Topic 11

Operational Amplifier (op-amp)

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An op-amp (operational amplifier) is a circuit with two inputs and one output.

\[ Y = A(V_+ - V_-) \]

The equivalent circuit of an op-amp is shown here.

For an ideal op-amp, \( R_i = \infty \), \( R_o = 0 \), \( A = \infty \)
Real Operational Amplifier

- Real op amp we use is a MCP6002 – it has two op amps in one package.

- Integrated circuit pins are numbered anti-clockwise from blob or notch (when looking from above).
Negative Feedback

- In a central heating system, if the temperature falls too low the thermostat turns on the heating, when it rises the thermostat turns it off again.
- **Negative feedback** is when the occurrence of an event causes something to happen that counteracts the original event.

- If op-amp output $Y$ falls then $V_-$ will fall by the same amount so $(V_+ - V_-)$ will increase.

- This causes $Y$ to rise since

  \[ Y = A (V_+ - V_-) . \]

  \[ Y = A (X - Y) \]

  \[ Y (1 + A) = AX \quad \Rightarrow \quad Y = \frac{1}{1+1/A} X \quad \rightarrow X \quad \text{for large} \ A \]

- If $Y = A (V_+ - V_-)$ then $V_+ - V_- = Y/A$ which, since $A \approx 10^5$, is normally very very small.

**Golden Rule:** **Negative feedback adjusts the output to make** $V_+ \approx V_-$. 
Analysing op-amp circuits

- Nodal analysis is simplified by making some assumptions.

Note: The op-amp often need two power supply connections. In our case, +5V and GND. These are almost always omitted from the circuit diagram. The currents only sum to zero (KCL) if all five connections are included.

1. **Check for negative feedback**: to ensure that an increase in \( Y \) makes \( (V_+ - V_-) \) decrease, \( Y \) must be connected (usually via other components) to \( V_- \).
2. **Assume** \( V_+ = V_- \): Since \((V_+ - V_-) = YA\), this is the same as assuming that \( A = \infty \). Requires negative feedback.
3. **Assume zero input current**: in most circuits, the current at the op-amp input terminals is much smaller than the other currents in the circuit, so we assume it is zero.
4. **Apply KCL at each op-amp input node separately** (input currents = 0).
5. **DO NOT apply KCL at output node** (output current is unknown).
Non-inverting amplifier

• Circuit has input voltage $X$ and output voltage $Y$.
• The circuit gain $\Delta \frac{Y}{X}$.
• Applying steps 1 to 3:
  1. Negative feedback OK.
  2. $V_- = V_+ = X$
  3. Zero input current at $V_-$ means $R_2$ and $R_1$ are in series ($\Rightarrow$ same current) and form a voltage divider. So $X = \frac{R_1}{R_1+R_2} Y$.

\[
Y = \frac{R_1+R_2}{R_1} X = \left( 1 + \frac{R_2}{R_1} \right) X = +4X
\]

• Non-inverting amplifier because the gain $Y/X$ is positive.
  • Consequence of $X$ connecting to $V_+$ input.
  • Can have any gain $\geq 1$ by choosing the ratio $R_2/R_1$.

• Cause/effect reversal: Potential divider causes
  • Feedback inverts this so that $Y = 4V_+$. 
Voltage Follower

- A special case of the non-inverting amplifier with $R_1 = \infty$ and/or $R_2 = 0$.

- Gain is $1 + R_2/R_1 = 1$.

- Output $Y$ “follows” input $X$.

- **Advantage**: Can supply a large current at $Y$ while drawing almost no current from $X$. Useful if the source supplying $X$ has a high resistance.

- **Without voltage follower**: $Y = 0.01U$.
- **With voltage follower**: $Y = U$.

- Although the **voltage gain** is only 1, the **power gain** is much larger.
Inverting Amplifier

- Negative feedback OK.
- Since \( V_+ = 0 \), we must have \( V_- = 0 \).
- KCL at \( V_- \) node:
  \[
  \frac{0-Y}{R_1} + \frac{0-Y}{R_2} = 0 \quad \Rightarrow \quad Y = -\frac{R_2}{R_1}X = -3X.
  \]

- **Inverting Amplifier** because gain \( Y/X \) is negative. Consequence of \( X \) connecting to the \( V_- \) input (via \( R_1 \)).
  - Can have any gain \( \leq 0 \) by choosing the ratio \( R_2/R_1 \).
  - Negative feedback holds \( V_- \) very close to \( V_+ \).
  - If \( V_+ = 0V \), then \( V_- \) is called a virtual earth or virtual ground.

- **Nodal Analysis**: Do KCL at \( V_+ \) and/or \( V_- \) to solve circuit. When analysing a circuit, you never do KCL at the output node of an op-amp because its output current is unknown. The only exception is if you have already solved the circuit and you want to find out what the op amp output current is (e.g. to check it is not too high).
Inverting Summing Amplifier

- We can connect several input signals to the inverting amplifier.

- As before, $V_\text{=} = 0$ is a virtual earth due to negative feedback and $V_+ = 0$.

- KCL at $V_-$ node:
  
  $$\frac{0-X_1}{R_1} + \frac{0-X_2}{R_2} + \frac{0-X_3}{R_3} + \frac{0-Y}{R_F} = 0$$

  $$\Rightarrow \quad Y = -\left(\frac{R_F}{R_1} X_1 + \frac{R_F}{R_2} X_2 + \frac{R_F}{R_3} X_3\right)$$

  $$\Rightarrow \quad Y = -(8X_1 + 4X_2 + 4X_3).$$

- $Y$ is a weighted sum of the input voltages with the weight of $X_i$ equal to

  $$\frac{R_F}{R_i} = G_i R_F.$$

- **Input Isolation**: The current through $R_I$ equals $\frac{X_1-0}{R_1}$ which is not affected by $X_2$ or $X_3$. Because $V_-$ is held at a fixed voltage, the inputs are isolated from each other.
Differential Amplifier

- A 2-input circuit combining inverting and non-inverting amplifiers.

- Linearity ⇒ \( Z = aX + bY \).

- Use superposition to find \( a \) and \( b \).

  - **Find** \( a \): Set \( Y = 0 \). KCL at \( V_+ \) node ⇒ \( V_+ = 0 \). We now have an inverting amplifier, so \( Z = -R_2/R_1 \) \( Z = -3X \) ⇒ \( a = -3 \).

  - **Find** \( b \): Set \( X = 0 \). We can redraw circuit to make it look more familiar: a potential divider followed by a non-inverting amplifier.

- \( R_3 \) and \( R_4 \) are a potential divider (since current into \( V_+ \) equals zero), so

  \[
  V_+ = \frac{R_4}{R_3+R_4} Y = \frac{3}{4} Y.
  \]

- The non-inverting amplifier has a gain of \( \frac{R_1+R_2}{R_1} = 4 \).

- The combined gain is \( b = \frac{R_4}{R_3+R_4} \times \frac{R_1+R_2}{R_1} = \frac{3}{4} \times 4 = +3 \).

- Combining the two gives \( Z = 3 \ (Y - X) \). The output of a differential amplifier is proportional to the difference between its two inputs.
Choosing Resistor Values

◆ The behaviour of an op-amp circuit depends on the ratio of resistor values:
◆ \( Gain = -\frac{R_2}{R_1}. \) How do you choose between 3Ω/1Ω, 3kΩ/1kΩ, 3MΩ/1MΩ and 3GΩ/1GΩ?

◆ Small resistors cause large currents.
  • If \( X = \pm 1V, \) then \( Y = \mp 3V, \)
    and \( I = \frac{Y-0}{R_2} = \mp 1 \text{ A}. \)
  • However typical op-amps can only supply \( \pm 5mA, \) so the circuit will not work.

◆ Large resistors increase sensitivity to interference and to op-amp input currents.
◆ If the bias current into \( V_\text{–} \) is \( I_B = 1 \text{ nA}, \) then KCL at \( V_\text{–} \) gives
  \[
  \frac{0-Y}{R_2} + \frac{0-X}{R_1} + I_B = 0 \Rightarrow Y = -\frac{R_2}{R_1}X + I_B R_2 = -3X + 3
  \]
  instead of \( Y = -3X. \)
◆ Within wide limits, the absolute resistor values have little effect. However you should avoid extremes.
Bandwidth of real op-amp

- The gain of an op-amp is very high at low frequency, but it decreases rapidly as the signal frequency increases as shown in the Gain vs Frequency plot for our op-amp.

- The gain at 1Hz is more than $10^5$.
- The corner frequency is around 10 Hz.
- The gain then drops off like a RC characteristic, at around -20dB/decade (or x 0.1 / decade).
- Op-amps are characterised by the frequency at which the gain becomes unity. This is known as the **unity gain bandwidth**.
- In the case of MCP6002, this is approximately 1MHz.
Benefits of negative feedback

- Using negative feedback in our op-amp circuit help to improve bandwidth.
- As the gain of the amplifier is reduced, the bandwidth is increased due to negative feedback.

- Without proving it on this course, for op-amp with negative feedback,

  \[ \text{GAIN} \times \text{BANDWIDTH} = \text{CONSTANT} \]

- This is known as the gain-bandwidth product of the op-amp.
- For MCP6002 opamp we use in Lab3, the gain-bandwidth product is around 1MHz.
- Since this product is constant, if the gain is reduced, the bandwidth is increased. This is shown in the graph here.
Summary

◆ Ideal properties:
  • Zero input current
  • Infinite gain
  • Do not use KCL at output (except to determine output current).

◆ Negative Feedback circuits:
  • Assume $V_+ = V_-$ and zero input current
  • Standard amplifier circuits:
    ➤ Non-inverting gain = $1 + R_2/R_1$
    ➤ Inverting gain = $-R_2/R_1$
    ➤ Summing amplifier
    ➤ Differential Amplifier

◆ Positive feedback circuits:
  • $VOUT = \pm V_{max}$ (no good for an amplifier)
  • Schmitt Trigger: switches when $V_+ = V_-.$

◆ Choosing resistors: not too low or too high.