

▷ **Revision Lecture 2:  
Phasors**

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**Basic Concepts**

**Reactive Components**

**Phasors**

**Phasor Diagram**

**Complex Power**

**Complex Power in  
Components**

# Revision Lecture 2: Phasors

# Basic Concepts

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#### Complex Power in Components

- **Phasors** and **Complex impedances** are only relevant to **sinusoidal** sources.
  - A DC source is a special case of a cosine wave with  $\omega = 0$ .
- For two sine waves, the **leading** one reaches its peak **first**, the **lagging** one reaches its peak **second**. So  $\sin \omega t$  lags  $\cos \omega t$ .
- If  $A \cos(\omega t + \theta) = F \cos \omega t - G \sin \omega t$ , then
  - $A = \sqrt{F^2 + G^2}$ ,  $\theta = \tan^{-1} \frac{G}{F}$ .
  - $F = A \cos \theta$ ,  $G = A \sin \theta$ .
  - In CMPLX mode, Casio fx-991ES can do complex arithmetic and can switch between the two forms with SHIFT,CMPLX,3 or SHIFT,CMPLX,4

# Reactive Components

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- Impedances:  $R, j\omega L, \frac{1}{j\omega C} = \frac{-j}{\omega C}$ .
  - Admittances:  $\frac{1}{R}, \frac{1}{j\omega L} = \frac{-j}{\omega L}, j\omega C$
- In a capacitor or inductor, the Current and Voltage are  $90^\circ$  apart :
  - CIVIL: Capacitor - current leads voltage; Inductor - current lags voltage
- **Average current** (or DC current) through a **capacitor** is always zero
- **Average voltage** across an **inductor** is always zero
- **Average power** absorbed by a **capacitor** or **inductor** is always zero

# Phasors

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A phasor represents a **time-varying sinusoidal waveform** by a **fixed complex number**.

### Waveform

$$x(t) = F \cos \omega t - G \sin \omega t$$

$$x(t) = A \cos(\omega t + \theta)$$

$$\max(x(t)) = A$$

### Phasor

$$X = F + jG$$

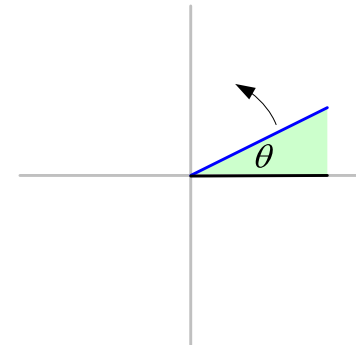
$$X = Ae^{j\theta} = A\angle\theta$$

$$|X| = A$$

[Note minus sign]

$x(t)$  is the projection of a rotating rod onto the real (horizontal) axis.

$X = F + jG$  is its starting position at  $t = 0$ .



# Phasor Diagram

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Draw phasors as vectors. Join vectors end-to-end to show how voltages in series add up (or currents in parallel).

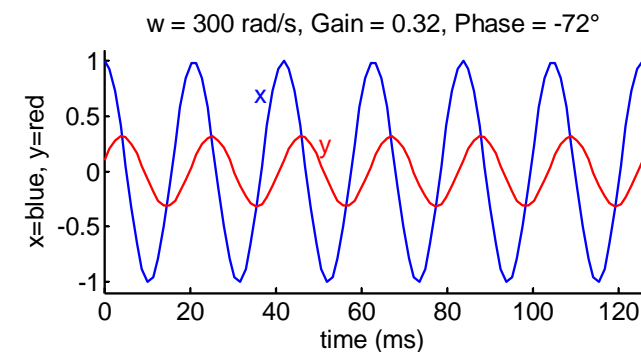
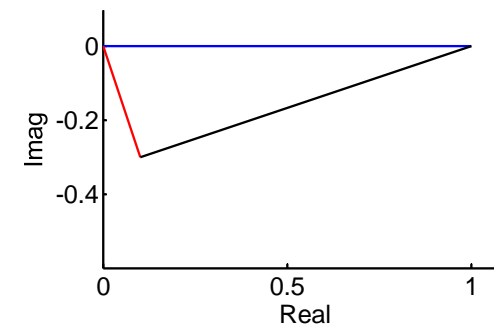
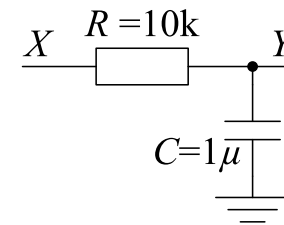
Find  $y(t)$  if  $x(t) = \cos 300t$ .

$$\begin{aligned}\frac{Y}{X} &= \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1} \\ &= \frac{1}{1 + 3j} \\ &= 0.1 - 0.3j = 0.32 \angle -72^\circ\end{aligned}$$

$$x(t) = \cos 300t \Rightarrow X = 1$$

$$\begin{aligned}Y &= X \times \frac{Y}{X} \\ &= 0.1 - 0.3j = 0.32 \angle -72^\circ\end{aligned}$$

$$\begin{aligned}y(t) &= 0.1 \cos 300t + 0.3 \sin 300t \\ &= 0.32 \cos(300t - 1.25) \\ &= 0.32 \cos(300(t - 4.2\text{ms}))\end{aligned}$$



# Complex Power

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If  $V = |V| \angle \theta_V$  is a phasor, we define  $\tilde{V} = \frac{1}{\sqrt{2}} \times V$  to be the corresponding r.m.s. phasor. The r.m.s. voltage is  $|\tilde{V}| = \frac{1}{\sqrt{2}} \times |V|$ .

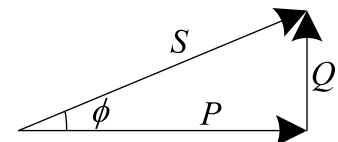
Power Factor	$\cos \phi = \cos (\theta_V - \theta_I)$	
Complex Power	$S = \tilde{V} \times \tilde{I}^* = P + jQ$	[* = complex conjugate]
Apparent Power	$ S  =  \tilde{V}  \times  \tilde{I} $	unit = VAs
Average Power	$P = \Re(S) =  S  \cos \phi$	unit = Watts $\rightarrow$ heat
Reactive Power	$Q = \Im(S) =  S  \sin \phi$	unit = VARs

**Conservation of power (Tellegen's theorem):** in any circuit the total complex power absorbed by all components sums to zero  
 $\Rightarrow P =$  average power and  $Q =$  reactive power sum separately to zero.

VARs are **generated** by **capacitors** and **absorbed** by **inductors**.

$\phi > 0$  for inductive impedance.

$\phi < 0$  for capacitive impedance.



# Complex Power in Components

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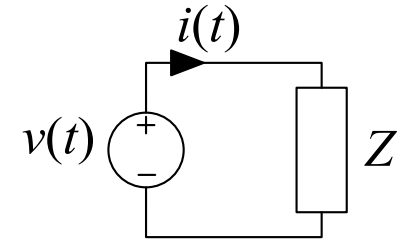
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$$S = \tilde{V}\tilde{I}^* = \frac{|\tilde{V}|^2}{Z^*} = |\tilde{I}|^2 Z \Rightarrow \angle S = \angle Z$$

**Resistor:** positive real (absorbs watts)

**Inductor:** positive imaginary (absorbs VARs)

**Capacitor:** negative imaginary (generates VARs)



## Power Factor Correction

$\tilde{V} = 230$ . Motor is  $5 || 7j \Omega$ .

$$\tilde{I} = 46 - 33j = 56.5 \angle -36^\circ$$

$$S = \tilde{V}\tilde{I}^* = 13 \angle 36^\circ \text{ kVA}$$

$$\begin{aligned} \cos \phi &= \frac{P}{|S|} \\ &= \cos 36^\circ = 0.81 \end{aligned}$$

**Add parallel capacitor:**  $300 \mu\text{F}$

$$\tilde{I} = 46 - 11j = 47 \angle -14^\circ$$

$$S = \tilde{V}\tilde{I}^* = 10.9 \angle 14^\circ \text{ kVA}$$

$$\cos \phi = \frac{P}{|S|} = \cos 14^\circ = 0.97$$

Current decreases by factor of  $\frac{0.81}{0.97}$ . Lower power transmission losses.

