Problem Sheet 1

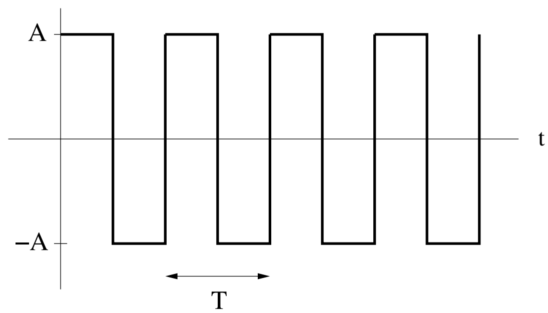
1. (a) From the definition for average power, show that the average power of a periodic signal $x(t)$ with period T_0 is

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 \, dt$$

   (b) Using this expression, verify that the average power of $A\cos(2\pi f t + \theta)$ is $A^2/2$.

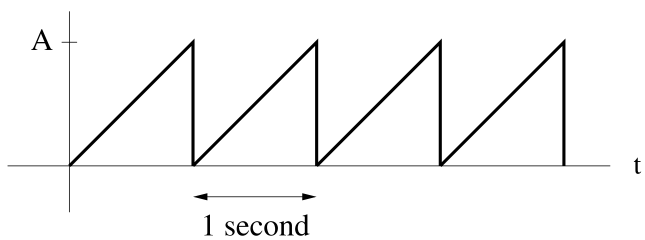
2. Determine the average power of each of the following periodic signals:

   (a)



[A^2]

   (b)



[$A^2/3$]

3. Draw the magnitude frequency spectrum of the following signals:

   (a) $x(t) = \cos^2(2\pi f_o t)$

   (b) $x(t) = m(t)\cos(2\pi f_c t)$, where $m(t) = \cos(2\pi f_m t)$

Problem Sheet 2

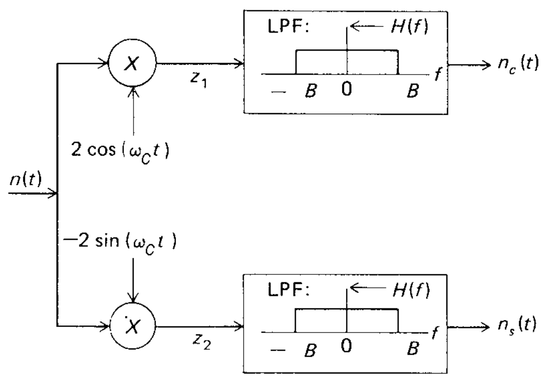
1. Consider the randomly-phased sinusoid

$$n(t) = A\cos(2\pi f_c t + \theta)$$

where A and f_c are constant amplitude and frequency, respectively, and θ is a random phase angle uniformly distributed over the range $[0, 2\pi]$. Calculate the mean and mean square of $n(t)$ using ensemble averages.

[0, $A^2/2$]

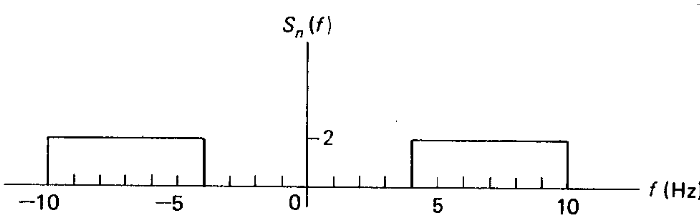
2. By evaluating the signals labeled z_1 and z_2, show that the following system could be used to obtain $n_c(t)$ and $n_s(t)$ from a bandpass noise signal $n(t)$. Assume that $f_c > 2B$.



3. Consider a bandpass noise signal having the power spectral density shown below. Draw the PSD of $n_c(t)$ if the center frequency is chosen as:

(a) $f_c = 7$ Hz

(b) $f_c = 5$ Hz



Problem Sheet 3

1. Consider a message signal with a bandwidth of 10 kHz and an average power of $P = 10$ watts. Assume the transmission channel attenuates the transmitted signal by 40 dB, and adds noise with a power spectral density of:

$$\mathbb{S}(f) = \begin{cases} N_o\left(1 - \frac{|f|}{200\times10^3}\right), & |f| < 200 \times 10^3 \\ 0, & \text{otherwise} \end{cases}$$

where $N_o = 10^{-9}$ watts/Hz.

What is the predetection SNR at the receiver if each of the following modulation schemes is used? Assume that a suitable filter is used at the input of the receiver to limit the out-of-band noise.

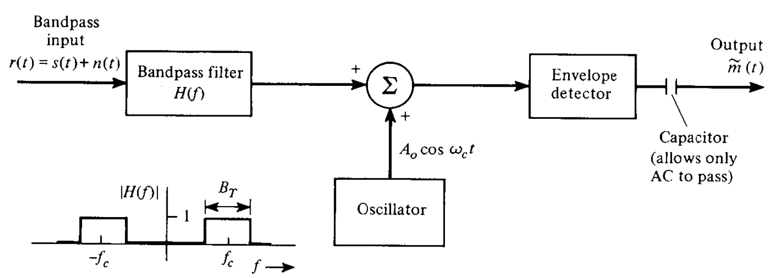
(a) baseband

(b) DSB-SC with a carrier frequency of 100 kHz and a carrier amplitude of $A_c = 1$ v

(c) DSB-SC with a carrier frequency of 150 kHz and a carrier amplitude of $A_c = 1$ v

[17.1 dB, 14 dB, 17 dB]

2. Consider the receiver shown below. Let the input be a DSB-SC signal plus white noise with a power spectral density of $N_o/2$.



(a) For A_o large, show that this receiver acts like a product detector.

(b) Derive an expression for the *output* signal-to-noise ratio when A_o is large.

3. For an AM envelope detector, what value of modulation index μ gives the maximum output SNR (assume that the noise power is small)? What is the resulting expression for output SNR?

Problem Sheet 4

1. (a) Consider a binary ASK modulated-carrier system, which employs coherent demodulation. Let the carrier amplitude at the detector input be 0.7 volts. Assume an additive white Gaussian noise channel with a standard deviation of 0.125 volts. If the binary source stream has equal probabilities of occurrence of a symbol 0 and a symbol 1, estimate the probability of detection error.

   (b) If PSK was used instead, what is the probability of error?

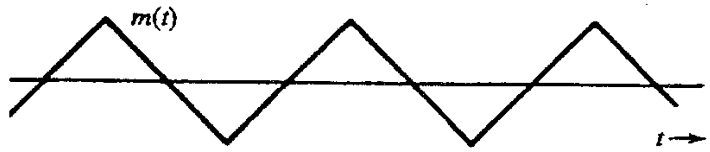
   [2.6×10^{-3}, 1.07×10^{-8}]

2. A PCM output is produced by a uniform quantizer that has 2^n levels. Assume that the input signal is a zero-mean Gaussian process with standard deviation σ.

   (a) If the quantizer range is required to be $\pm 4\sigma$, show that the quantization signal-to-noise ratio is $6n - 7.3$ dB.

   (b) Write down an expression for the probability that the input signal will overload the quantizer.

3. The input to a uniform n-bit quantizer is the periodic triangular waveform shown below, which has a period of $T = 4$ seconds, and an amplitude that varies between $+1$ and -1 volt.



   Derive an expression for the signal-to-noise ratio (in decibels) at the output of the quantizer. Assume that the dynamic range of the quantizer matches that of the input signal.

   [$\text{SNR}_{\text{dB}} = 6.02n$]

4. Consider a binary source alphabet where a symbol 0 is represented by 0 volts, and a symbol 1 is represented by 1 volt. Assume these symbols are transmitted over a baseband channel having uniformly distributed noise with a probability density function:

$$p(n) = \begin{cases} \frac{1}{2}, & |n| < 1 \\ 0, & \text{otherwise.} \end{cases}$$

   Assume that the decision threshold T is within the range of 0 to 1 volt. If the symbols are equally likely, derive an expression for the probability of error.

Problem Sheet 5

1. By what fraction is it theoretically possible to compress a binary bit stream if the probability of occurrence of one of the symbols is:

   (a) 0.5

   (b) 0.2

   (c) 0.01

2. Using the Huffman coding procedure, construct a coding scheme for each of the 5-symbol alphabets having the following probabilities. Also calculate the average codeword length and compare it with the source entropy.

   (a) 0.5, 0.25, 0.125, 0.0625, 0.0625

   (b) 0.4, 0.25, 0.2, 0.1, 0.05

3. Assume that a computer has 110 characters on its keyboard, and that each character is represented using binary words.

   (a) How many bits are required to represent each character?

   (b) How fast can the characters be sent (in characters per second) over a telephone channel with a bandwidth of 3.2 kHz to obtain a signal-to-noise ratio at the receiver of 20 dB ?

   [7 bits, 3.04×10^3 characters / second]

4. A discrete source produces the symbols A and B with probabilities $p_A = \frac{3}{4}$ and $p_B = \frac{1}{4}$ at a rate of 100 symbols/second. The symbols are grouped into blocks of two and encoded as follows:

Grouped symbols	Binary code
AA	1
AB	01
BA	001
BB	000

   Is this code optimum? If not, how efficient is it?

   [No; 96% efficient]