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Research Letter

An Algorithm to Generate Representations of System Identification Errors

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An algorithm to generate representations of system identification (SI) errors, which enables systematic testing of the performance of system equalization techniques, is proposed. With this algorithm, the normalized projection misalignment (NPM) of the generated error representation can be chosen to suit the particular characteristics of the application under test. Additionally, the generated error representation can represent all the error vectors corresponding to different scaling factors in the estimates of the system impulse response (SIR), without influencing the signal-to-distortion ratio (SDR) of the equalized impulse response.

1. INTRODUCTION

It is commonly known that the outcome of system identification (SI) usually includes some estimation errors which can cause problems in the equalization of the system. Suppose we have an estimate $\hat{\mathbf{h}} = [\hat{h}_0 \ \hat{h}_1 \ \cdots \ \hat{h}_{L-1}]^T$ of some system impulse response (SIR) $\mathbf{h} = [h_0 \ h_1 \ \cdots \ h_{L-1}]^T$. An equalization system $\mathbf{g}$ can be designed based on $\hat{\mathbf{h}}$ to equalize the true SIR $\mathbf{h}$. Since $\hat{\mathbf{h}}$ includes errors, we cannot obtain in general an exact equalization system. The quality of $\mathbf{g}$ depends, to a large extent, on the robustness of the equalization system design method to the estimation errors induced by the SI. Therefore, to evaluate equalization systems and their design methods efficiently, we need to generate error vectors, which can be added to known $\mathbf{h}$, to represent the estimate $\hat{\mathbf{h}}$. Particularly, we need to generate error vectors of some desired levels, so that the performance of equalization techniques can be tested with errors of different magnitude. A well-recognized measure of the SI error level is the normalized projection misalignment (NPM) [1]:

$$\text{NPM} = \min_{\beta} \frac{||\mathbf{h} - \beta\hat{\mathbf{h}}||^2}{||\mathbf{h}||^2}. \quad (1)$$

Another characteristic of SI is that the outcome of many SI algorithms is determined only up to a multiplicative scaling factor [2]. Estimates of different scaling factors correspond to different error vectors. A natural evaluation measure for system equalization quality is the signal-to-distortion ratio (SDR) [3]. It can be shown that for most of the equalization techniques, the SDR of the equalized impulse response is independent of the scaling factor. That is to say that, applying equalization systems designed based on estimates of different scaling factors to the true SIR $\mathbf{h}$ will give the same equalization quality in the sense of SDR.

In this letter, we will first show the independence of the SDR to scaling factors. Then, we will present an algorithm with which error representation of desired NPM can be generated, so that the performance of equalization techniques can be tested with SI errors at particular NPM. Since the SDR is independent of the scaling of $\hat{\mathbf{h}}$, we can use the error vector generated by the proposed algorithm to represent all error vectors corresponding to different scaling factors in $\hat{\mathbf{h}}$, without influencing the evaluation result of SDR.

2. PROBLEM FORMULATION

We formulate the problem in Figure 1. For illustration, we reduce this problem to 2 dimensions although extension to

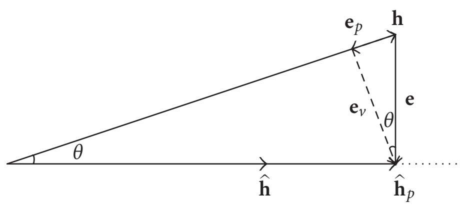


Figure 1: Illustration of the error representation.

higher dimensionality is straightforward. Consider the two vectors, the SIR $\mathbf{h}$, and its estimate

$$\hat{\mathbf{h}} = \alpha\vec{\hat{h}}, \tag{2}$$

where $\vec{\hat{h}}$ is the unit vector in the direction of $\hat{\mathbf{h}}$. The estimate $\hat{\mathbf{h}}$ is normally used as a basis for designing the equalization system. The projection of $\mathbf{h}$ onto $\hat{\mathbf{h}}$ is denoted as

$$\hat{\mathbf{h}}_p = \beta\hat{\mathbf{h}} = \beta\alpha\vec{\hat{h}}, \tag{3}$$

where $\beta = \hat{\mathbf{h}}^T\mathbf{h}/(\hat{\mathbf{h}}^T\hat{\mathbf{h}})$. Suppose that we have an equalization system

$$\mathbf{g} = \gamma\vec{g} \tag{4}$$

such that

$$\mathbf{g}*\hat{\mathbf{h}} = \gamma\vec{g}*\alpha\vec{\hat{h}} = \delta(n-\tau) + \Delta, \tag{5}$$

where $*$ denotes convolution, $\vec{g}$ is the unit vector in the direction of $\mathbf{g}$, τ is an arbitrary delay, and Δ is included to represent any inversion error in the case that $\mathbf{g}$ is not a perfect inverse of $\hat{\mathbf{h}}$. However, Δ vanishes when the inverse filter is perfect, such as in MINT [4]. Then, the equalization system $\mathbf{g}$ is used to equalize the SIR $\mathbf{h}$, giving the result

$$\mathbf{g}*\mathbf{h} = \mathbf{g}*(\hat{\mathbf{h}}_p - \mathbf{e}), \tag{6}$$

where $\mathbf{e}$ denotes the projection error vector representing the difference between $\mathbf{h}$ and $\hat{\mathbf{h}}_p$.

In fact, (6) can be rewritten as

$$\begin{aligned}\mathbf{g}*\mathbf{h} &= \beta\mathbf{g}*\hat{\mathbf{h}} - \beta\alpha\tan\theta\mathbf{g}*\vec{e}\\ &= \beta\delta(n-\tau) + \beta(\Delta - \alpha\tan\theta\mathbf{g}*\vec{e}),\end{aligned} \tag{7}$$

where $\vec{e}$ is the unit vector in the direction $\mathbf{e}$. The first term in (7) is the desired equalization result, and the second term gives rise to distortions in the equalized impulse response. The SDR, that is, the energy ratio between these two terms can be written as

$$\text{SDR} = \frac{||\beta\delta(n-\tau)||^2}{||\beta(\Delta - \alpha\tan\theta\mathbf{g}*\vec{e})||^2}. \tag{8}$$

Using (4), (5), (8) together gives

$$\text{SDR} = \frac{||\vec{\hat{h}}*\vec{g}||^2}{||\Delta(||\vec{\hat{h}}*\vec{g}||) - \tan\theta(||\delta(t-\tau)+\Delta||)\vec{g}*\vec{e}||^2}. \tag{9}$$

Above, we have shown the process of using an equalization system $\mathbf{g}$ designed based on $\hat{\mathbf{h}}$, of scaling factor α, to equalize $\mathbf{h}$. It can be seen in (9) that in the resultant SDR of the equalized impulse response $\mathbf{g}*\mathbf{h}$, the performance of the equalization system designed based on $\hat{\mathbf{h}}$ is independent of the scaling factor α in $\hat{\mathbf{h}}$, subject to the assumption that Δ does not depend on α, which is often the case. The equalization system designed based on $\hat{\mathbf{h}}_p$, which corresponds to the same NPM as $\hat{\mathbf{h}}$ of any α, performs as well as that designed based on $\hat{\mathbf{h}}$. Therefore, we can represent the SI error vectors of same NPM, but corresponding to different α, by the projection error vector $\mathbf{e}$ in Figure 1. In Section 3, we will propose an algorithm to generate $\mathbf{e}$ of desired NPM.

3. ERROR VECTOR GENERATION

The most commonly used scaling independent evaluation measure of SI errors is NPM [1] which corresponds, in terms of Figure 1, only to the angle θ between $\mathbf{h}$ and $\hat{\mathbf{h}}$. It can be written as

$$\text{NPM} = \left(\frac{||\mathbf{e}||}{||\mathbf{h}||}\right)^2 = \sin^2\theta. \tag{10}$$

Now we build the link between $\mathbf{h}$ and $\mathbf{e}$ so that, for a given $\mathbf{h}$, we can generate SI errors of specific NPM. The error vector $\mathbf{e}$ can be decomposed into two components, of which one is parallel to $\mathbf{h}$ and the other is normal to $\mathbf{h}$. The magnitude of the parallel component $\mathbf{e}_p$ is $\|\mathbf{e}_p\| = \sin^2\theta\|\mathbf{h}\|$. For the component $\mathbf{e}_v$ normal to $\mathbf{h}$, we can write

$$\mathbf{h}^T\mathbf{e}_v = 0, \tag{11}$$

$$||\mathbf{e}_v|| = \|\mathbf{h}\|\sin\theta\cos\theta, \tag{12}$$

and substituting (10) into (12) gives

$$||\mathbf{e}_v|| = \|\mathbf{h}\|\sqrt{\text{NPM}(1-\text{NPM})}. \tag{13}$$

It can be seen that the direction of $\mathbf{e}_v$ is constrained by (11) and its length is determined from (13).

The ensuing procedure is first to generate a random vector orthogonal to $\mathbf{h}$, and then adjust it to the desired length. The error vector can be generated following the steps below.

(1) Generate an i.i.d. random vector written as $\mathbf{y} = [y(0)\ \ y(1)\ \ \cdots\ \ y(L-1)]^T$, the elements of which are uniformly distributed on $(-0.5, 0.5)$, where L is the dimension of $\mathbf{h}$.

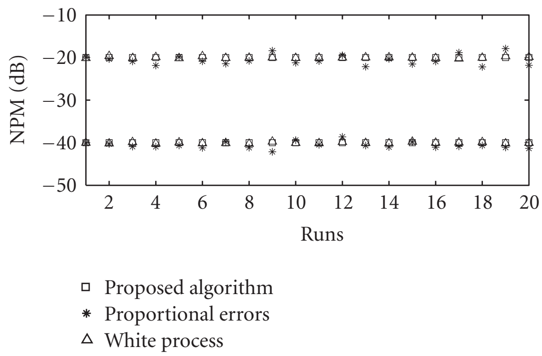
(2) Test if the vector $\mathbf{y}$ is parallel to $\mathbf{h}$. If yes, go to step (1); if not, go to step (3).

(3) Apply Gram-Schmidt orthogonalization [5] to $\mathbf{h}$ and the random vector to obtain a new vector $\mathbf{y}_\perp$ which is orthogonal to $\mathbf{h}$.

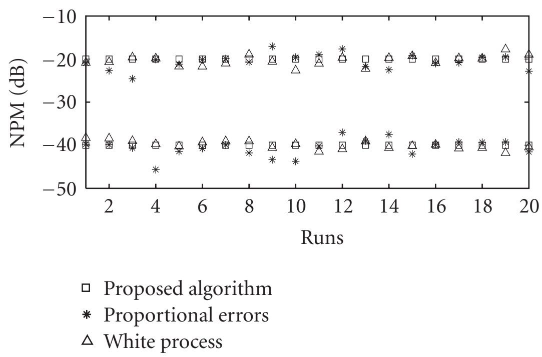
(4) Adjust the length of $\mathbf{y}_\perp$ according to (13) to obtain $\mathbf{e}_v$.

(5) Generate $\mathbf{e}_p = -\sin^2\theta\mathbf{h}$.

(6) Sum $\mathbf{e}_v$ and $\mathbf{e}_p$ to obtain $\mathbf{e}$.



(a) RIR, $L = 1600$



(b) random channel, $L = 32$

Figure 2: NPMs of generated errors with different algorithms.

Table 1: SDR of different scaling factors. (a) −20 dB, (b) −40 dB, (c) $L_i = 512$, (d) $L_i = 1024$.

MINT	(a)	α	−0.43	−1.67	0.13
		SDR	4.48	4.48	4.48
	(b)	α	1.19	1.18	−0.04
		SDR	23.62	23.62	23.62
LS	(c)	α	−0.19	0.73	−0.59
		SDR	2.38	2.38	2.38
	(d)	α	0.11	1.07	0.60
		SDR	2.74	2.74	2.74

The projection error vectors of same NPM can be in different directions in the L-dimension space. Since the vector is randomly generated in step (1), the generality of the direction of $\mathbf{e}$ is not lost.

4. EXPERIMENT RESULTS

The following results show some illustrative examples of the use of our algorithm in the context of SI and equalization of room impulse responses (RIRs). Firstly, we compare the algorithm proposed above with existing methods. Secondly, the SDR results are shown when using MINT and least-square (LS) [4] inverse systems to perform equalization of the RIRs with different scaling factors in $\widehat{\mathbf{h}}$.

Two existing error generation methods are (i) proportional errors [6], in which error samples amplitudes are proportional to $h(n)$,

$$\xi(n) = \epsilon(n) \times h(n), \tag{14}$$

with $\epsilon(n)$ being white Gaussian noise (WGN) of variance corresponding to the desired NPM, and (ii) additive errors such that WGN is added to $\mathbf{h}$. These error generation methods will be compared to our approach.

Figure 2(a) shows the NPM results of the generated errors for an RIR of $L = 1600$ generated with image method [7] for 20 experimental runs. The desired NPMs are −20 and −40 dB. The results show that with the proposed algorithm, the errors of the desired NPM can be exactly generated. The proportional errors are less accurate and less consistent over different runs. Although the white Gaussian process is more accurate and consistent than the proportional errors, it does not match the desired NPM exactly. In Figure 2(b), we used a random channel of $L = 32$, and it can be seen that for the short-impulse response, both proportional errors and white Gaussian process are not accurate and not consistent.

Table 1 shows the SDR results with the equalization systems designed with MINT and LS. The 2-channel RIRs used in the MINT experiments are from the image method with reverberation time (T_{60}) 0.2 s ($L = 1600$) and sampling frequency 8 kHz. The RIR used in the LS experiments is one of the above two channels and it is truncated to $L = 512$. The SDR results of (a) group in Table 1 are obtained with MINT and errors of −20 dB NPM, and the 3 values of SDR correspond to 3 random scaling factors α in $\widehat{\mathbf{h}}$. The (b) group is obtained with −40 dB NPM. It can be seen that the SDR results obtained using our approach are independent of scaling of $\widehat{\mathbf{h}}$, as desired. The NPM in the LS experiments is −20 dB. The (c) group is obtained with inverse filter of length $L_i = 512$, and the (d) group is obtained with $L_i = 1024$. For LS, the Δ in (5) cannot be zero since LS inverse filters are generally not exact, and different L_i give different Δ. However, the SDR results still show independence of scaling of $\widehat{\mathbf{h}}$.

5. CONCLUSIONS

We have presented an algorithm for the generation of SI errors. We have shown by experiments that the generated error representations can meet the desired NPM exactly. We have also shown that the generated error representation can represent all the error vectors corresponding to different scaling factors in the system estimates, without influencing the SDR of the equalized impulse response. Therefore, we conclude that the proposed error generation algorithm facilitates reliable and repeatable testing of system equalization methods.

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