A FAMILY OF SELECTIVE-TAP ALGORITHMS FOR STEREO ACOUSTIC ECHO CANCELLATION

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ABSTRACT

The use of adaptive filters employing tap-selection for stereophonic acoustic echo cancellation is investigated. We propose to employ subsampling of the tap-input vector, that is intrinsic to partial update schemes, to improve the conditioning of the tap-input autocorrelation matrix hence improving convergence. We investigate the effect of MMax tap-selection on the convergence rate for the single channel case by proposing a new measure which is then used as an optimization parameter in the development of our tap-selection scheme in the two channel case. The resultant exclusive maximum tap-selection is then applied to two channel NLMS, AP and RLS algorithms. Although our main motivation is *not* the reduction of complexity of SAEC, the proposed tap-selection nevertheless brings significant computation savings in additional to an improved rate of convergence over algorithms using only a non-linear preprocessor.

1. INTRODUCTION

Stereophonic teleconferencing systems as shown in Fig. 1, are becoming increasingly popular [1][2]. For such systems, stereophonic acoustic echo cancellers (SAECs) are required to suppress the echo returned to the transmission room to allow undisturbed communication between the rooms.

In SAEC, the solutions for the adaptive filters $\widetilde{\mathbf{h}}_1(n)$ and $\widetilde{\mathbf{h}}_2(n)$ can be non-unique [2]. Defining L and W as the lengths of the adaptive filters and transmission room's impulse response respectively and $\mathbf{R_{xx}}(n) = \mathbf{x}(n)\mathbf{x}^T(n)$ where $\mathbf{x}(n) = [\mathbf{x}_1^T(n) \ \mathbf{x}_2^T(n)]^T$ as in [1], two cases have been described for a noiseless system:

case 1 : $\quad L \geq W \Rightarrow \mathbf{R_{xx}}(n)$ is singular $\forall \ n$
case 2 : $\quad L < W \Rightarrow \mathbf{R_{xx}}(n)$ is ill $-$ conditioned.

For case 1, it has been shown [2] that there are non-unique solutions which depend on the impulse responses of the transmission and receiving rooms. In the practical case 2, the problem of non-uniqueness is ameliorated to some degree by the 'tail' effect [2]. However, direct application of standard adaptive filtering is not normally successful due to the high interchannel coherence between $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ [2] which leads to slow convergence. This is known as the misalignment problem. Several approaches including [2], which uses a non-linear preprocessor (NL), and [3] solve these problems with the common aim of achieving interchannel decorrelation without affecting speech quality and stereophonic perception.

In recent years, selective-tap schemes such as [4][5][6] were introduced to reduce computational complexity of, in particular,
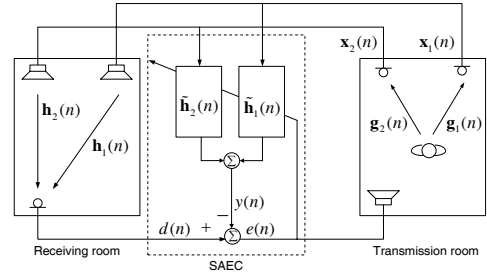
Fig. 1. Schematic diagram of stereophonic acoustic echo cancellation (after [2]). Only one channel of the return path is shown for reason of simplicity.

the normalized least mean squares (NLMS) algorithm by updating only a subset of taps at each iteration. These techniques allow implementation of single-channel echo cancellation with performance close to that of the full update NLMS algorithm. In this paper, our main motivation is *not* to reduce the complexity of SAEC algorithms. Instead, we propose to employ tap-selection as a means of improving the conditioning of $\mathbf{R_{xx}}(n)$, hence addressing problem case 2.

In Section 2, we review the single channel MMax-NLMS algorithm and propose a new measure, $\mathcal{M}$ to examine the effect of tap-selection on convergence rate in the single channel case. For the stereo case, Section 3 presents the exclusive maximum (XM) tap-selection technique which jointly maximizes $\mathcal{M}$ and minimizes the interchannel coherence. The proposed XM tap-selection was applied with the NL preprocessor to NLMS in [7]. We now further extend the XM tap-selection to the affine projection (AP) and recursive least squares (RLS) algorithms. Additionally, we formulate an explanation of the improvements obtained in terms of the conditioning of $\mathbf{R_{xx}}$ due to XM tap-selection. Section 4 presents the resultant XMNL-NLMS, XMNL-AP and XMNL-RLS algorithms and discusses the computational complexity. Section 5 presents comparative simulation results while Section 6 concludes this work.

2. SINGLE CHANNEL MMAX-NLMS

In the MMax-NLMS algorithm [5], for an adaptive filter of length L, only taps corresponding to the M largest magnitude tap-inputs are updated at each iteration such that

$$\widetilde{\mathbf{h}}(n+1) = \widetilde{\mathbf{h}}(n) + \mathbf{Q}(n)\frac{\mu\mathbf{x}(n)e(n)}{\|\mathbf{x}(n)\|^2} \qquad (1)$$

where $\mathbf{Q}(n) = diag\{\mathbf{q}(n)\}$ is the tap-selection matrix with elements given by

$$q_i(n) = \begin{cases} 1 & |x_i(n)| \in \{M \text{ maxima of } |\mathbf{x}(n)|\} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

for $i = 1, 2, \ldots, L$, and the adaptive step-size is μ. The error signal is given by $e(n) = d(n) - \widetilde{\mathbf{h}}^T(n)\mathbf{x}(n)$.

The penalty incurred due to tap-selection in MMax-NLMS is a degradation in convergence rate for a given step-size μ. We proposed a new measure $\mathcal{M}(n)$ as the ratio of the energy of the M selected tap-inputs to the energy of the full tap-input vector [8],

$$\mathcal{M}(n) = \frac{\|\mathbf{Q}(n)\mathbf{x}(n)\|^2}{\|\mathbf{x}(n)\|^2}. \quad (3)$$

This measure quantifies the 'closeness' of the MMax tap-selection to the full tap-input vector such that $\mathcal{M}(n) = 1$ corresponds to full update adaptation. Figure 2(a) shows how $\mathcal{M}$ varies with the size of tap-selection M for zero mean, unit variance white Gaussian noise (WGN) at a particular time iteration n. We note that $\mathcal{M}$ exhibits only a modest reduction for $0.5L \leq M < L$. Defining misalignment $\zeta(n)$ as

$$\zeta(n) = \frac{\|\mathbf{h} - \widetilde{\mathbf{h}}(n)\|^2}{\|\mathbf{h}\|^2}, \quad (4)$$

Fig. 2(b) shows the number of iterations for MMax-NLMS to achieve -20 dB misalignment for various $\mathcal{M}$. This verifies our expectation that, over the range $0.5L \leq M < L$, only a graceful reduction in convergence rate is exhibited as compared to full update adaptation [8]. Since convergence rate can be seen to increase monotonically with $\mathcal{M}$, as shown by a reduction in T_{20}, we propose that any degradation in convergence due to subselection of taps can be minimized by selecting taps so as to maximize $\mathcal{M}$.

3. EXCLUSIVE MAXIMUM (XM) TAP-SELECTION

3.1. Formulation

Selective-tap adaptation is now applied to SAEC. We note that direct application of MMax tap-selection will not serve to decorrelate the two tap-input vectors because, since $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ are themselves highly correlated, nearly identical tap-indices will be selected in both filters. We therefore formulate the exclusive maximum (XM) tap-selection criterion which aims jointly to maximize $\mathcal{M}(n)$ and minimize interchannel coherence at each iteration. In this two channel case, $\mathcal{M}(n)$ is then defined similarly to (3) except now $\mathbf{x}(n) = [\mathbf{x}_1^T(n) \ \mathbf{x}_2^T(n)]^T$ and $\mathbf{Q}(n) = diag[\mathbf{q}_1(n) \ \mathbf{q}_2(n)]$. The XM tap-selection addresses the minimum coherence condition by constraining tap-selections to be exclusive such that the same coefficient index may not be selected in both channels.

Although an exhaustive search of all exclusive tap-selections could be used to find the selection set which maximizes $\mathcal{M}$ [7], a more efficient method can be found by considering

$$\mathbf{p}(n) = |\mathbf{x}_1(n)| - |\mathbf{x}_2(n)|. \quad (5)$$

The exclusive tap-selection with maximum $\mathcal{M}(n)$ can then be found efficiently by sorting $\mathbf{p}(n)$. Consider as a simple example an SAEC system with channels $k = 1, 2$, adaptive filters each of length $L = 4$ and tap-input vectors $\mathbf{x}_k(n) = [x_{k,1} \ x_{k,2} \ x_{k,3} \ x_{k,4}]^T$. Also consider the example case $p_3 > p_2 >$
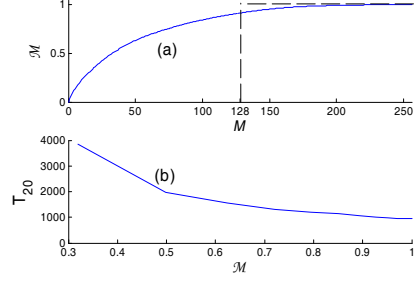
Fig. 2. (a) Variation of $\mathcal{M}$ with subselection parameter M, (b) Dependence of convergence rate on $\mathcal{M}$.

$p_1 > p_4$, for a particular time instance. Since $p_3 + p_2 > \ldots > p_1 + p_4$, it can be shown using (5) that $|x_{1,3}| + |x_{1,2}| + |x_{2,1}| + |x_{2,4}| > \ldots > |x_{1,1}| + |x_{1,4}| + |x_{2,2}| + |x_{2,3}|$ where $\ldots$ refers to all other pair-wise combinations of p_i, $i = 1, 2, 3, 4$. Thus the tap-selection corresponding to inputs $x_{1,3}, x_{1,2}, x_{2,1}$ and $x_{2,4}$ maximizes $\mathcal{M}(n)$ with the minimum coherence constraint satisfied by the exclusivity. Consequently, the XM tap-selection matrix is $\mathbf{Q}(n)$ such that at each iteration n, element u of $\mathbf{q}_1(n)$ and element v of $\mathbf{q}_2(n)$ are defined for $u, v = 1, 2, \ldots, L$ as

$$q_{1,u} = \begin{cases} 1 & p_u \in \{M \text{ maxima of } \mathbf{p}\} \\ 0 & \text{otherwise} \end{cases}$$

$$q_{2,v} = \begin{cases} 1 & p_v \in \{M \text{ minima of } \mathbf{p}\} \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

3.2. Effect of XM tap-selection on the autocorrelation matrix

The exclusive tap-selection can be seen as a method for improving the conditioning of the input autocorrelation matrix [8]. Defining $E[\ ]$ as the mathematical expectation operator, the two channel autocorrelation matrix for the stereo case can be expressed as

$$\begin{aligned} \mathbf{R}_{\mathbf{xx}}(n) &= E[\mathbf{x}(n)\mathbf{x}^T(n)] \\ &= \begin{bmatrix} \mathbf{R}_{11}(n) & \mathbf{R}_{12}(n) \\ \mathbf{R}_{21}(n) & \mathbf{R}_{22}(n) \end{bmatrix}. \end{aligned} \quad (7)$$

After exclusive tap-selection, the resulting sparse vectors $\widetilde{\mathbf{x}}_1(n) = \mathbf{Q}_1(n)\mathbf{x}_1(n)$ and $\widetilde{\mathbf{x}}_2(n) = \mathbf{Q}_2(n)\mathbf{x}_2(n)$ give rise to $\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}(n) = E[\widetilde{\mathbf{x}}(n)\widetilde{\mathbf{x}}^T(n)]$ in which the diagonals and some off-diagonal elements of $\mathbf{R}_{12}(n)$ and $\mathbf{R}_{21}(n)$ are zero. This improves the conditioning of $\mathbf{R}_{\mathbf{xx}}(n)$ and in the limit where $\widetilde{\mathbf{x}}_1(n)$ and $\widetilde{\mathbf{x}}_2(n)$ are perfectly uncorrelated, the autocorrelation matrix is diagonal, $\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}(n) = diag[\sigma_1^2 \ldots \sigma_1^2 \ \sigma_2^2 \ldots \sigma_2^2]$ with a condition number of $\|\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}\|\|\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}^{-1}\| = \frac{\max(\sigma_1^2, \sigma_2^2)}{\min(\sigma_1^2, \sigma_2^2)}$ where σ_k^2 is the kth channel subselected tap-input variance.

To illustrate the improvement in conditioning of $\mathbf{R}_{\mathbf{xx}}$, impulse response $\mathbf{g}_1$ was first generated using method of images [9] with $\mathbf{g}_2 = \gamma\mathbf{g}_1 + (1 - \gamma)\mathbf{b}$ where $0 \leq \gamma \leq 1$ and $\mathbf{b}$ is a zero mean independent WGN sequence. The autocorrelation matrix $\mathbf{R}_{\mathbf{xx}}$ was formed from $\mathbf{x}_1$ and $\mathbf{x}_2$ generated by convolving a WGN sequence with $\mathbf{g}_1$ and $\mathbf{g}_2$ while $\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}$ was formed from $\widetilde{\mathbf{x}}_1$ and $\widetilde{\mathbf{x}}_2$. Figure 3 shows the variation of mean condition number of time-averaged autocorrelation matrices $\mathbf{R}_{\mathbf{xx}}$ and $\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}$ as a function of γ. For each case of γ, the average condition number for 50 trials is plotted in Fig. 3(a) and (b) for $\mathbf{R}_{\mathbf{xx}}$ and $\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}$ respectively. For small γ, $\mathbf{x}_1$ and $\mathbf{x}_2$ are less correlated and the condition number of $\mathbf{R}_{\mathbf{xx}}$

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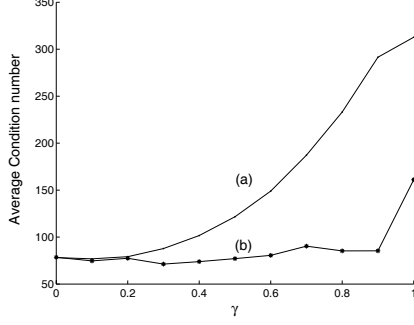
Fig. 3. Effect of exclusive tap-selection on mean condition number for WGN sequence (a) without tap-selection (b) with exclusive tap-selection.

is seen to be correspondingly small. For each case of γ, $\mathbf{R}_{\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}}$ has a lower mean condition number than $\mathbf{R}_{\mathbf{xx}}$ and hence improved convergence performance for case 2 is obtained using exclusive tap-selection.

4. EXCLUSIVE MAXIMUM (XM) ALGORITHMS

The XM selective-tap criterion is now applied to the NLMS, AP and RLS adaptive algorithms. As has been shown, the XM selective-tap criterion intrinsically improves the conditioning of $\mathbf{R}_{\mathbf{xx}}$ but relies on the existence of a unique solution achieved using, for example, the NL preprocessor [2] as described below.

4.1. XMNL-NLMS Algorithm

The non-linear (NL) preprocessor [2] is one of the most effective methods of achieving signal decorrelation without affecting stereo perception by using $0 < \alpha \leq 0.5$ as the non-linearity constant such that

$$\mathbf{x}_1'(n) = \mathbf{x}_1(n) + 0.5\alpha[\mathbf{x}_1(n) + |\mathbf{x}_1(n)|] \tag{8}$$
$$\mathbf{x}_2'(n) = \mathbf{x}_2(n) + 0.5\alpha[\mathbf{x}_2(n) - |\mathbf{x}_2(n)|]. \tag{9}$$

Several algorithms in combination with the NL preprocessor have been proposed [1][10][11] to enhance misalignment performance. A combined algorithm, XMNL-NLMS, employing the XM tap-selection to improve the conditioning of the autocorrelation matrix in combination with the NL preprocessor has been proposed in [7]. The XMNL-NLMS algorithm is given in (1), (6), (8) and (9).

4.2. XMNL-AP Algorithm

The affine projection (AP) algorithm [12] incorporates multiple projections by concatenating past input vectors from time iteration n to time iteration $n - K + 1$ where K is defined as the projection order. We first define $\widetilde{\mathbf{x}}'(n) = \mathbf{Q}(n)\mathbf{x}'(n)$ where $\mathbf{x}'(n) = [\mathbf{x}_1'^T(n)\ \ \mathbf{x}_2'^T(n)]^T$, the subselected and full tap-input matrices are then denoted respectively as

$$\widetilde{\mathbf{X}}'(n) = [\widetilde{\mathbf{x}}'(n)\ \widetilde{\mathbf{x}}'(n-1)\ \ldots\ \widetilde{\mathbf{x}}'(n-K+1)]^T \tag{10}$$
$$\mathbf{X}'(n) = [\mathbf{x}'(n)\ \mathbf{x}'(n-1)\ \ldots\ \mathbf{x}'(n-K+1)]^T. \tag{11}$$

The tap-update for the XMNL-AP algorithm is given as

$$\widetilde{\mathbf{h}}(n+1) = \widetilde{\mathbf{h}}(n) + \mu\widetilde{\mathbf{X}}'^T(n)[\mathbf{X}'(n)\mathbf{X}'^T(n)]^{-1}\mathbf{e}(n) \tag{12}$$

where $\mathbf{e}(n) = [e(n)\ e(n-1)\ \ldots\ e(n-K+1)]^T$. Thus for $K = 1$, XMNL-AP is equivalent to XMNL-NLMS.

4.3. XMNL-RLS Algorithm

The tap-update equation of the RLS algorithm is given as $\widetilde{\mathbf{h}}(n) = \widetilde{\mathbf{h}}(n-1) + \mathbf{k}(n)e(n)$ where $\mathbf{k}(n)$ is the Kalman gain. Direct extension of the XM tap-selection approach achieved by sorting the magnitude difference of the $\mathbf{k}(n)$ will not achieve the desired convergence because $\mathbf{k}(n)$ depends on previous values of time-averaged correlation matrix $\Psi(n) = \sum_{i=1}^{n} \lambda^{n-i}\mathbf{x}(i)\mathbf{x}^T(i)$ where $0 < \lambda < 1$ is the forgetting factor. Our approach will be to improve the condition of $\Psi(n)$ using $\widetilde{\mathbf{x}}'(n) = \mathbf{Q}(n)\mathbf{x}'(n)$ which ensures that the subsampled input vectors propagate consistently through the memory of the algorithm. Following the approach in [8], the XMNL-RLS tap-update equation is given by

$$\widetilde{\mathbf{h}}(n+1) = \widetilde{\mathbf{h}}(n) + \widetilde{\mathbf{k}}(n)e(n) \tag{13}$$

where $\widetilde{\mathbf{k}}(n) = [\widetilde{\mathbf{k}}_1^T(n)\ \widetilde{\mathbf{k}}_2^T(n)]^T$ is the modified Kalman gain such that

$$\widetilde{\mathbf{k}}(n) = \frac{\lambda^{-1}\widetilde{\Psi}'^{-1}(n)\widetilde{\mathbf{x}}'(n)}{1 + \lambda^{-1}\widetilde{\mathbf{x}}'^T(n)\widetilde{\Psi}'^{-1}(n)\widetilde{\mathbf{x}}'(n)} \tag{14}$$

and using the matrix inversion lemma [12], we have

$$\widetilde{\Psi}'^{-1}(n+1) = \frac{1}{\lambda}[\widetilde{\Psi}'^{-1}(n) - \widetilde{\mathbf{k}}(n)\widetilde{\mathbf{x}}'^T(n)\widetilde{\Psi}'^{-1}(n)]. \tag{15}$$

4.4. Computational Complexity

As in MMax-NLMS, the XM tap-selection employs the SORT-LINE algorithm [13] which requires at most $2\log_2 L + 2$ comparisons. Thus the XMNL-NLMS requires at most $1.5L + 2\log_2 L + 3$ operations (multiplications or comparison) for each filter per sample period with $M = 0.5L$ compared to $2L$ for NL-NLMS. The XMNL-AP algorithm requires at most $1.5LK + 7K^2 + 2 + 2\log_2 L$ compared to $2LK + 7K^2$ for AP algorithm. The XMNL-RLS algorithm requires at most $2.5L(L+1) + 3 + 2\log_2 L$ per adaptive filter compared to $4L^2 + 3L + 2$ multiplications for RLS. Although complexity reduction is not the main aim of this work, the XM selective-tap updating nevertheless brings significant computation savings.

5. SIMULATION RESULTS

In these simulations, all room impulse responses were generated using the method of images [9] with the microphones placed one meter apart and the source positioned one meter away from each of the microphones in the transmission room. For generality, all simulations were performed using different speech signals. Both the transmission and receiving room's responses were of length $W = N = 800$ and adaptive filters were of length $L = 256$ and $M = 128$. Figure 4 compares the convergence of NL-NLMS, XMNL-NLMS and NL-RLS [2]. Forgetting factor $\lambda = 1 - \frac{1}{10L}$ and step-size $\mu = 0.7$ were used for the NL-RLS and XMNL-NLMS algorithms respectively. It can be seen that the performance of XMNL-NLMS exceeds that of NL-NLMS by around 5 to 10 dB and is close to that of NL-RLS. The XMNL-NLMS algorithm however has lower complexity than the NL-RLS algorithm.

Figure 5 shows the misalignment plot for AP-based algorithms where the projection order was $K = 2$ with $\mu = 0.7$. It can be seen that the rate of convergence of XMNL-NLMS is close to that of the NL-AP. Additionally, XMNL-AP achieves an improvement of approximately 6 to 8 dB misalignment compared to that of the NL-AP for this speech signal.

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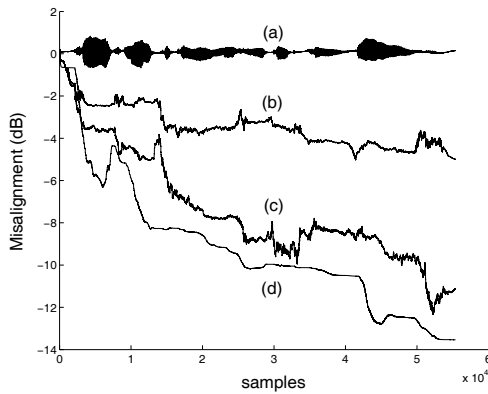
Fig. 4. (a) Speech and Misalignment plot for (b) NL-NLMS, (c) XMNL-NLMS and (d) NL-RLS.
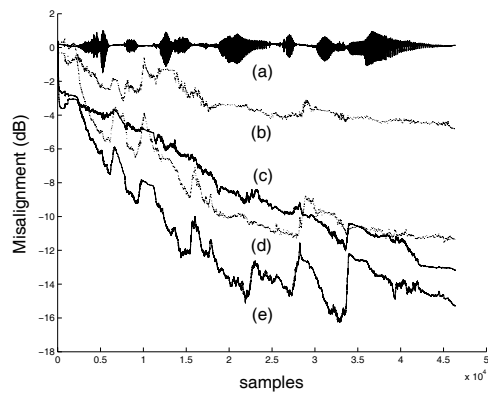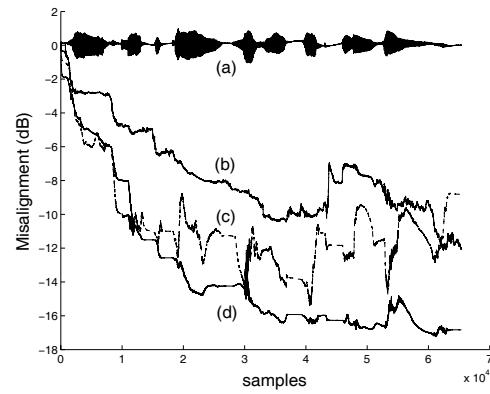

Fig. 5. (a) Speech and Misalignment plot for (b) NL-NLMS, (c) NL-AP, (d) XMNL-NLMS and (e) XMNL-AP.

Figure 6 compares the rate of convergence between the XMNL-RLS algorithm and the NL-RLS algorithm using the same experimental setup as the previous experiment but using a different speech signal. We see that there is a significant improvement in misalignment of 3 to 9 dB for the XMNL-RLS compared to that of the NL-RLS.

6. CONCLUSION

We have formulated the XM tap-selection technique and employed it in the proposed XMNL-NLMS, XMNL-AP and the XMNL-RLS algorithms. These algorithms achieve the required decorrelation of the tap-input vectors, hence improving the conditioning of $\mathbf{R_{xx}}$, in SAEC using this novel selective-tap scheme and give a significant improvement in performance over and above the use of the NL preprocessor alone. Although direct application of NLMS is not normally satisfactory for SAEC because of its poor convergence, relatively good performance close to that of RLS-based schemes can be obtained nevertheless through the use of the proposed XM tap-selection approach. XMNL-NLMS has the benefits of low complexity and robustness compared to least squares approaches. Additionally, a significant increase in convergence rate can be seen for XMNL-AP and XMNL-RLS as compared to that obtained from NL-AP and NL-RLS respectively.

Fig. 6. (a) Speech and Misalignment plot for (b) XMNL-NLMS, (c) NL-RLS and (d) XMNL-RLS.

7. REFERENCES

[1] P. Eneroth, S. L. Gay, T. Gansler, and J. Benesty, "A real-time implementation of a stereophonic acoustic echo canceller," *IEEE Trans. Speech Audio Processing*, vol. 9, no. 5, pp. 513–523, Jul. 2001.

[2] J. Benesty, D. R. Morgan, and M. M. Sondhi, "A better understanding and an improved solution to the specific problems of stereophonic acoustic echo cancellation," *IEEE Trans. Speech Audio Processing*, vol. 6, no. 2, pp. 156–165, Mar. 1998.

[3] T. Hoya, Y. Loke, J. Chambers, and P. A. Naylor, "Application of the leaky extended LMS algorithm in stereophonic acoustic echo cancellation," *Signal Processing*, vol. 64, pp. 87–91, 1998.

[4] S. C. Douglas, "Adaptive filters employing partial updates," *IEEE Trans. Circuits Syst. II*, vol. 44, no. 3, pp. 209–216, Mar. 1997.

[5] T. Aboulnasr and K. Mayyas, "Complexity reduction of the NLMS algorithm via selective coefficient update," *IEEE Trans. Signal Processing*, vol. 47, no. 5, pp. 1421–1424, 1999.

[6] P. A. Naylor and W. Sherliker, "A short-sort M-max NLMS partial update adaptive filter with applications to echo cancellation," in *Proc. IEEE Int. Conf. Acoustics Speech Signal Processing*, vol. 5, 2003, pp. 373–376.

[7] A. W. H. Khong and P. A. Naylor, "Reducing inter-channel coherence in stereophonic acoustic echo cancellation using partial update adaptive filters," in *Proc. Eur. Signal Process. Conf.*, 2004, pp. 405–408.

[8] ——, "Selective-tap adaptive algorithms in the solution of the non-uniqueness problem for stereophonic acoustic echo cancellation," *accepted for publication in IEEE Signal Processing Letters*, 2004.

[9] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Amer.*, vol. 65, no. 4, pp. 943–950, Apr. 1979.

[10] T. Gansler and J. Benesty, "An adaptive nonlinearity solution to the uniqueness problem of stereophonic echo cancellation," in *Proc. IEEE Int. Conf. Acoustics Speech Signal Processing*, vol. 2, 2002, pp. 1885–1888.

[11] K. Mayyas, "Stereophonic acoustic echo cancellation using lattice orthogonalization," *IEEE Trans. Speech Audio Processing*, vol. 10, no. 7, pp. 517–525, Oct. 2002.

[12] S. Haykin, *Adaptive Filter Theory*, 4th ed., ser. Information and System Science.   Prentice Hall, 2002.

[13] I. Pitas, "Fast algorithms for running ordering and max/min calculation," *IEEE Trans. on Circuits and Systems*, vol. 36, no. 6, pp. 795–804, Jun. 1989.