

▷ **Revision Lecture 1:
Nodal Analysis**

Exam

Basic Concepts

Ohm's Law

Nodal Analysis

Nodal Analysis

Example

**Voltage and Current
Dividers**

Superposition

**Superposition
Example**

Thévenin and Norton

Op Amps

Block Diagrams

Diodes

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Exam Format

Question 1 (40%): eight short parts covering the whole syllabus.

Questions 2 and 3: single topic questions (answer both)

Syllabus

Does include: Everything in the notes.

Does not include: Two-port parameters (2008:1j), Gaussian elimination (2007:2c), Application areas (2008:3d), Nullators and Norators (2008:4c), Small-signal component models (2008:4e), Gain-bandwidth product (2006:4c), Zener Diodes (2008/9 syllabus), Non-ideal models of L, C and transformer (2011/12 syllabus), Transmission lines VSWR and crank diagram (2011/12 syllabus).

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- All the points in a circuit joined together with zero-resistance wires make up a single **node**. The voltage is the same at any point in a node.
- Choose one node in a circuit to be the **reference node**. Measure all node voltages relative to this one.
- Voltage across a component is the difference between the voltages at the two ends. $V_{AB} = V_A - V_B$.
- **Passive sign convention**: Current and voltage arrows for a 2-terminal component are in opposite directions.
- Components are **in parallel** if they are joined together at both ends. Components in parallel have the **same voltage** across them.
- Components are **in series** if the **same current** is forced to flow through each one.
- **KCL (Kirchoff's Current Law)**: The total current going out of a node (or any collection of nodes) sums to zero.

Ohm's Law

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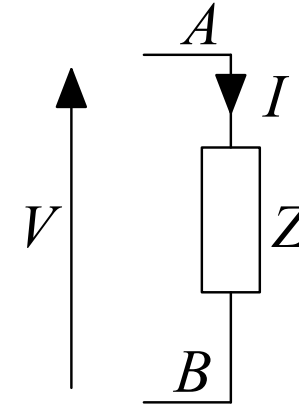
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Ohm's law relates the voltage drop across a resistor to the current flowing in it:

$$V = IZ$$



Passive Sign Convention: V and I arrows must point in opposite directions (otherwise you need a $-$ sign in Ohm's law)

Voltage arrow means $V = V_{tip} - V_{base} = V_{AB}$ (subscript order opposite from vectors)

Ohm's law still true if **all currents** are in mA and **all impedances** in $k\Omega$.

Ohm's law still true if **all currents** and **all voltages** are RMS values: $\tilde{V} = \tilde{I}Z$ where $\tilde{V} = \frac{V}{\sqrt{2}}$ for a voltage phasor V .

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- (1) Pick any node as the voltage reference. Label its voltage as 0 V. Label any dependent sources with V_S , I_S , ...
- (2) If any voltage sources are connected to a labelled node, label their other ends by adding the value of the source onto the voltage of the labelled end.
- (3) Pick an unlabelled node and label it with X , Y , ..., then loop back to step (2) until all nodes are labelled.
- (4) For each **dependent source**, write down an equation that expresses its value in terms of other node voltages.
- (5) Write down a KCL equation for each “normal” node (i.e. one that is not connected to a floating voltage source).
- (6) Write down a KCL equation for each “super-node”. A super-node consists of a set of nodes that are joined by floating voltage sources and includes any other components linking these nodes.
- (7) Solve the set of simultaneous equations that you have written down.

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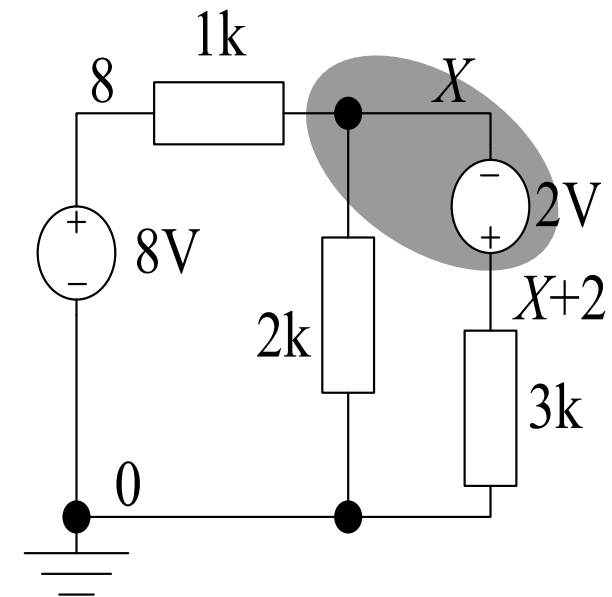
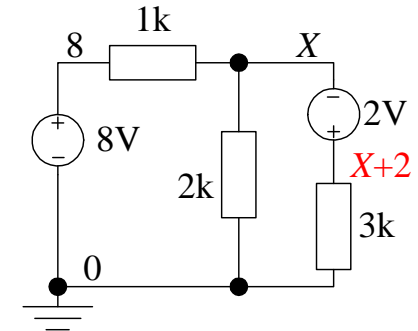
Diodes

- (1) Pick reference node.
- (2) Label nodes: 8, X and $X + 2$ since it is joined to X via a voltage source.
- (3) Write KCL equations but count all the nodes connected via floating voltage sources as a single "super-node" giving one equation

$$\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0$$

Ohm's law always involves the difference between the voltages at **either end of a resistor**. (Obvious but easily forgotten)

- (4) Solve the equations: $X = 4$



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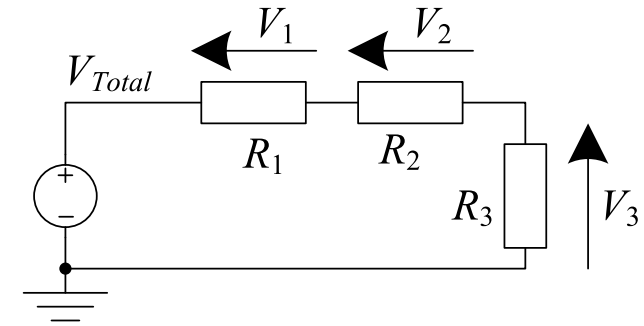
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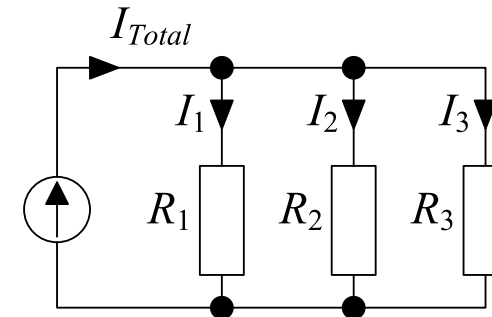
Components in **series** form a **voltage divider**: Voltages $\propto R$.

$$\frac{V_k}{V_{Total}} = \frac{R_k}{R_{Total}}$$



Components in **parallel** form a **current divider**: Currents $\propto G$.

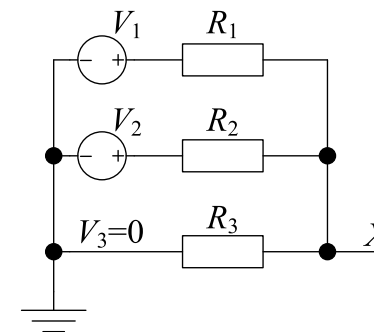
$$\frac{I_k}{I_{Total}} = \frac{G_k}{G_{Total}}$$



Summing Junction: X is a **weighted average** with weights $\propto G$.

$$X = \frac{G_1 V_1 + G_2 V_2 + G_3 V_3}{G_{Total}}$$

Also works if V_k is zero (less obvious).



Superposition

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Any voltage or current in a circuit is the sum of the effects of the sources taken one at a time.

To use superposition:

- Set all the sources to zero except for the first one and find the voltage at each node
 - A zero-valued **voltage** source is a **short circuit**
 - A zero-valued **current** source is an **open circuit**
- Repeat for each of the sources and add up the results to get a formula for the voltage at each node.
- Express all **dependent** source values in terms of node voltages and eliminate them from the formulae. A source is **dependent** if its value is determined by the voltages/currents elsewhere in the circuit.

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Problem: Find X in terms of U_1 and U_2 .

- If we set $U_2 = 0$ then the current source becomes an *open circuit* and now the 3 k plays no part in the circuit.

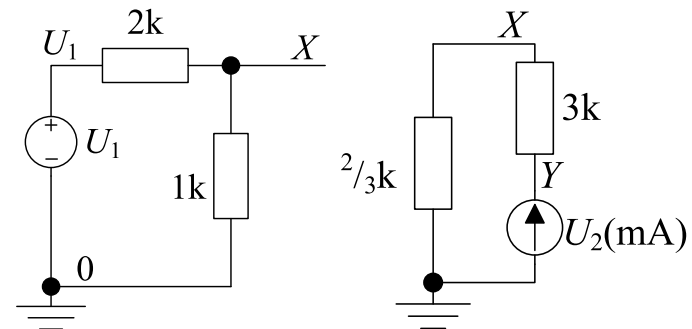
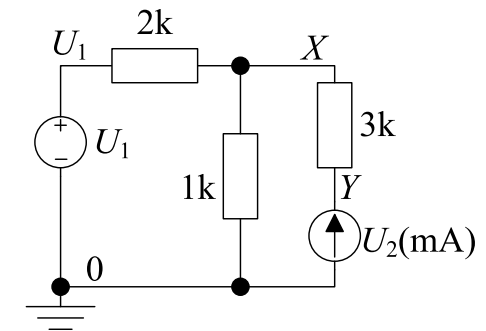
2 k and 1 k form a potential divider and so $X = \frac{1\text{ k}}{2\text{ k} + 1\text{ k}} U_1$.

- If we set $U_1 = 0$ then the voltage source becomes a *short circuit* and the 2 k and 1 k are in parallel.

$$2\text{ k} || 1\text{ k} = \frac{2\text{ k} \times 1\text{ k}}{2\text{ k} + 1\text{ k}} = \frac{2}{3}\text{ k}.$$

$$\text{Now } X = \frac{2}{3} U_2.$$

- Combining these two gives $X = \frac{1}{3} U_1 + \frac{2}{3} U_2$.



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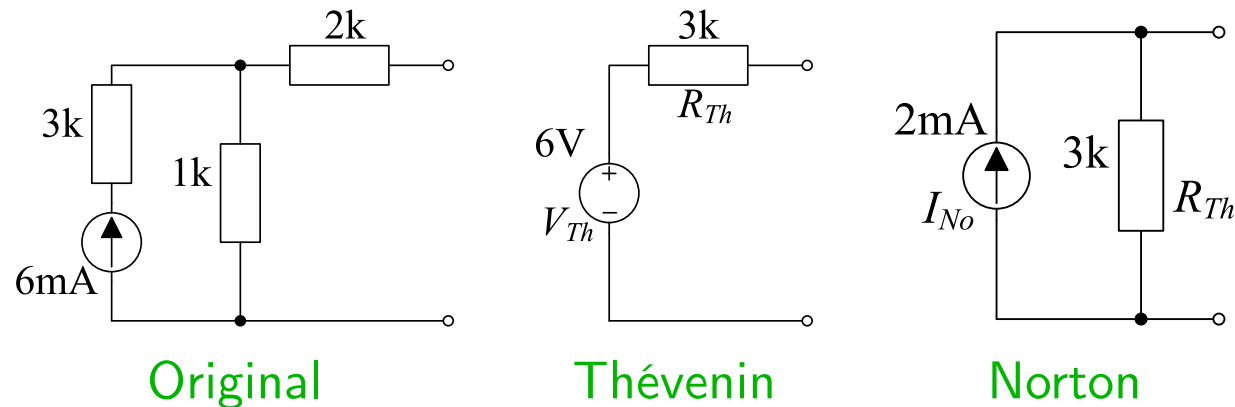
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Any complicated linear circuit with two terminals is **exactly equivalent** to either of two simpler circuits with only two components. Any measurement will give the same result on any of the three circuits.



Common measurements are:

- **Thévenin Resistance**, R_{Th} : Resistance between the terminals when all sources are set to zero (same for Thévenin and Norton).
- **Open circuit voltage** (= voltage when drawing zero current from the terminals): $= V_{Th} = I_{No} R_{Th}$.
- **Short circuit current**: $= I_{No} = \frac{V_{Th}}{R_{Th}}$.

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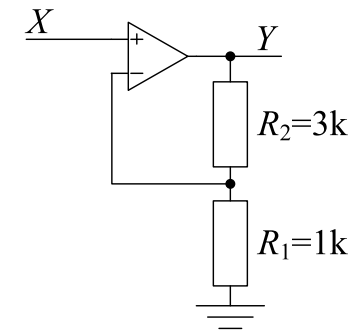
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Diodes

- **Ideal Op Amp:** (a) Zero input current, (b) Infinite gain
(b) $\Rightarrow V_+ = V_-$ provided the circuit has **negative feedback**.
- **Negative Feedback:** An increase in V_{out} causes $(V_+ - V_-)$ to decrease.

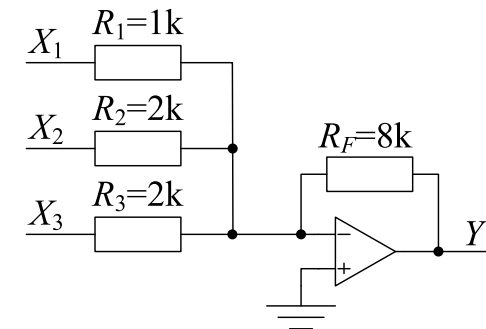
Non-inverting amplifier

$$Y = \left(1 + \frac{3}{1}\right) X$$



Inverting amplifier

$$Y = \frac{-8}{1} X_1 + \frac{-8}{2} X_2 + \frac{-8}{2} X_3$$



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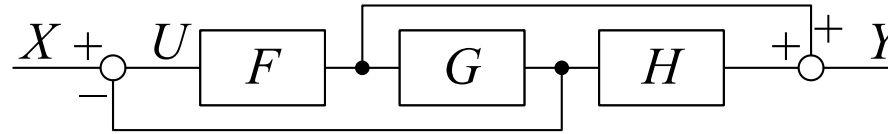
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Blocks are labelled with their gains and connected using adder/subtractors shown as circles. Adder inputs are marked + for add or – for subtract.



To analyse:

1. Label the inputs, the outputs and the output of each adder.

2. Write down an equation for each variable:

- $U = X - FGU, \quad Y = FU + FGHU$
- Follow signals back through the blocks until you meet a labelled node.

3. Solve the equations (eliminate intermediate node variables):

- $U(1 + FG) = X \quad \Rightarrow \quad U = \frac{X}{1+FG}$
- $Y = (1 + GH)FU = \frac{(1+GH)F}{1+FG} X$

[Note: “Wires” carry information not current: KCL not valid]

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Each diode in a circuit is in one of two modes; each has an **equality** condition and an **inequality** condition:

- Off: $I_D = 0, V_D < 0.7$
- On: $V_D = 0.7, I_D > 0$

- (a) Guess the mode
- (b) Do nodal analysis assuming the equality condition
- (c) Check the inequality condition. If the inequality condition fails, you made the wrong guess.

- Assume Diode Off

$$X = 5 + 2 = 7$$

$$V_D = 2 \quad \text{Fail: } V_D > 0.7$$

- Assume Diode On

$$X = 5 + 0.7 = 5.7$$

$$I_D + \frac{0.7}{1\text{k}} = 2\text{ mA} \quad \text{OK: } I_D > 0$$

