EXPERIMENT: AM1

Experimental Handout:

Array Communications & Processing

Supervisor:
Dr. A. Manikas (Room:1102, Ext.: 46266)

Equipment:
Any Computer with Audio Card; MATLAB;
In-house Array Processing Toolbox for MATLAB.

Aims & Outline:
By distributing a number of receiving elements (from now on called sensors) in a 3-dimensional Cartesian space, an array is formed; the region over which the sensors are distributed is called the aperture of the array. The general array processing problem is the obtaining of information about a signal environment from the waveforms received at the array elements, where the signal environment consists of a number of emitting sources plus noise. These emitting sources, in the case of radar-based systems, are often targets which either reflect transmitted signals (as in active radars) or emit their own signals, (as in passive radars). Other typical examples of applications of arrays of sensors are:

- radio telescopy where the sensors are antennas and the emitting sources are radio sources;
- sonar where the sensors are hydrophones and the emitting sources are ships (ship noise);
- geophysics where the sensors are seismometers and the sources are earthquakes etc.;

However, a new 'hot' application of *arrays* is in the area of digital communications where *space-time* and not just *time* information associated with the communication signals can be utilized to provide more sophisticated and powerful communication systems. Thus, by integrating array theory with digital communication theory, new communication system architectures are being proposed which have a considerable impact, for instance, on the capacity and performance of a mobile communication network. These improvements can be achieved in a number of different ways, for example, by suppressing co-channel interferences, combating fading effects, locating/tracing the mobile users, estimating other signal environment parameters, etc.

This experiment aims to provide a theoretical framework for handling various problems in a number of different applications. Emphasis will be given to the so called "superresolution" approaches and three general problems of great interest will be investigated:

- the detection problem;
- the directions of arrival (DOA) estimation problem.
- the signal parameter estimation problem.

With respect to detection problems the array estimates the number of emitting sources (co-channel signals) present in the array environment. On the other hand, Spatial Spectral Estimation techniques are concerned with the DOA problem. Classical spatial estimation techniques are based on the Fourier Transform (Conventional Beamformer). The main drawback of the Fourier-type methods is that they offer limited resolving capabilities. Thus, in the last decade the so called Superresolution methods have been introduced, their main objective being to improve the resolving capabilities by using a model for the signals better than that used by Fourier methods. These methods have given fresh impetus to the array processing problem by dealing with the question of resolution of the arrays in such a way that there is elimination of the effects of the Signal-to-Noise Ratio (SNR) on resolution, in contrast to the conventional methods where the resolution is limited by noise.

Thus, this experiment is concerned with a compact examination of the above concepts and provides a tutorial mechanism regarding array processing techniques with main objectives to:

- estimate the number of the emitting sources;
- provide complete information about the directions of the emitting sources;
- investigate the use of an array of sensors for detecting and rejecting in-band interferences;
- estimate relative powers, cross-correlations, etc.

Tasks:

Consider a linear array of 5 sensors uniformly distributed along the x-axis in the 3 dimensional real space.

type:
array=[-2 0 0; -1 0 0; 0 0 0; 1 0 0; 2 0 0];

The array receives noisy data from three sources which transmit at the same frequency band. The directions of the sources are (30°,0°), (35°,0°), and (90°,0°), where the first number represents the *azimuth* and the second the *elevation* angles of arrival.

type: $\mathbf{directions = [30, 0;\ 35, 0\ ;\ 90, 0]}$

Using a multi-cross-correlator the second order statistics of the received array-signal, that is the covariance matrix $\mathbb{R}_{xx}$, can estimated.

By using the matrix $\mathbb{R}_{xx}$ solutions to the following three basic problems will be addressed:

- detection problem
- estimation problem
- interference suppression problem.

The experiment will involve three covariance matrices. The first will be the theoretical covariance matrix for three equi-power sources, the second will involve three audio signals while the third three colour images.

1. Form the *pattern* of the above array.

type:

```
Z=pattern(array);
plot2d3d(Z,[0:180],0,'gain in dB','initial  pattern');
```

Check the gain provided by the array for the three sources.

2. Theoretical Covariance Matrix Formation:

- For each source there is a vector, called the *Source Position Vector* (*or manifold vector*), which is a function of the position of the source and the geometry of the array. This means if you know the location of the sources and the array you may form the SPVs. Consider the matrix $\mathbb{S}$ with columns the Source Position Vectors $\underline{S}_i$ i=1,2,3. This matrix can be formed by typing:

  ```
  𝕊=spv(array, directions)
  ```

  Comment on the form of the vectors $\underline{S}_i$

- Consider that you know that the above mentioned three sources are *uncorrelated* and of *equal* powers (say 1, normalised). Form the covariance matrix of the 3 sources $\mathbb{R}_{mm}$.

- Assume that the noise present in the array is: additive isotropic noise, uncorrelated with the transmitting sources and of power σ^2=40dB below the power of the signals

  i.e. type: `sigma2=0.0001`

- Form the covariance matrix of the signals at the input of the array $\mathbb{R}_{xx,\text{theoretical}}$. This is known as the theoretical covariance matrix and is given by

  $$\mathbb{R}_{xx,\text{theoretical}} = \mathbb{S}.\,\mathbb{R}_{mm}.\mathbb{S}^{\mathrm{H}} + \sigma^2\,\mathbb{I}_5$$

  i.e. type

  ```
  Rxx_theoretical=S*Rmm*S'+sigma2*eye(5,5)
  ```

- Comment on the form of the matrix $\mathbb{R}_{xx}$. Check if your comments are valid for any array geometry.

3. Practical Covariance Matrix:

- In this case the three signal are initially three audio signals and then three images which are transmitted at the same frequency band in the presence of noise. The received array-signal is the matrix **𝕏_au** for the audio signals and the matrix **𝕏_im** for the images

  type:

  ```
  load 𝕏audio;
  load 𝕏image;
  ```

  Listen to the audio signal associated with the 2nd antenna by using your audio facilities.

  type:

  ```
  soundsc(real(X_au(2,:)), 11025);
  ```

  Then observe the image at the output of the second antenna. (see Figure 1).

  type:

  ```
  displayimage(X_im(2,:),image_size, 201,
  'The received signal at the 2nd antenna');
  ```

- Form the covariance matrix of the received signal.

  i.e. type

  ```
  Rxx_au=𝕏_au*𝕏_au'/length(𝕏_au(1,:));
  Rxx_im=𝕏_im*𝕏_im'/length(𝕏_im(1,:));
  ```

- N.B.: Remember that **ℝxx_theoretical** is the theoretical covariance matrix while **ℝxx_au** and **ℝxx_im** are two practical covariance matrices for the situations described above.

4. Forget that you know that there are 3 sources present and in addition forget their directions and their powers

i.e. type:

```
directions=[];
ℝmm=[];
𝕊=[];
sigma2=[];
```

5. Detection Problem:

5.1. Consider that the only information provided to you is the matrix $\mathbb{R}_{xx,\text{theoretical}}$ at the input of the above array. Form the eigendecomposition of the matrix $\mathbb{R}_{xx,\text{theoretical}}$

i.e. type:

```
eig(ℝxx_theoretical)
```

Is there any way to estimate

- the power of noise, and
- the number of sources which are present

simply by observing the eigenvalues of $\mathbb{R}_{xx,\text{theoretical}}$?

5.2.Write your conclusions from 5.1 as a formal STATEMENT (or THEOREM).

5.3. Repeat instructions 5.1 & 5.2 but this time use the practical covariance matrices $\mathbb{R}$**xx_au** and $\mathbb{R}$**xx_im** rather than the theoretical covariance.

6. Estimation Problem - Conventional Approach:

- Let us consider that the source located at 90° is the 'desired' source and this direction is known to you while the remaining two sources (i.e. 30°, 35°), known as interferences, are completely unknown.

- Let us estimate firstly the SPV $\underline{S}_d$ which corresponds to the desired source

  i.e. type: **Sd=spv(array,[90,0])**

- Next let us weight the array elements by the following vector $\underline{w}_{opt} = \mathbb{R}^{-1}_{xx,\text{theoretical}}\underline{S}_d$
  i.e. type
  wopt=α*inv($\mathbb{R}$xx_theoretical)*Sd

  where α is a constant (gain factor).
  This is the optimum Wiener-Hopf solution.

- Estimate once again the array pattern of the array when it is weighted by w_{opt}

  i.e. type:

  ```
      Z=pattern(array, wopt);
      plot2d3d(Z, [0:180], 0, ’gain in dB’,
                           ’W-H  array pattern’);
  ```

- Can you distinguish the directions of the two interferences from that array pattern?

7. Repeat instructions 2, 4, 5.1 and 6 but with noise level 10dB below the level of the sources.

8. What conclusions can be drawn from 6 and 7 ?

9. Estimation Problem - Superresolution Approach: Study and implement the MuSIC algorithm [1] in order to estimate the directions of the three sources.

N.B. please use the following format

```
Z=music(array, Rxx_theoretical, numofsources);
plot2d3d(Z,[0:180],0,’dB’, ’MuSIC spectrum’);
```

10. Repeat instruction 9 by using $\mathbb{R}$**xx_au** and $\mathbb{R}$**xx_im** rather than $\mathbb{R}$**xx_theoretical**.

11. Multipaths - Coherent Sources:

- Consider the cases where two sources i.e. 30°, 35° are fully correlated (or coherent). Repeat instructions 5.1, 5.2 and 9.
- Compare your results from the uncorrelated and correlated cases.
- Is it possible to overcome the problem resulting when the sources are coherent? The answer is YES by applying the so called spatial Smoothing technique [2,5] before using the MuSIC algorithm.
  Can you apply this technique? Write a small program in matlab.

12. Estimations & Reception Problem - Superresolution Techniques:

12.1. Repeat instruction 6 but this time use $\mathbb{R}$**xx_au** rather than $\mathbb{R}$**xx_theoretical.**

Listen to the 90° signal at the output of the array (i.e signal **y(t))** by typing

```
yt=wopt’*𝕏_au;
soundsc(real(yt), 11025);
```

and describe it in your log-book.

12.2. Repeat instruction 6 but this time use $\mathbb{R}$**xx_im** rather than $\mathbb{R}$**xx_theoretical.**

Observe the 90° signal at the output of the array (i.e signal **y(t))** by typing

```
yt=wopt’*𝕏_im;
displayimage(yt, image_size, 202,
’The received signal at o/p of W-H
 beamformer’);
```

and describe it in your log-book.

12.3. Using the directions given by the MuSIC algorithm in instruction 10, is it possible to receive one of the transmitted signals and, at the same time, to suppress completely the effects of the rest of the transmitted signals (i.e. to provide complete interference cancellation)? Design a multi-beam beamformer to receive the three signals using the $\mathbb{R}$**xx_theoretical**.

N.B.: Study the material provided in Appendix-1 in order to give a reasonable answer to this question.

12.3. repeat 12.3 but this time using the $\mathbb{R}$**xx_im.** Observe the three images at the output of the beamformer using the **displayimage** command.

12.4. Comment on the results of 12.1, 12.2 and 12.3.

13. Practical Detection Criteria:

Simulate 250 snapshots for the environment decsribed in instructions 1 and 2. Then form the practical covariance matrix $\mathbb{R}_{xx}$.

Apply the detection criterion (Question 5) in order to detect the three sources. Identify any problem and write it down in your Log-book together with your conclusions. Then search the EEED Library on Level 6 to find papers associated with the following criteria:

- AIC (Akaike Information Criterion)
- MDL (Minimum Description Length)

Apply these criteria to detect the three sources.

What conclusions can be drawn from these criteria?

14. Design an adaptive beamformer using the following two adaptive algorithms: LMS and RLS

Appendices:

Appendix-1: "Support Notes - Experiment AM1", Department of Electrical & Electronic Engineering, Imperial College, 1994.

Appendix-2: "Some Introductory Notes on Diversity Techniques", Department of Electrical & Electronic Engineering, Imperial College, 1989.

References:

1. Schmidt R.O., "Multiple Emitter Location and Signal Parameter Estimation", *IEEE Transactions on Antennas and Propagation*, Vol. AP-34, No.3, pp 276-280, March 1986.

2. Shan T., Wax M., Kailath T., "On Spatial Smoothing for Direction-of-Arrival Estimation of Coherent Signals", *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. ASSP-33, No.4, pp 806-811, August 1985.

3. Monzinco A.A., Miller T.W., Introduction to Adaptive Arrays, John-Wiley, 1980.

4. Hudson J.E, Adaptive Array Principles, IEE-Peter Peregrinus, 1981.

5. Pillai S.U., Array Signal Processing, Springer-Verlag, 1989.

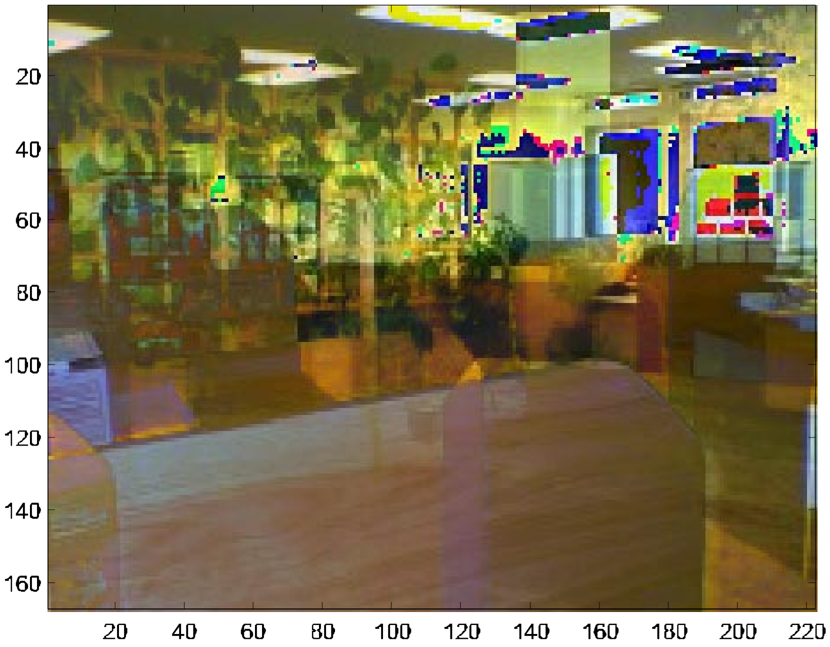

Figure 1: Signal received at the 2nd antenna of a linear array of 5 antennas operating in the presence of 3 cochannel signals (images) of bearings 30°, 35° and 90°.