IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2017

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

Engineering Analysis EA 2.3 – Electronics 2

3rd May 2017: 10.00 to 11.30 (one hour thirty minutes)

This paper contains EIGHT questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

This is a CLOSED BOOK Examination.
1. A signal $x(t)$ can be modelled mathematically as:

$$x(t) = 10 \cos (100t + \frac{\pi}{3}) + 2$$

(i) What is the dc value of $x(t)$? [2]

(ii) What is the frequency in Hz and phase angle in degrees of $x(t)$ at $t = 0$? [2]

(iii) Rewrite $x(t)$ in exponential form. [4]

(iv) Sketch the amplitude spectrum $|X(j\omega)|$ of the signal. [4]

2. An electronic system converts continuous time analogue signals into discrete time digital signals for processing in a microprocessor. The analogue to digital converter (ADC) and the digital to analogue converter (DAC) both have a sampling frequency of 10 kHz. Sketch the amplitude spectrum of the signal between 0 Hz and the Nyquist frequency for the following signals if the microprocessor system simply passes signals from input to output. Briefly explain and justify your answers.

(i) $x(t) = 0.5e^{-j6283t} + 0.5e^{j6283t}$ [5]

(ii) $x(t) = 2.5 \sin(11000\pi t) + 0.4 \cos(3142t + \pi/9)$ [8]

3. Explain the meaning of “polling” and “interrupt” in an embedded electronics system. What are the relative merits of these two techniques when used in a real-time system? [8]
4. It is given that a second order system has a transfer function of the form:

\[ H(s) = K \frac{\omega_0^2}{s^2 + 2\zeta \omega_0 s + \omega_0^2} \]

where \( \zeta \) is the damping factor and \( \omega_0 \) is the natural (resonant) frequency.

(i) Find the pole locations for a system with the following transfer function and derive the system’s damping factor and natural frequency.

\[ H(s) = \frac{2600}{s^2 + 10s + 2600} \]

(ii) If the system is now modified so that its natural frequency remains the same but the system is now critically damped, write down its transfer function.

5. The following differential equation describes the relationship between the output \( y(t) \) and the input \( x(t) \) of a second-order linear system:

\[ \frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 9y(t) = 2 \frac{dx}{dt} + 9x(t) \]

Given that \( Y(s) \) and \( X(s) \) are the Laplace Transforms of \( y(t) \) and \( x(t) \) respectively, write down the transfer function \( H(s) = Y(s)/X(s) \) for the system.

6. A discrete-time shift-invariant system is specified by the following difference equation:

\[ y[n] - 0.5y[n - 1] = x[n] \]

(i) Derive the output sequence \( y[0] \ldots y[9] \) given that the input \( x[i] = 1 \), for \( i = 1 \) and 2, and \( x[i] = 0 \) for all other values of \( i \). You may assume that \( y[-1] = 0 \).

(ii) Find the transfer function \( H(z) \) for this discrete-time system. Hence, derive its frequency response.
7. Figure 1 shows a complementary filter used to combine the pitch angle measurements made by an accelerometer and a gyroscope in an Inertia Measurement Unit (IMU).

(i) Explain why neither the accelerometer nor the gyroscope measurements alone is sufficient to provide an accurate pitch angle.

(ii) Derive the relationship between the filtered pitch angle \( p[n] \) and the two input signals: the accelerometer pitch angle \( a[n] \) and the gyroscope pitch angle \( g[n] \). Your answer should be in the form of a difference equation.
8. Figure 2 shows a proportional-integral-derivative (PID) controller with a transfer function of $C(s)$ controlling the behaviour of a first order system with a transfer function of $P(s)$ in a close-loop feedback arrangement. It is known that:

$$P(s) = \frac{1}{1+s}$$

(i) Write down the transfer function of $C(s)$ in terms of the proportional gain $K_p$, the derivative gain $K_d$ and the integral gain $K_i$ gains of the controller.

(ii) If $K_d = K_i = 0$, derive the close-loop transfer function $H(s)$ of the system where

$$H(s) = \frac{Y(s)}{R(s)}.$$  

(iii) Explain the effect of the proportional gain $K_p$, the derivative gain $K_d$ and the integral gain $K_i$ gains of the controller on the close-loop response of the system.