Revision Lecture 2: Transients & Lines

- Transients: Basic Ideas
- Steady States
- Determining Time Constant
- Determining Transient Amplitude
- Transmission Lines Basics
- Reflections
- Sinewaves and Phasors
- Standing Waves
Transients: Basic Ideas

- Transients happen in response to a sudden change
  - Input voltage/current abruptly changes its magnitude, frequency or phase
  - A switch alters the circuit
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  - Steady States: find with nodal analysis or transfer function
    - Note: Steady State is not the same as DC Level
    - Need steady states before and after the sudden change
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    - Time Constant: \( \tau = RC \) or \( \frac{L}{R} \) where \( R \) is the Thévenin resistance at the terminals of \( C \) or \( L \)
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    - Find transient amplitude, \( A \), from continuity since \( V_C \) or \( I_L \) cannot change instantly.
    - \( \tau \) and \( A \) can also be found from the transfer function.
A **steady-state** output assumes the input frequency, phase and amplitude are constant forever. You need to determine two $y_{SS}(t)$ steady state outputs: one for **before** the transient ($t < 0$) and one **after** ($t \geq 0$).
**Steady States**

A **steady-state** output assumes the input frequency, phase and amplitude are constant forever. You need to determine two $y_{SS}(t)$ steady state outputs: one for before the transient ($t < 0$) and one after ($t \geq 0$).

At $t = 0$, $y_{SS}(0-) = 0$ means the first one and $y_{SS}(0+) = 0$ means the second.
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**Method 1: Nodal analysis**

Input voltage is DC ($\omega = 0$)

⇒ $Z_L = 0$ (for capacitor: $Z_C = \infty$)

So $L$ acts as a short circuit
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Potential divider: $y_{SS} = \frac{1}{2}x$
Steady States

A steady-state output assumes the input frequency, phase and amplitude are constant forever. You need to determine two $y_{SS}(t)$ steady state outputs: one for before the transient ($t < 0$) and one after ($t \geq 0$). At $t = 0$, $y_{SS}(0-) = 1$, $y_{SS}(0+) = 3$.

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$Y(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$
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$\frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$

set $\omega = 0$: $\frac{Y}{X}(0) = \frac{1}{2}$
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Sinusoidal input $\Rightarrow$ Sinusoidal steady state $\Rightarrow$ use phasors.

Then convert phasors to time waveforms to calculate the actual output voltages $y_{SS}(0-) \text{ and } y_{SS}(0+) \text{ at } t = 0.$
Determining Time Constant

Method 1: Thévenin

![Circuit Diagram]

- Determining Time Constant
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(a) Remove the capacitor/inductor
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\[ R_{Th} = 8R || 4R || (6R + 2R) = 2R \]
Determining Time Constant

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Method 2: Transfer function
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Method 2: Transfer function
(a) Calculate transfer function using nodal analysis
Determining Time Constant

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Method 2: Transfer function
(a) Calculate transfer function using nodal analysis
KCL @ V: \[ \frac{V - X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V - Y}{2R} = 0 \]
KCL @ Y: \[ \frac{V - X}{2R} + \frac{Y - X}{6R} = 0 \]
Determining Time Constant

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→ Eliminate V to get transfer Function: \[ \frac{Y}{X} = \frac{8j\omega RC + 13}{32j\omega RC + 16} \]
Determining Time Constant

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(a) Remove the capacitor/inductor
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(c) Calculate the Thévenin resistance between the capacitor/inductor terminals:
\[ R_{Th} = 8R || 4R || (6R + 2R) = 2R \]
(d) Time constant: \( \tau = \frac{R_{Th}C}{L} \)
\[ \tau = R_{Th}C = 2RC \]

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(a) Calculate transfer function using nodal analysis
KCL @ V: \[ \frac{V - X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V - Y}{2R} = 0 \]
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→ Eliminate V to get transfer Function: \[ \frac{Y}{X} = \frac{8j\omega RC + 13}{32j\omega RC + 16} \]
(b) Time Constant = \( \frac{1}{\text{Denominator corner frequency}} \)
**Determining Time Constant**

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(b) Time Constant = \( \frac{1}{\text{Denominator corner frequency}} \)
\( \omega_d = \frac{16}{32RC} \Rightarrow \tau = \frac{1}{\omega_d} = 2RC \)
After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$. 
Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

$\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$
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Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get $A$
Determining Transient Amplitude

After an input change at \( t = 0 \), \( y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}} \).

\[ y(0^+) = y_{SS}(0^+) + A \Rightarrow A = y(0^+) - y_{SS}(0^+) \]

Method: (a) calculate true output \( y(0^+) \), (b) subtract \( y_{SS}(0^+) \) to get \( A \)

(i) Version 1: \( v_C \) or \( i_L \) continuity

![Diagrams showing Circuit Analysis](#)
After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

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Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get $A$

(i) Version 1: $v_C$ or $i_L$ continuity

$x(0-) = 2$
After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

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$x(0-) = 2 \Rightarrow i_L(0-) = 1 \text{ mA}$
Determining Transient Amplitude

After an input change at \( t = 0 \), \( y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}} \).

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\[ x(0-) = 2 \Rightarrow i_L(0-) = 1 \text{ mA} \]

Continuity \( \Rightarrow i_L(0+) = i_L(0-) \)
Determining Transient Amplitude

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Continuity $\Rightarrow i_L(0+) = i_L(0-)$

Replace $L$ with a $1\ mA$ current source
Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

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$x(0-) = 2 \Rightarrow i_L(0-) = 1 \text{ mA}$

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Replace $L$ with a 1 mA current source

$y(0+) = x(0+) - iR = 6 - 1 = 5$
After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$. 
\[ y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+) \]

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(i) Version 2: Transfer function
Determining Transient Amplitude

After an input change at \( t = 0 \), \( y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}} \).

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Continuity \( \Rightarrow i_L(0^+) = i_L(0-) \)

Replace \( L \) with a 1 mA current source

\[ y(0^+) = x(0^+) - iR = 6 - 1 = 5 \]

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\[ H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L} \]
Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

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$y(0+) = x(0+) - iR = 6 - 1 = 5$

(ii) Version 2: Transfer function

$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R + j\omega L}{2R + j\omega L}$

Input step, $\Delta x = x(0+) - x(0-) = +4$
Determining Transient Amplitude

After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

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Replace $L$ with a $1 \text{ mA}$ current source

$y(0+) = x(0+) - iR = 6 - 1 = 5$

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Input step, $\Delta x = x(0+) - x(0-) = +4$

$y(0+) = y(0-) + H(j\infty) \times \Delta x$
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$= 1 + \Delta y = 1 + 1 \times 4 = 5$
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$y(0+) = x(0+) - iR = 6 - 1 = 5$

(ii) $A = y(0+) - y_{SS}(0+) = 5 - 3 = 2$

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Input step, $\Delta x = x(0+) - x(0-) = +4$

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After an input change at \( t = 0 \), \( y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}} \).

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Continuity \( \Rightarrow i_L(0+) = i_L(0-) \)

Replace \( L \) with a 1 mA current source

\[ y(0+) = x(0+) - iR = 6 - 1 = 5 \]

(ii) \( A = y(0+) - y_{SS}(0+) = 5 - 3 = 2 \)

(iii) \( y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}} \)
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Replace $L$ with a 1 mA current source

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(iii) $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$

$= 3 + 2e^{-\frac{t}{2\mu}}$
Transmission Lines Basics

Transmission Line: constant $L_0$ and $C_0$ : inductance/capacitance per metre.

Forward wave travels along the line: $f_x(t) = f_0 \left(t - \frac{x}{u}\right)$.

Velocity $u = \sqrt{\frac{1}{L_0 C_0}} \approx \frac{1}{2} c = 15 \text{ cm/ns}$
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\[ f(t-0/u) \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \text{ Time (ns)} \]

\[ v_S(t) \quad v_R(t) \]

\[ x=0 \quad x=45 \quad x=90 \quad 90 \text{ cm} \]

\[ f(t-0/u) \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \text{ Time (ns)} \]
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Knowing $f_x(t)$ for $x = x_0$ fixes it for all other $x$. 
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Backward wave: $g_x(t)$ is the same but travelling $\leftarrow$: $g_x(t) = g_0 \left( t + \frac{x}{u} \right)$. 

\[ f(t-0/u) f(t-45/u) f(t-90/u) \]
Transmission Line Basics

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Voltage and current are: $v_x = f_x + g_x$ and $i_x = \frac{f_x - g_x}{Z_0}$ where $i_x$ is positive in the $+x$ direction ($\rightarrow$) and $Z_0 = \sqrt{\frac{L_0}{C_0}}$.
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Waveforms of $f_x$ and $g_x$ are determined by the connections at both ends.
Reflections

- Transients: Basic Ideas
- Steady States
- Determining Time Constant
- Determining Transient Amplitude
- Transmission Lines Basics
- Reflections
- Sinewaves and Phasors
- Standing Waves

\[ v_x = f_x + g_x \]
\[ i_x = \frac{f_x - g_x}{Z_0} \]
Reflections

At $x = L$, Ohm's law $\Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L-Z_0}{R_L+Z_0} \times f_L(t)$. 

$\begin{align*}
v_x &= f_x + g_x \\
i_x &= \frac{f_x-g_x}{Z_0}
\end{align*}$
Reflections

\[ v_x = f_x + g_x \]
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At \( x = L \), Ohm’s law \( \Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t) \).

Reflection coefficient: \( \rho_L = \frac{g_L(t)}{f_L(t)} = \frac{R_L - Z_0}{R_L + Z_0} \)
Reflections

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\( \rho_L \in [-1, +1] \) and increases with \( R_L \)
**Reflections**

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Knowing \( f_x(t) \) for \( x = x_0 \) now tells you \( f_x, g_x, v_x, i_x \ \forall x \)
Reflections

\[ v_x = f_x + g_x \]
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At \( x = L \), Ohm’s law \( \Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t) \).

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\( \rho_L \in [-1, 1] \) and increases with \( R_L \)

Knowing \( f_x(t) \) for \( x = x_0 \) now tells you \( f_x, g_x, v_x, i_x \) \( \forall x \)

At \( x = 0 \):
\[ f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t) \]
At \( x = L \), Ohm's law \( \Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L-Z_0}{R_L+Z_0} \times f_L(t) \).

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At \( x = 0: \)

\[
    f_0(t) = \frac{Z_0}{R_S+Z_0} v_S(t) + \frac{R_S-Z_0}{R_S+Z_0} g_0(t) = \tau_0 v_S(t) + \rho_0 g_0(t)
\]
Reflections

\[ v_x = f_x + g_x \]
\[ i_x = \frac{f_x - g_x}{Z_0} \]

At \( x = L \), Ohm’s law \( \frac{v_L(t)}{i_L(t)} = R_L \) \( \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t) \).

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Wave bounces back and forth getting smaller with each reflection:
Reflections

\[ v_s(t) \quad i_0(t) \quad v_0(t) \quad Z_0 = 100 \quad i_L(t) \quad v_L(t) \]

At \( x = L \), Ohm’s law ⇒ \( \frac{v_L(t)}{i_L(t)} = R_L \) ⇒ \( g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t) \).

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\( \rho_L \in [-1, +1] \) and increases with \( R_L \)

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Wave bounces back and forth getting smaller with each reflection:

\[ v_S(t) \times \tau_0 \rightarrow f_0(t) \]
Reflections

At $x = L$, Ohm's law $\Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t)$.

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Wave bounces back and forth getting smaller with each reflection:

$v_S(t) \xrightarrow{\tau_0} f_0(t) \xrightarrow{\rho_L} g_0(t + \frac{2L}{u})$
Reflections

\[ v_x = f_x + g_x \]
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At \( x = L \), Ohm’s law \( \Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t) \).

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Knowing \( f_x(t) \) for \( x = x_0 \) now tells you \( f_x, g_x, v_x, i_x \) for all \( x \).

At \( x = 0 \): \( f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t) = \tau_0 v_S(t) + \rho_0 g_0(t) \)

Wave bounces back and forth getting smaller with each reflection:

\[ v_S(t) \times \tau_0 \rightarrow f_0(t) \times \rho_L \rightarrow g_0(t + \frac{2L}{u}) \times \rho_0 \rightarrow f_0(t + \frac{2L}{u}) \]
Reflections

At $x = L$, Ohm’s law $\Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t)$.

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Wave bounces back and forth getting smaller with each reflection:

$$v_S(t) \xrightarrow{\times \tau_0} f_0(t) \xrightarrow{\times \rho_L} g_0(t + \frac{2L}{u}) \xrightarrow{\times \rho_L} f_0(t + \frac{2L}{u}) \xrightarrow{\times \rho_L} g_0(t + \frac{4L}{u})$$
Reflections

At \( x = L \), Ohm’s law \( \Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t) \).

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\( v_S(t) \xrightarrow{\tau_0} f_0(t) \xrightarrow{\rho_L} g_0(t + \frac{2L}{u}) \xrightarrow{\rho_L} f_0(t + \frac{2L}{u}) \xrightarrow{\rho_L} g_0(t + \frac{4L}{u}) \xrightarrow{\rho_L} \cdots \)
Reflection coefficient: $\rho_L = \frac{g_L(t)}{f_L(t)} = \frac{R_L-Z_0}{R_L+Z_0}$

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Knowing $f_x(t)$ for $x = x_0$ now tells you $f_x$, $g_x$, $v_x$, $i_x \forall x$

At $x = L$, Ohm’s law $\Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L-Z_0}{R_L+Z_0} \times f_L(t)$.

Wave bounces back and forth getting smaller with each reflection:

$\tau_0 v_S(t) \xrightarrow{\times \rho_0} f_0(t) \xrightarrow{\times \rho_L} g_0(t + \frac{2L}{\omega}) \xrightarrow{\times \rho_0} f_0(t + \frac{2L}{\omega}) \xrightarrow{\times \rho_L} g_0(t + \frac{4L}{\omega}) \xrightarrow{\times \rho_0} \ldots$

Infinite sum:

$f_0(t) = \tau_0 v_S(t) + \tau_0 \rho_L \rho_0 v_S(t - \frac{2L}{\omega}) + \ldots$
Reflections

At $x = L$, Ohm’s law $\Rightarrow \frac{v_L(t)}{i_L(t)} = R_L \Rightarrow g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t)$.

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Wave bounces back and forth getting smaller with each reflection:

$v_S(t) \xrightarrow{\times \tau_0} f_0(t) \xrightarrow{\times \rho_L} g_0(t + \frac{2L}{u}) \xrightarrow{\times \rho_0} f_0(t + \frac{2L}{u}) \xrightarrow{\times \rho_L} g_0(t + \frac{4L}{u}) \xrightarrow{\times \rho_0} \ldots$

Infinite sum:

$f_0(t) = \tau_0 v_S(t) + \tau_0 \rho_L \rho_0 v_S(t - \frac{2L}{u}) + \ldots = \sum_{i=0}^{\infty} \tau_0 \rho_L^i \rho_0^i v_S(t - \frac{2Li}{u})$
Sinewaves and Phasors

Sinewaves are easier because:

1. **Use phasors to eliminate $t$:**

2. **Time delays are just phase shifts:**
Sinewaves and Phasors

Sinewaves are easier because:

1. **Use phasors to eliminate** $t$: $f_0(t) = A \cos (\omega t + \phi) \iff F_0 = Ae^{j\phi}$

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Sinewaves and Phasors

Sinewaves are easier because:

1. **Use phasors to eliminate** $t$: 
   \[ f_0(t) = A \cos(\omega t + \phi) \leftrightarrow F_0 = Ae^{j\phi} \]

2. **Time delays are just phase shifts:**
   \[ f_x(t) = A \cos\left(\omega\left(t - \frac{x}{u}\right) + \phi\right) \leftrightarrow F_x = Ae^{j\left(\phi - \frac{\omega}{u}x\right)} = F_0e^{-jkx} \]
Sinewaves and Phasors

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   \( k = \frac{\omega}{u} = \frac{2\pi}{\lambda} \) is the **wavenumber**: radians per metre (c.f. \( \omega \) in rad/s)
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   \]

   \[
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As before: \( V_x = F_x + G_x \) and \( I_x = \frac{F_x - G_x}{Z_0} \).
Sinewaves are easier because:

1. **Use phasors to eliminate \( t \):** 
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As before: \( V_x = F_x + G_x \) and \( I_x = \frac{F_x - G_x}{Z_0} \)

As before: 
\[ G_L = \rho_L F_L \]
\[ F_0 = \tau_0 V_S + \rho_0 G_0 \]
Sinewaves are easier because:

1. **Use phasors to eliminate** $t$: 
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2. **Time delays are just phase shifts:**
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As before:

\[ G_L = \rho_L F_L \]
\[ F_0 = \tau_0 V_S + \rho_0 G_0 \]

But $G_0 = F_0 \rho_L e^{-2jkL}$: roundtrip delay of $\frac{2L}{u}$ + reflection at $x = L$. 
Sinewaves and Phasors

Sinewaves are easier because:

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2. **Time delays are just phase shifts**:
   \[
   f_x(t) = A \cos \left( \omega \left( t - \frac{x}{u} \right) + \phi \right) \iff F_x = Ae^{j\left(\phi - \frac{\omega x}{u}\right)} = F_0 e^{-jkx}
   \]
   \[
   k = \frac{\omega}{u} = \frac{2\pi}{\lambda}
   \]
   is the **wavenumber**: radians per metre (c.f. $\omega$ in rad/s)

As before: $V_x = F_x + G_x$ and $I_x = \frac{F_x - G_x}{Z_0}$

---

For a network with:

- $V_S$ and $R_S = 20$
- $V_0$
- $Z_0 = 100$
- $I_0$
- $R_L = 300$
- $I_L$
- $V_L$

But $G_0 = F_0 \rho_L e^{-2jkL}$: roundtrip delay of $\frac{2L}{u}$ + reflection at $x = L$.
Substituting for $G_0$ in source end equation: $F_0 = \tau_0 V_S + \rho_0 F_0 \rho_L e^{-2jkL}$

As before:

- $G_L = \rho_L F_L$
- $F_0 = \tau_0 V_S + \rho_0 G_0$
Sinewaves are easier because:

1. **Use phasors to eliminate** \( t \): 
   \[
   f_0(t) = A \cos(\omega t + \phi) \iff F_0 = A e^{j\phi}
   \]

2. **Time delays are just phase shifts**:
   
   \[
   f_x(t) = A \cos(\omega \left(t - \frac{x}{u}\right) + \phi) \iff F_x = A e^{j\left(\phi - \frac{\omega}{u}x\right)} = F_0 e^{-jkx}
   \]

   \[
   k = \frac{\omega}{u} = \frac{2\pi}{\lambda}
   \]

   is the wavenumber: radians per metre (c.f. \( \omega \) in rad/s)

As before: 

\[
V_x = F_x + G_x \quad \text{and} \quad I_x = \frac{F_x - G_x}{Z_0}
\]

As before:

\[
G_L = \rho_L F_L
\]

\[
F_0 = \tau_0 V_S + \rho_0 G_0
\]

But \( G_0 = F_0 \rho_L e^{-2jkL} \): roundtrip delay of \( \frac{2L}{u} \) + reflection at \( x = L \).

Substituting for \( G_0 \) in source end equation:

\[
F_0 = \tau_0 V_S + \rho_0 F_0 \rho_L e^{-2jkL}
\]

\[
\Rightarrow F_0 = \frac{\tau_0}{1 - \rho_0 \rho_L \exp(-2jkL)} V_S
\]

so no infinite sums needed 😊
Standing waves arise whenever a wave meets its reflection: at positions where the two waves are in phase their amplitudes add but where they are anti-phase their amplitudes subtract.
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At any point \( x \), delay of \( \frac{x}{u} \) ⇒
\[
F_x = F_0 e^{-j k x}
\]
Standing waves arise whenever a wave meets its reflection:

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At any point $x$,

\[ F_x = F_0 e^{-jkx} \]

**Backward wave:**

\[ G_x = \rho_L F_x e^{-2jk(L-x)} \]
Standing Waves

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At any point $x$,

delay of $\frac{x}{u} \implies F_x = F_0 e^{-jkx}$

Backward wave: $G_x = \rho_L F_x e^{-2jk(L-x)}$: reflection + delay of $\frac{2(L-x)}{u}$
Standing waves arise whenever a wave meets its reflection:

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Backward wave: $G_x = \rho L F_x e^{-2jk(L-x)}$: reflection + delay of $2\frac{L-x}{u}$

Voltage at $x$: $V_x = F_x + G_x$
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At any point $x$,

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Voltage at $x$: $V_x = F_x + G_x = F_0 e^{-jkx} \left(1 + \rho_L e^{-2jk(L-x)}\right)$
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Voltage at $x$:

\[ V_x = F_x + G_x = F_0 e^{-jkx} \left(1 + \rho_L e^{-2jk(L-x)}\right) \]

Voltage Magnitude:

\[ |V_x| = |F_0| \left|1 + \rho_L e^{-2jk(L-x)}\right| \]
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Voltage Magnitude: $|V_x| = |F_0| \left|1 + \rho_L e^{-2 j k (L-x)}\right|$: depends on $x$
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Voltage Magnitude:

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|V_x| = |F_0| \left| 1 + \rho_L e^{-2jk(L-x)} \right|
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If \( \rho_L \geq 0 \), **max magnitude** is \((1 + \rho_L) |F_0|\) whenever \( e^{-2jk(L-x)} = +1 \)
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Voltage Magnitude: $|V_x| = |F_0| |1 + \rho_L e^{-2jk(L-x)}|$: depends on $x$

If $\rho_L \geq 0$, max magnitude is $(1 + \rho_L) |F_0|$ whenever $e^{-2jk(L-x)} = +1$

$\Rightarrow x = L$ or $x = L - \frac{\pi}{k}$ or $x = L - \frac{2\pi}{k}$ or ...
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Min magnitude is $(1 - \rho_L) |F_0|$ whenever $e^{-2 j k (L-x)} = -1$
Standing waves arise whenever a wave meets its reflection:

at positions where the two waves are in phase their amplitudes add
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F_x = F_0 e^{-jkx}
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Backward wave: \( G_x = \rho_L F_x e^{-2jk(L-x)} \): reflection + delay of \( 2\frac{L-x}{u} \)

Voltage at \( x \): \( V_x = F_x + G_x = F_0 e^{-jkx} \left( 1 + \rho_L e^{-2jk(L-x)} \right) \)

Voltage Magnitude: \( |V_x| = |F_0| \left| 1 + \rho_L e^{-2jk(L-x)} \right| \): depends on \( x \)

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Min magnitude is \( (1 - \rho_L) |F_0| \) whenever \( e^{-2jk(L-x)} = -1 \)
\( \Rightarrow x = L - \frac{\pi}{2k} \) or \( x = L - \frac{3\pi}{2k} \) or \( x = L - \frac{5\pi}{2k} \) or \ldots