Revision Lecture 2: Phasors

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  - $F = A \cos \theta, \qquad G = A \sin \theta$.
  - In CMPLX mode, Casio fx-991ES can do complex arithmetic and can switch between the two forms with SHIFT,CMPLX,3 or SHIFT,CMPLX,4

Reactive Components

- Impedances: R, $j\omega L$, $\frac{1}{j\omega C} = \frac{-j}{\omega C}$.
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- Average power absorbed by a capacitor or inductor is always zero

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Waveform	Phasor
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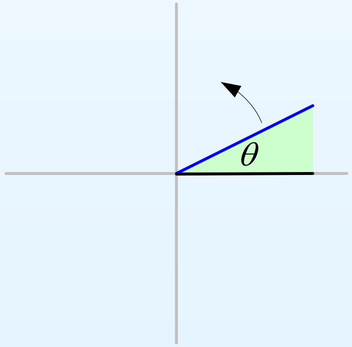
$x(t)$ is the projection of a rotating rod onto the real (horizontal) axis.

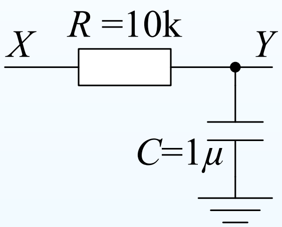
$X = F + jG$ is its starting position at $t = 0$.



Phasor Diagram

Draw phasors as vectors. Join vectors end-to-end to show how voltages in series add up (or currents in parallel).

Find $y(t)$ if $x(t) = \cos 300t$.



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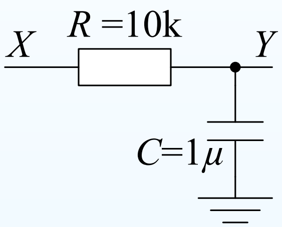

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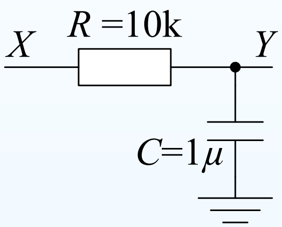

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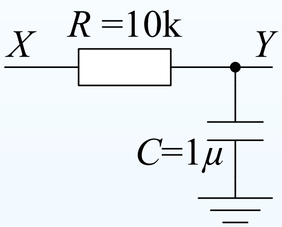

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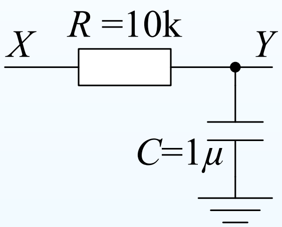

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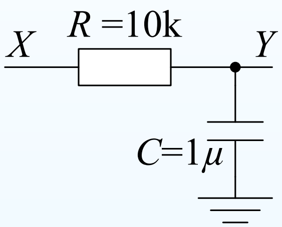

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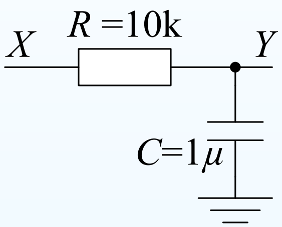

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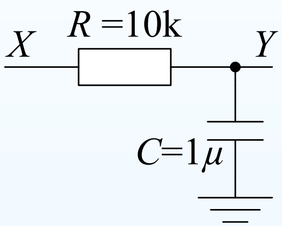

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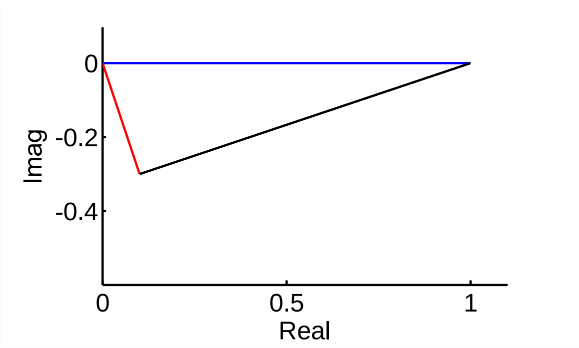

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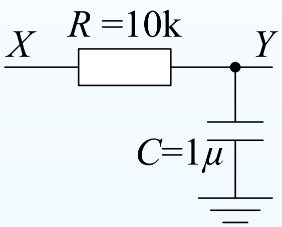

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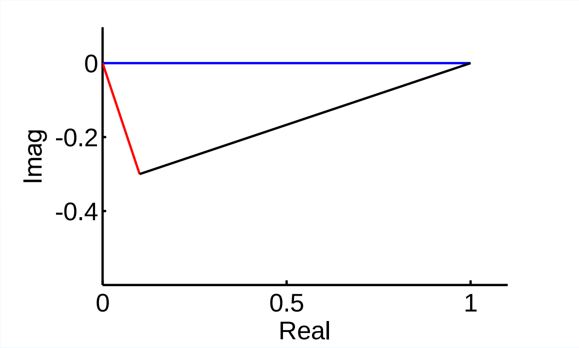

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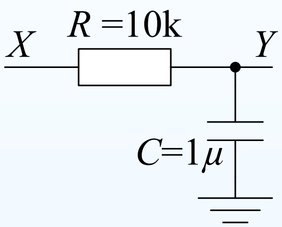

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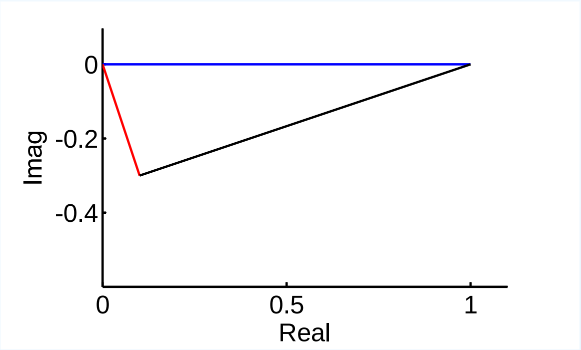

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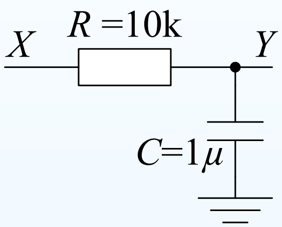

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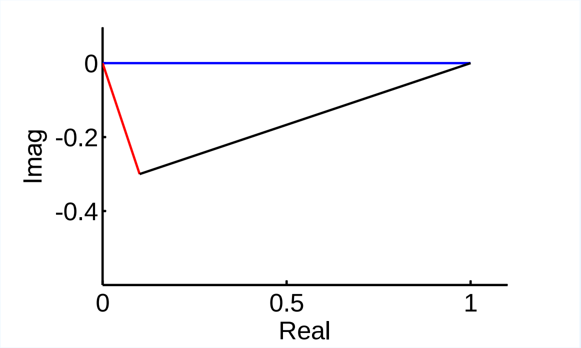

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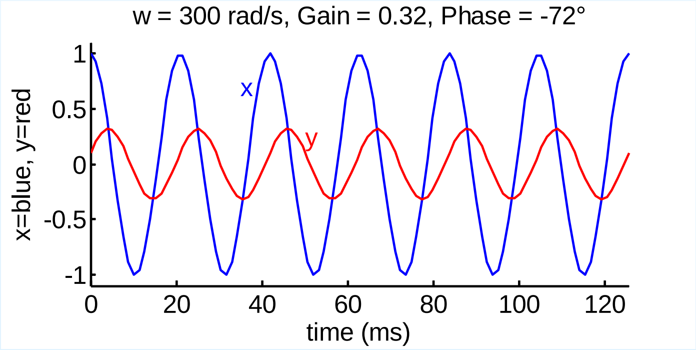

Complex Power

If $V = |V| \angle\theta_V$ is a phasor, we define $\widetilde{V} = \frac{1}{\sqrt{2}} \times V$ to be the corresponding r.m.s. phasor. The r.m.s. voltage is $\left|\widetilde{V}\right| = \frac{1}{\sqrt{2}} \times |V|$.

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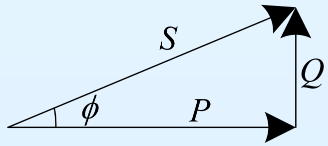

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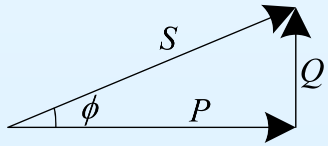

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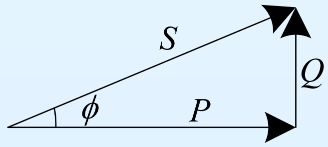

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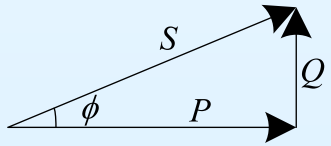

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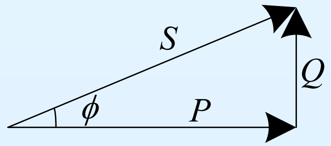

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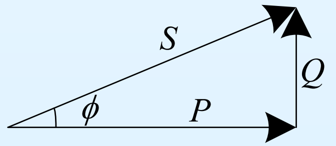
VARs are generated by capacitors and absorbed by inductors.

$\phi > 0$ for inductive impedance.
$\phi < 0$ for capacitive impedance.



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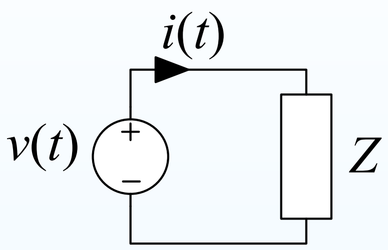

Resistor: positive real (absorbs watts)
Inductor: positive imaginary (absorbs VARs)
Capacitor: negative imaginary (generates VARs)

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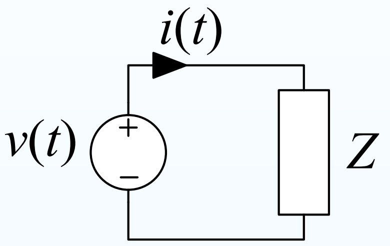

Resistor: positive real (absorbs watts)

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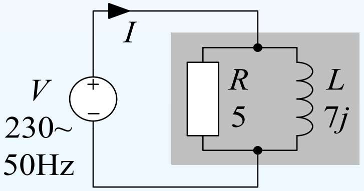
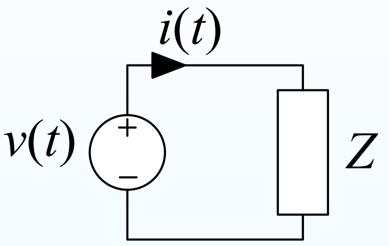
Capacitor: negative imaginary (generates VARs)

Power Factor Correction



Complex Power in Components

$$S = \widetilde{V}\widetilde{I}^* = \frac{\left|\widetilde{V}\right|^2}{Z^*} = \left|\widetilde{I}\right|^2 Z \Rightarrow \angle S = \angle Z$$



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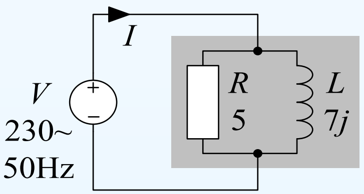

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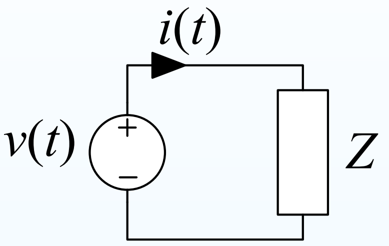

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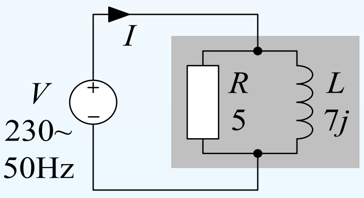

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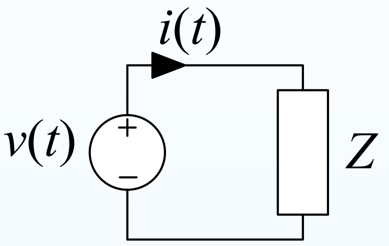

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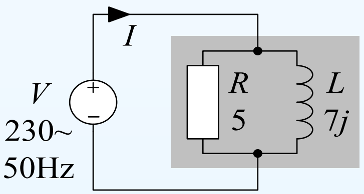
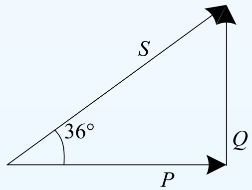
$S = \widetilde{V}\widetilde{I}^* = 13\angle 36°$ kVA





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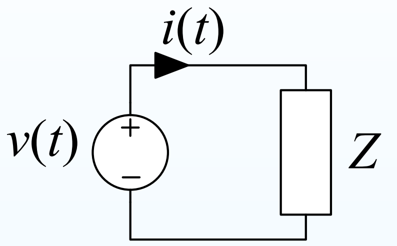

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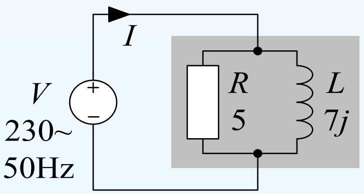
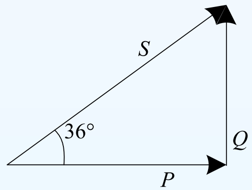
$\cos\phi = \frac{P}{|S|}$

$= \cos 36° = 0.81$





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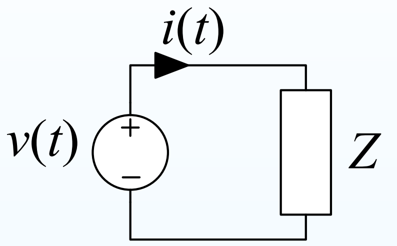

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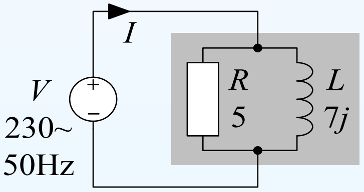
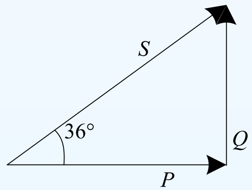
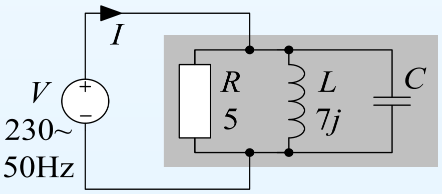
Add parallel capacitor: $300\ \mu$F







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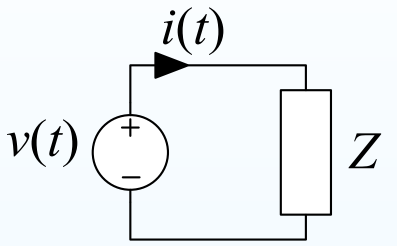

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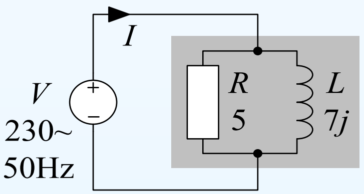
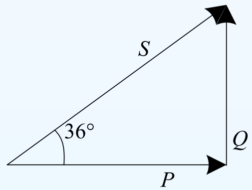
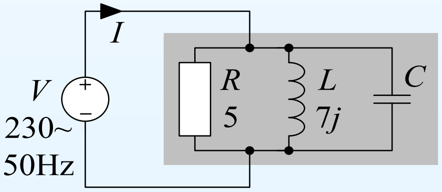




Add parallel capacitor: $300\,\mu$F

$\widetilde{I} = 46 - 11j = 47\angle - 14°$



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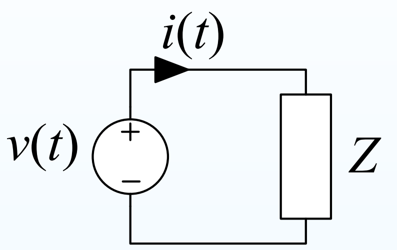

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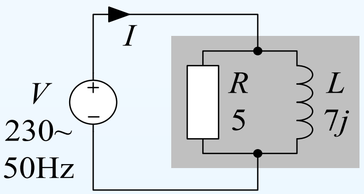

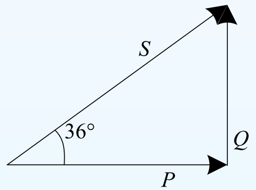

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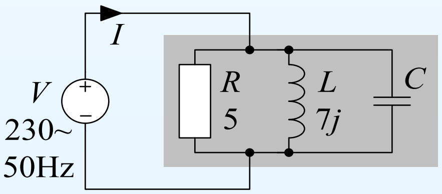

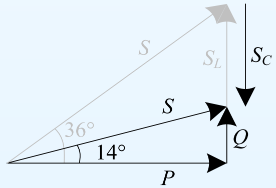

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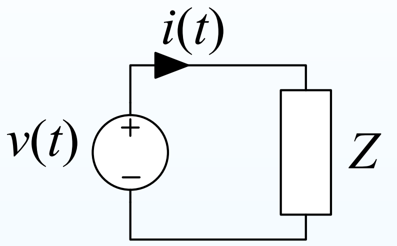

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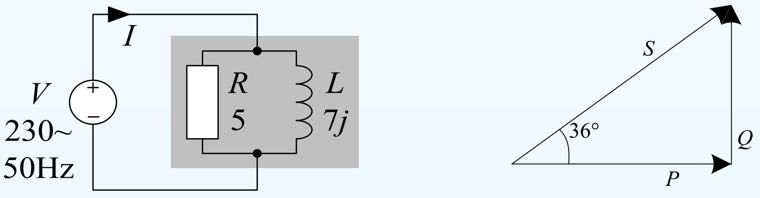

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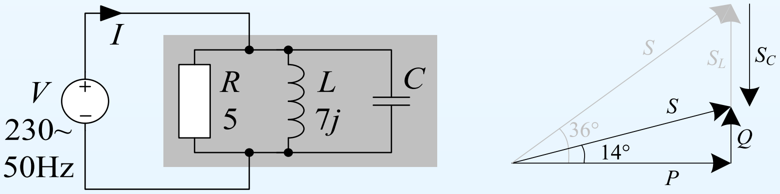
$\cos\phi = \frac{P}{|S|} = \cos 14° = 0.97$



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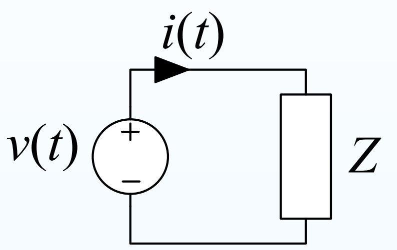

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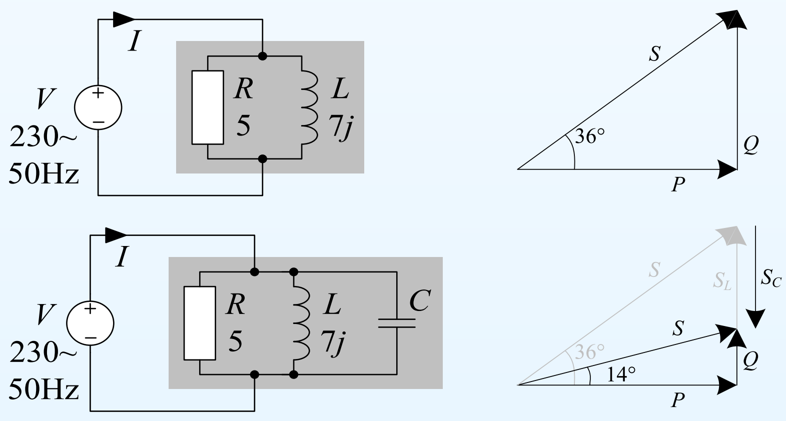

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Current decreases by factor of $\frac{0.81}{0.97}$. Lower power transmission losses.