

**Revision Lecture 4:
Transients & Lines**

- Transients: Basic Ideas
- Steady States
- Determining Time Constant
- Determining Transient Amplitude
- Transmission Lines Basics
- Reflections
- Sinewaves and Phasors
- Standing Waves

Revision Lecture 4: Transients & Lines

Transients: Basic Ideas

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 - Input voltage/current abruptly changes its magnitude, frequency or phase
 - A switch alters the circuit

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 - Steady States: find with nodal analysis or transfer function
 - Note: Steady State is not the same as DC Level
 - Need steady states before and after the sudden change

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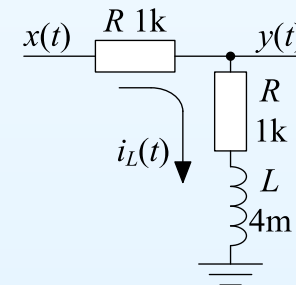
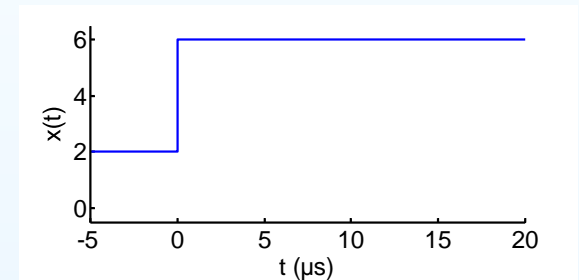
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 - τ and A can also be found from the transfer function.

Steady States

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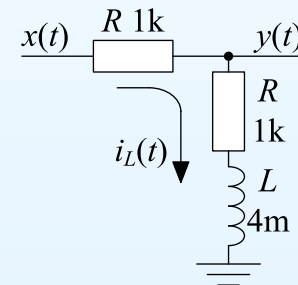
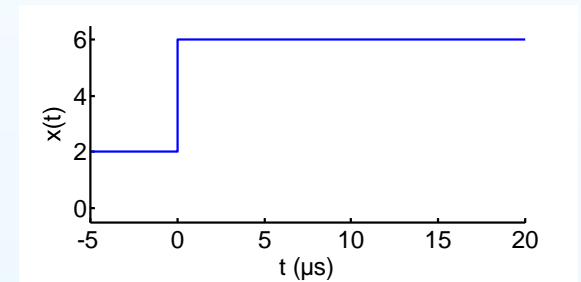


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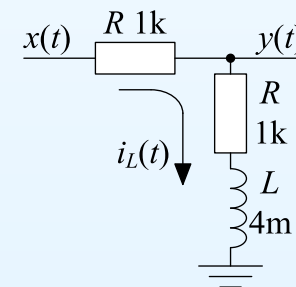
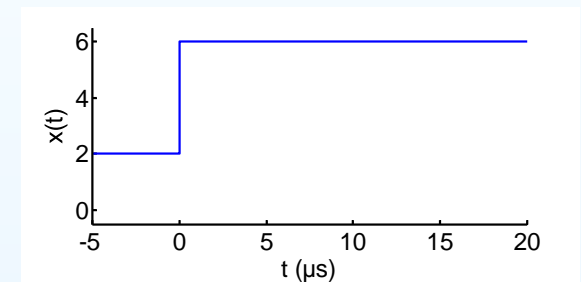
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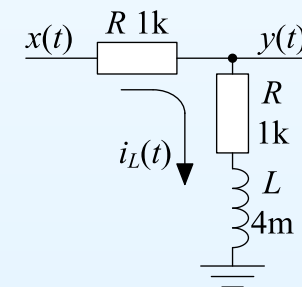
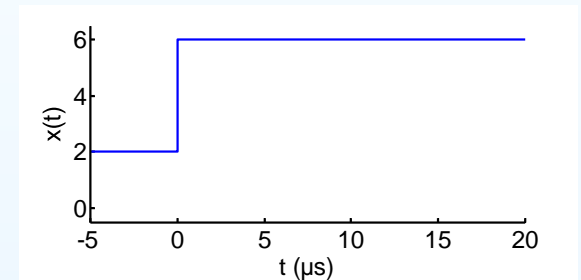
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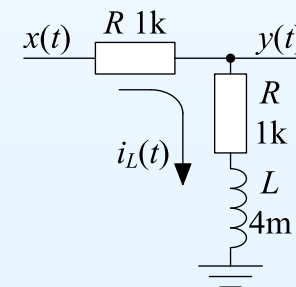
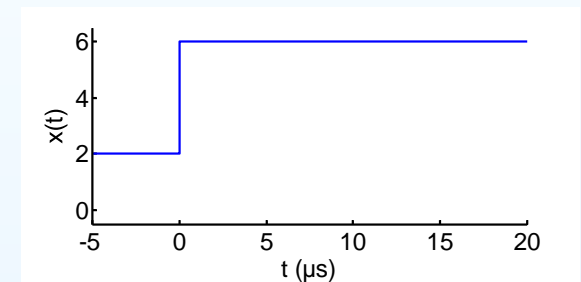
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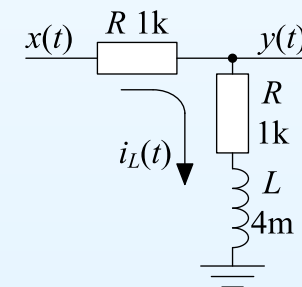
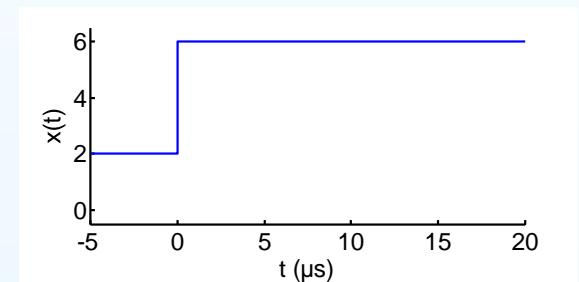
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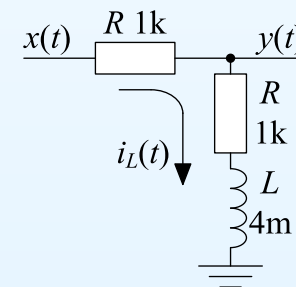
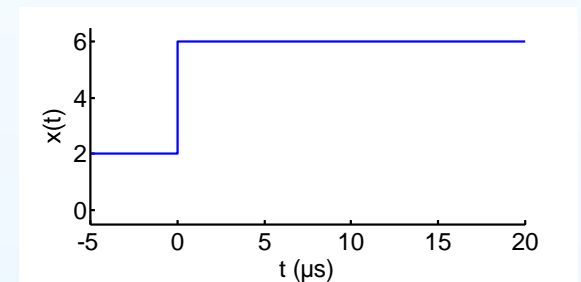
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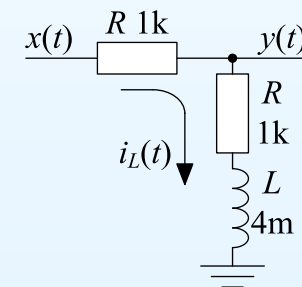
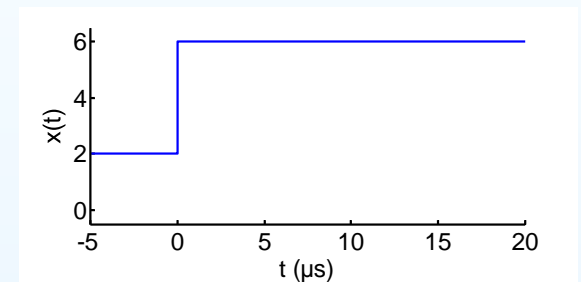
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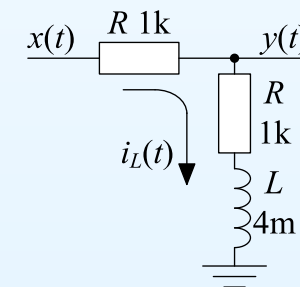
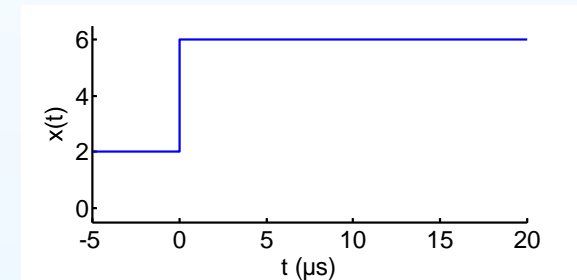
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Sinusoidal input \Rightarrow **Sinusoidal steady state** \Rightarrow use phasors.

Then convert phasors to time waveforms to calculate the actual output voltages $y_{SS}(0-)$ and $y_{SS}(0+)$ at $t = 0$.

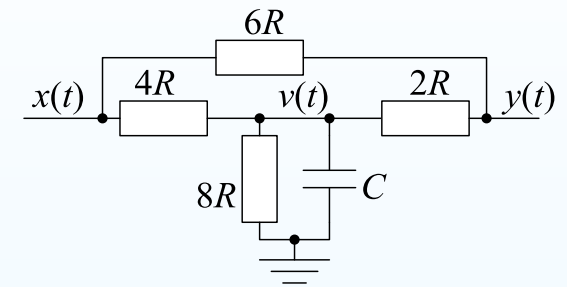


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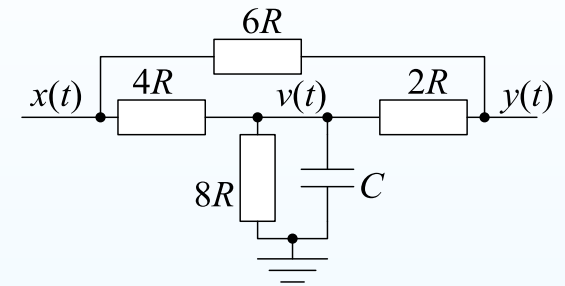
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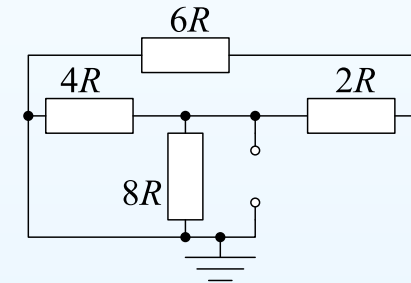
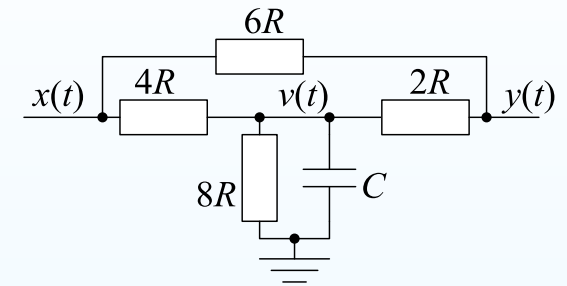
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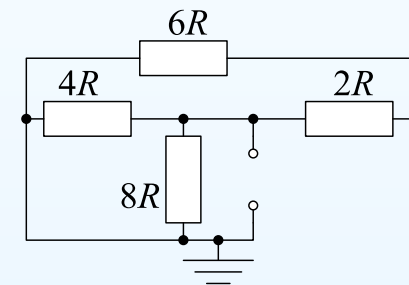
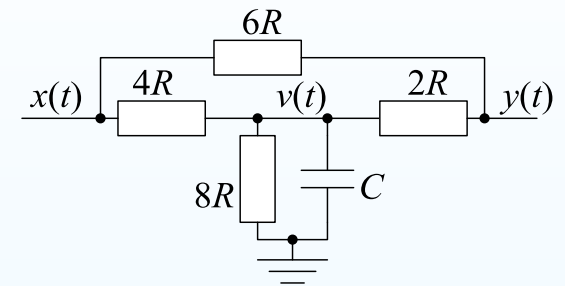
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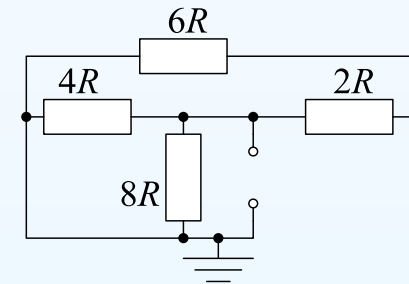
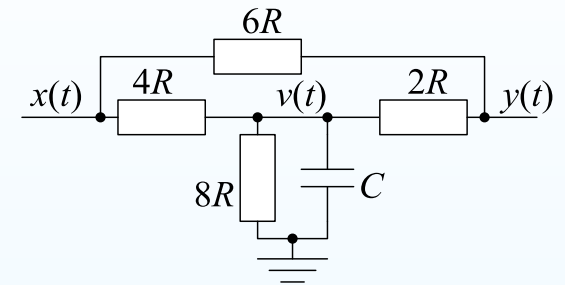
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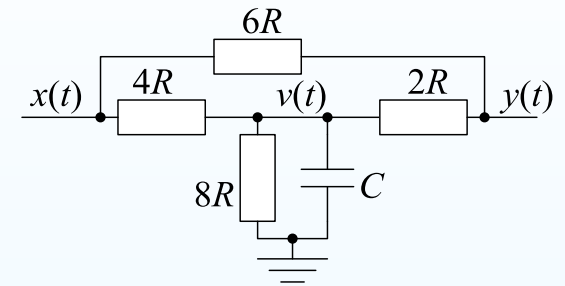
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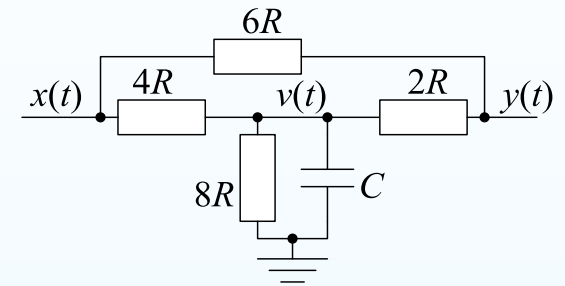
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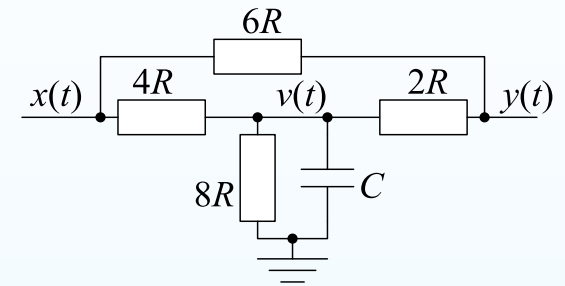
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$$\text{KCL @ V: } \frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$$

$$\text{KCL @ Y: } \frac{Y-V}{2R} + \frac{Y-X}{6R} = 0$$



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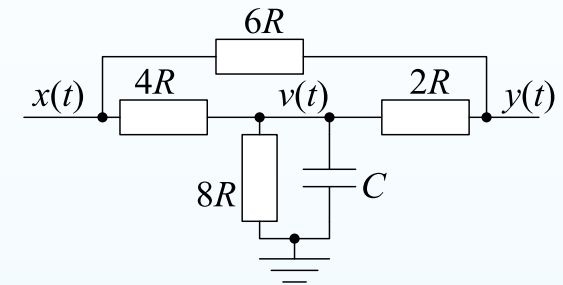
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$$\text{KCL @ } Y: \frac{Y-V}{2R} + \frac{Y-X}{6R} = 0$$

$$\rightarrow \text{Eliminate } V \text{ to get transfer Function: } \frac{Y}{X} = \frac{8j\omega RC + 13}{32j\omega RC + 16}$$

Determining Time Constant

Revision Lecture 4: Transients & Lines

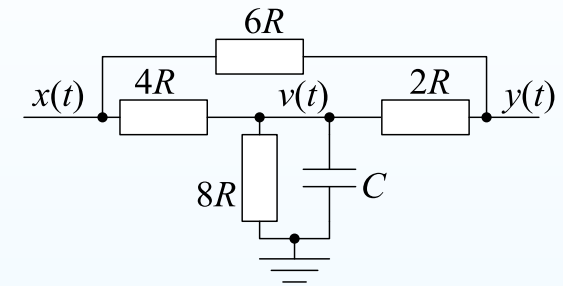
- Transients: Basic Ideas
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Method 1: Thévenin

- Remove the capacitor/inductor
- Set all sources to zero (including the input voltage source). Leave output unconnected.
- Calculate the Thévenin resistance between the capacitor/inductor terminals:

$$R_{Th} = 8R || 4R || (6R + 2R) = 2R$$

- Time constant: $= R_{Th}C$ or $\frac{L}{R_{Th}}$
 $\tau = R_{Th}C = 2RC$



Method 2: Transfer function

- Calculate transfer function using nodal analysis

$$\text{KCL @ V: } \frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$$

$$\text{KCL @ Y: } \frac{Y-V}{2R} + \frac{Y-X}{6R} = 0$$

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- Time Constant = $\frac{1}{\text{Denominator corner frequency}}$

Determining Time Constant

Revision Lecture 4: Transients & Lines

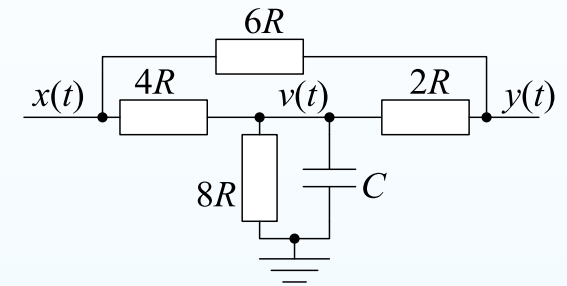
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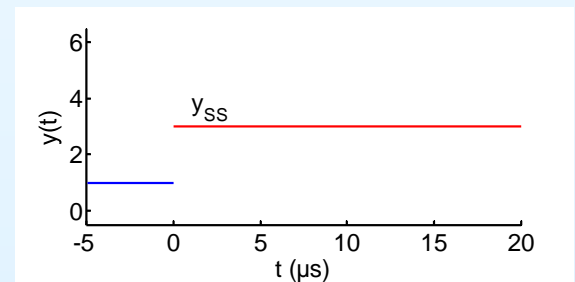
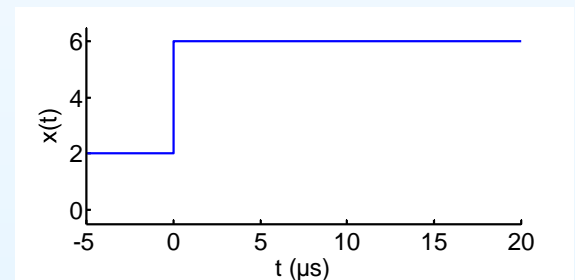
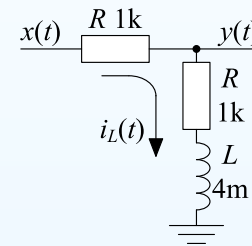
$$\omega_d = \frac{16}{32RC} \Rightarrow \tau = \frac{1}{\omega_d} = 2RC$$

Determining Transient Amplitude

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After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

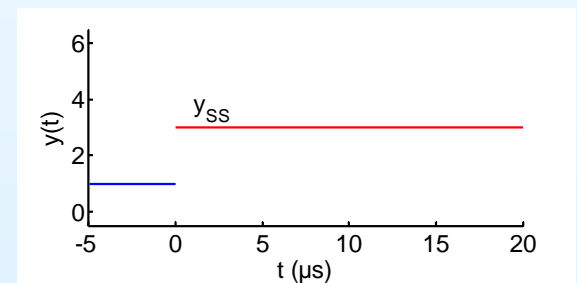
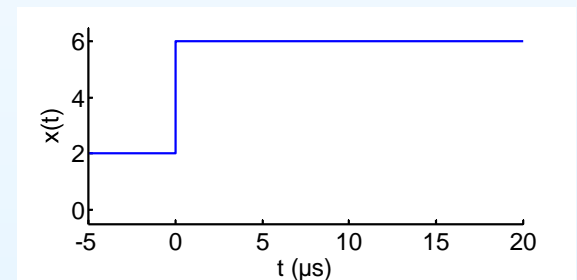
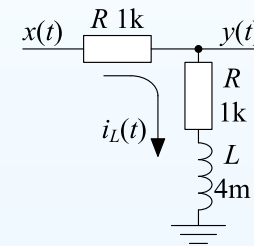


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After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.
 $\Rightarrow y(0+) = y_{SS}(0+) + A \Rightarrow A = y(0+) - y_{SS}(0+)$



Determining Transient Amplitude

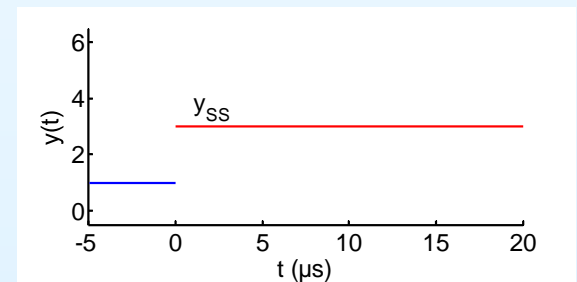
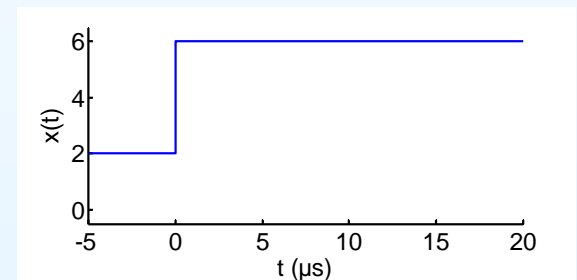
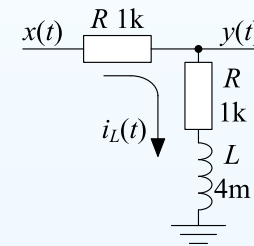
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After an input change at $t = 0$, $y(t) = y_{SS}(t) + Ae^{-\frac{t}{\tau}}$.

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Determining Transient Amplitude

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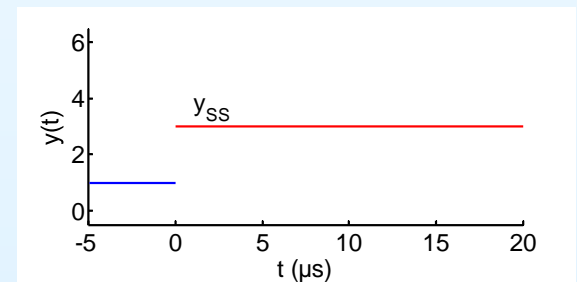
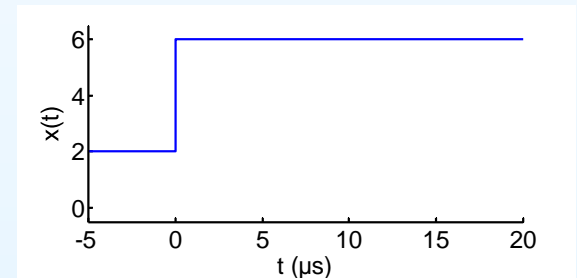
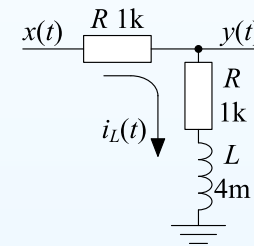
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Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A

(i) Version 1: v_C or i_L continuity



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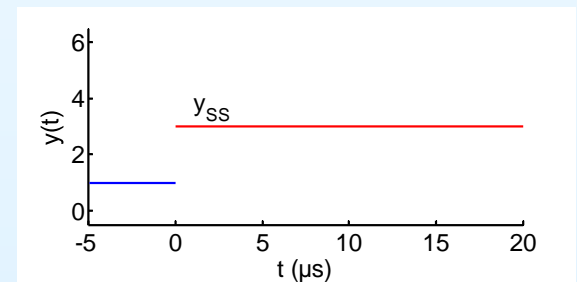
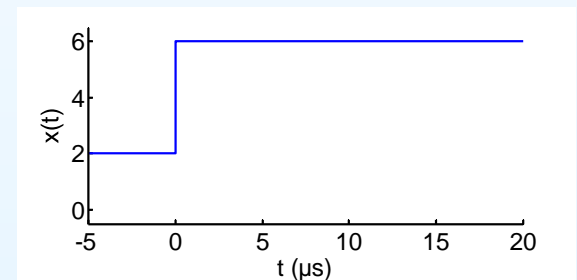
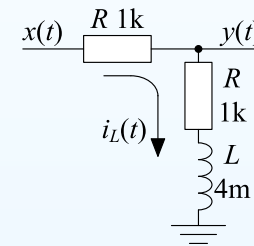
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Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A

(i) **Version 1: v_C or i_L continuity**

$$x(0-) = 2$$



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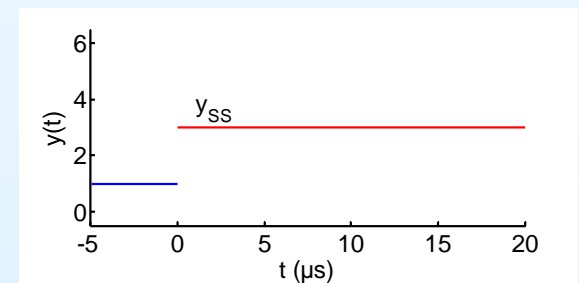
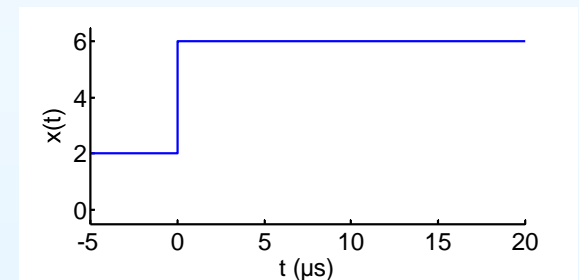
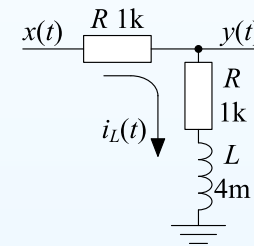
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$$x(0-) = 2 \Rightarrow i_L(0-) = 1 \text{ mA}$$



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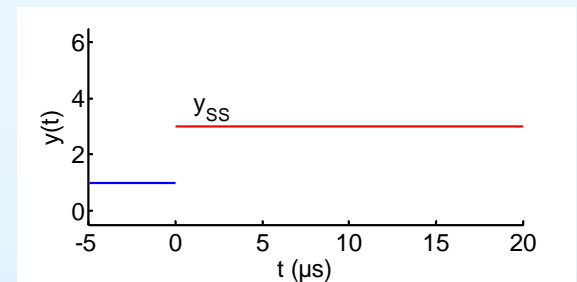
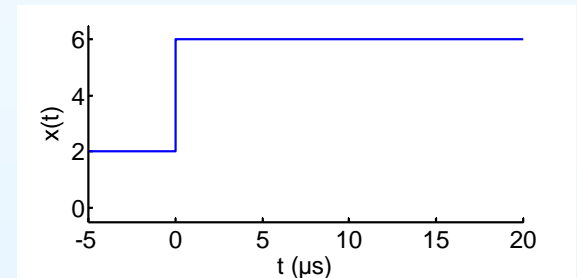
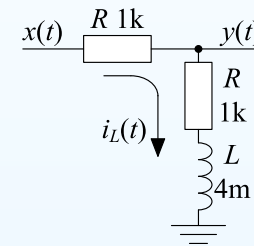
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(i) **Version 1: v_C or i_L continuity**

$$x(0-) = 2 \Rightarrow i_L(0-) = 1 \text{ mA}$$

$$\text{Continuity} \Rightarrow i_L(0+) = i_L(0-)$$



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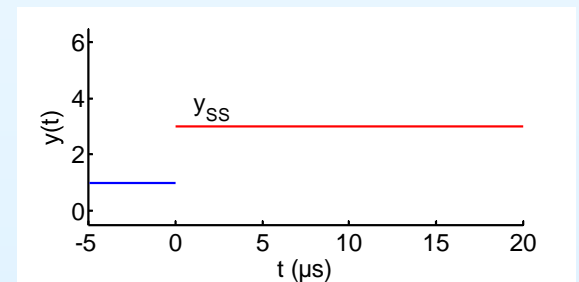
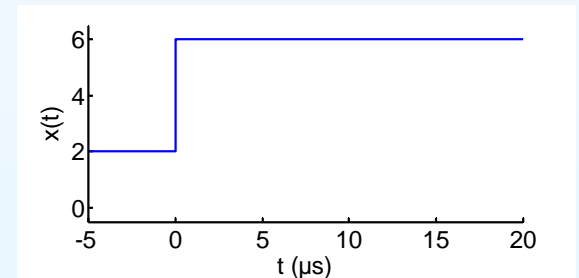
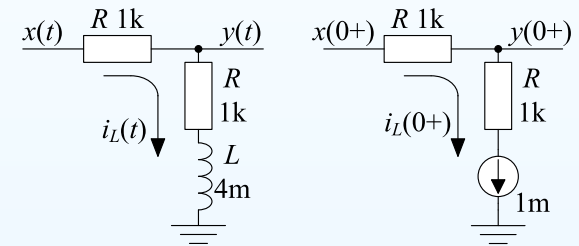
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Replace L with a 1 mA current source



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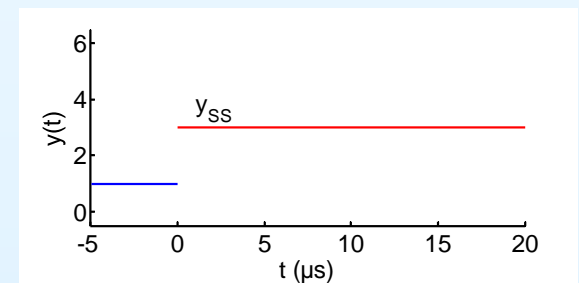
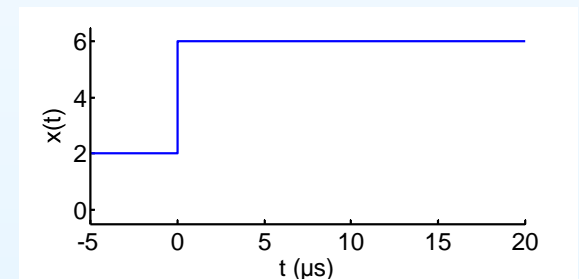
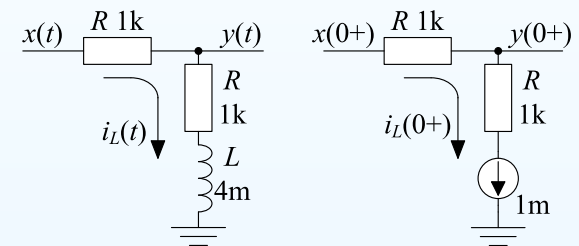
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Replace L with a 1 mA current source

$$y(0+) = x(0+) - iR = 6 - 1 = 5$$



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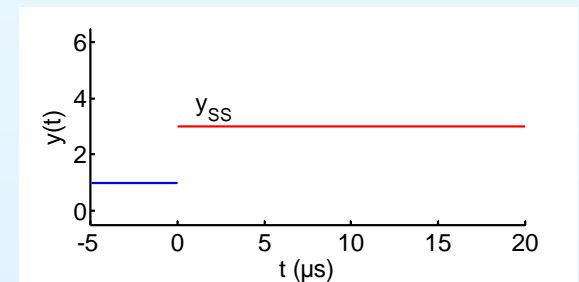
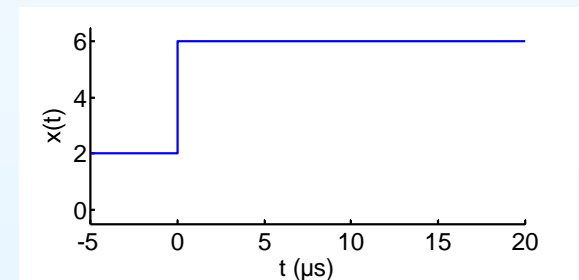
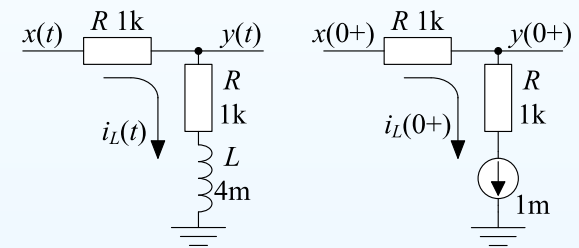
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Replace L with a 1 mA current source

$$y(0+) = x(0+) - iR = 6 - 1 = 5$$

(i) **Version 2: Transfer function**



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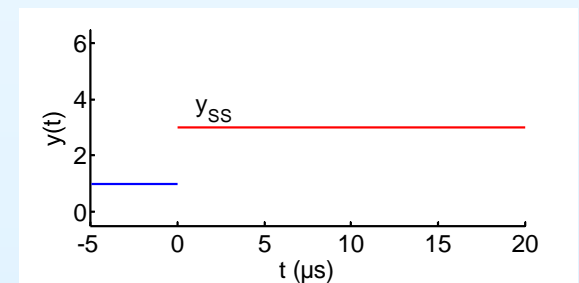
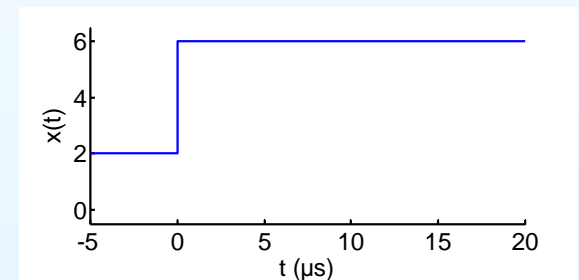
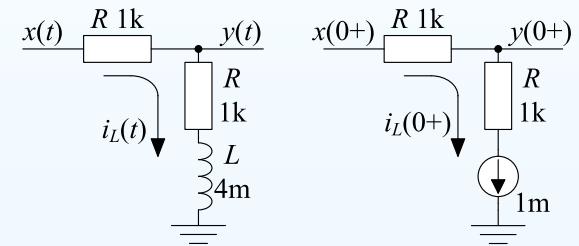
$$\text{Continuity} \Rightarrow i_L(0+) = i_L(0-)$$

Replace L with a 1 mA current source

$$y(0+) = x(0+) - iR = 6 - 1 = 5$$

(i) Version 2: Transfer function

$$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$$



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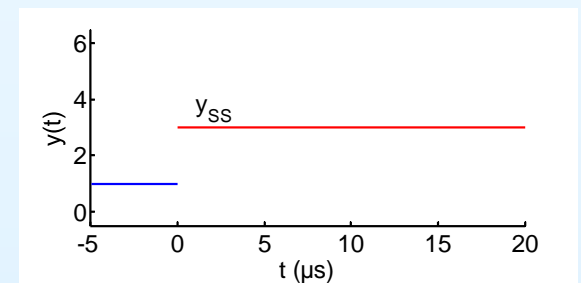
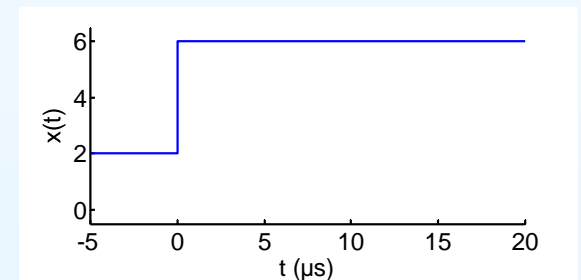
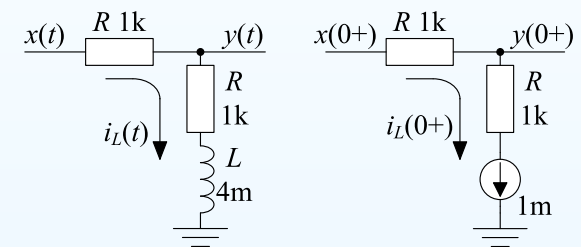
Replace L with a 1 mA current source

$$y(0+) = x(0+) - iR = 6 - 1 = 5$$

(i) Version 2: Transfer function

$$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$$

$$\text{Input step, } \Delta x = x(0+) - x(0-) = +4$$



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Replace L with a 1 mA current source

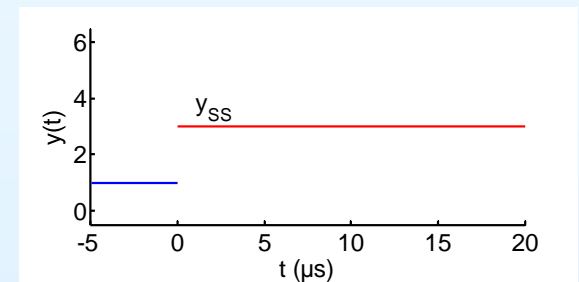
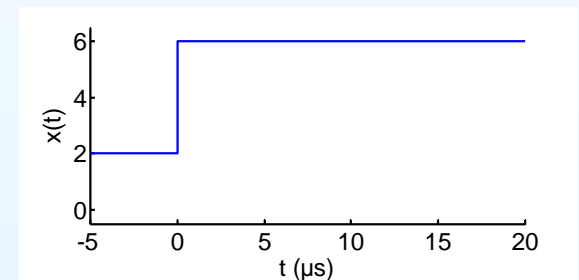
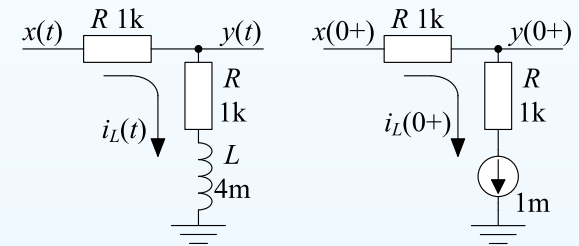
$$y(0+) = x(0+) - iR = 6 - 1 = 5$$

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$$H(j\omega) = \frac{Y}{X}(j\omega) = \frac{R+j\omega L}{2R+j\omega L}$$

Input step, $\Delta x = x(0+) - x(0-) = +4$

$$y(0+) = y(0-) + H(j\infty) \times \Delta x$$



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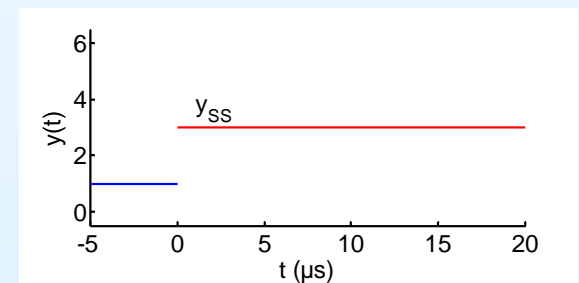
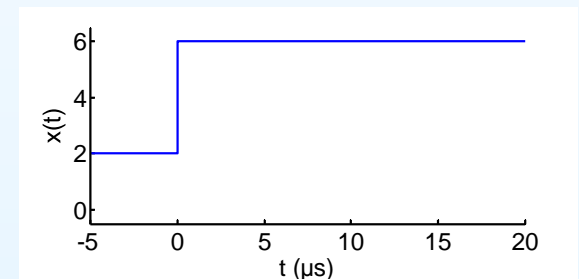
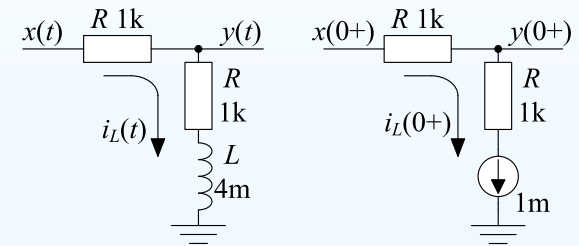
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$$y(0+) = y(0-) + H(j\infty) \times \Delta x \\ = 1 + 1 \times 4 = 5$$



Determining Transient Amplitude

Revision Lecture 4: Transients & Lines

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Method: (a) calculate true output $y(0+)$, (b) subtract $y_{SS}(0+)$ to get A

(i) Version 1: v_C or i_L continuity

$$x(0-) = 2 \Rightarrow i_L(0-) = 1 \text{ mA}$$

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Replace L with a 1 mA current source

$$y(0+) = x(0+) - iR = 6 - 1 = 5$$

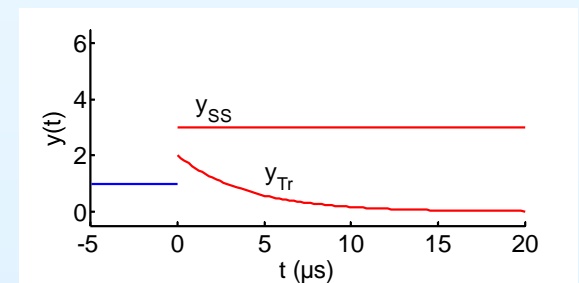
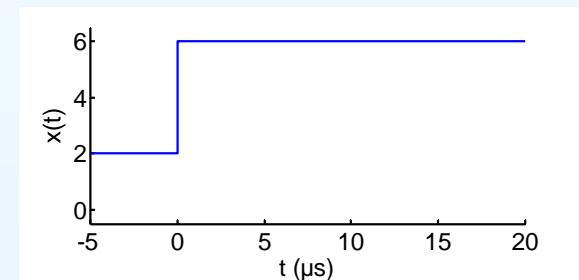
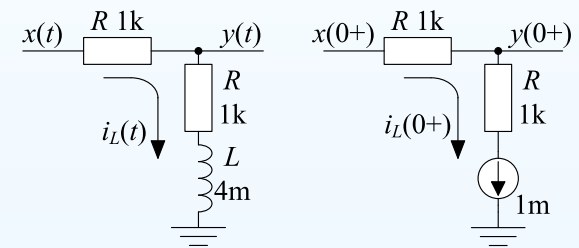
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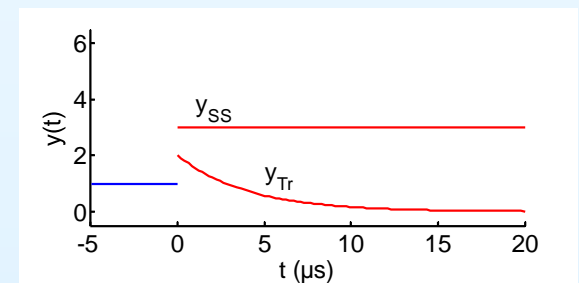
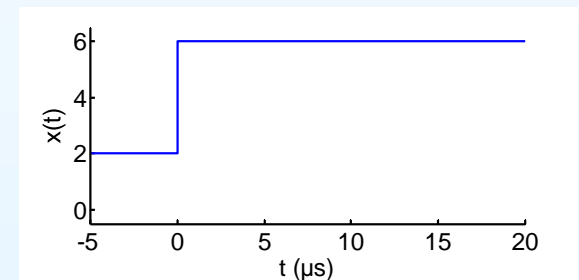
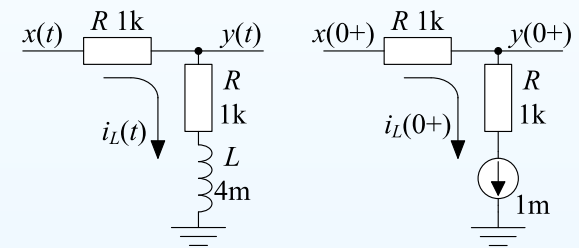
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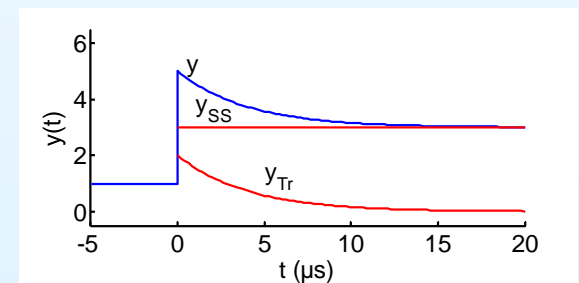
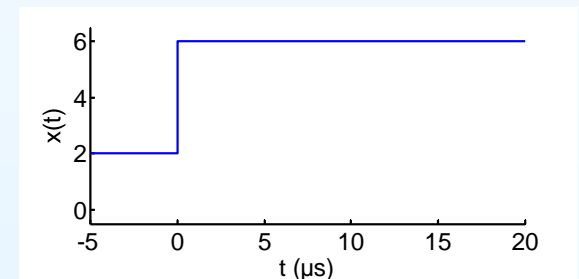
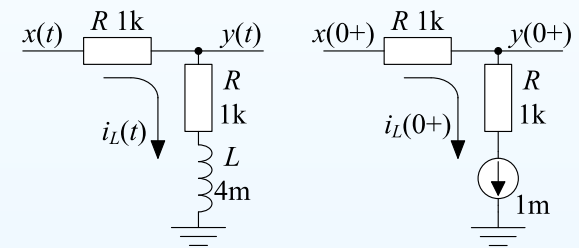
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Transmission Lines Basics

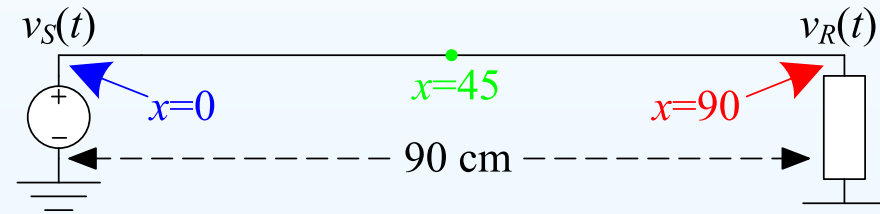
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Forward wave travels along the line: $f_x(t) = f_0 \left(t - \frac{x}{u} \right)$.

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Transmission Lines Basics

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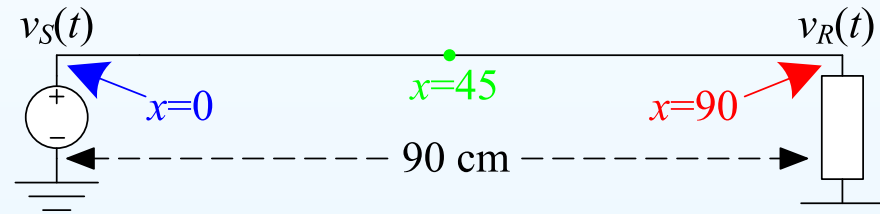
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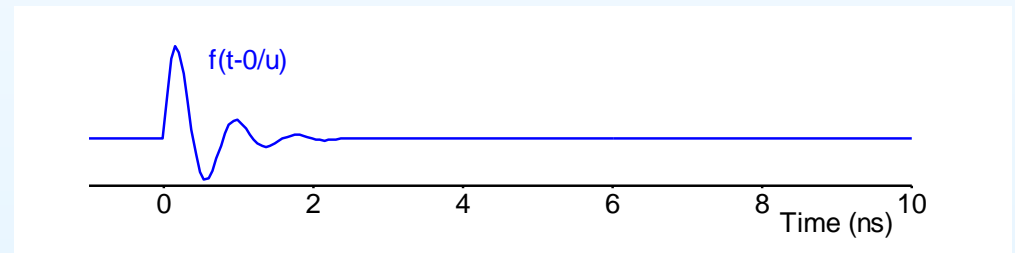
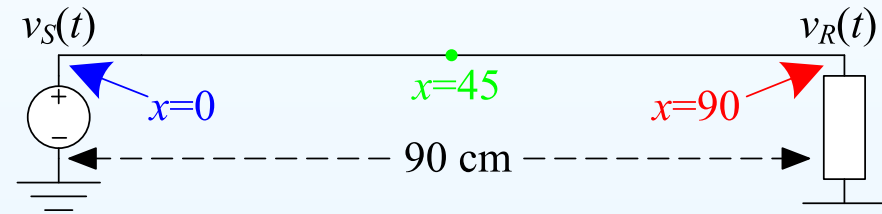
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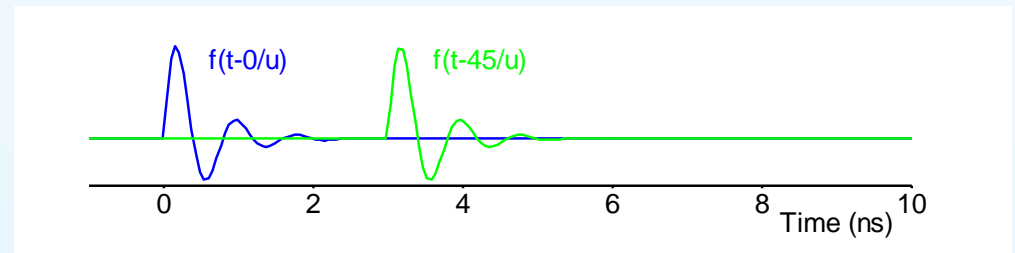
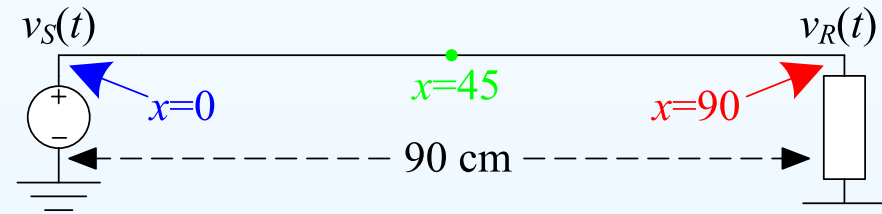
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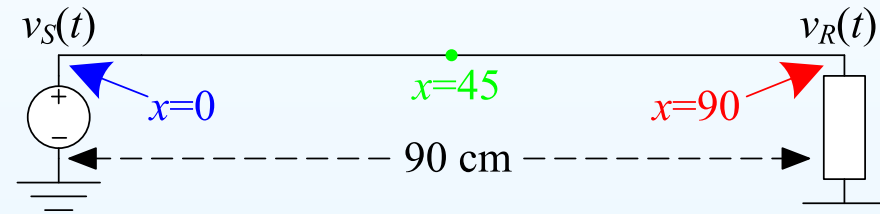
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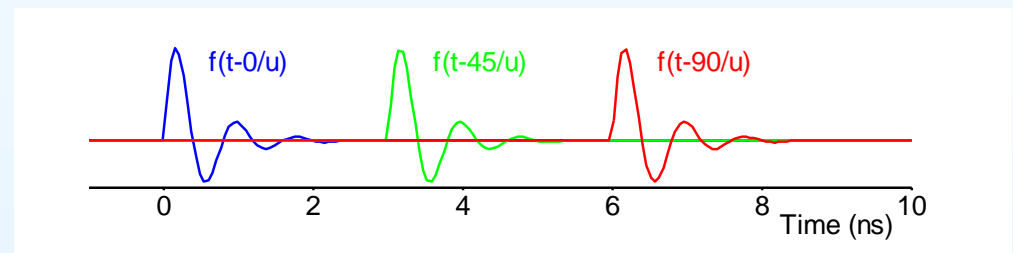
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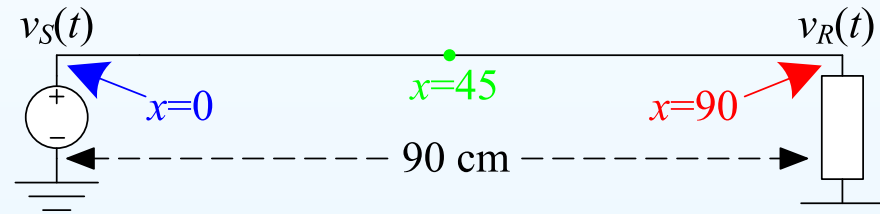
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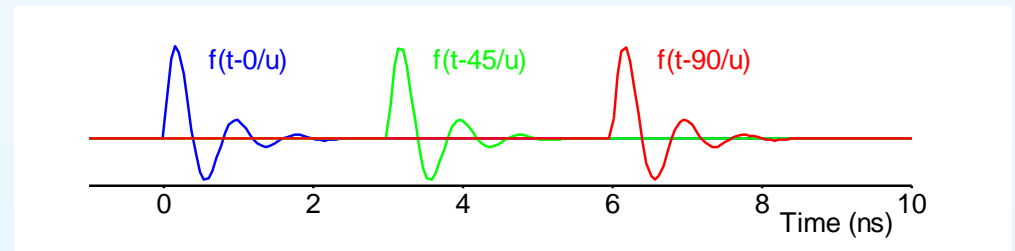
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Transmission Lines Basics

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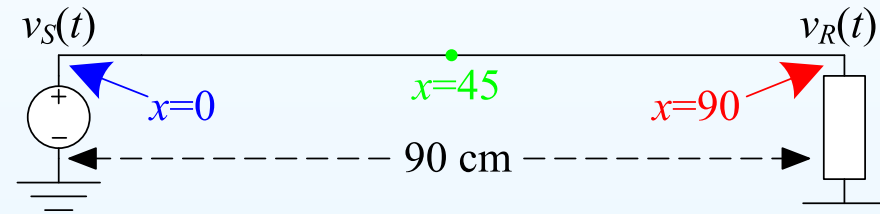
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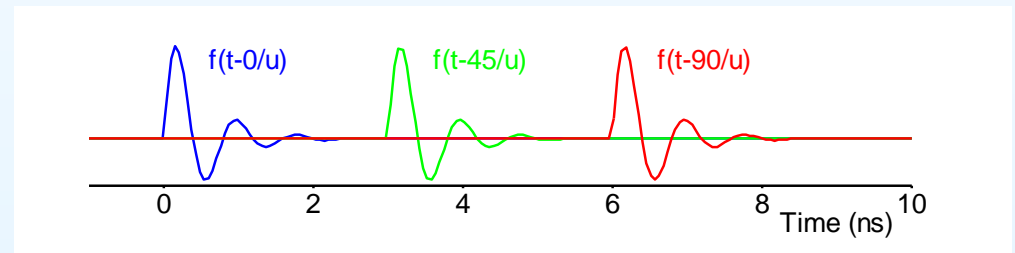
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Transmission Lines Basics

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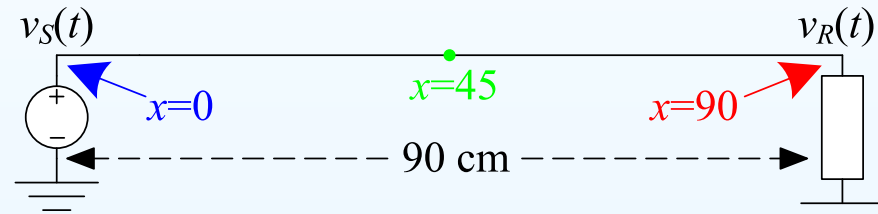
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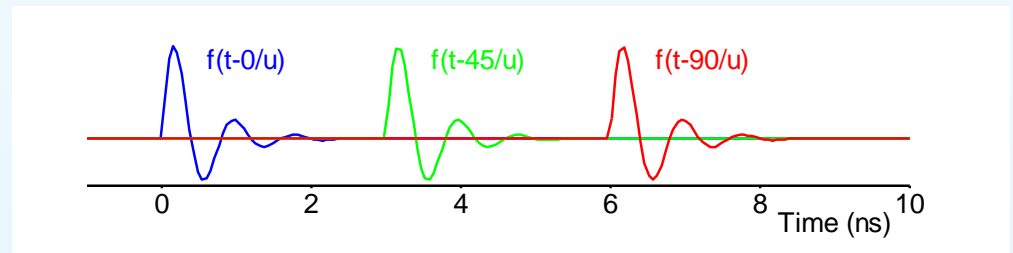
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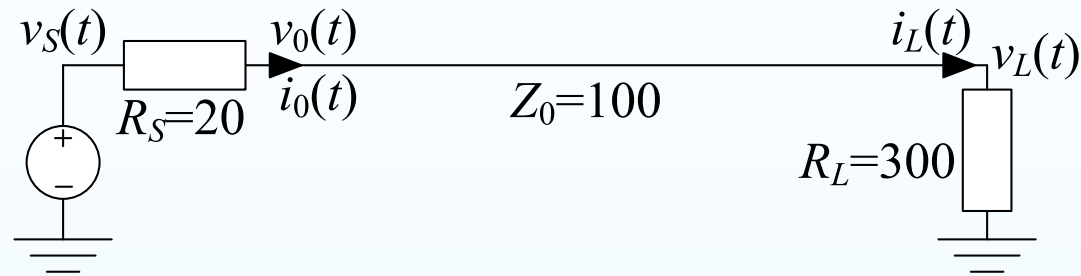
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Waveforms of f_x and g_x are determined by the connections at both ends.

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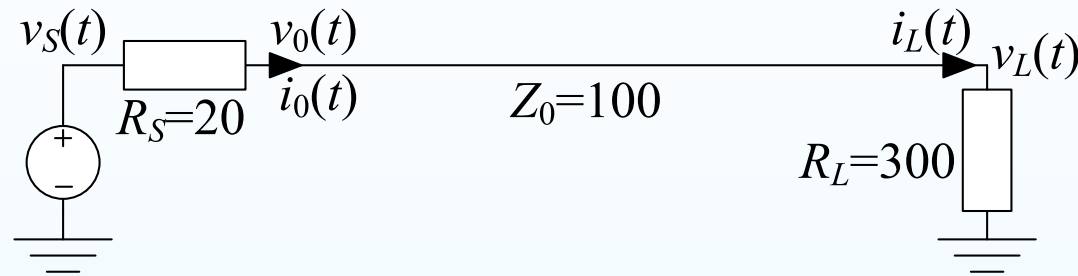


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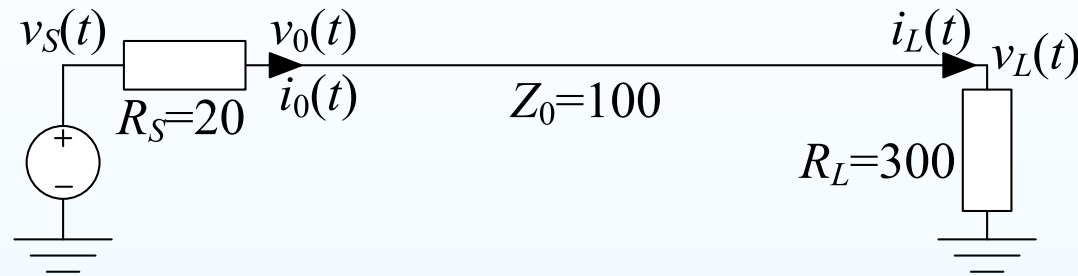
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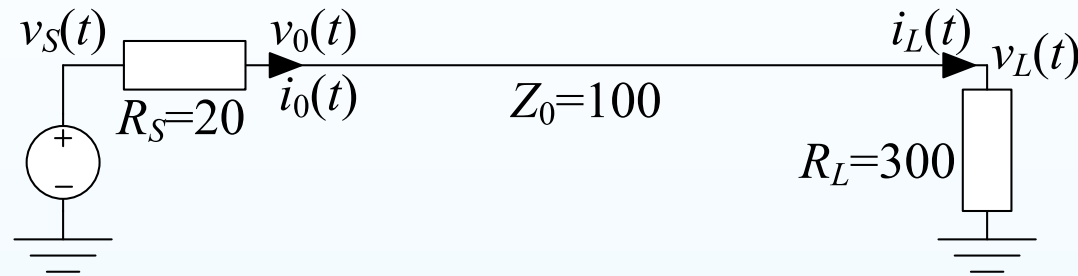
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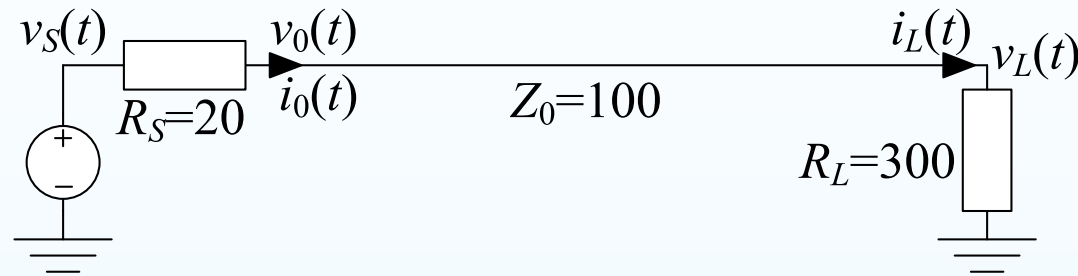
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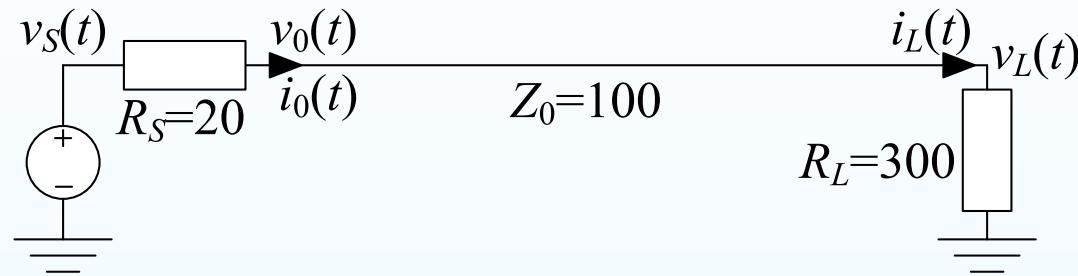
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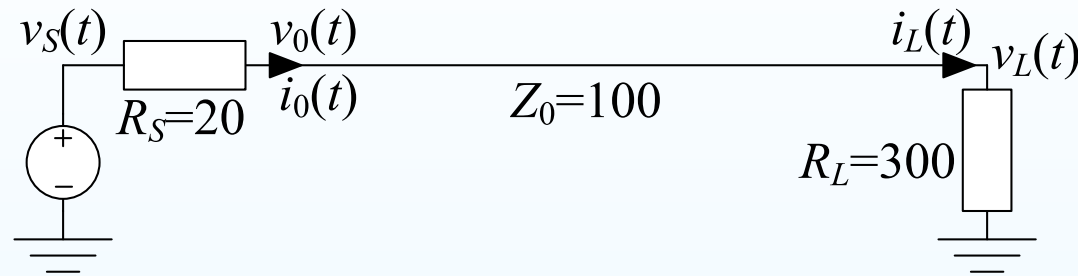
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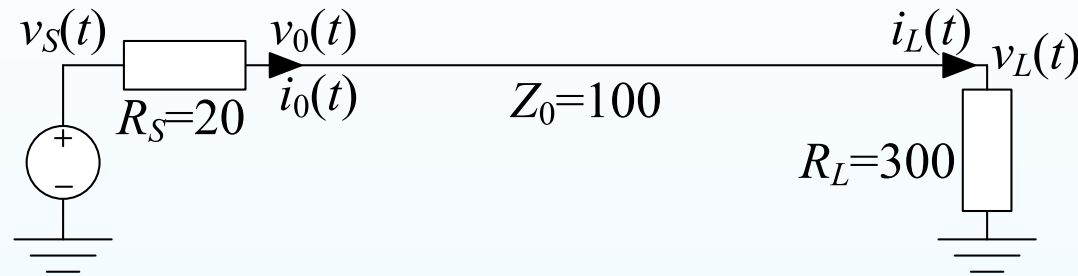
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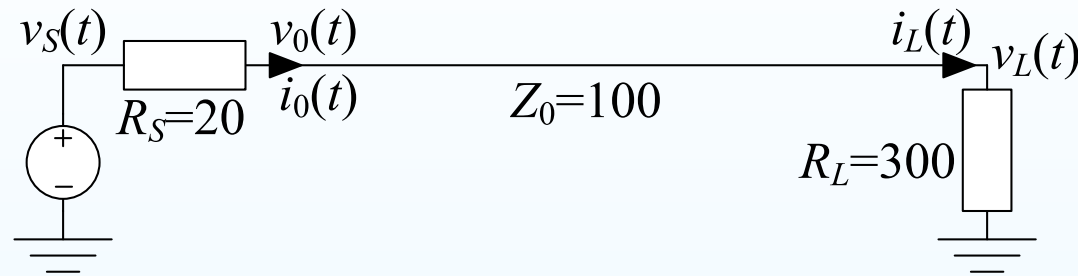
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Wave bounces back and forth getting smaller with each reflection:

Reflections

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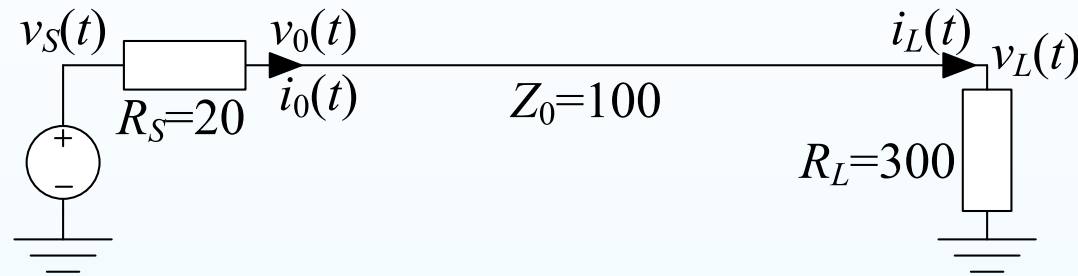
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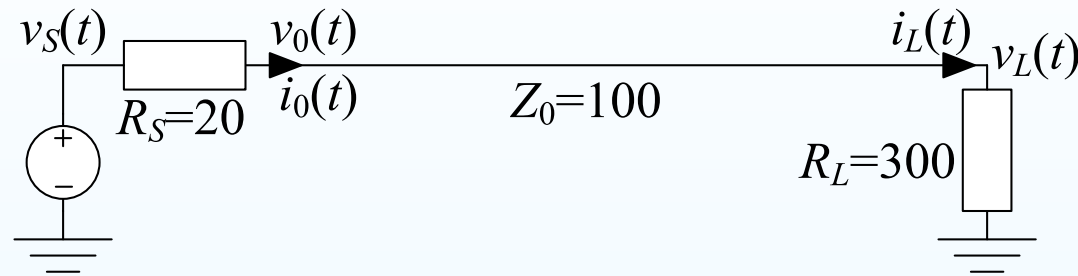
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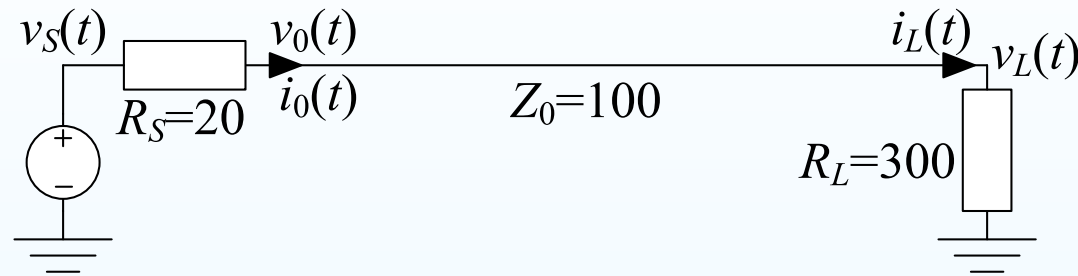
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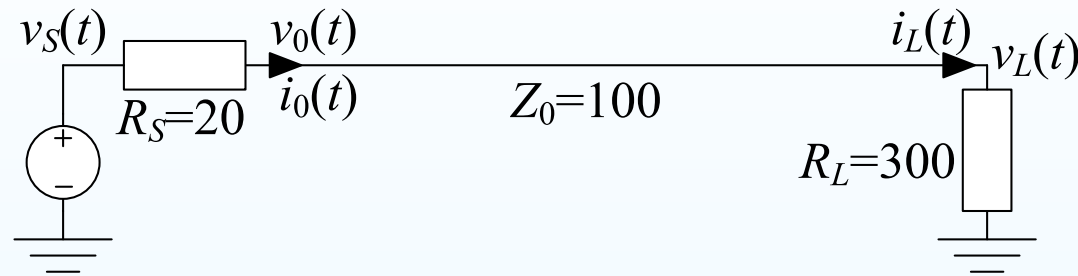
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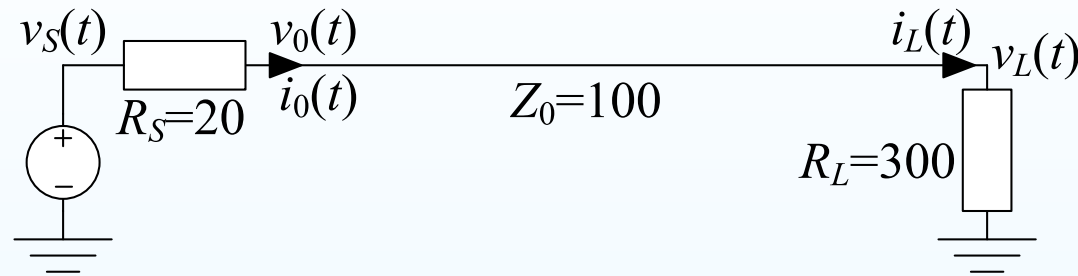
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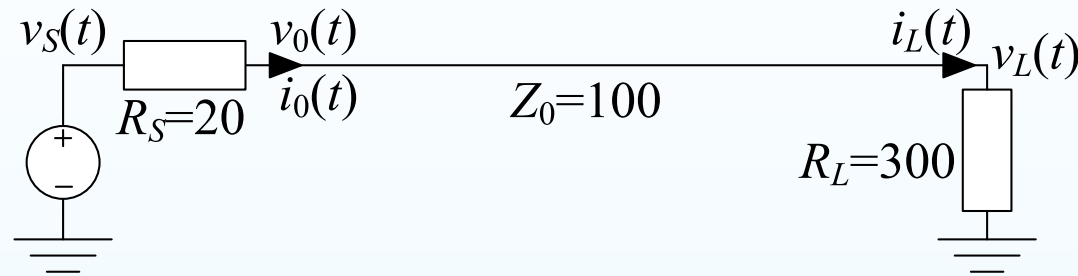
Infinite sum:

$$f_0(t) = \tau_0 v_S(t) + \tau_0 \rho_L \rho_0 v_S(t - \frac{2L}{u}) + \dots$$

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Infinite sum:

$$f_0(t) = \tau_0 v_S(t) + \tau_0 \rho_L \rho_0 v_S(t - \frac{2L}{u}) + \dots = \sum_{i=0}^{\infty} \tau_0 \rho_L^i \rho_0^i v_S(t - \frac{2Li}{u})$$

Sinewaves and Phasors

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1. Use phasors to eliminate t :
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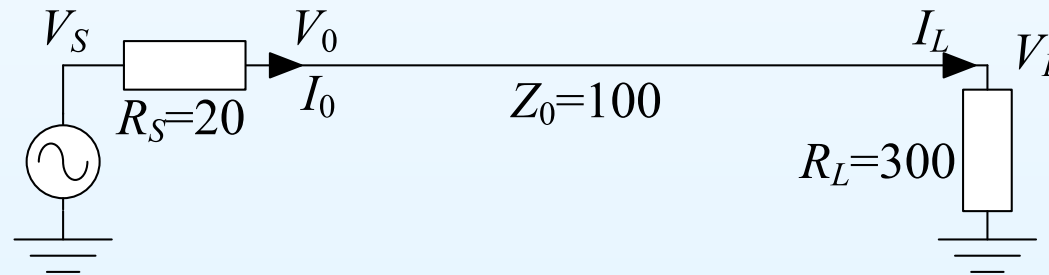
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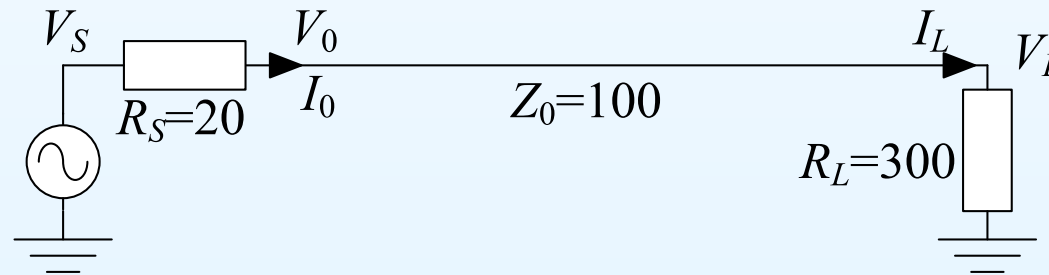
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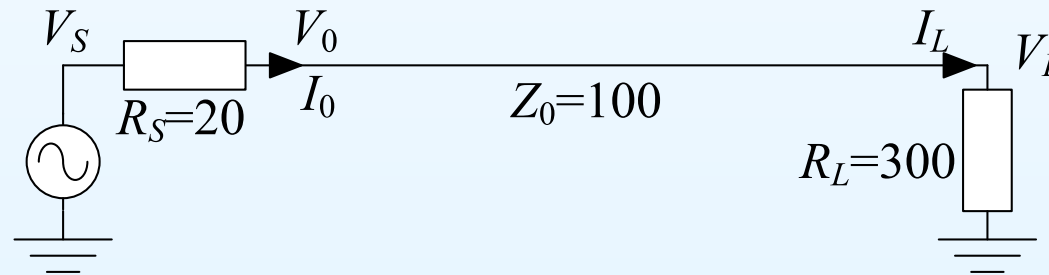
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Substituting for G_0 in source end equation: $F_0 = \tau_0 V_S + \rho_0 F_0 \rho_L e^{-2jkL}$

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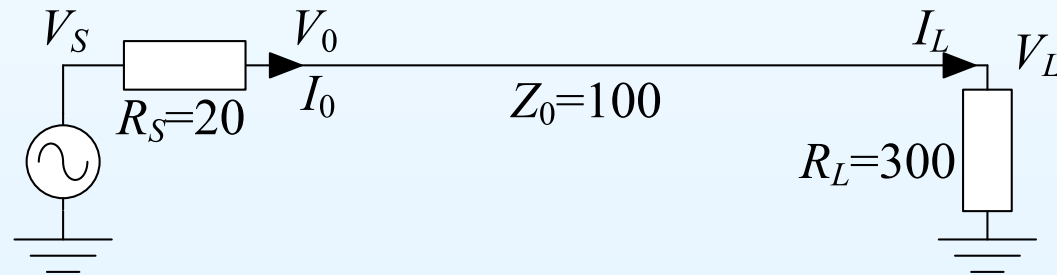
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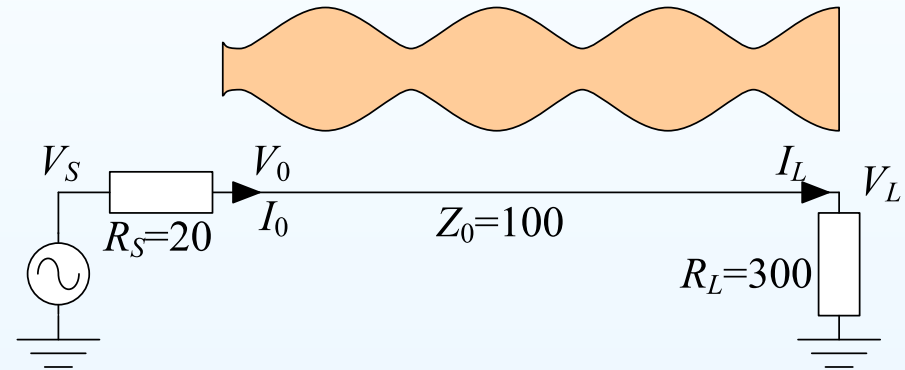
$$\Rightarrow F_0 = \frac{\tau_0}{1 - \rho_0 \rho_L \exp(-2jkL)} V_S \text{ so no infinite sums needed } \textcircled{\smile}$$

Standing Waves

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Standing waves arise whenever a wave meets its reflection:
at places where the two waves are **in phase** their amplitudes add
but where they are **anti-phase** their amplitudes subtract.



Standing Waves

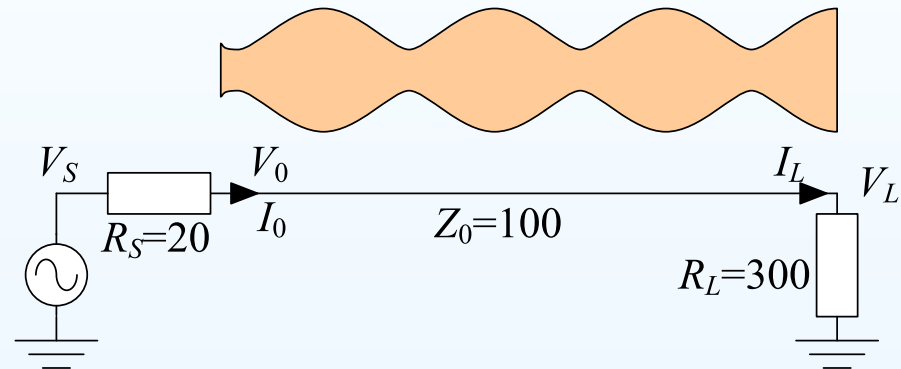
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At any point x ,

$$\text{delay of } \frac{x}{u} \Rightarrow$$
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Standing Waves

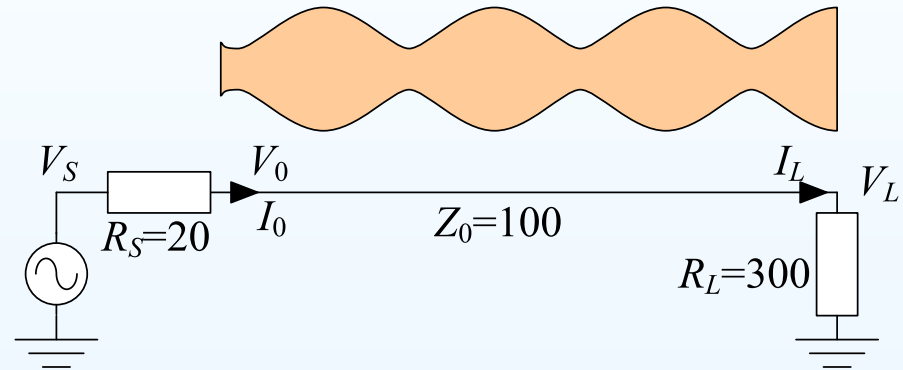
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Standing Waves

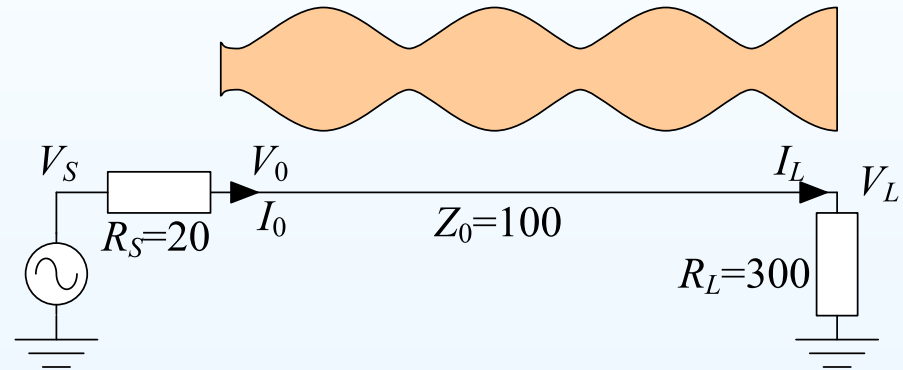
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Standing Waves

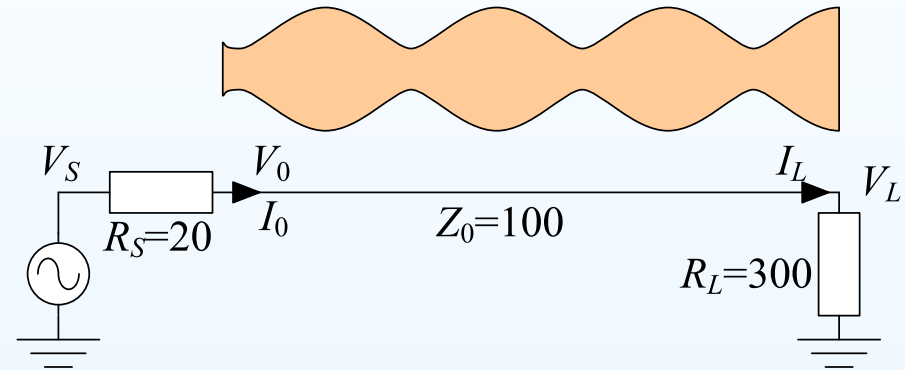
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- Transients: Basic Ideas
- Steady States
- Determining Time Constant
- Determining Transient Amplitude
- Transmission Lines Basics
- Reflections
- Sinewaves and Phasors
- Standing Waves

Standing waves arise whenever a wave meets its reflection:
at places where the two waves are **in phase** their amplitudes add
but where they are **anti-phase** their amplitudes subtract.

At any point x ,

$$\text{delay of } \frac{x}{u} \Rightarrow F_x = F_0 e^{-jkx}$$



Backward wave: $G_x = \rho_L F_x e^{-2jk(L-x)}$: reflection + delay of $2 \frac{L-x}{u}$

Voltage at x : $V_x = F_x + G_x$

Standing Waves

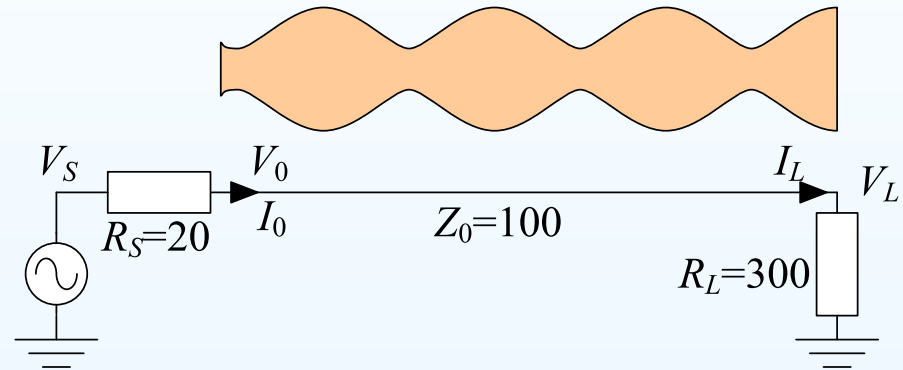
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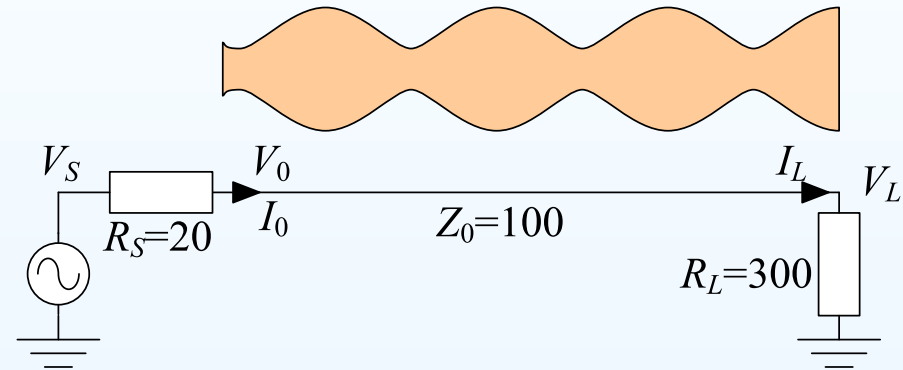
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Standing Waves

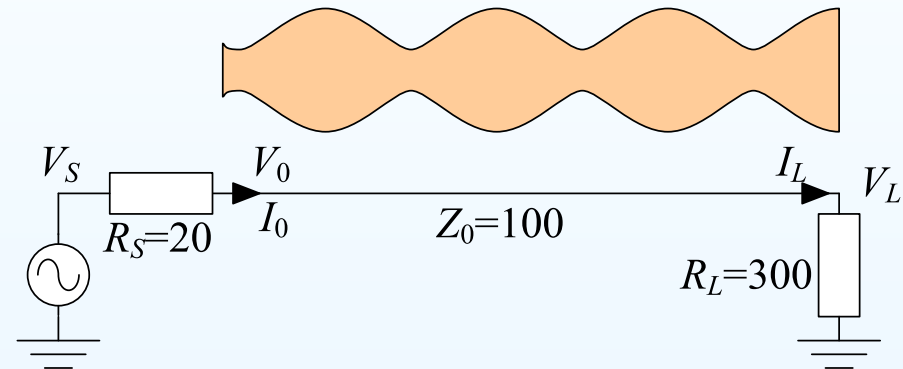
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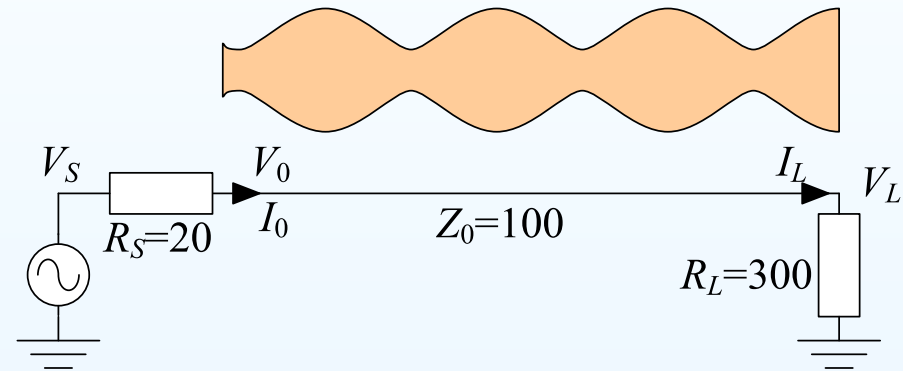
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Standing Waves

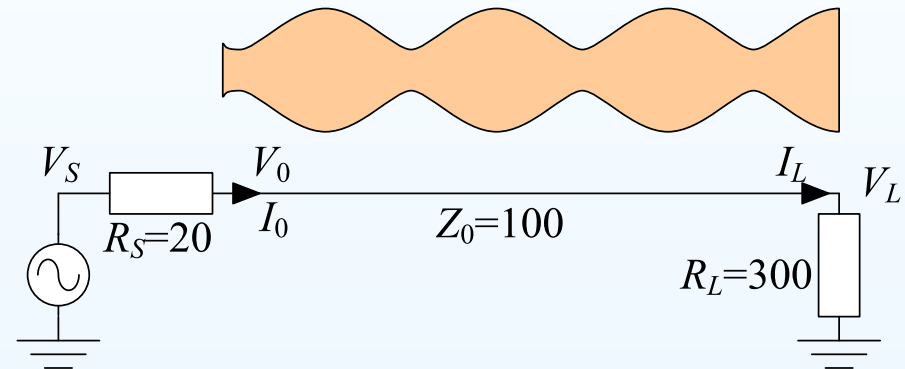
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Standing Waves

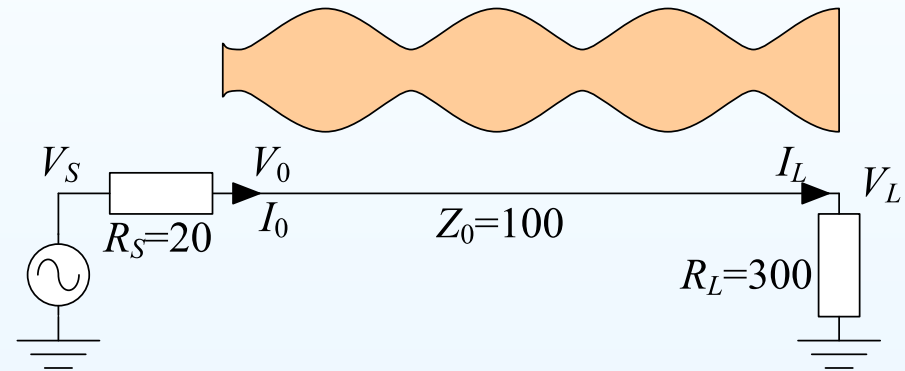
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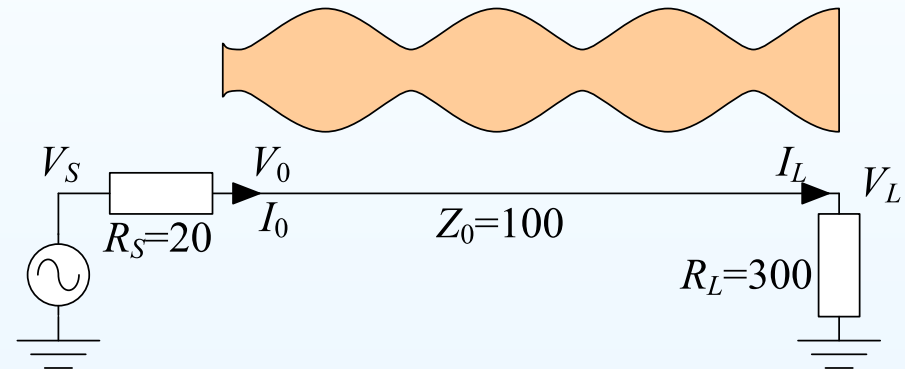
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