This lecture will be about an additional component – the capacitor. Unlike resistor, the behaviour of the current flowing through a capacitor and the voltage across a capacitor depends on whether the signal is a dc voltage source, an ac voltage source (e.g. a sine wave) or a step signal (such as a square or clock signal).
Besides resistors, capacitors are one of the most common electronic components that you will encounter. Sometimes capacitors are components that one would deliberately add to a circuit. Other times, capacitors are side effects that come about even if we don’t want them.

The simplest capacitor is formed by an insulating material (known as dielectric) sandwiched between two parallel conducting plates. When a voltage potential is applied to the two ends, charge accumulates on the plates.

In capacitors, voltage \( v \) is proportional to the charged stored \( q \). The constant of proportionality is the capacitance \( C \). Since current \( i \) is the rate of change of charge (i.e. the flow of charge), the relationship between \( v \) and \( i \) involves differentiation or integration.

Capacitance is measured in Farads. However, a 1F capacitor is considered large! Most of the capacitors you will use in this module are of the order of microFarad (\( \mu \)F).
Since there is an insulating layer between the two conducting plates of a capacitor, DC current cannot flow through a capacitor. So always remember: **A CAPACITOR IN SERIES BLOCKS DC part of a signal.** However, alternating or changing current can flow through a capacitor. The best analogy is the flow of air from inside to outside of the building. Assuming that the window is completely sealed, air inside the building cannot flow to the outside in spite of the pressure difference between the two sides. The pressure difference is analogous to the voltage potential at the two end of the capacitor. The air flow is like DC current.

However, if the air pressure difference is alternating, there can be air movement on both sides as shown in the diagram.
Connecting two capacitors in parallel results in their capacitances ADD-ed together (just like resistors in series).

Connecting two capacitors in series results in their capacitances combined in a product/sum manner, similar to two parallel resistances.
Before we embark on circuits using capacitors, let us examine one of the signals that you explored in Lab 1 in the past week – the exponential signal.

Exponential signals are interesting. Here the rate of change is shown in terms of the time constant $\tau$ (tau).

The following facts are worth remembering:
1. For exponential rise, the signal reaches 63% at one $\tau$, and 95% at $3\tau$.
2. For exponential fall, the signal reaches 37% at one $\tau$, and 5% at $3\tau$.
For the circuit shown here, assume the capacitor has zero charge (and 0v) at \( t = 0 \). The switch is closed, connecting the circuit to the constant voltage source \( V_s \). Initially, the voltage drop across the resistor is \( V_s \). A current of \( \frac{V_s}{R} \) flows from the source to capacitor. However, as \( V \) increases, the current \( I \) decreases. This results in the exponential drop of changing current and an exponential rise of the capacitor voltage. We will examine mathematically how \( i(t) \) and \( V(t) \) changes over time later.
For now, it is important to consider the parameter known as the time constant. If R is large, the charging current I is small, and it takes longer to charge the capacitor. For a given R, if C is large, it can store more charge for a given voltage, therefore the time needed to charge a capacitor to a certain voltage is proportional to the produce R x C. RC is known as the time constant of this circuit.
We will now derive the exponential equation formally.

For that you need to be familiar with solving first-order differential equations from your maths lectures.

We want to solve: 

\[ RC \frac{dv}{dt} + v = V_s \]

\[ \frac{dv}{dt} = \frac{V_s - v}{RC} \]

\[ \Rightarrow \frac{dt}{RC} = \frac{dv}{V_s - v} \]

Integrate both sides, we get:

\[ \frac{t}{RC} = -\ln(V_s - v) + A, \text{ where A constant of integration} \]

Use boundary condition: when \( t = 0, v = 0 \):

\[ \frac{0}{RC} = -\ln(V_s - 0) + A \]

\[ \Rightarrow A = \ln V_s \]

Therefore

\[ \frac{t}{RC} = -\ln(V_s - v) + \ln V_s = \ln \frac{V_s}{V_s - v} \]

\[ \Rightarrow e^{-\frac{t}{RC}} = \frac{V_s}{V_s - v} \Rightarrow v = V_s(1 - e^{-\frac{t}{RC}}) \]
Let us now consider what happens if we charge up the capacitor, then at $t = 0$, discharges it. The equations is also easy to solve and it is clear that the discharge profiles in $V$ and $I$ also follow the exponential curves.
We will now consider some common use of capacitors in electronic circuit. The first application of a capacitor is to remove or “block” DC part of an electrical signal.

Consider the circuit above. The input signal Vin has a 3V DC voltage on which is a sinewave. By choosing the correct value for C and R, you can obtain an output signal, Vout, which is has 0V DC (i.e. DC is blocked) and only the sinusoidal signal remains.

The capacitor is acting like a window of an airplane. The constant pressure on outside the airplane (DC value) is not affecting the pressure inside the cabin. However, vibration of the window (sinusoidal component) can pass through to the cabin if the window is sufficient flexible relative to the "resistance" of the cabin air.
Let us take another view of this circuit besides its DC blocking (or AC coupling) quality.

If C = 0.1μF and R = 10kΩ, you will find that the circuit will strongly suppress a 5Hz sinewave, but if the input signal has a much higher frequency (say 100 Hz), the signal is passed through the capacitor with little or no reduction.

If you plot signal gain (or attenuation), i.e. Vout/Vin, vs Frequency, you can a characteristic as shown in the slide. The low the Y-axis value, the lower the gain and stronger the suppression. In fact if you project the plot toward frequency = 0 (that is, to DC), the gain is –infinity (dB). That’s why this circuit can “BLOCK” DC signal.

Note that the plot here has both axes in logarithmic scale. The frequency axis is clearly log scale. How about the y-axis? It is plotting the gain of the circuit in dB or decibel. dB is also in log scale as will be shown in the next slide.
In electronics, we often ask the question: what is the ratio of the output to input signal? This ratio is important. If it is larger than 1, the electronic circuit provides gain (we also call this amplification as will be seen in a later lecture). If the ratio is lower than 1, then the circuit provides attenuation (or suppression).

However, we often express this ratio or gain not just as such a ratio, but in logarithmic scale. Why? It turns out that expressing such ratio in log scale provide us with much higher dynamic range. For example, our human perception is generally in log scale, not linear scale. Our hearing and seeing sensitivity is not linear, but logarithmic.

Log - base 10 of the ratio is known as a bel. Scaling this further up by a factor of 10 is known as decibel. That ratio is generally considering the ratio of output power to input power (not voltage). However, since power is voltage square, we found that the common equation to find voltage gain (ratio) in decibel is given by the equation:

\[ A \text{ (in } dB) = 20 \log_{10} \left| \frac{V_{out}}{V_{in}} \right| \]

This is an equation that you must commit to memory – very useful for many things!
There are many different types of capacitors depending on the method of construction and the materials used. The most common three types are: polystyrene, ceramic and electrolytic. You will choose which to use depending on the operating signal frequency. Polystyrene and ceramic capacitors are good for a wide frequency range, particularly at very high frequencies (say over 1MHz). However, they only come in fairly low capacitance value.

For large capacitances, one would use electrolytic capacitors. Electrolytic capacitors are only good for fairly low frequencies. Furthermore, they have polarity, i.e. it has a positive and a negative terminal. You must connect the capacitor +ve terminal to the more positive voltage than the –ve terminal.
The take-home message that you must remember is that:

**Capacitor tries to keep its voltage constant.**
Summary

- **Capacitor:**
  - \( i = C \frac{dv}{dt} \)
  - Parallel capacitors add in value
  - \( v \) across a capacitor never changes instantaneously
  - When charging a capacitor with a constant DC voltage through a resistor, the capacitor voltage rises exponentially
  - The time constant of the exponential is the product of \( R \) and \( C \).