Revision Lecture 3:
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  - It is a complex number that depends on frequency.
  - Calculate using nodal analysis.
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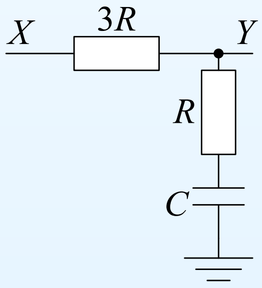

Plotting Frequency Responses

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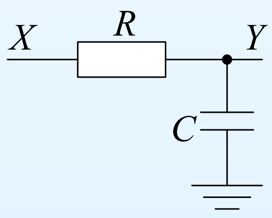

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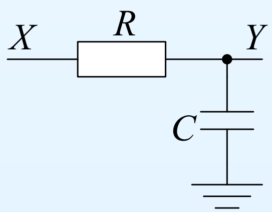

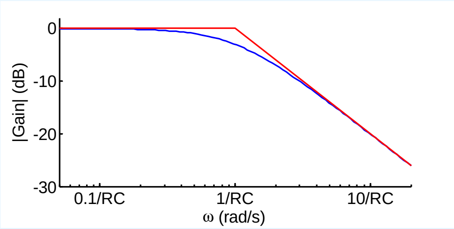

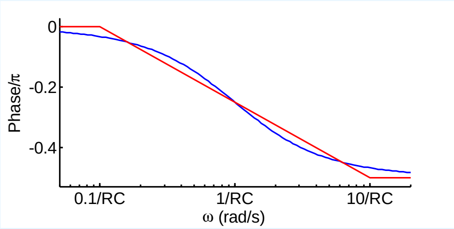

$$\frac{Y}{X} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{j\omega RC+1} = \frac{1}{\frac{j\omega}{\omega_c}+1} \text{ where } \omega_c = \frac{1}{RC}$$

LF and HF Asymptotes

- Frequency response is always a ratio of two polynomials in $j\omega$ with real coefficients that depend on the component values.
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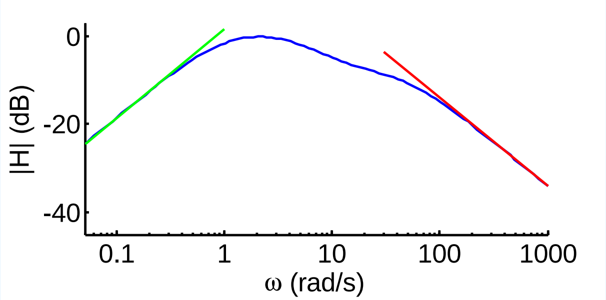

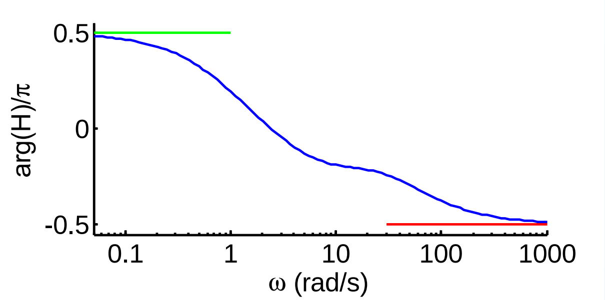

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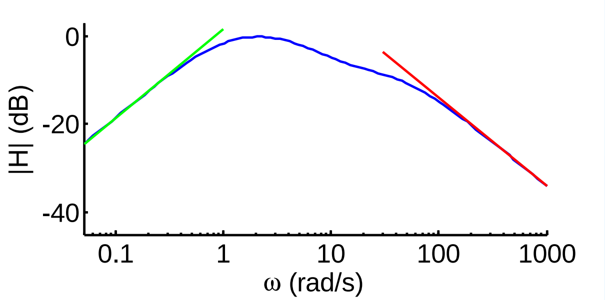

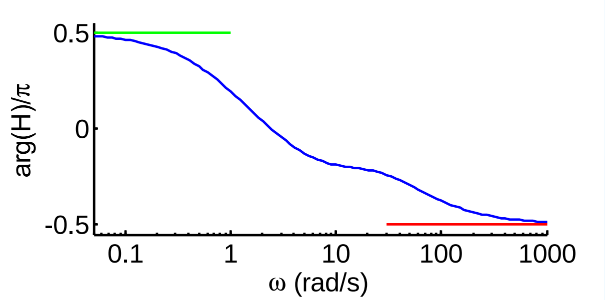

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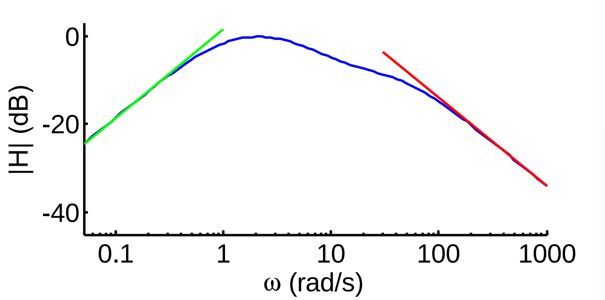

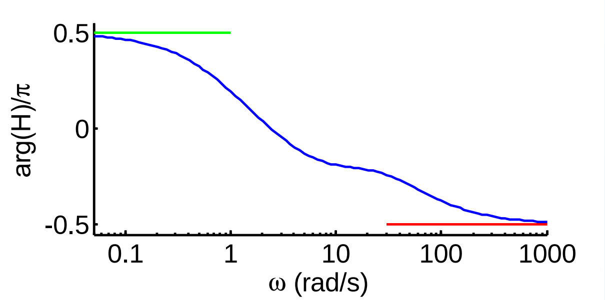

LF: $H(j\omega) \simeq 1.2j\omega$

HF: $H(j\omega) \simeq 20\,(j\omega)^{-1}$

Corner frequencies (linear factors)

- We can factorize the numerator and denominator into linear terms of the form $(aj\omega + b) \simeq \begin{cases} b & \omega < \left|\frac{b}{a}\right| \\ aj\omega & \omega > \left|\frac{b}{a}\right| \end{cases}$.

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- The corner frequencies are the absolute values of the roots (values of $j\omega$) of the numerator and denominator polynomials.

Sketching Magnitude Responses (linear factors)

1. Find corner frequencies: (a) factorize the numerator/denominator polynomials or (b) find their roots
2. Find LF and HF asymptotes. $A\,(j\omega)^k$ has a slope of k; substitute the value of ω_c to get the value at first/last corner frequency.
3. At a corner frequency, the gradient of the magnitude response changes by ± 1 (± 20 dB/decade). $+$ for numerator (top line) and $-$ for denominator (bottom line).
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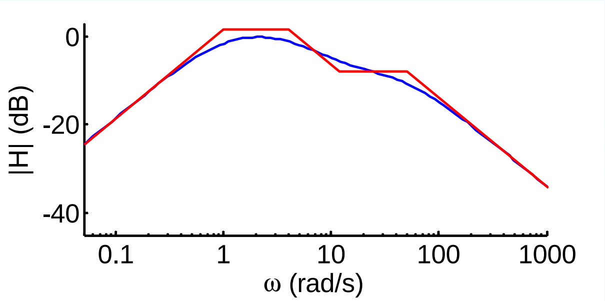

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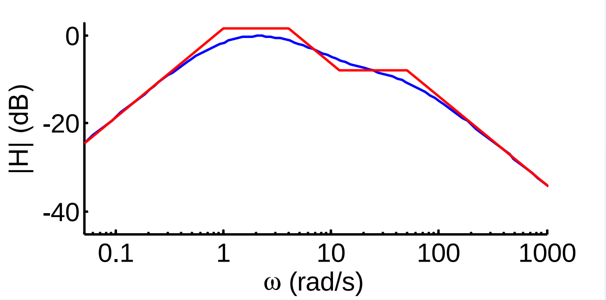

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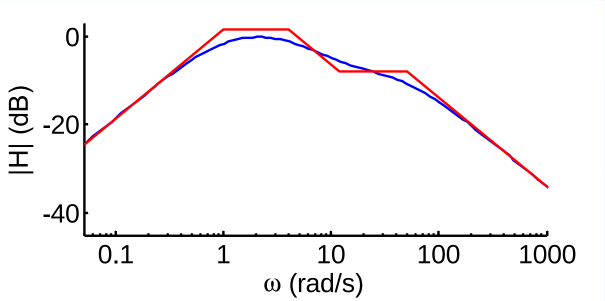

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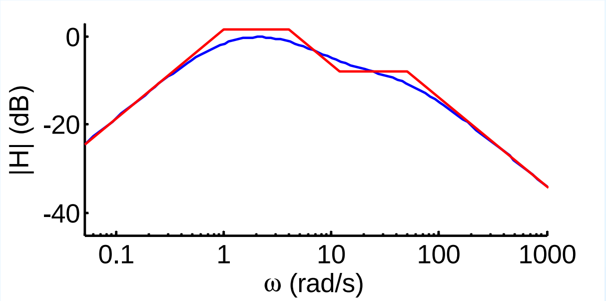

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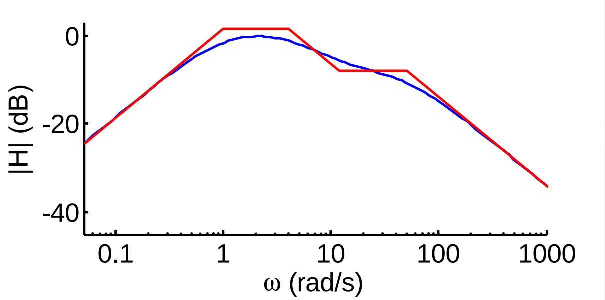

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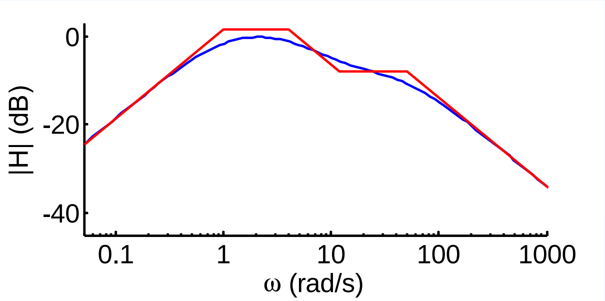

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    - a numerator factor has a positive gradient change at $\omega = 0.1\left|\frac{b}{a}\right|$ and negative at $\omega = 10\left|\frac{b}{a}\right|$.

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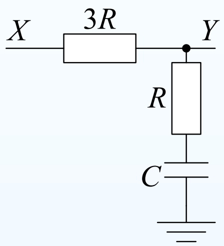

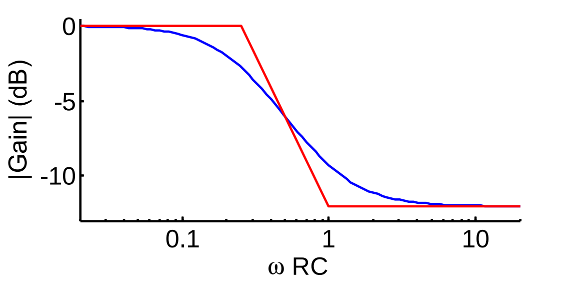

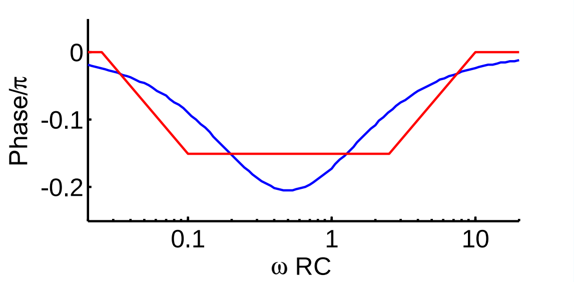

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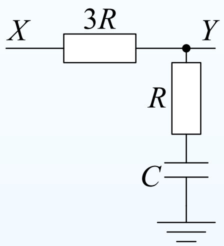

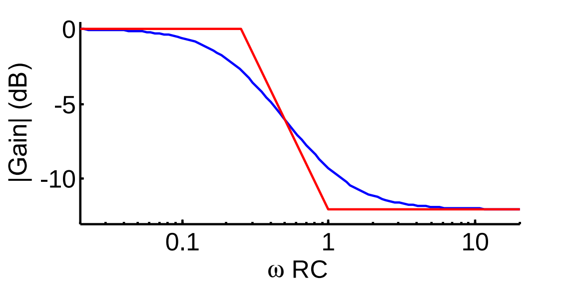

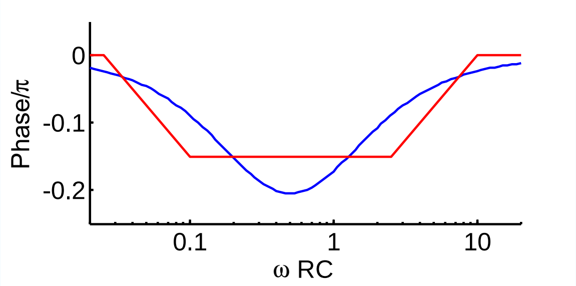

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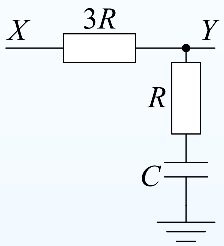

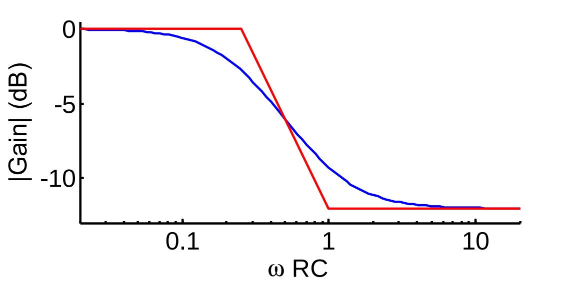

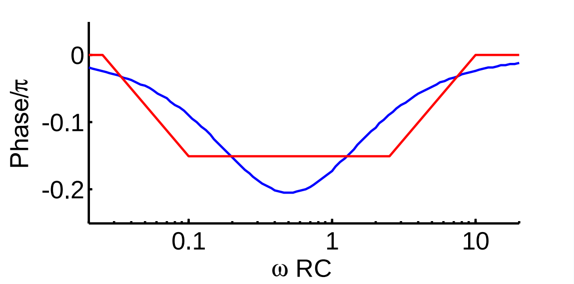

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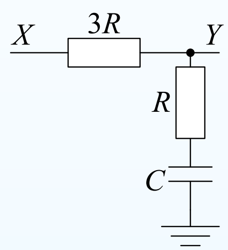

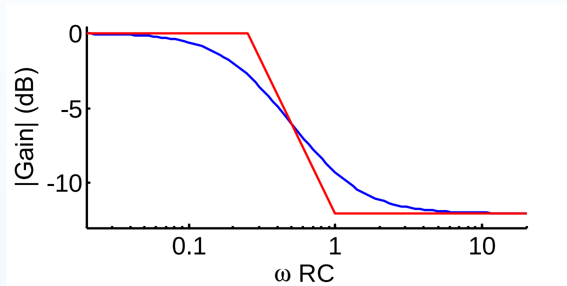

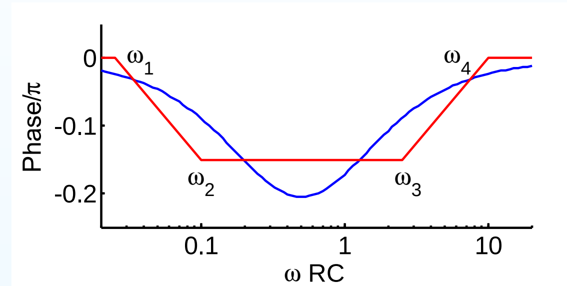

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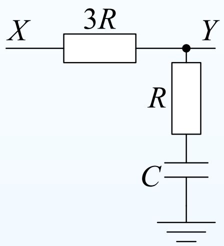

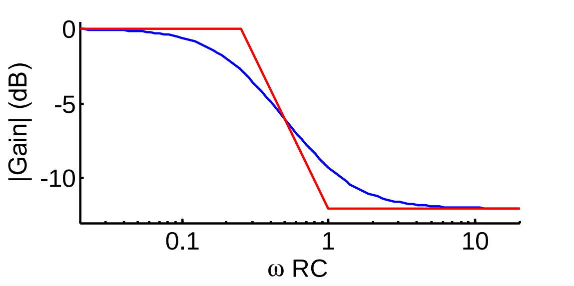

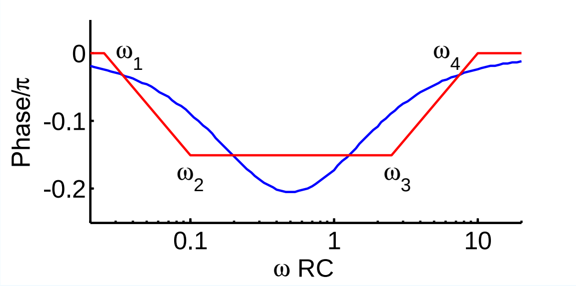

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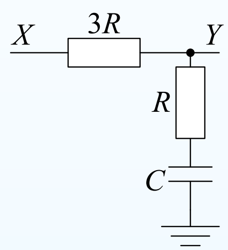

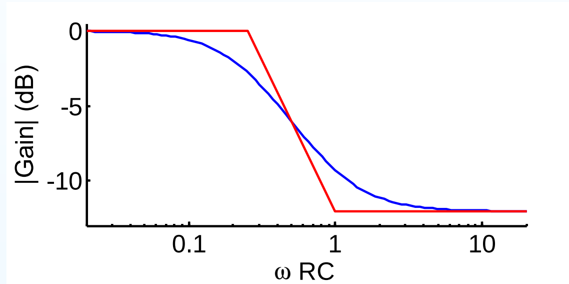

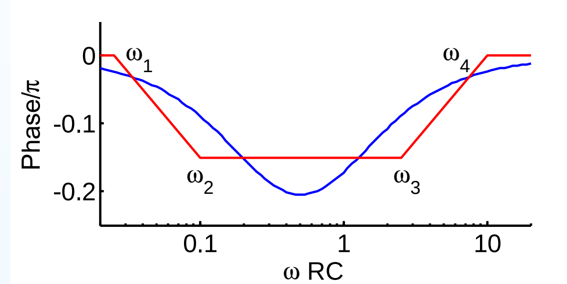

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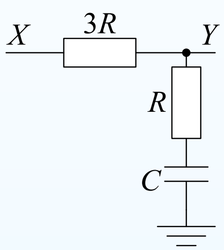

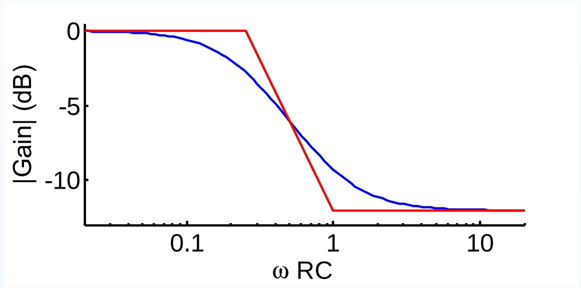

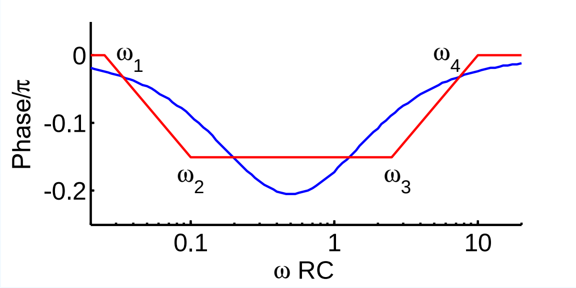

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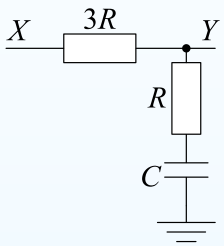

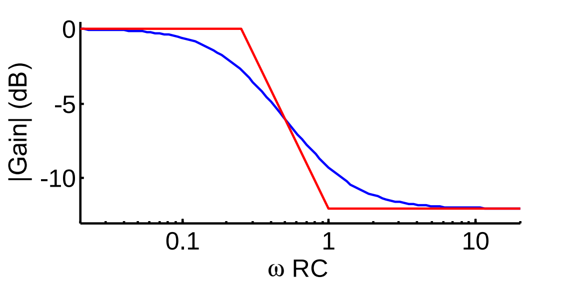

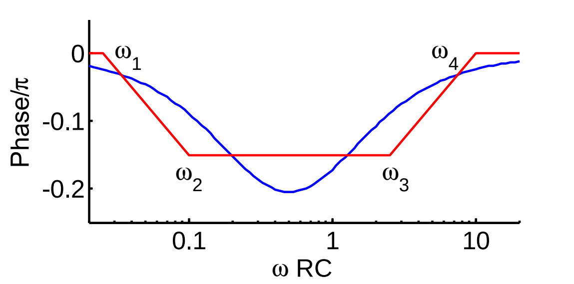

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Filters

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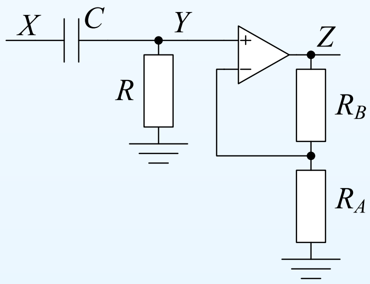

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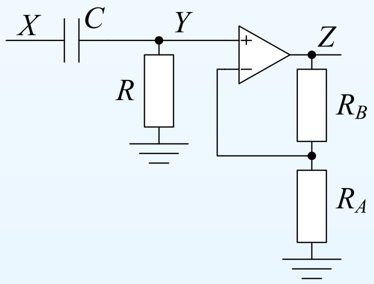

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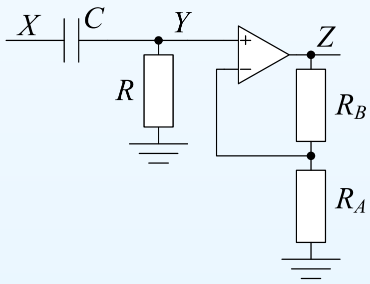

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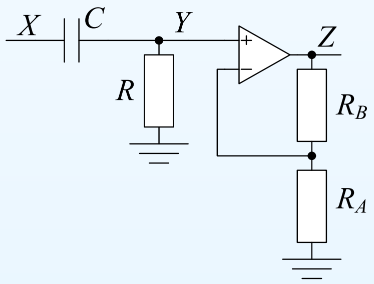

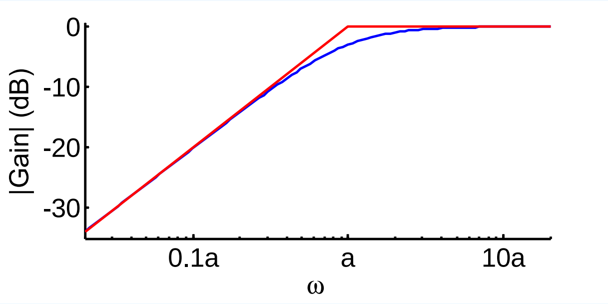

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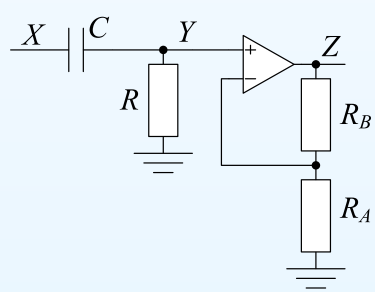

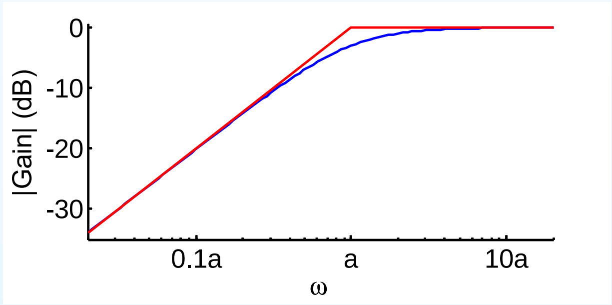

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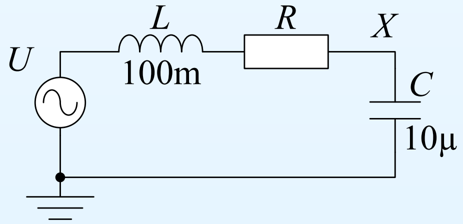

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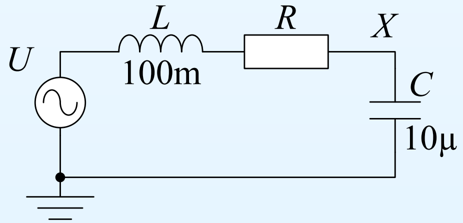

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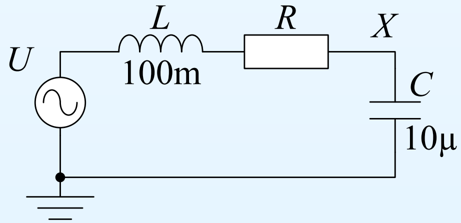
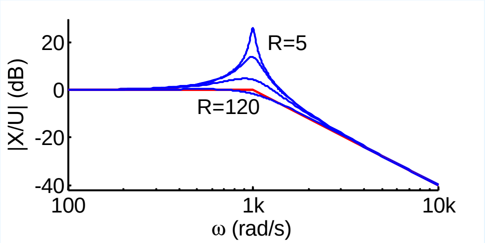
$$\zeta = \frac{1}{40}, \frac{1}{10}, \frac{3}{10}, \frac{6}{10}$$





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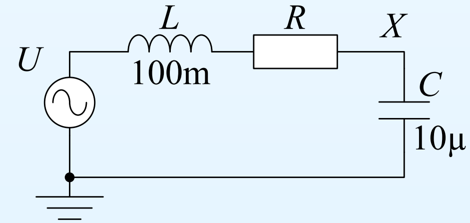

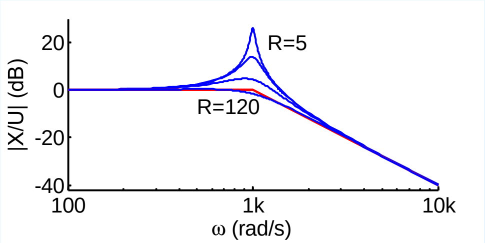

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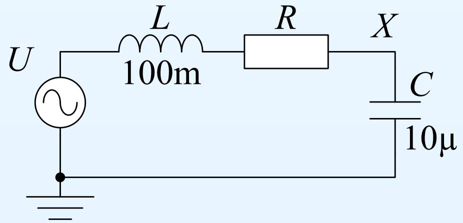

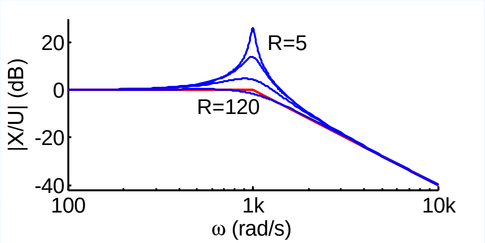

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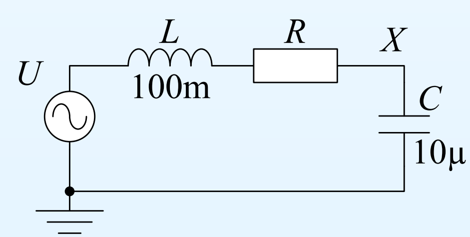

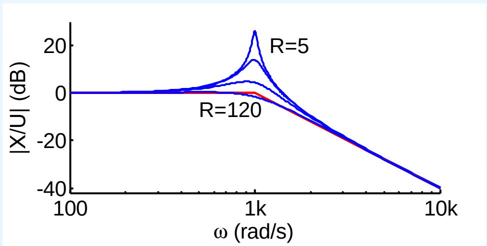

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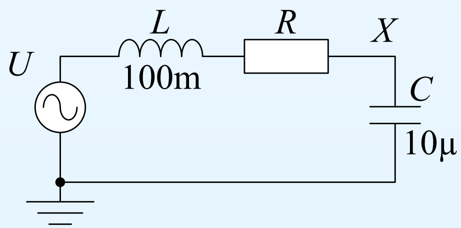

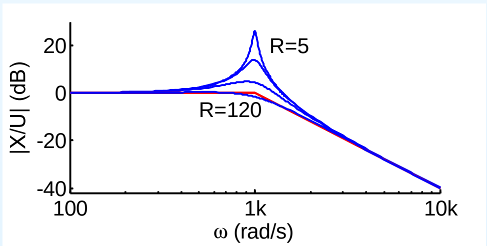

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