Autocorrelation model-based identification method for ARMA systems in noise

M.K. Hasan, N.M. Hossain and P.A. Naylor

Abstract: A novel method for parameter estimation of minimum-phase autoregressive moving average (ARMA) systems in noise is presented. The ARMA parameters are estimated using a damped sinusoidal model representation of the autocorrelation function of the noise-free ARMA signal. The AR parameters are obtained directly from the estimates of the damped sinusoidal model parameters with guaranteed stability. The MA parameters are estimated using a correlation matching technique. The simulation results show that the proposed method can estimate the ARMA parameters with better accuracy as compared to other reported methods, in particular for low SNRs.

1 Introduction

The problem of identifying an autoregressive moving average (ARMA) system with unknown white noise input excitation from its noisy observations arises in many areas of science and engineering, such as speech processing, radar, time-series analysis, spectral estimation, economics, seismology, communications, and biomedical signal processing [1–4]. However, the widespread use of ARMA systems is particularly limited by the difficulty involved in the MA parameter estimation.

The problem becomes even more difficult when noise corrupts the observations. As an approximate alternative, autoregressive (AR) systems are more frequently used to bypass this difficulty. Many effective techniques have been reported in the literature for identification of AR systems in noise [5–11]. In contrast, only a limited number of results are available for the identification of ARMA systems corrupted by noise [9, 12–14], though many effective techniques exist for noise-free ARMA system identification [15–18]. The Yule-Walker method and the lattice filter (LF) are commonly used in both noisy and noise-free cases. The LF in the estimation of ARMA parameters presents several advantages [19]. The LF parameters, called partial autocorrelation (PARCOR) coefficients, provide an alternative parametrisation of ARMA systems. In [12], an overfitting LF that estimates the ARMA parameters from noisy measurements has been reported. The performance of these methods, however, is satisfactory above a moderate SNR but they have been found to fail at a low SNR. Moreover, stability of the noise-compensated LF (NCLF) is not guaranteed [9].

In this paper, we investigate a new method of ARMA parameter estimation from noisy observations. A damped sinusoidal model for the autocorrelation function of the noise-free signal is adopted for ARMA parameter estimation. The parameters of the sinusoidal model are estimated using the given noisy observations and the desired AR system parameters are then directly obtained from this model's parameters. For MA parameter estimation we propose an iterative method that selects the desired parameters from a set of over-determined parameters using a correlation-matching technique. The method proposed here may be viewed as an extension of the method reported by the authors in [11] for identification of AR systems in noise. This extension is important as ARMA modelling is a more generalised form for representation of signals using linear systems.

IEE Proceedings online no. 20045042

doi: 10.1049/ip-vis:20045042

Paper first received 29th May 2004 and in revised form 21st January 2005. Originally published online 5th July 2005

M.K. Hasan and P.A. Naylor are with the Communications and Signal Processing Group, Department of Electrical and Electronic Engineering, Imperial College London, London, SW7 2AZ, UK

N.M. Hossain is with the Department of Electrical and Electronic Engeering, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh

E-mail: khasan@eee.buet.ac.bd, p.naylor@imperial.ac.uk

2 Problem formulation

Consider the following ARMA(p, q) system:

$$x(n) = -\sum_{i=1}^{p} a_i x(n-i) + \sum_{j=0}^{q} b_j u(n-j) \qquad (1)$$

where $x(n)$ is the output signal of the minimum-phase ARMA system, which is excited by a sequence of white noise $u(n)$ with distribution $\mathcal{N}(0, \sigma_u^2)$, p and q are the known AR and MA orders, respectively. If the signal $x(n)$ is contaminated by a white noise process $v(n)$ with distribution $\mathcal{N}(0, \sigma_v^2)$, the observed signal $y(n)$ is obtained as

$$y(n) = x(n) + v(n) \qquad (2)$$

The observation noise $v(n)$ is assumed to be independent of the input noise $u(n)$, i.e. $E[u(n)v(n-m)] = 0$ for all m, where $E[\cdot]$ denotes the statistical expectation operator.

This paper estimates the AR (a_i, $i = 1, 2, \ldots, p$) and MA (b_j, $j = 1, 2, \ldots, q$) parameters using only the noisy signal $y(n)$.

3 Estimation of AR parameters

To develop a new mathematical model for estimation of the autocorrelation function of $x(n)$ from a finite set of observations of the noisy signal $y(n)$ we introduce an alternative representation for $x(n)$. The model we introduce here may be viewed as an extension of the model reported in [11] for identification of AR systems.

The transfer function of an ARMA(p, q) system in the z-domain can be expressed as

$$H(z) = \frac{B(z)}{A(z)} = \sum_{k=1}^{p} \frac{\Gamma_k}{1 - z_k z^{-1}} \tag{3}$$

where $A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_p z^{-p}$, $B(z) = 1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_q z^{-q}$, z_k denotes the kth pole of the ARMA system and Γ_k is the partial fraction coefficient corresponding to the kth pole. In the above expansion it is assumed that the ARMA system has no multiple-order poles.

The unit impulse response $h(n)$ of the causal ARMA system described in (3) can be expressed as

$$h(n) = \sum_{k=1}^{p} \Gamma_k z_k^n, \quad n = 0, 1, 2, \ldots, \infty \tag{4}$$

If this relaxed ARMA system is excited by $u(n)$, then the response $x(n)$ is given by

$$x(n) = u(n) * h(n) = \sum_{l=0}^{n} u(l) h(n-l) \tag{5}$$

Using (4), (5) can be written as

$$x(n) = \sum_{k=1}^{p} \sum_{l=0}^{n} \Gamma_k u(l) z_k^{n-l} \tag{6}$$

Note that (1) is the difference equation implementation of $x(n)$ using the system parameters and (6) is the convolution sum implementation of $x(n)$ using the system roots. Using (6), the autocorrelation of the noise-free signal $x(n)$ can be obtained as [20]

$$R_{xx}(m) = \sum_{k=1}^{p} \beta_k z_k^m \tag{7}$$

where

$$\beta_k = \sigma_u^2 \left[\frac{\Gamma_k^2}{1 - z_k^2} + \sum_{i=1, i \neq k}^{p} \frac{\Gamma_k \Gamma_i}{1 - z_k z_i} \right] \tag{8}$$

The coefficient β_k may be real or complex depending on whether the pole k is real or complex. Let the number of real poles be p_r and the remaining $p_c = (p - p_r)$ poles be complex, occurring in $p_{cc} = p_c/2$ complex conjugate pairs if $x(n)$ is real. As p_{cc} cannot be a fraction, for a pth order AR system with p being odd, p_r must be odd. Separating the terms with real poles from the terms with complex poles, we can write (7) as

$$R_{xx}(m) = \sum_{k_c=1}^{p_c} \beta_{k_c} z_{k_c}^m + \sum_{k_r=p_c+1}^{p_c+p_r} \beta_{k_r} z_{k_r}^m \tag{9}$$

For a pair of complex conjugate poles, the corresponding β will also be a complex conjugate pair. Let us consider a pair of complex conjugate poles $z_1 = r_1 \exp(j\omega_1)$ and $z_2 = r_1 \exp(-j\omega_1)$ with corresponding $\beta_1 = \zeta_1 \exp(j\phi_1)$ and $\beta_2 = \zeta_1 \exp(-j\phi_1)$. We can write

$$\beta_1 z_1^m + \beta_2 z_2^m = G_1^c r_1^m \cos(\omega_1 m + \phi_1) \tag{10}$$

where $G_1^c = 2\zeta_1$ is a constant that depends on β_1. Hence the sum of terms with complex poles in (9) can be expressed as

$$\sum_{k_c=1}^{p_c} \beta_{k_c} z_{k_c}^m = \sum_{j_c=1}^{p_{cc}} G_{j_c} r_{j_c}^m \cos(\omega_{j_c} m + \phi_{j_c}) \tag{11}$$

where $G_{j_c}^c = 2\zeta_{j_c}$ is a constant that depends on β_{j_c}. As in the case of a complex pole, a real pole can be expressed as $z_k = r_k \exp(j\omega_k)$, where ω is 0 or π, with corresponding $\beta_k = \zeta_k \exp(j\phi_k)$, where ϕ_k is also 0 or π. Hence the sum of terms with real poles in (9) can be expressed as

$$\begin{aligned} \sum_{k_r=p_c+1}^{p_c+p_r} \beta_{k_r} z_{k_r}^m &= \sum_{j_r=1}^{p_r} \beta_{j_r} z_{j_r}^m \\ &= \sum_{j_r=1}^{p_r} G_{j_r}^r r_{j_r}^m \cos(\omega_{j_r} m + \phi_{j_r}) \end{aligned} \tag{12}$$

where $G_{j_r}^r = \zeta_{j_r}$ is a constant that depends on β_{j_r}. Substituting (11) and (12), we can write (9) as

$$R_{xx}(m) = \sum_{j=1}^{p_{cc}+p_r} G_j r_j^m \cos(\omega_j m + \phi_j) \tag{13}$$

where G_j is a constant. Equation (13) can be further expanded as

$$R_{xx}(m) = \sum_{j=1}^{p_{cc}+p_r} r_j^m [P_j \cos(\omega_j m) + Q_j \sin(\omega_j m)], \quad \text{for } m \geq 0 \tag{14}$$

where $P_j = G_j \cos(\phi_j)$ and $Q_j = -G_j \sin(\phi_j)$ are constants that depend on β_j. In general, r_j governs the decay rate of the ARMA system response and ω_j determines the angular position of a pole of the ARMA system in the z-plane.

The damped sinusoidal functions constituting $R_{xx}(m)$ as shown in (14) can be estimated sequentially by defining a residue function from the autocorrelation function $R_{yy}(m)$ of the observed noisy data $y(n)$. The jth residue function is defined as

$$\Re_j(m) = \begin{cases} R_{yy}(m), & j = 0 \\ \Re_{j-1}(m) - r_j^m F_j(m), & j = 1, 2, \ldots, p_{cc} + p_r - 1 \end{cases} \tag{15}$$

where $F_j(m) = P_j \cos(\omega_j m) + Q_j \sin(\omega_j m)$, and $R_{yy}(m)$ is calculated as

$$R_{yy}(m) = \frac{1}{N} \sum_{n=0}^{N-1-|m|} y(n) y(n + |m|) \tag{16}$$

The parameters ω_j, r_j, P_j, and Q_j of the jth component function $\{r_j^m [P_j \cos(\omega_j m) + Q_j \sin(\omega_j m)\}$ are chosen such that the sum-squared error (SSE) between the $(j-1)$th residue function and the jth component function is minimised. The SSE, $J_j^{(i)}$, is defined as

$$J_j^{(i)} = \sum_{m=1}^{M} \left| \Re_{j-1}(m) - \left(r_j^{(i)}\right)^m F_j^{(i)}(m) \right|^2, \quad j = 1, 2, \ldots, p_{cc} + p_r - 1 \tag{17}$$

where M denotes the number of autocorrelation lags to be used in the minimisation process. Since the proposed method is iterative, the superscript '(i)' denotes the iteration index, i.e. $\omega_j^{(i)}$ denotes the angle of the jth pole at iteration i.

To estimate the kth component function of the damped sinusoidal model, if ω_k and r_k are searched in their entire domain, e.g. $[0, \pi]$ and [0, 1], respectively, with an acceptable resolution, the computational cost will be extremely high. Instead, we use a high-order AR model to obtain a solution space for the underlying problem. It is known that the noisy process $y(n)$ can be more accurately characterised by a higher-order AR process containing both noise and system poles [5]. Then this mixture of noise plus system poles can be used as candidate solutions for (17) and the desired system poles are to be extracted as the ones that minimise the cost function J_j. Using a higher-order AR model and then extracting the desired roots based on a root matching technique has also been reported in [10] for autoregressive spectral estimation from noisy observations. In contrast with the main aim of computational complexity reduction, we first estimate poles of a higher-order AR model fitted to the observed noisy process by using the stable noise-uncompensated lattice filter (NULF) [19]. The order $\tilde{p}$ of the over-fitted AR model may be determined using a standard technique [21]. Now, instead of scanning the entire range of ω_k and r_k, only the angular positions $\omega_l,\ l=1,2,\ldots,\tilde{p}$, and magnitude $r_l,\ l=1,2,\ldots,\tilde{p}$, of these poles are searched, so as to select p (out of $\tilde{p}$) poles for which $J_j^{(i)}$ is minimum. The selected p poles are then used for computing an estimate $\hat{a}_i$ of the desired AR parameters, a_i. Note that, unlike conventional noise-compensation techniques [7], the proposed method is inherently stable for minimum phase systems.

4 Estimation of MA parameters

In this work, for MA parameter estimation we first fit a higher-order ARMA $(\tilde{q},\tilde{q})$ model to the observed noisy process and estimate the ARMA parameters using the stable NULF. The MA$(\tilde{q}),\tilde{b}_i,\ i=1,2,\ldots,\tilde{q}$, parameters thus obtained and the AR(p) parameters as estimated in Section 3 are then used for estimating the desired MA(q) parameters, b_j, as follows. We note that AR$(\tilde{q})$ parameters of the ARMA $(\tilde{q},\tilde{q})$ model are not used in MA parameter calculation. Moreover, the higher-order AR model parameters obtained using the NULF in Section 3 are not employed in the MA part estimation in this Section.

The MA$(\tilde{q})$ polynomial consisting of $\tilde{b}_i,\ i=1,2,\ldots,\tilde{q}$, parameters is given by

$$\tilde{B}(z)=1+\tilde{b}_1z^{-1}+\tilde{b}_2z^{-2}+\cdots+\tilde{b}_{\tilde{q}}z^{-\tilde{q}} \qquad (18)$$

The roots of $\tilde{B}(z)$ are denoted as $\tilde{r}_i$. Suppose that in $\tilde{r}_i$ there are $\tilde{p}_c$ complex roots, denoted as $\tilde{r}_{k_c}$ that occur in complex conjugate pairs, and $\tilde{p}_r$ real roots, denoted as $\tilde{r}_{k_r}$. Thus the number of complex conjugate roots is $\tilde{p}_{cc}=\tilde{p}_c/2$. Note that q roots are required to be selected from $\tilde{q}$ roots, for which we describe a rule below. The roots of the MA part can be complex, real, or a mixture of complex and real roots. For q being odd or even, and for any possible combinations of real and complex roots, the MA polynomial can be constructed as

$$B^l(z)=\prod_{k_r=1}^{q_{r\gamma}}(1-\tilde{r}_{k_r}z^{-1})\prod_{k_c=1}^{q_{c\gamma}}(1-\tilde{r}_{k_c}z^{-1}) \times\left(1-\tilde{r}_j^{c^*}z^{-1}\right) \qquad (19)$$

where $q_{r\gamma}=2(\gamma-1),\ \gamma=1,2,\ldots,(q+2)/2$ and $q_{c\gamma}=(q-q_{r\gamma})/2$, for q even, and $q_{r\gamma}=2\gamma-1,\ \gamma=1,2,\ldots,(q+1)/2$ and $q_{c\gamma}=(q-q_{r\gamma})/2$, for q odd. Note that γ gives the choices available to select different numbers of real and complex roots to construct the polynomial $B^l(z)$. For each of the selected numbers of real $(q_{r\gamma})$ and complex $(q_{c\gamma})$ roots, the polynomial $B^l(z)$ can be constructed in ${}^{\tilde{p}_r}C_{q_{r\gamma}}\times{}^{\tilde{p}_{cc}}C_{q_{c\gamma}}$ different ways. Now, suppose that there are L possible combinations of roots taking q roots from $\tilde{r}_i,\ i=1,2,\ldots,\tilde{q}$, at a time. Note that in such a combination if a complex root is included then its complex conjugate part is also included. Then L is given by

$$L=\begin{cases}\sum_{\gamma=1}^{(q+2)/2}{}^{\tilde{p}_r}C_{q_{r\gamma}}\times{}^{\tilde{p}_{cc}}C_{q_{c\gamma}}, & \text{for } q \text{ even}\\ \sum_{\gamma=1}^{(q+1)/2}{}^{\tilde{p}_r}C_{q_{r\gamma}}\times{}^{\tilde{p}_{cc}}C_{q_{c\gamma}}, & \text{for } q \text{ odd}\end{cases} \qquad (20)$$

where ${}^nC_r=n!/r!(n-r)!$, but it is assumed to be zero if $n<r$. Therefore, there are a total of L different MA(q) polynomials, $B^l(z),\ l=1,2,\ldots,L$, from which the desired MA polynomial is to be selected using a minimisation technique described later in this Section.

In this work, the autocorrelation function of the noise-free ARMA signal $R_{xx}(m)$ given in (7) for certain lags (m) is first estimated to determine the MA part of the ARMA system under consideration. A possible estimate of $H(z)$ is obtained as

$$\tilde{H}^l(z)=\frac{B^l(z)}{\hat{A}(z)}=\sum_{k=1}^{p}\frac{\tilde{\Gamma}_k^l}{1-\hat{z}_kz^{-1}},\qquad l=1,2,\ldots,L \qquad (21)$$

where $\hat{z}_k$ denotes the estimated kth pole of the ARMA system as in Section 3 and $\tilde{\Gamma}_k^l$ is the partial fraction coefficient corresponding to the kth pole. Now, using (21) an estimate of the partial fraction constant $\tilde{\Gamma}_k^l$ can be obtained. The estimated values of $\tilde{\Gamma}_k^l,\ \hat{z}_k$ and $\hat{\sigma}_u^2$ are used to find β_k using (8). Note that for complex conjugate pair of poles the corresponding β_k also comes in complex conjugate pair form. The values of β_k thus obtained are used to find $R_{xx}^l(m)$ where the superscript 'l' is used to indicate that it is estimated using the model in (7) for a particular $B^l(z)$. Now, the objective is to choose a $B^l(z)$ such that the distance between $R_{xx}^l(m)$ and $R_{yy}(m)$ is minimum. An objective function is then defined as

$$D(B^l)=\sum_{m=1}^{M'}\left|R_{xx}^l(m)-R_{yy}(m)\right|^2 \qquad (22)$$

where M' denotes the number of lags to be used in the minimisation process. An estimate of the desired MA(q) polynomial $(\hat{B}(z))$ can be obtained as

$$\hat{B}(z)=\arg(\min_{B^l}(D(B^l))) \qquad (23)$$

5 Simulation results

In this Section, we evaluate the performance of the proposed ARMA system identification method by presenting several numerical examples. Data were generated according to (1) and (2). In all the simulations, the excitation noise power σ_u^2 is assumed to be known, and $N=4000$ samples of the noisy data were used. As explained in Section 3, we calculate the desired AR parameters of the ARMA system from the damped sinusoidal model parameters and in all these simulations we have used $R_{yy}(m)$ for $m=1,2,\ldots,M$ with $M=100$. The MA parameters of the ARMA system were estimated using the iterative method as explained in Section 4 with $M'=100$ (see (22)).

The following four ARMA systems were used in the experiments. The systems were selected with poles

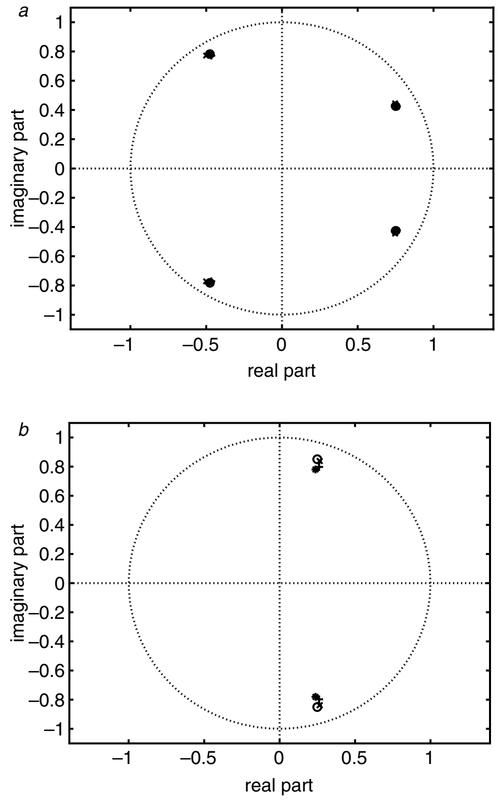

Fig. 1 *Parameter estimation results at SNR = 20 dB for ARMA System 1*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

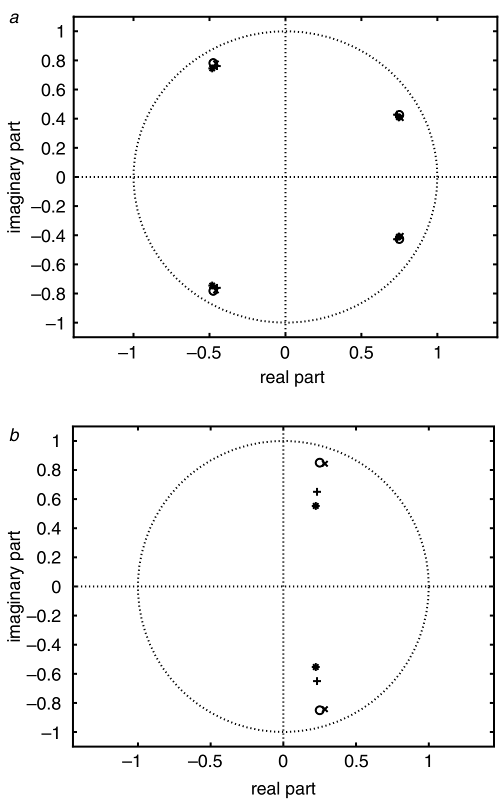

Fig. 2 *Parameter estimation results at SNR = 10 dB for ARMA System 1*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

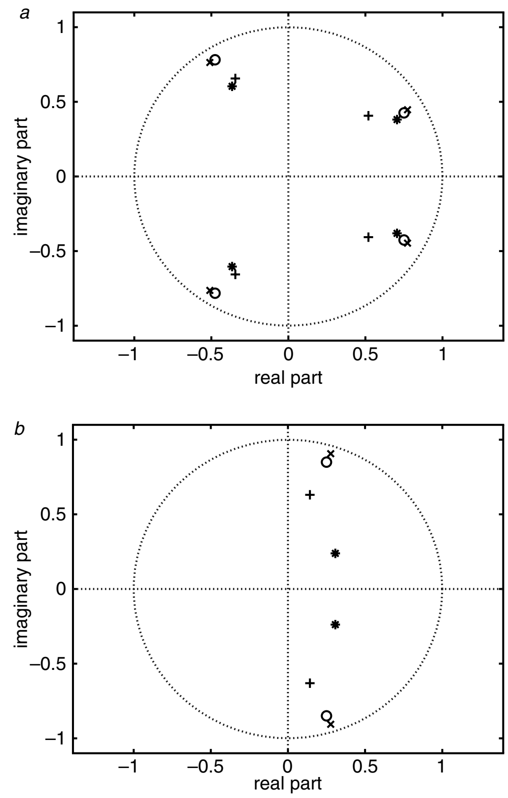

Fig. 3 *Parameter estimation results at SNR = 0 dB for ARMA System 1*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

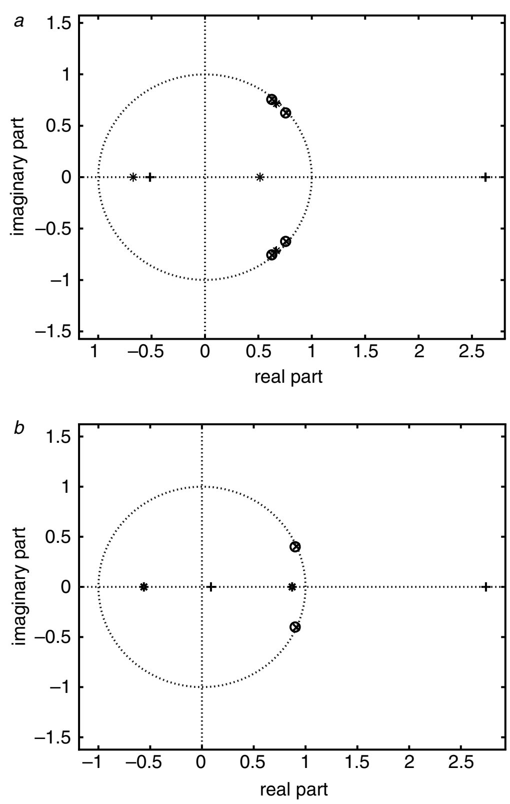

Fig. 4 *Parameter estimation results at SNR = 20 dB for ARMA System 2*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

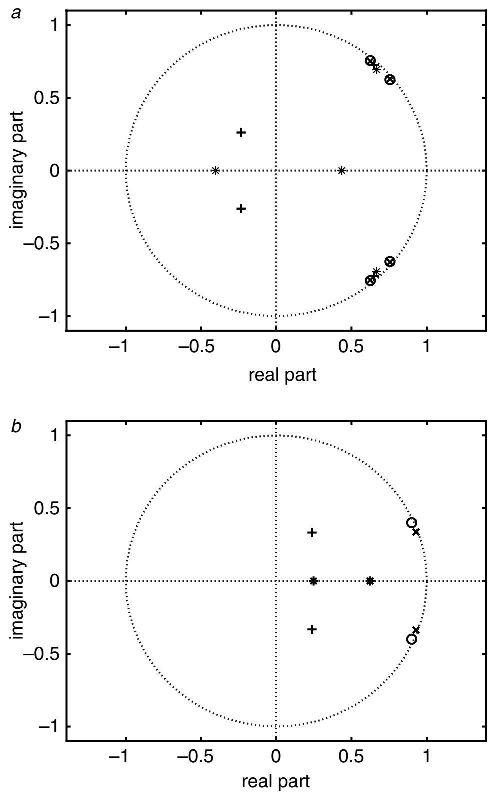

Fig. 5 *Parameter estimation results at SNR = 10 dB for ARMA System 2*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

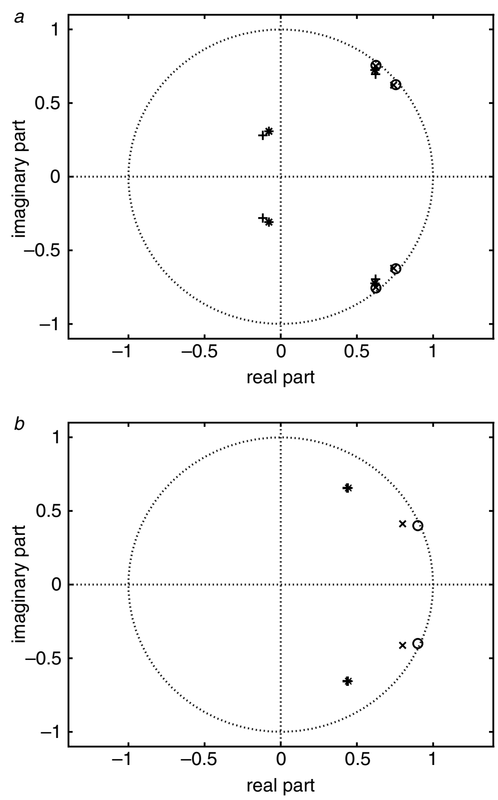

Fig. 6 *Parameter estimation results at SNR = 0 dB for ARMA System 2*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

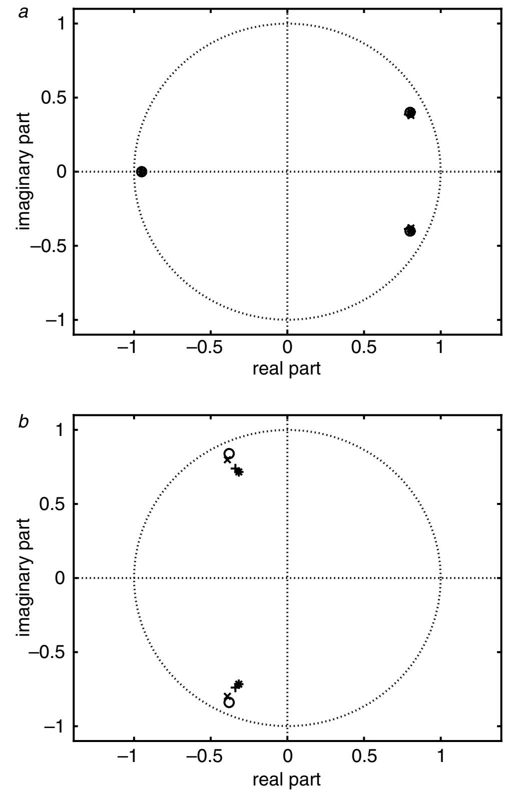

Fig. 7 *Parameter estimation results at SNR = 30 dB for ARMA System 3*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

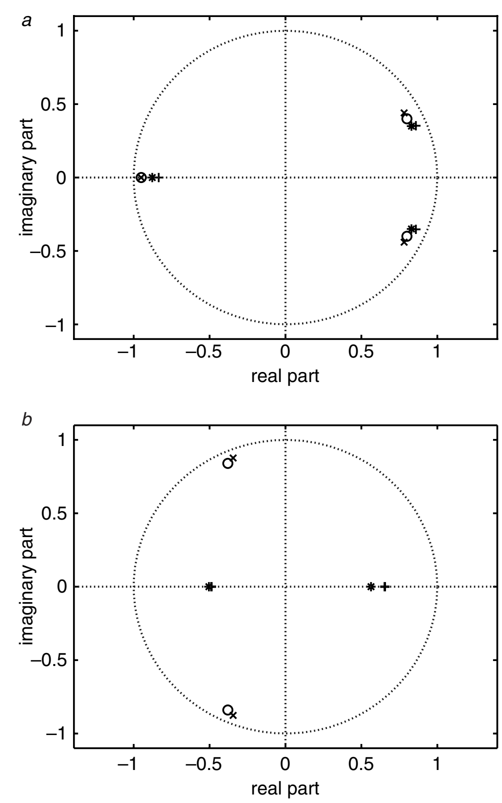

Fig. 8 *Parameter estimation results at SNR = 10 dB for ARMA System 3*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

and zeros distributed in different regions inside the unit circle in the z-plane.

System 1:

$$x(n) - 0.55x(n-1) + 0.155x(n-2) - 0.5495x(n-3) + 0.6241x(n-4) = u(n) - 0.5u(n-1) + 0.785u(n-2) \quad (24)$$

System 2:

$$x(n) - 2.7607x(n-1) + 3.8106x(n-2) - 2.6535x(n-3) + 0.9238x(n-4) = u(n) - 1.8u(n-1) + 0.97u(n-2) \quad (25)$$

System 3:

$$x(n) - 0.65x(n-1) - 0.72(n-2) + 0.76x(n-3) = u(n) - 0.76u(n-1) + 0.85u(n-2) \quad (26)$$

System 4:

$$x(n) - 2.2990x(n-1) + 2.1262(n-2) - 0.7604x(n-3) = u(n) + 0.87u(n-1) + 0.92u(n-2) \quad (27)$$

The noisy signal $y(n)$ was generated by adding Gaussian distributed white noise to $x(n)$, although the probability density function (PDF) of the noise is not significant. The variance of the random noise $v(n)$ was adjusted so as to give different levels of signal-to-noise ratio (SNR).

The results at different SNRs obtained using the proposed method are compared in Figs. 1–12 to the prediction error

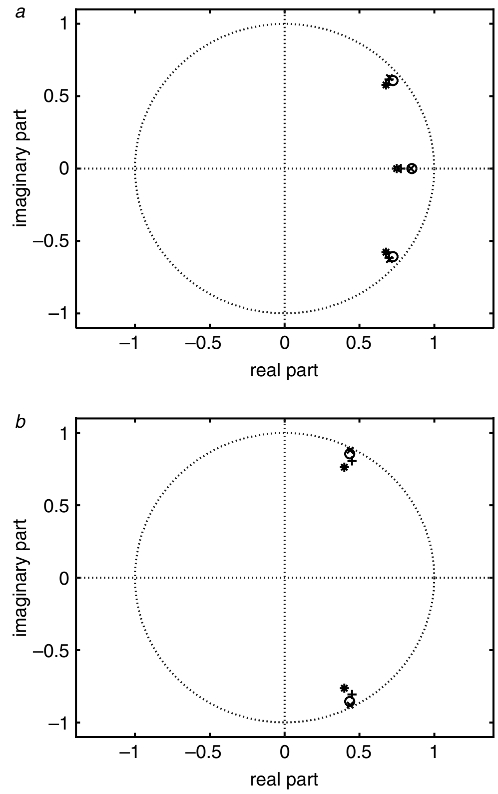

Fig. 10 *Parameter estimation results at SNR = 20 dB for ARMA System 4*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

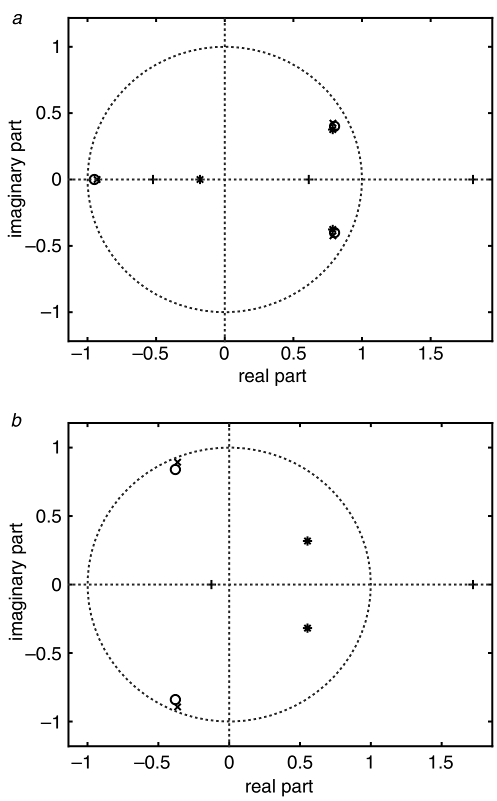

Fig. 9 *Parameter estimation results at SNR = 0 dB for ARMA System 3*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

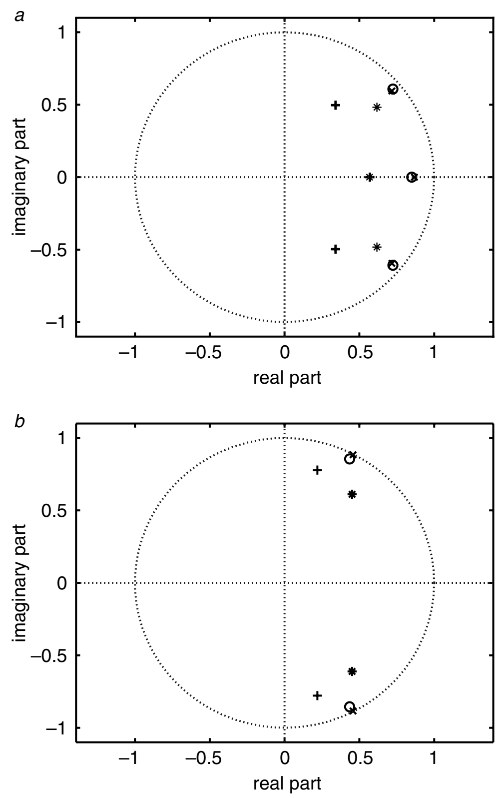

Fig. 11 *Parameter estimation results at SNR = 10 dB for ARMA System 4*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

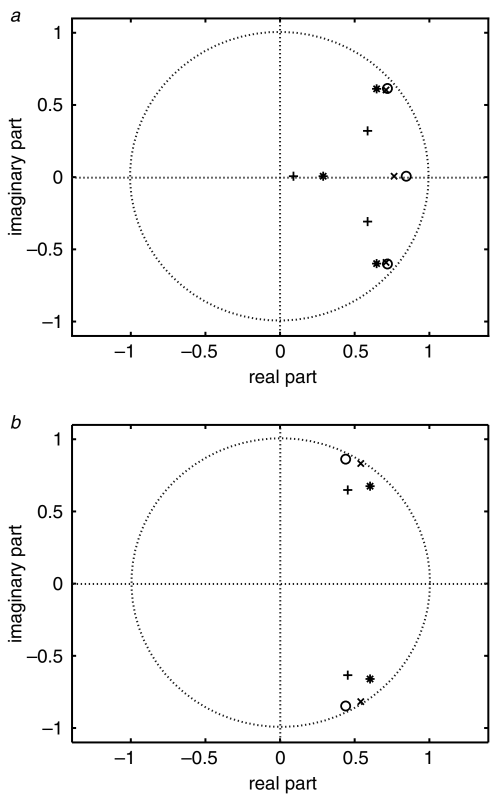

Fig. 12 *Parameter estimation results at SNR = 0 dB for ARMA System 4*

○: true, +: [13], *: PEM, ×: proposed
a Roots of $A(z)$
b Roots of $B(z)$

method (PEM) for the estimation of an ARMAX model [15], and the noise-compensated Yule-Walker equations based method reported in [13]. It is evident from Figs. 1, 4, 7 and 10 that, at 20 dB or a higher SNR, all the methods perform with good accuracy. However, it can be seen in Figs. 2, 5, 8 and 11 that, when more noise is introduced to give 10 dB SNR, the accuracy of the proposed method is significantly better than that of the other methods. The effectiveness of the proposed method is also tested at a low SNR (e.g. 0 dB). From Figs. 3, 6, 9 and 12 it can be seen that, for the proposed algorithm, the estimation accuracy for the 0 dB case is comparable to the accuracy for the 10 and 20 dB cases. In contrast, for the other algorithms, accuracy of the estimates is poor. From all the results presented it can be inferred that the proposed method is a good candidate for ARMA system identification from noisy observations.

6 Conclusions

In this paper, a novel method for the identification of ARMA systems in noise using a damped sinusoidal model of the autocorrelation function has been proposed. It has been shown that the proposed method is able to estimate the AR and MA parameters with good accuracy as compared to other reported methods. The conventional correlation-based techniques fail to estimate accurately the ARMA parameters below a certain threshold SNR (e.g. 10 dB) owing to inaccurate estimation of the autocorrelation function from a finite set of noisy observations. These results have shown that the calculation of the autocorrelation function based on a damped sinusoidal model can overcome the problem of identification of ARMA systems, even at low SNRs.

7 References

1 Haykin, S., and Steinhart, A.: 'Adaptive radar detection and estimation' (John Wiley & Sons, New York, USA, 1992)
2 Bruce, E.N.: 'Biomedical signal processing and signal modeling' (John Wiley & Sons, New York, USA, 2001)
3 de Waele, S., and Broersen, P.M.T.: 'Modeling radar data with time series models'. Proc. EUSIPCO, Tappere, Finland, 2000, Re. 116
4 Li, J., Miyashita, K., Kato, T., and Miyazaki, S.: 'GPS time series modeling by autoregressive moving average method; Application to the crustal deformation in central Japan', *Earth Planets Space*, 2000, **52**, pp. 155–162
5 Kay, S.M.: 'Noise compensation for autoregressive spectral estimates', *IEEE Trans. Acoust. Speech Signal Process.*, 1980, **28**, (3), pp. 292–303
6 Kanai, H., Abe, M., and Kido, K.: 'Accurate autoregressive spectrum estimation at low signal-to-noise ratio using a phase matching technique', *IEEE Trans. Acoust. Speech Signal Process.*, 1987, **35**, (9), pp. 1264–1272
7 Hu, H.T.: 'Linear prediction analysis of speech signals in the presence of white Gaussian noise with unknown variance', *IEE Proc., Vis. Image Signal Process.*, 1998, **145**, (4), pp. 303–308
8 Zheng, W.X.: 'A least-squares based method for autoregressive signals in the presence of noise', *IEEE Trans. Circuits Syst. II*, 1999, **46**, (1), pp. 81–85
9 Yahagi, T., and Hasan, M.K.: 'Estimation of noise variance from noisy measurements of AR and ARMA systems: application to blind identification of linear time-invariant systems', *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, 1994, **E77**, (5), pp. 847–855
10 Hasan, M.K., Ahmed, K.I.U., and Yahagi, T.: 'Further results on autoregressive spectral estimation from noisy observations', *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, 2001, **E84-A**, (2), pp. 577–588
11 Hasan, M.K., Fattah, S.A., and Khan, M.R.: 'Identification of noisy AR systems using damped sinusoidal model of autocorrelation function', *IEEE Signal Process. Lett.*, 2003, **10**, (6), pp. 1303–1313
12 Sun, W., and Yahagi, T.: 'A new overfitting lattice filter for ARMA parameter estimation with additive noise', *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, 1992, **E75-A**, (2), pp. 247–254
13 Hasan, M.K., Yahagi, T., and Henriques, M.A.A.: 'An approach to ARMA model identification from noise corrupted output measurements', *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, 1994, **E77-A**, (4), pp. 726–730
14 Rojo-Álvarez, J.L., Martínez-Ramón, M., de Prado-Cumplido, M., Artés-Rodríguez, A., and Figueiras-Vidal, A.R.: 'Support vector method for robust ARMA system identification', *IEEE Trans. Signal Process.*, 2004, **52**, (1), pp. 155–164
15 Ljung, L.: 'System Identification: theory for the user'. (Prentice Hall, Englewood Cliffs, NJ, USA, 1999, 2nd edn.)
16 Zhang, X.-D., and Takeda, H.: 'An approach to time series analysis and ARMA spectral estimation', *IEEE Trans. Acoust. Speech. Signal Process.*, 1987, **35**, (9), pp. 1303–1313
17 Li, S., and Dickinson, B.W.: 'Application of the lattice filter to robust estimation of AR and ARMA models', *IEEE Trans. Acoust. Speech Signal Process.*, 1988, **36**, (4), pp. 502–512
18 Ribeiro, M.I., and Moura, J.M.F.: 'LD^2-ARMA identification algorithm', *IEEE Trans. Signal Process.*, 1991, **39**, (8), pp. 1822–1835
19 Li, S., and Dickinson, B.W.: 'An efficient method to compute consistent estimates of the AR parameters of an ARMA model', *IEEE Trans. Autom. Control*, 1986, **AC-31**, (3), pp. 275–278
20 Fattah, S.A.: 'Identification of autoregressive systems at a very low SNR using damped cosine model of autocorrelation function'. MSc Engg. Thesis, BUET, 2002
21 Djurić, P.M., and Kay, S.M.: 'Order selection of autoregressive models', *IEEE Trans. Signal Process.*, 1992, **40**, (11), pp. 2829–2833