i_L(0+) = i_L(0-)$

Replace L with a $1\,\mathrm{mA}$ current source







Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

$\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$

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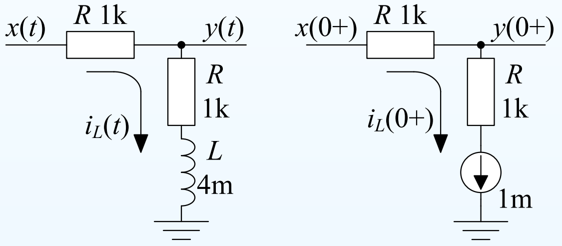
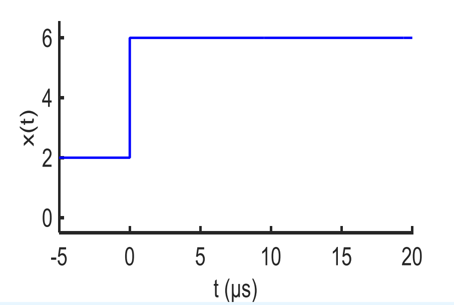
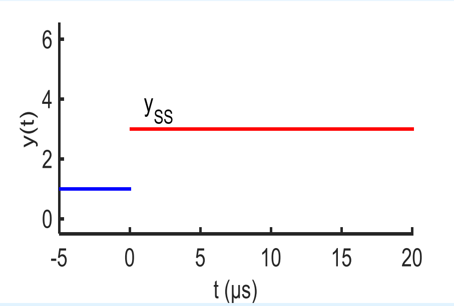
Continuity $\Rightarrow i_L(0+) = i_L(0-)$

Replace L with a $1\,\mathrm{mA}$ current source

$y(0+) = x(0+) - iR = 6 - 1 = 5$







Determining Transient Amplitude

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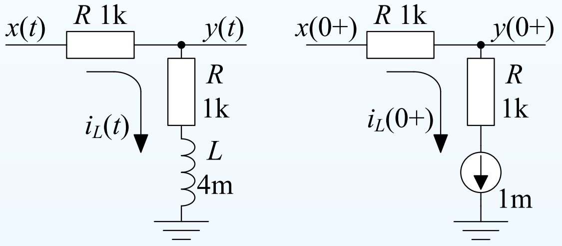
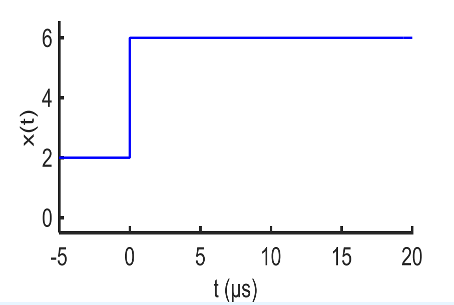
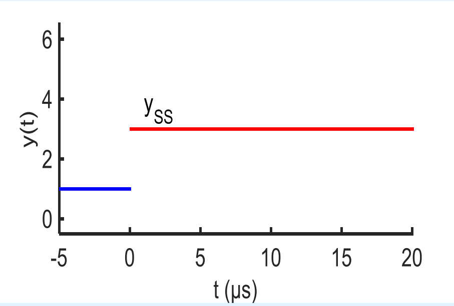
Continuity $\Rightarrow i_L(0+) = i_L(0-)$
Replace L with a $1\,\text{mA}$ current source
$y(0+) = x(0+) - iR = 6 - 1 = 5$

(i) Version 2: Transfer function







Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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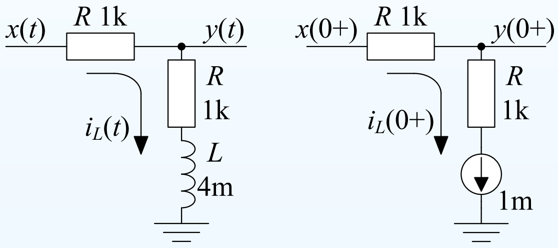
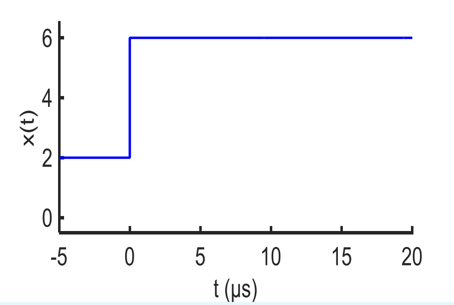
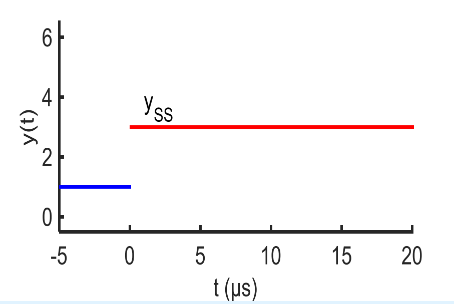
Continuity $\Rightarrow i_L(0+) = i_L(0-)$

Replace L with a $1\,\text{mA}$ current source

$y(0+) = x(0+) - iR = 6 - 1 = 5$







(i) Version 2: Transfer function

$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R + j\omega L}{2R + j\omega L}$

Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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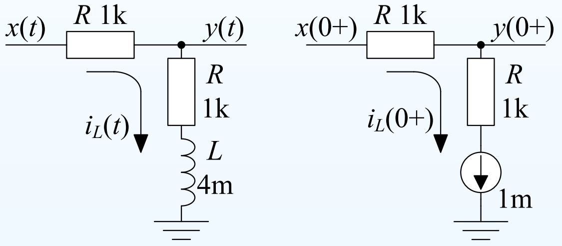
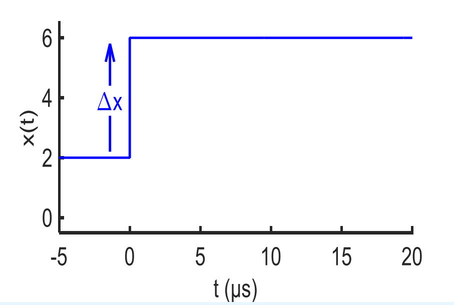
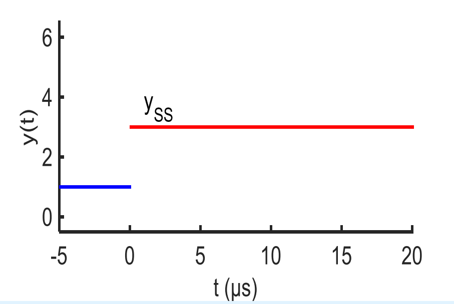


(i) Version 2: Transfer function

$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$
Input step, $\Delta x = x(0+) - x(0-) = +4$





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After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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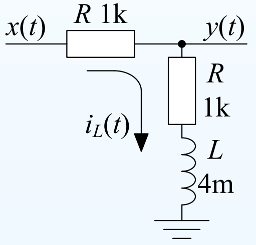

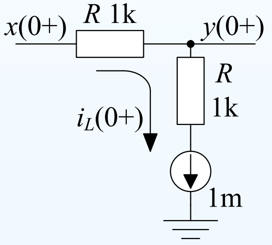

(i) Version 2: Transfer function

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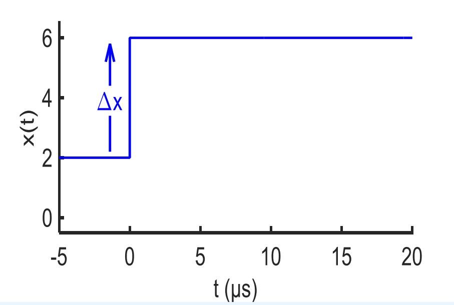
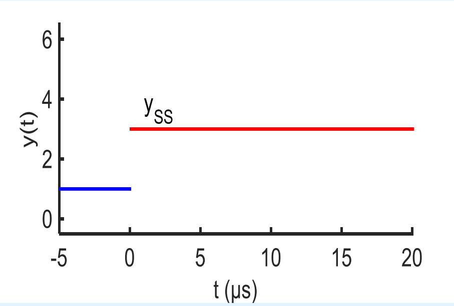
Input step, $\Delta x = x(0+) - x(0-) = +4$
$y(0+) = y(0-) + H(j\infty) \times \Delta x$





Determining Transient Amplitude

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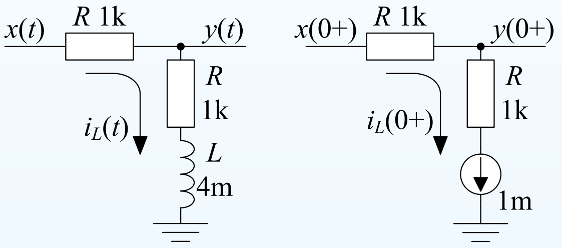
(i) Version 1: v_C or i_L continuity

$x(0-) = 2 \Rightarrow i_L(0-) = 1\,\mathrm{mA}$
Continuity $\Rightarrow i_L(0+) = i_L(0-)$
Replace L with a $1\,\mathrm{mA}$ current source
$y(0+) = x(0+) - iR = 6 - 1 = 5$



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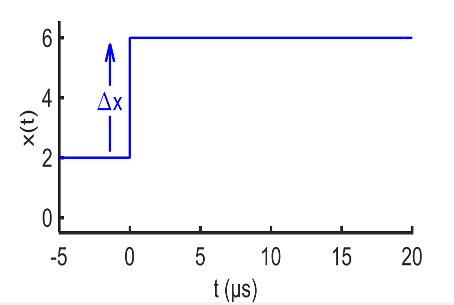
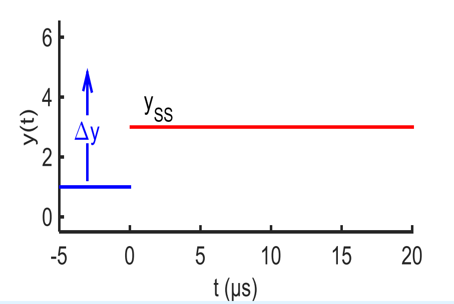
$y(0+) = y(0-) + H(j\infty) \times \Delta x$
$= 1 + \Delta y = 1 + 1 \times 4 = 5$





Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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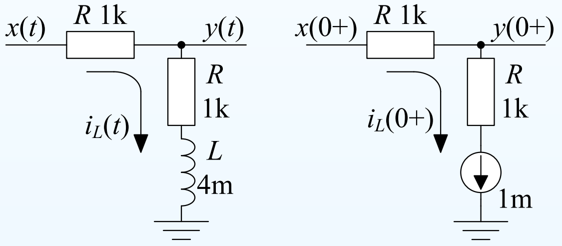
Continuity $\Rightarrow i_L(0+) = i_L(0-)$
Replace L with a $1\,\text{mA}$ current source
$y(0+) = x(0+) - iR = 6 - 1 = 5$



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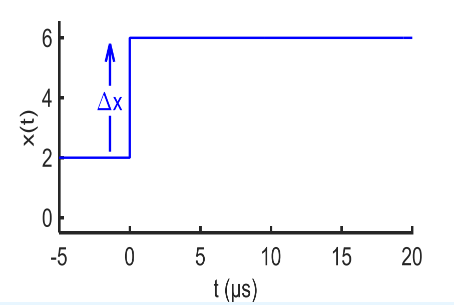

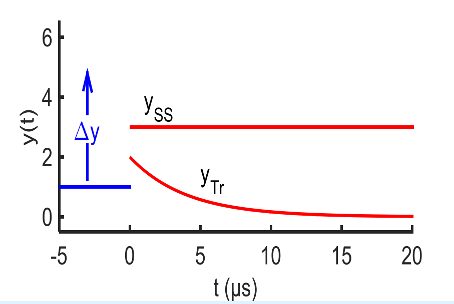

(ii) $A = y(0+) - y_{SS}(0+) = 5 - 3 = 2$

Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

$\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$

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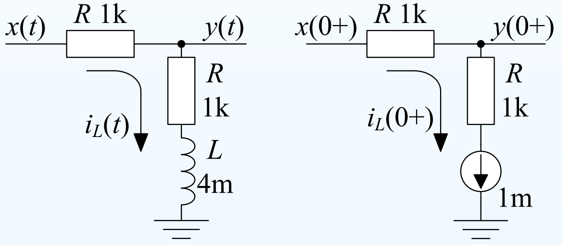
Continuity $\Rightarrow i_L(0+) = i_L(0-)$

Replace L with a $1\,\text{mA}$ current source

$y(0+) = x(0+) - iR = 6 - 1 = 5$



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$y(0+) = y(0-) + H(j\infty) \times \Delta x$

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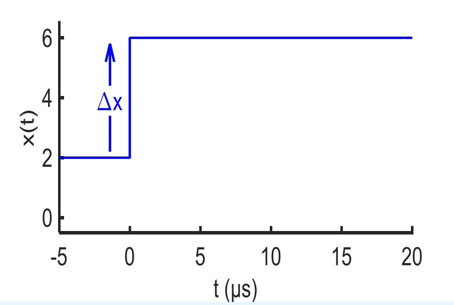
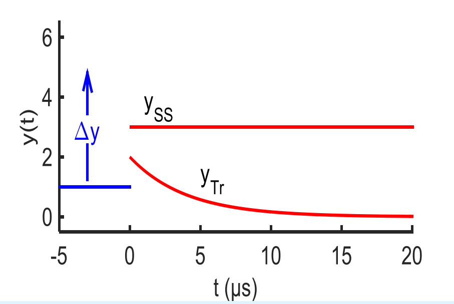


(ii) $A = y(0+) - y_{SS}(0+) = 5 - 3 = 2$

(iii) $y(t) = y_{SS}(t) + Ae^{-t/\tau}$



Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
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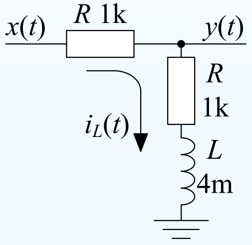
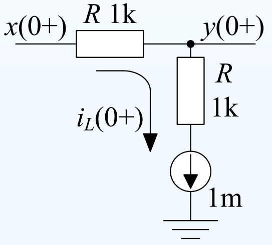
(i) Version 1: v_C or i_L continuity

$x(0-) = 2 \Rightarrow i_L(0-) = 1\,\text{mA}$
Continuity $\Rightarrow i_L(0+) = i_L(0-)$
Replace L with a $1\,\text{mA}$ current source
$y(0+) = x(0+) - iR = 6 - 1 = 5$



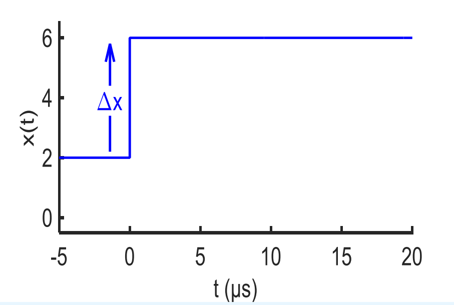


(i) Version 2: Transfer function

$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$

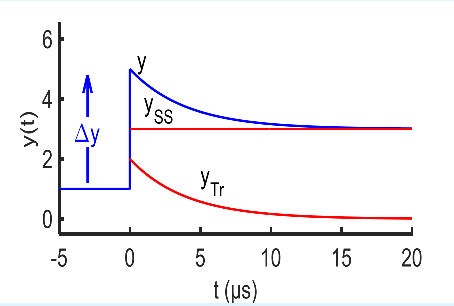
Input step, $\Delta x = x(0+) - x(0-) = +4$

$y(0+) = y(0-) + H(j\infty) \times \Delta x$
$= 1 + \Delta y = 1 + 1 \times 4 = 5$



(ii) $A = y(0+) - y_{SS}(0+) = 5 - 3 = 2$

(iii) $y(t) = y_{SS}(t) + Ae^{-t/\tau}$
$= 3 + 2e^{-t/2\mu}$



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Transmission Line: constant L_0 and C_0 : inductance/capacitance per metre.

Forward wave travels along the line: $f_x(t) = f_0\left(t - \frac{x}{u}\right)$.

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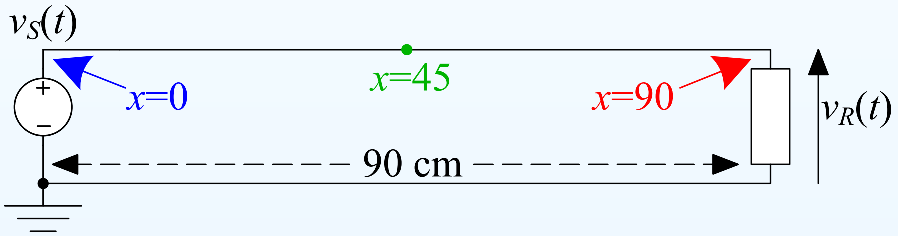

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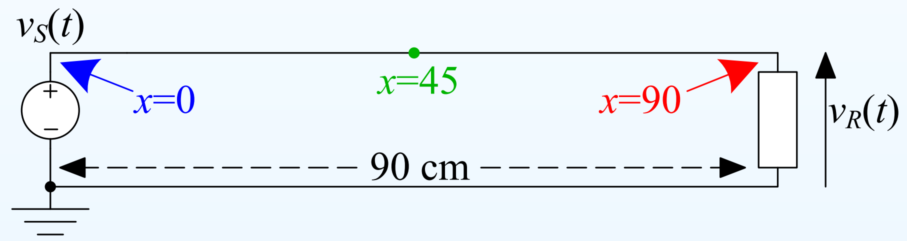

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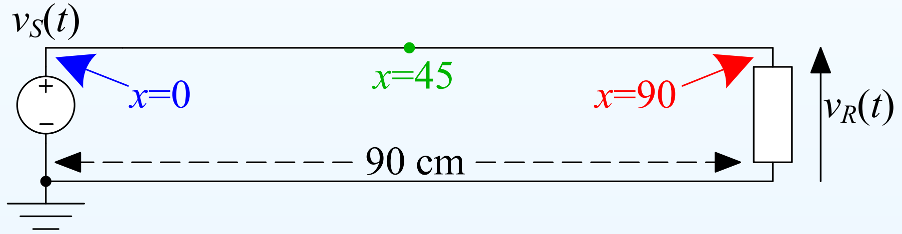

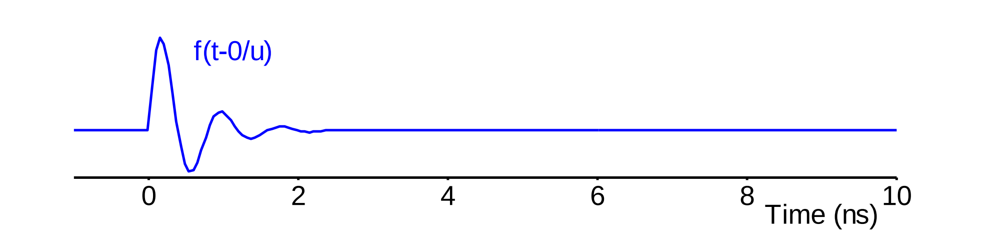

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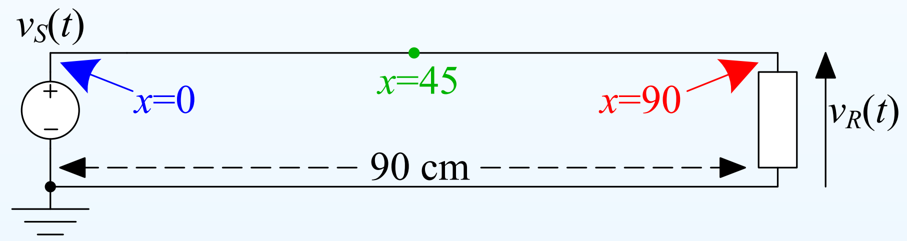

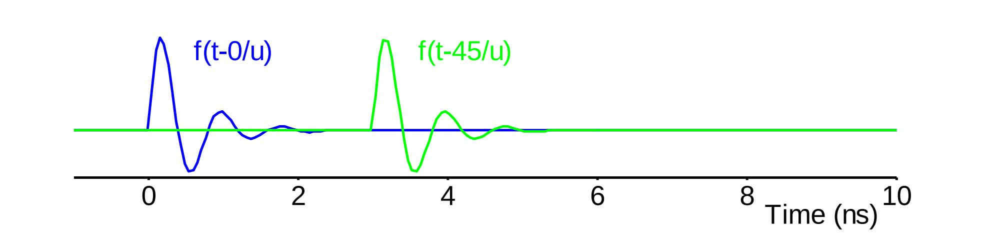

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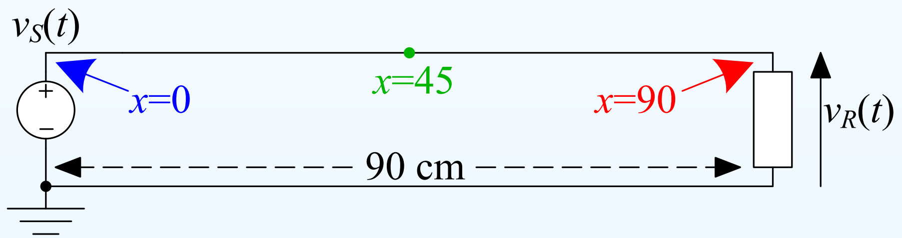

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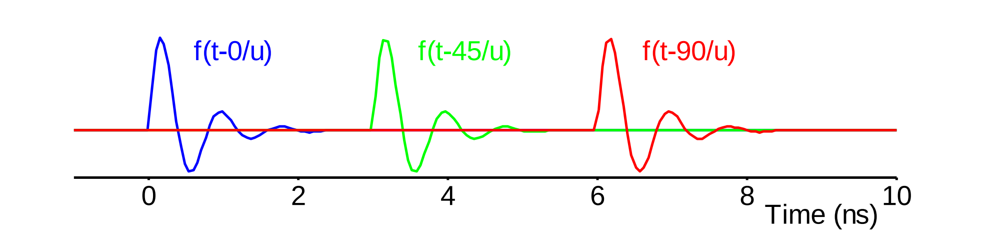

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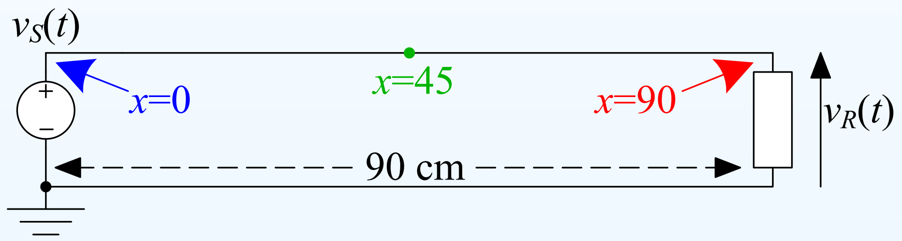

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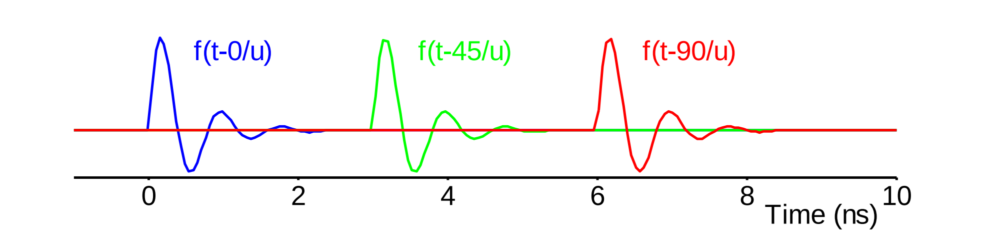

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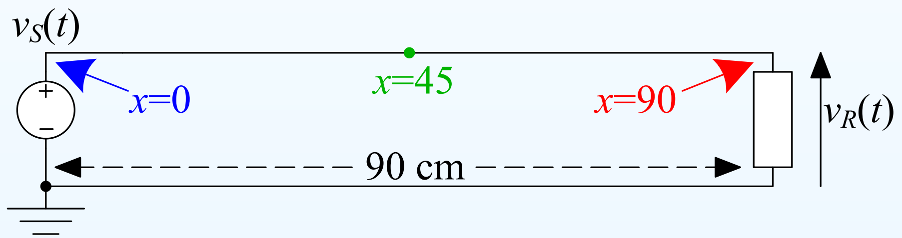

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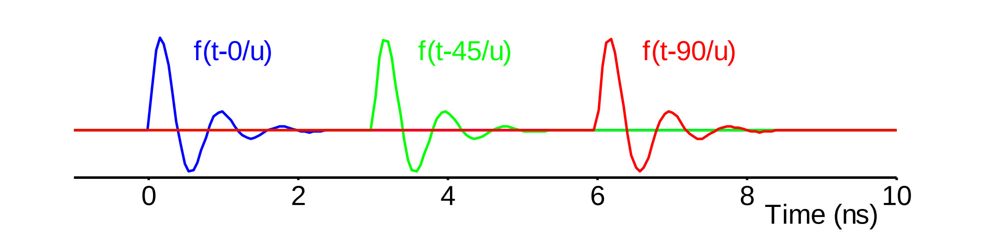

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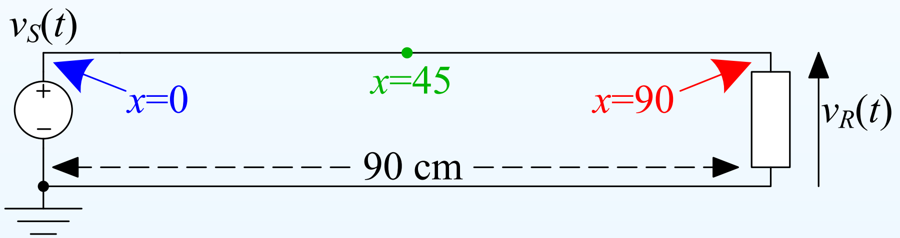

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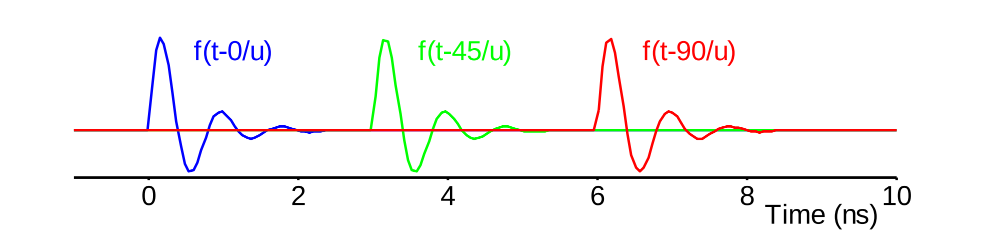

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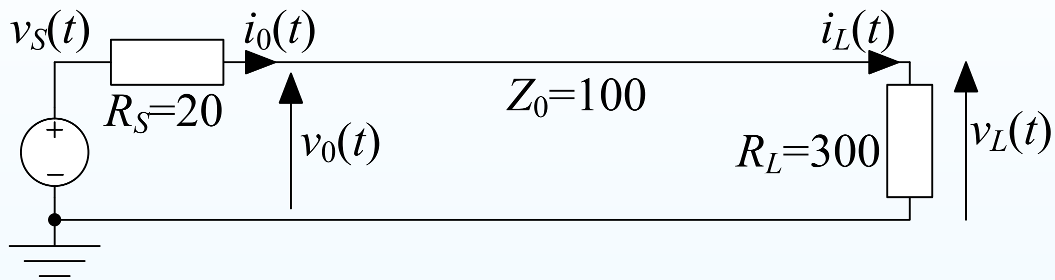
Waveforms of f_x and g_x are determined by the connections at both ends.

Reflections



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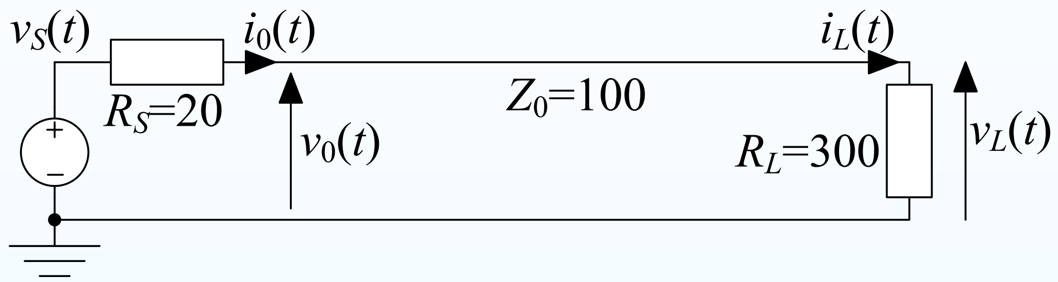

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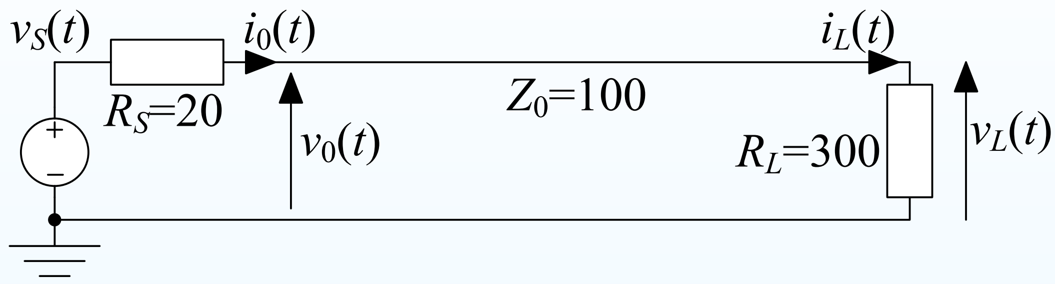

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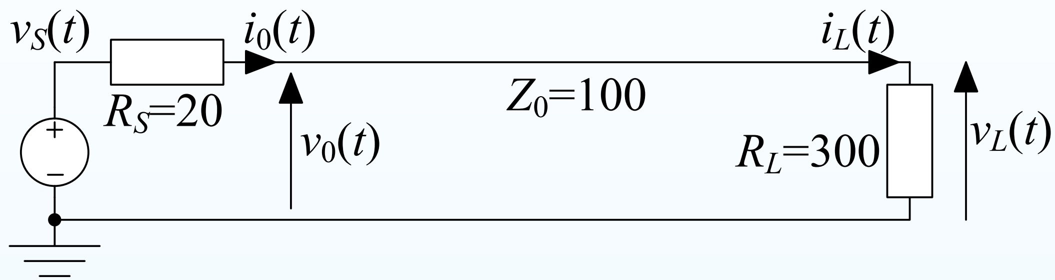

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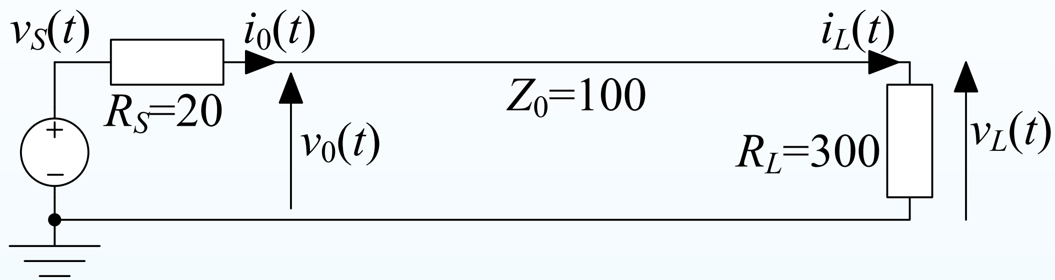

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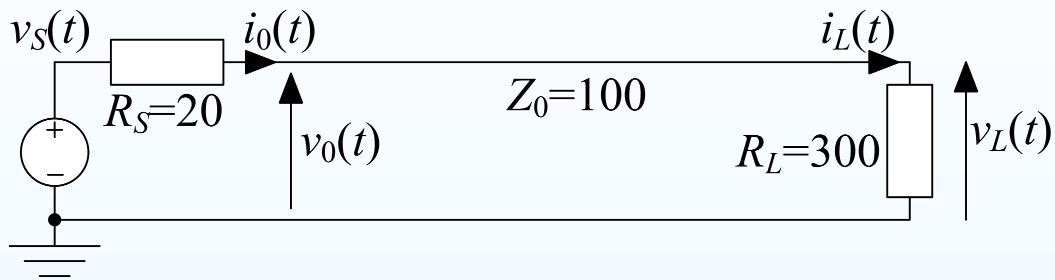

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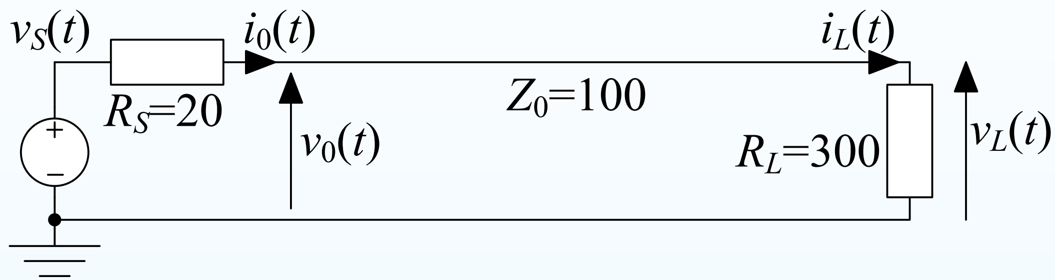

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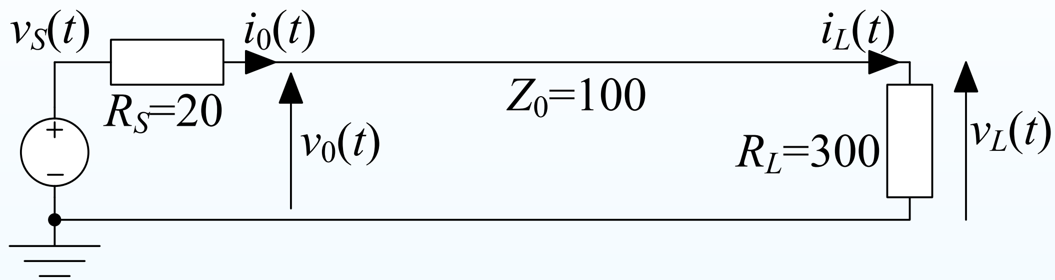

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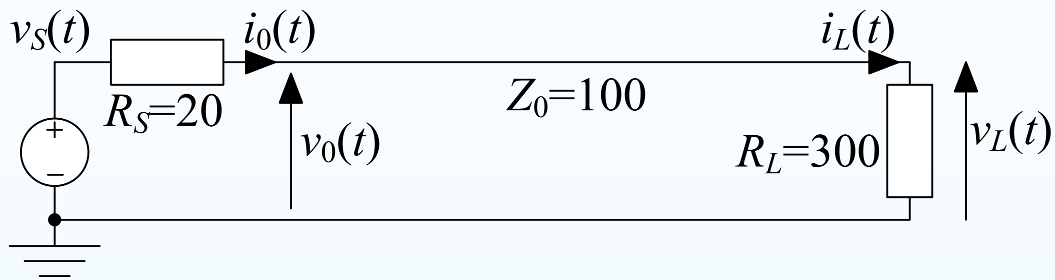

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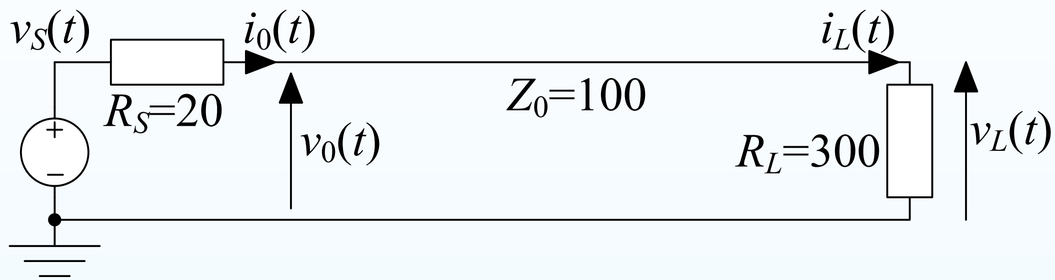

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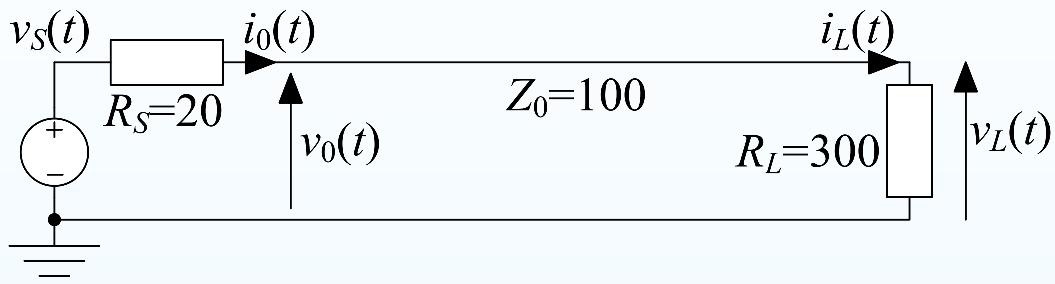

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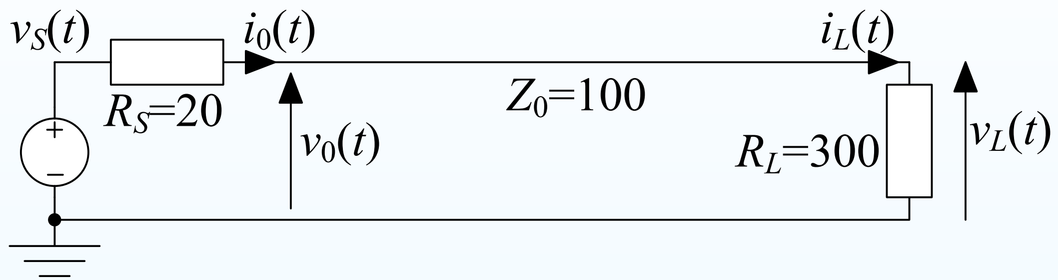

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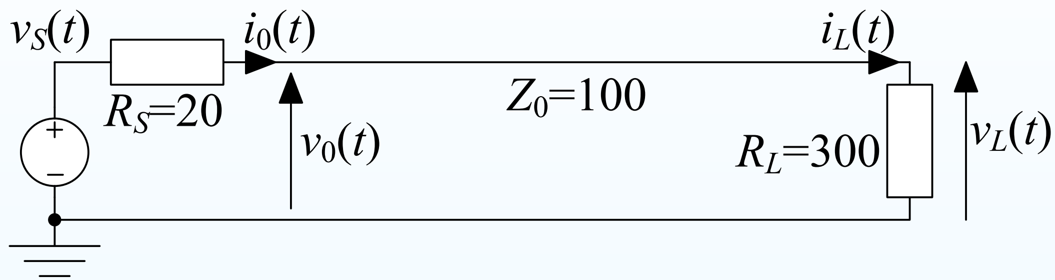
Reflections



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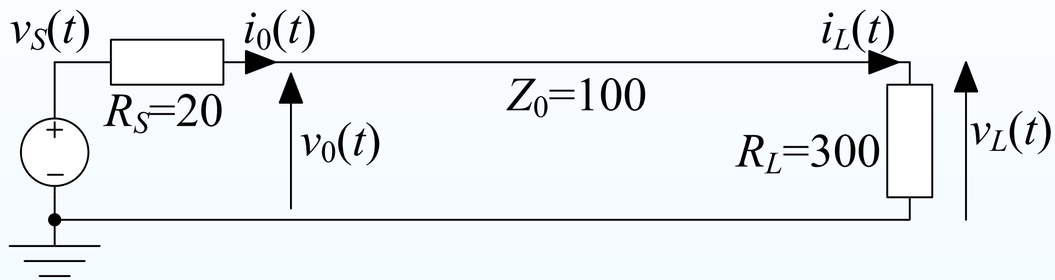

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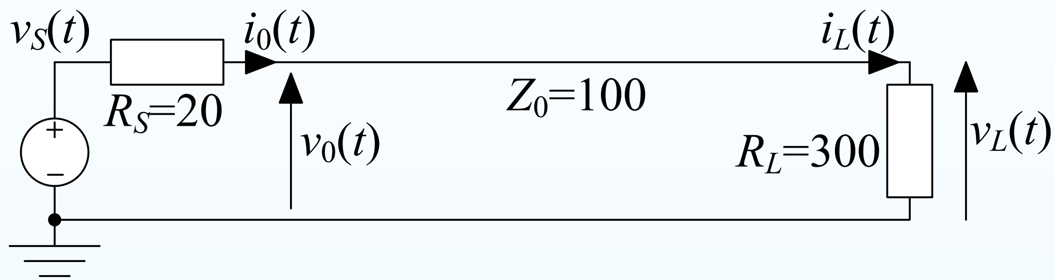

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1. Use phasors to eliminate t:

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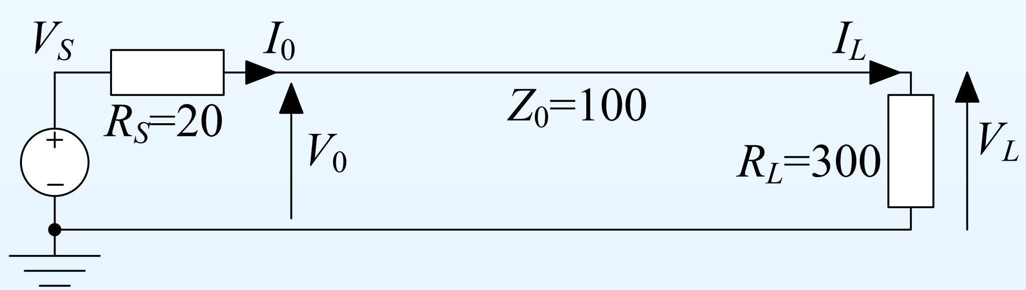

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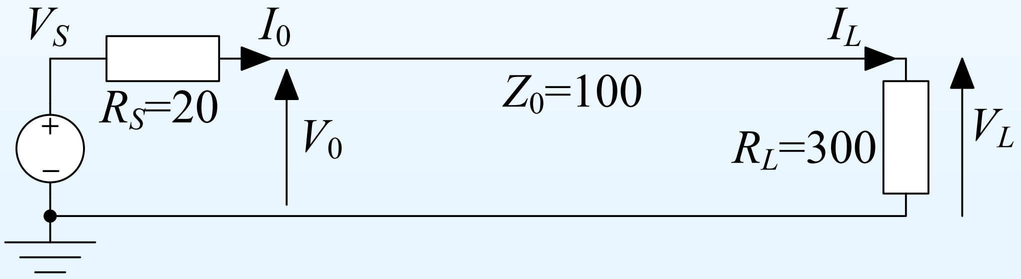

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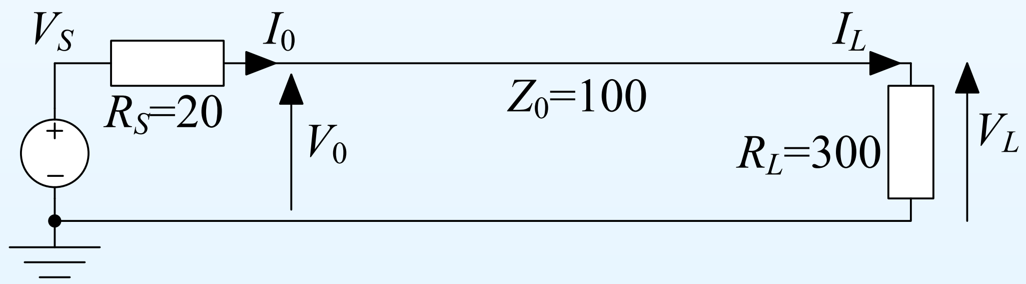

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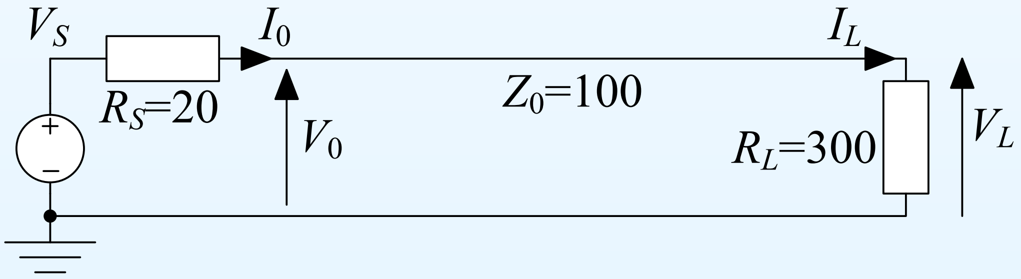

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$\Rightarrow F_0 = \frac{\tau_0}{1 - \rho_0 \rho_L \exp(-2jkL)} V_S$ so no infinite sums needed ☺

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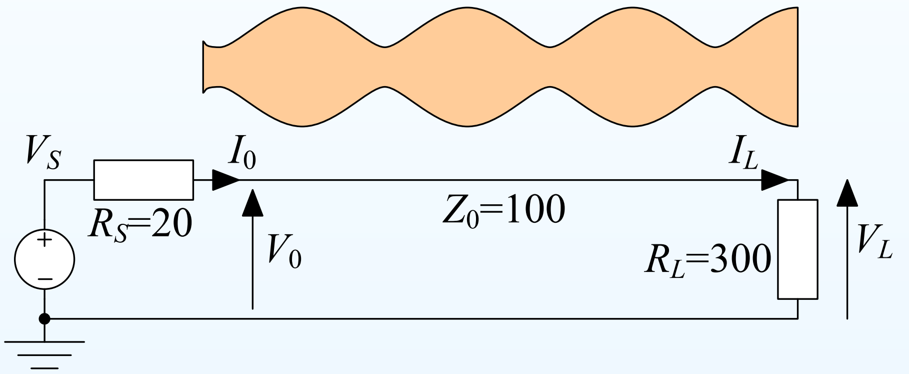
Standing waves arise whenever a wave meets its reflection:

at positions where the two waves are in phase their amplitudes add but where they are anti-phase their amplitudes subtract.



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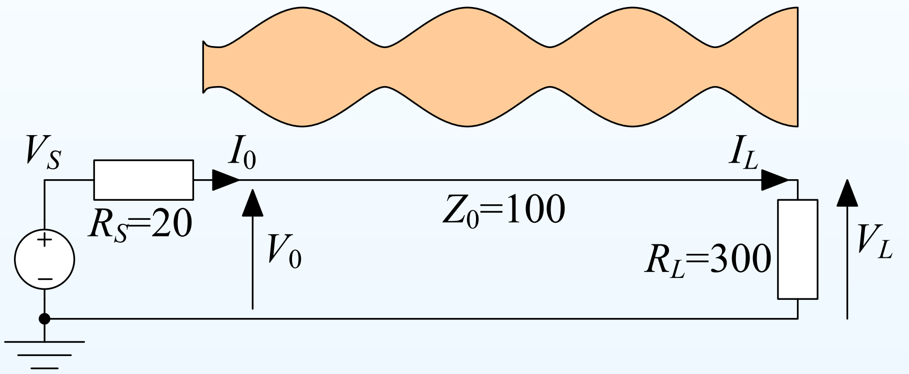

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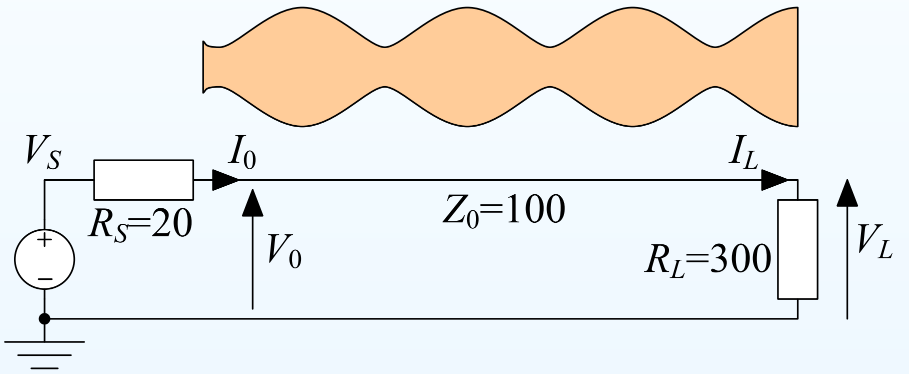

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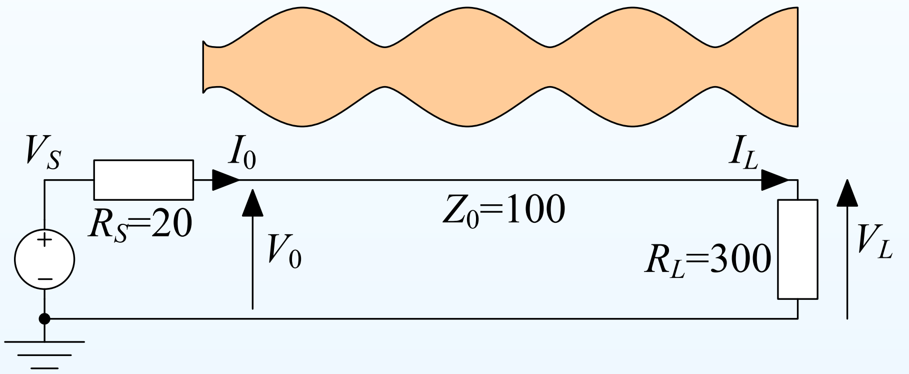

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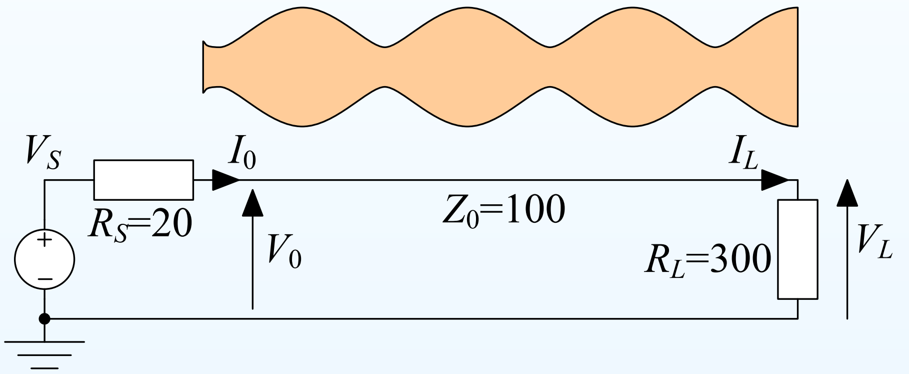

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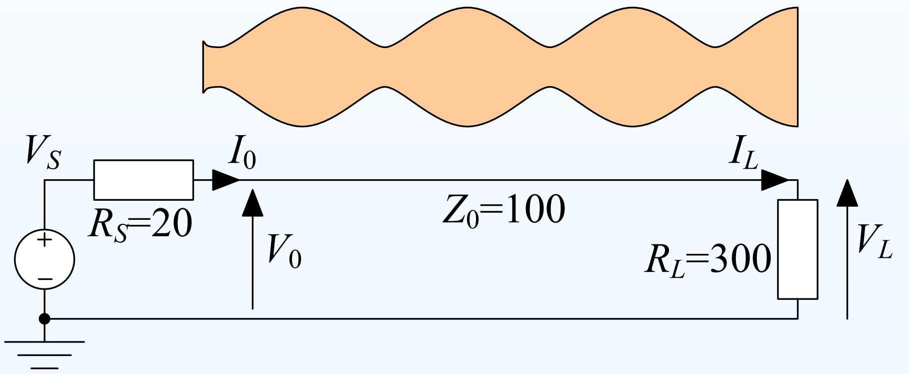

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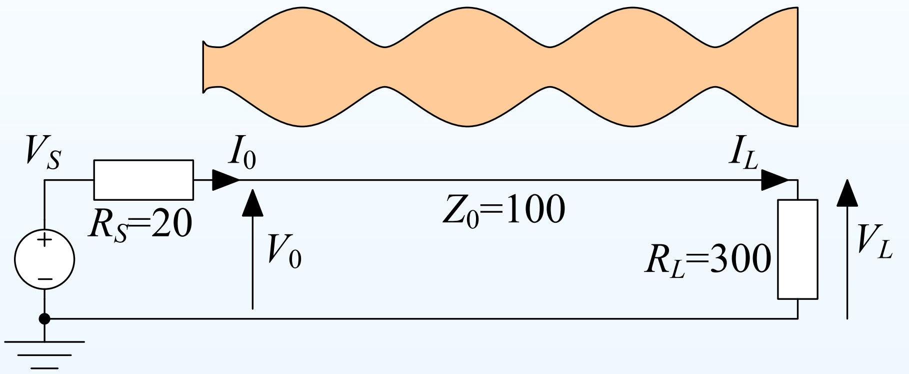

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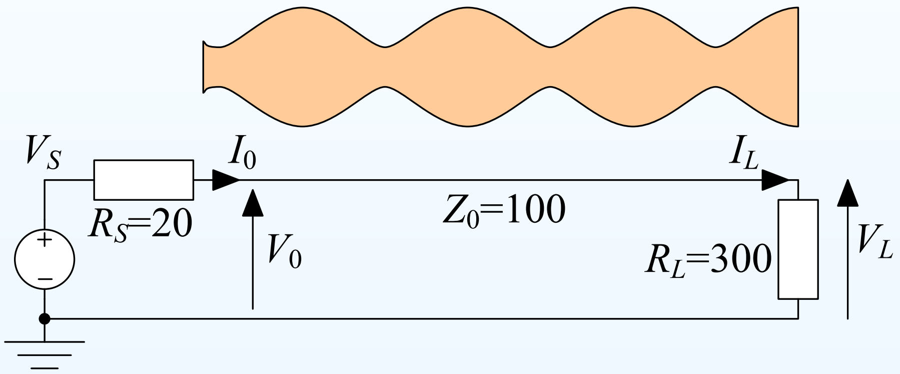

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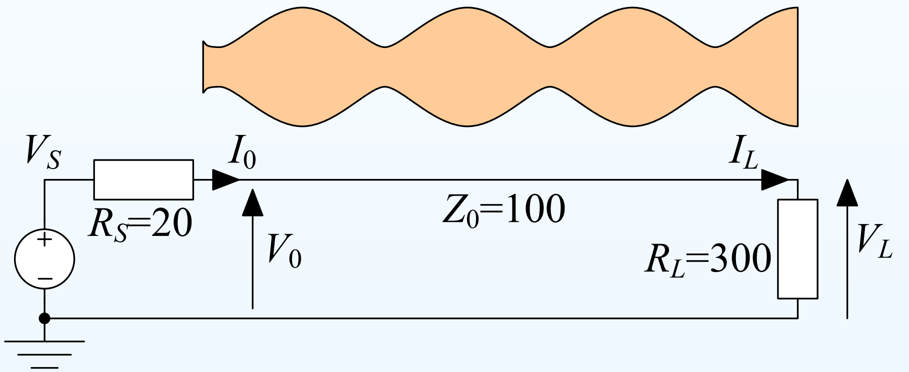

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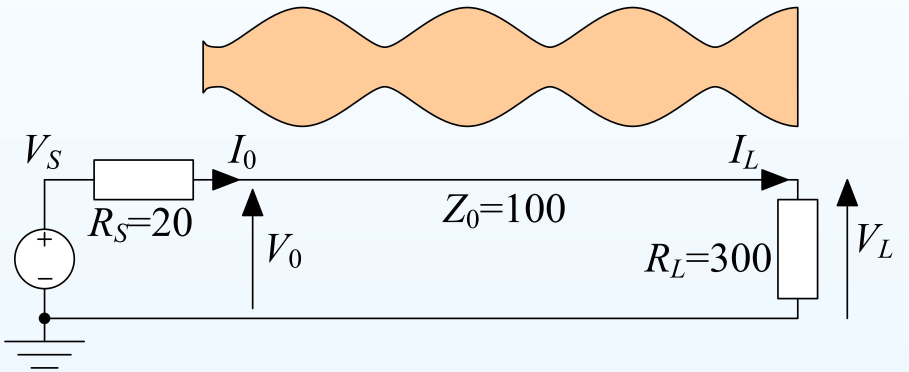

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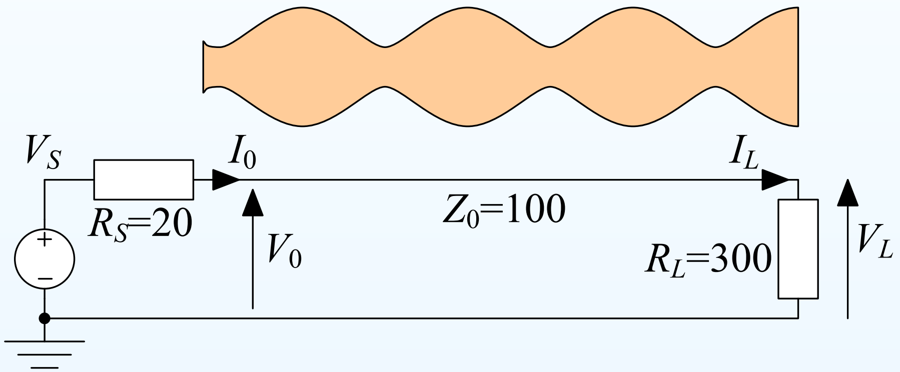

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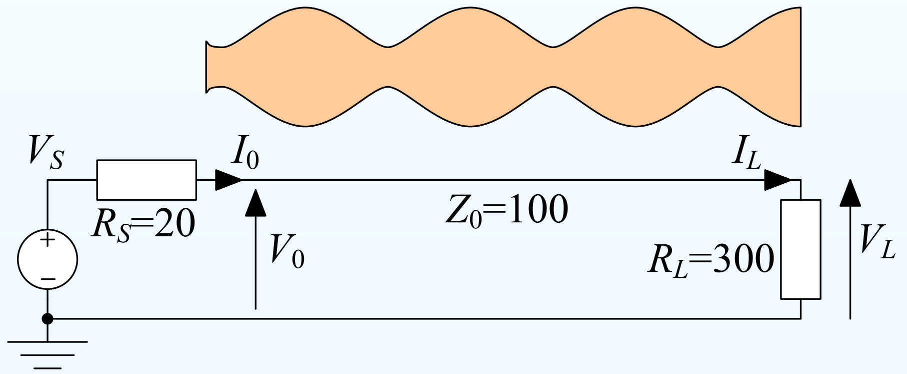

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