Blind Adaptive Channel Equalization for OFDM Using the Cyclic Prefix Data

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Abstract— **In an Orthogonal Frequency Division Multiplexing (OFDM) system using DQPSK, such as DAB, channel equalization is considered to be unnecessary. The reason for this is that there is no relative phase rotation due to the multipath channel in successive OFDM symbols provided the channel delays are all shorter than the guard interval of the OFDM symbol. However, delays greater than the guard interval result in inter-carrier and inter-symbol interference to the OFDM system. In this paper, we present a blind adaptive equalization algorithm which uses the cyclic prefix data of the OFDM signals to equalize the multipath channel. This algorithm is ideal for systems such as DAB in which no pilots are available for the channel estimation yet the long delays nevertheless can cause significant performance degradation if no channel equalization is done. Our simulation results shows improved BER performance for the equalized system over the non-equalized system when long channel delays are present.**

I. Introduction

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has been adopted in many communication standards including broadband ADSL modems, digital video broadcasting (DVB), and digital audio broadcasting (DAB) for digital radio. In general OFDM can be made immune to multipath fading by using a frequency domain one-tap equalizer if the delay of the longest multipath is less than the guard interval (T_g). The purpose of this equalizer is to correct the amplitude and the phase of each sub-carrier of OFDM signal by simply multiplying the OFDM spectrum by the value of the channel impulse response at the sub-carrier frequency [1]. It can be shown that in the case of differential PSK coding, such as DQPSK used in DAB, the need for the one tap equalizer is also eliminated because the channel channel impulse response for a particular sub-carrier is almost constant for consecutive OFDM symbols provided that the fading is slow (i.e. Doppler spread is small compared with the sub-carrier spacing). However, broadcast DAB signals in the UK have been observed to have multipath delays that exceed the guard interval resulting in significant performance degradation due to both inter-symbol (ISI) and inter-subcarrier interference (ISCI). For instance, in DAB transmission mode I, signals received from transmitters that are further than 74 km cause delays longer than $T_g(= 246\ \mu s)$ at the receiver.

There are several ways to combat the ISI and ISCI caused by time delays greater than T_g,

- Channel Equalization
- Error Correction Codes [2], or adaptive OFDM [3] (employing different modulation schemes on individual OFDM subcarriers according to the channel conditions)
- Increased Guard interval length so that the delays are less than T_g

The first of the three approaches is the most attractive since it does not reduce the throughput of the channel being used. The various equalization approaches that have been presented in the literature for the equalization of OFDM can be categorized according to whether they operate in the frequency domain or the time domain.

Sub-channel equalization in the frequency domain is an effective way of implementing a frequency domain equalizer. Adaptive sub-channel equalizers are capable of cancelling both ISCI and ISI. However, these sub-channel equalizers have a high computational complexity and a slow convergence rate [2], [4]. Time domain equalizers generally use one of two approaches. The first approach is to use conventional methods to estimate the time domain impulse response of the channel transfer function and then to cancel the effect of channel by deconvolving the received time domain signal [4]. The second approach is to use a short (relative to the channel impulse response) finite impulse response (FIR) adaptive filter to shorten the overall impulse response of the channel and the filter to be less than T_g [4], [5]. In doing so, one exploits the presence of a guard interval in OFDM and eliminate both ISI and ISCI by using a shorter filter than otherwise would have needed. Both time and frequency domain sub-space methods of blind channel equalization have also been developed recently [6], [7], however, these subspace methods are computationally inefficient for online applications. The blind channel equalization method developed in [8] uses an eigenvalue decomposition method to estimate the channel and a time domain equalizer to equalize the channel and it is therefore computationally cumbersome.

In this paper we present an efficient and simple to implement adaptive blind equalization algorithm for an OFDM system which is capable of equalizing the long delays and short delays up to a scalar constant. In this algorithm, equalizer impulse response adaptation is only done during the portion of the signal corresponding to the guard interval. Therefore the additional computational complexity involved, other than

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the convolution operation in the equalizer, is small.

This paper is organized as follows: In section II, we develop the proposed equalizer for OFDM. In section III, through MATLAB simulations, we compare the performance of the new equalizer against a non-equalized system. Concluding comments are made in section IV.

II. Cyclic Prefix Equalizer for OFDM

Assume that the length of the equalizer is L and that the number of samples in the useful OFDM symbol and in the cyclic prefix (CP) are respectively N_u and N_g as shown in Fig. 1. We also assume that $L < N_u$. For the

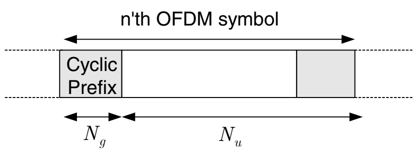

Fig. 1. OFDM Signal Diagram.

system with the equalizer, we assume that we have perfect knowledge of the symbol synchronization with respect to the reference multipath, normally the strongest. When determining the OFDM window for the FFT in the non-equalized receiver, we pick the OFDM symbol window such that the multipaths with delays less than the guard interval contains the most possible energy. Throughout this section, we use the indexing $u_n(i) = u((n-1)(N_u + N_g) + i)$ to denote the i'th sample value within the n'th OFDM symbol when $1 \leqslant i \leqslant N_u + N_g$.

If the i'th sample value of the n'th OFDM symbol at the output of the equalizer is denoted as $y_n(i) = y((n-1)(N_u + N_g) + i)$, then the equalization is achieved when $e_n(i) = y_n(i) - y_n(i - N_u)$ is zero for $N_u < i \leqslant N_u + N_g$, in a noise free environment. Therefore we define the cost function as,

$$J(i) = E\left\{|e_n(i)|^2\right\} \tag{1}$$

We use the instantaneous estimate of $\hat{J}(i) = |e_n(i)|^2$ to derive a steepest descent algorithm to estimate the weight vector $\mathbf{w}$ to minimize the cost function. Adaptation is only done when the delayed input $u(i - N_u)$ lies within a cyclic prefix. Fig. 2 shows the configuration of the algorithm. For $N_u < i \leqslant$

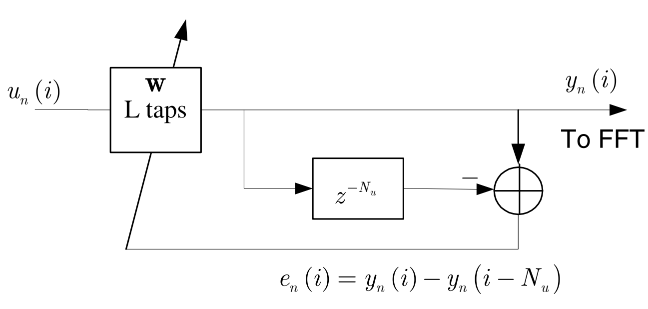

Fig. 2. Block Diagram of the Proposed Equalizer.

$N_u + N_g$,

$$\begin{aligned}\hat{J}(i) &= |e_n(i)|^2 = |y_n(i) - y_n(i - N_u)|^2 \\ &= \left|\mathbf{w}^H \mathbf{u}_n(i+k) - \mathbf{w}^H \mathbf{u}_n(i+k-N_u)\right|^2\end{aligned} \tag{2}$$

where $\mathbf{u}_n(i) = [u_n(i), \cdots, u_n(i - L + 1)]$ and k is a positive integer representing the length of the non-causal portion of the input to the equalizer.

The gradient of the cost function can be calculated as $\nabla \hat{J}_{\mathbf{w}} = 2\frac{\partial \hat{J}}{\partial \mathbf{w}^*}$ [9]. Therefore,

$$\nabla \hat{J}_{\mathbf{w}} = 2e_n^*(i)(\mathbf{u}_n(i+k) - \mathbf{u}_n(i+k-N_u)) \tag{3}$$

and the weight update equation can be written as,

$$\begin{aligned}\mathbf{w}((n-1)N_g + i) &= \mathbf{w}((n-1)N_g + i - 1) \\ &\quad - 2\mu e_n^*(i)(\mathbf{u}_n(i+k) - \mathbf{u}_n(i+k-N_u))\end{aligned} \tag{4}$$

where μ is the adaptation gain, which has to be chosen to ensure the convergence of the algorithm. An upper bound for μ is given by $\mu < \frac{1}{2L\sigma_u^2}$ where σ_u^2 is the energy of the input signal, u, to the equalizer.

If the above algorithm is allowed to converge in an unconstraint environment, the weight vector gradually goes to zero and reduces the output energy at the equalizer causing underflow in a practical implementation. In order to avoid this problem, we modify the above algorithm by constraining one of the weight coefficients to 1. For example, constraining $w(k)$ to be 1 is equivalent to the linearly constrained least mean square (LMS) algorithm given in [10] with the linear constraint being $\mathbf{c}^H \mathbf{w} = a$ where $\mathbf{c}$ is a constant unit vector with a single 1 at position k and $a = 1$. Simplifying the algorithm formulation given in [10] for this particular value of a and $\mathbf{c}$ gives the modified weight update equation,

$$\begin{aligned}\mathbf{w}'((n-1)N_g + i) &= \mathbf{w}'((n-1)N_g + i - 1) \\ &\quad - 2\mu e_n(i)(\mathbf{u}_n'(i+k) - \mathbf{u}_n'(i+k-N_u))\end{aligned} \tag{5}$$

where $\mathbf{w}'$ and $\mathbf{u}_n'$ are equal to $\mathbf{w}$ and $\mathbf{u}_n$ with the k'th element omitted.

We start the algorithm by initializing the $\mathbf{w}$ to be a unit vector with $w(k) = 1$. By initializing $\mathbf{w}$ as above and forcing the k'th element to be always 1, we are constraining the equalizer filter to be a non-causal filter where the filter output depends on k non-causal inputs and $L - k$ causal inputs, provided we have achieved perfect OFDM symbol synchronization with respect to the reference multipath of the signal. This in effect allows us to initialize the algorithm in the region of attraction (location of the channel cursor - the peak magnitude of the channel impulse response) associated with the MMSE optimal system delay [11]. Having a non-causal equalizer also enables us to use this algorithm in both minimum phase and non-minimum phase channel conditions. Table I summarises the CP equalizer algorithm.

TABLE I

SUMMARY OF CYCLIC PREFIX (CP) EQUALIZER

Initialization: Initialize $\mathbf{w}$ to a unit vector with 1 at position k

For each new OFDM symbol arriving at the equalizer

For $i = N_u + 1$ to $N_u + N_g$

Calculate the Output:

$$y_n(i) = \mathbf{w}^H(n)\,\mathbf{u}_n(i+k)$$
$$y_n(i - N_u) = \mathbf{w}^H(n)\,\mathbf{u}_n(i+k-N_u)$$

Eestimate error:

$$e_n(i) = y_n(i) - y_n(i - N_u)$$

Update the weight vector:

$$\mathbf{w}'(n+1) = \mathbf{w}'(n) - 2\mu e_n(i)^*\left(\mathbf{u}_n(i+k) - \mathbf{u}_n(i+k-N_u)\right)$$

End

End

III. RESULTS AND DISCUSSION

MATLAB simulations were carried out to compare the raw BER performance of the equalized and non equalized DAB receiver operating in transmission mode III [12]. The channel was modelled as a time invariant multipath channel with several short delays and a single long delay. We examine the BER performance for two different channel impulse responses. The first, shown in Fig. 3, has a long delay that is 1.5 times the length of the guard interval and the second, shown in Fig. 4, has a long delay that is 3 times the length of the guard interval. In channel impulse response graphs, x-axis shows the channel delay normalized with respect to the guard interval and y-axis shows the amplitude of the multipaths. Fig. 5 and 6 show

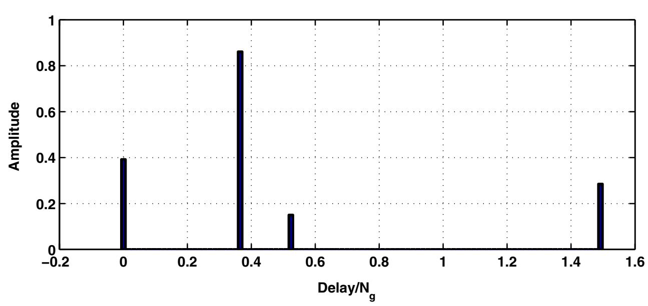

Fig. 3. Amplitude of the Channel Impulse Response with long delay of 1.5 N_g.

the raw BER performance before the Viterbi decoding of the receiver both with and without the channel equalization for the first and second simulations respectively. The performance of the minimum mean square error (MMSE) equalizer [13], in which the perfect knowledge of the channel impulse response and the SNR is assumed, is also shown in the graphs as a BER performance bound.

A comparison of the non-equalized BER curves in the two graphs indicate a reduced performance for the longer long

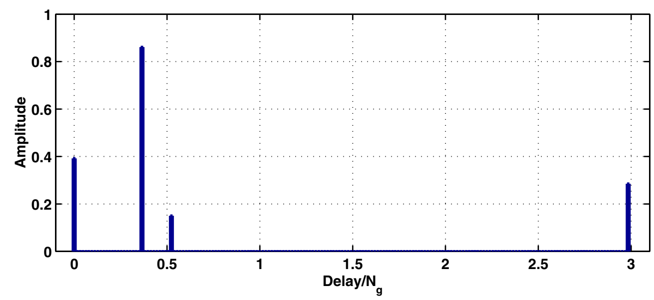

Fig. 4. Amplitude of the Channel Impulse Response with long delay of 3 N_g.

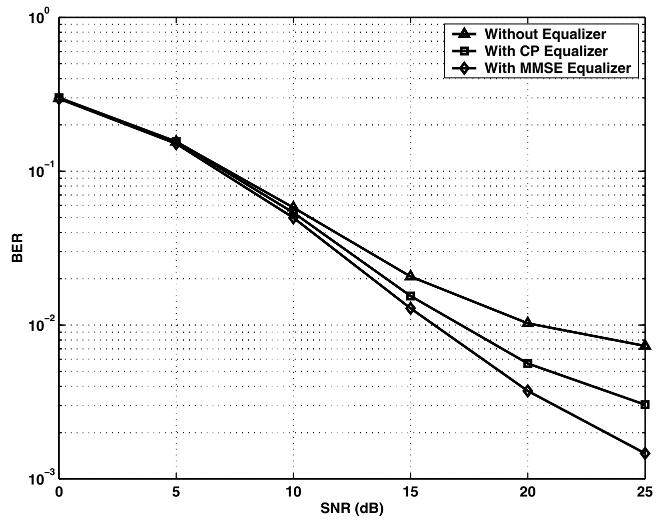

Fig. 5. Raw BER for the Equalized and Non-equalized OFDM systems with long delay of 1.5 N_g.

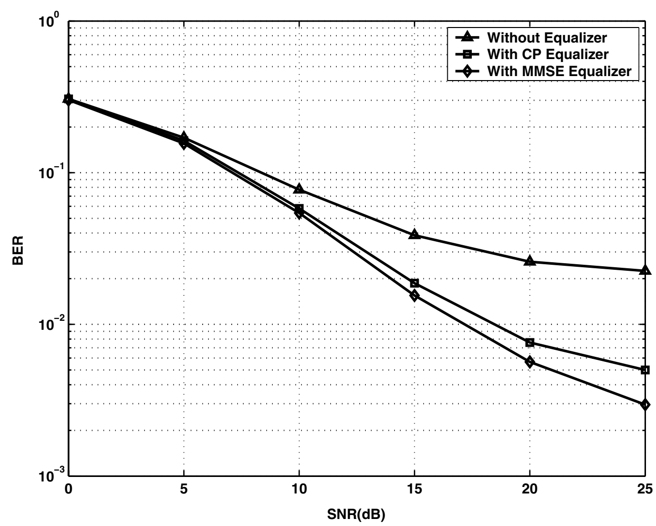

Fig. 6. Raw BER for the Equalized and Non-equalized OFDM systems with long delay of 3 N_g.

delay simulation. In both simulations, equalized system gives equal or better performance in the given signal to noise ratio (SNR) range. Performance of our blind adaptive equalizer is

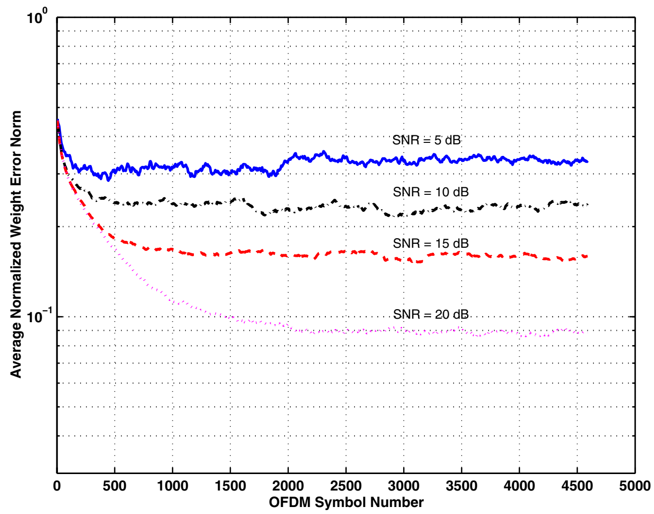

Fig. 7. Average weight error vector norm plotted against the OFDM symbol number.

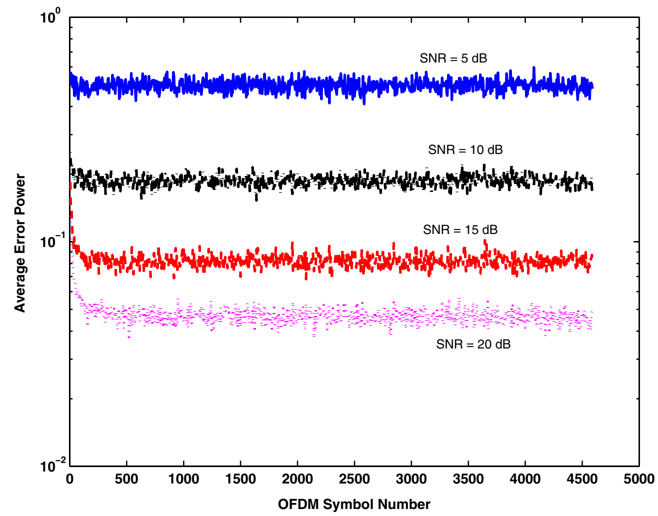

Fig. 8. Average error power plotted against the OFDM symbol number.

also favourable against that of the MMSE equalizer. However, the MMSE assumes perfect knowledge of the channel impulse response and the SNR and is therefore expected to outperform any blind equalization algorithm. It is also clear from the two BER graphs that the performance gain obtainable is larger for the longer long delay. Minimum SNR where the equalizer is capable of giving improved performance is also lower for the larger long delay. For the first simulation, it was about 7 dB whereas for the second simulation it was about 5 dB. Our results also show that the equalized system gives an increased performance gain with the increase in SNR.

In DAB, the threshold of audibility for errors usually occurs with a post Viterbi BER of around 5×10^{-5} with a service target edge of 10^{-4} [14]. For the Gaussian channel environment, a post Viterbi BER of 5×10^{-5} using equal error protection (EEP) level 3 [12] corresponds to a SNR of 8 dB [14]. For the same protection levels, multipath channel with coloured noise also requires similar SNR value to obtain the required post Viterbi BER performance [15]. Furthermore, using lower error protection levels, such as EEP level 4 in DAB, means that this SNR threshold can be even higher than 8 dB. The raw BER performance gain obtained using the proposed equalization algorithm could enable us to lower this SNR threshold of performance.

We used the normalized weight error vector norm of the equalizer impulse response and the error power plots to examine the convergence characteristics of our algorithm. The normalised weight error vector norm, $\eta(n)$, is defined as,

$$\eta(n) = \frac{\|\mathbf{w}(n) - \mathbf{w}_0\|}{\|\mathbf{w}_0\|} \qquad (6)$$

where $\mathbf{w}_0$ is the converged value of the weight vector in a noise free environment.

Fig. 7 and Fig. 8 shows the average weight error vector norm and error power plots for the second experiment. In all cases, convergence was achieved within several DAB Frames, each of which is 153 OFDM symbols long in DAB Mode III. As with any LMS algorithm, we can trade off the convergence rate with the weight error by changing the value of the adaptation gain, μ. However, using a larger μ value to increase the convergence rate results in reduced BER performance due to higher weight error. Both the weight error and the energy left at the cyclic prefix after the equalizer reduces with the increase in SNR. Error energy accounts for both the noise energy and the energy of the residue multipath delays after the equalizer.

IV. Conclusion

We have presented a low complexity blind channel equalization algorithm for an OFDM system which can be used in systems such as the DAB to combat the long delays. The advantages of this algorithm includes not requiring the use of pilot data for the channel estimation and being able to switch off for the majority of OFDM symbol duration and therefore saving computations. Simulation results shows better performance in the equalized system compared to the non-equalized system. In the next step of the research, we plan to adopt this algorithm for the equalization of fading multipath channels in OFDM systems.

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