IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2017

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

Engineering Analysis EA 2.3 – Electronics 2

SOLUTIONS

This paper contains EIGHT questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

SOLUTION to Q1

This question tests student's understanding of the frequency domain view of signal.

(i) DC value = 2V because the cosine signal has 0 dc.

[2]

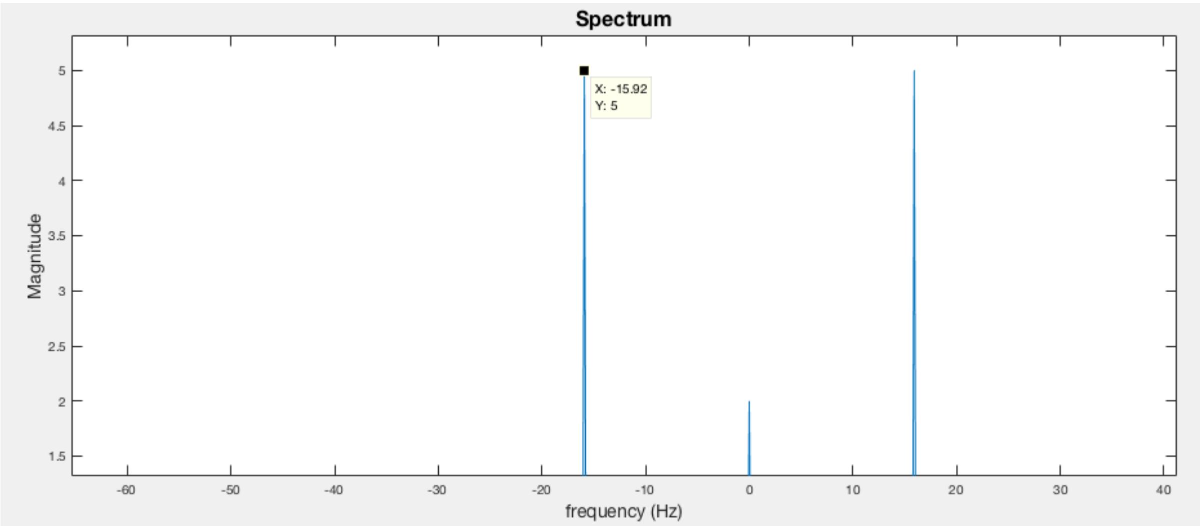
(ii) The frequency $f = \frac{100}{2\pi} = 15.92$ Hz. Phase of the signal is 60 degrees.

[2]

(iii) $x(t) = 5 \times \left(e^{-j(100t+\frac{\pi}{3})} + e^{j(100t+\frac{\pi}{3})}\right) + 2$

[4]

(iv)



Student may give only positive spectrum, which is correct if amplitude is shown as 10. Note that I expect DC component at 2 to be shown. Amplitude of the full spectrum with positive and negative frequency is 5. Phase of pi/3 has no impact on the amplitude spectrum of the signal.

[4]

Generally, student did well in this in this question. A few people made the mistake of leaving out the bracket in the negative frequency term in part (iii), i.e. $-j(100t + \frac{\pi}{3})$. Some students got confused about frequency in rad/sec vs Hz. This is one of easiest questions in the paper.

SOLUTION to Q2

This question tests student's understanding of: 1) the Euler formula and the exponential representation of sinusoids; 2) the Sampling theorem and 3) the impact of aliasing.

(i) $x(t) = 0.5e^{-j6283t} + 0.5e^{+j6283t}$

ω = 6283 rad/sec is the same as 1000Hz. Using Euler's formula,

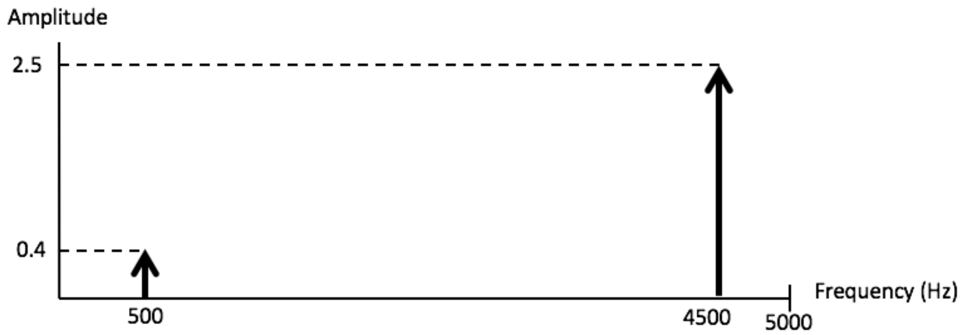
$x(t) = \cos(2\pi \times 1000t)$.

Since sampling frequency is 10KHz, Nyquist frequency is half sampling frequency = 5KHz. There is no aliasing. The question asks for one-sided (i.e. only positive frequency) representation. Hence:

[5]

(ii) $x(t) = 2.5\sin(11000\pi t) + 0.4\cos(3142t + \pi/9)$

$\omega = 11000\pi$ rad/sec is the same as 5500Hz. Since the sampling frequency is 5000Hz, this frequency component is aliased back to 4500Hz. The second frequency component is at 500Hz. Hence the spectrum will be:



[8]

Feedback comments:

Student scored poorest in this question. Many did not understand the principle of aliasing and how it affects the frequency spectrum. Nyquist frequency is ½ x sampling frequency, and is NOT dependent on the signal frequency.

However, some students scored full marks for this question, demonstrating that this question should be do-able.

SOLUTION to Q3:

This question tests student's understanding of interrupt vs polling in a real-time embedded system and their advantages and disadvantages. It is mostly BOOKWORK.

Polling is a technique where the status of a variable is tested repeatedly until some condition is satisfied. For example, the time out flag of a timer may be tested until the desired time has elapsed. Then the program falls through the loop and proceed to other instructions. The disadvantage of using polling are:

1. The program can only check one status at a time, therefore it is difficult to perform multiple tasks;
2. When in a polling loop, the processor is not performing useful computation;
3. If the condition being tested is transient, polling may miss detecting that condition (for example, it is like checking if someone is calling you on the phone without a ringer, and you must check periodically and manually);

The advantage is that polling is easy to implement and the behaviour of the program is predictable.

Interrupt is a hardware technique where an event force the processor to stop whatever it is doing and branch off elsewhere to deal with the interruption by executing the interrupt service routine, called a "call back function". Once the interrupt is serviced, the processor return to the main program and continue with its normal execution.

The advantages of using interrupt in an embedded system are:

1. It is highly efficient – the interrupt service routine is only executed when needed, and there is no polling loop required;
2. You can have multiple interrupts, therefore there can be different background tasks which will be dealt with when necessary. It allows multitasking;

The disadvantage of interrupts are that it can cause unpredictable behaviour. An interrupt service routine can itself be interrupted, and this may cause synchronisation errors in the program.

[8]

Feedback comments:

Some students do not fully understand the meaning of polling and interrupt in a microcontroller. Their answers were often very brief. A few students did not even attempt this question – they are the ones who probably missed the lecture and did not read the notes for this topic. Most students did attempt the question and at least walked away with a few marks.

SOLUTION to Q4:

This question tests student's ability to associate physical meaning to a transfer function of a system.

(i)

$$H(s) = \frac{2600}{s^2 + 10s + 2600} = \frac{2600}{(s - (-5 + j51))(s - (-5 - j51))}$$

Therefore the system poles are at -5 ± j51. Since

$$H(s) = K \frac{\omega_0{}^2}{s^2 + 2\zeta\omega_0 s + \omega_0{}^2}$$

and the damping factor is 0.1 and natural frequency is 51 rad/sec.

[8]

(ii)

If the system is critically damped, $\zeta = 1$. Therefore, the system's transfer function is now:

$$H(s) = \frac{2600}{s^2 + 102s + 2600}$$

[4]

Feedback comments:

Most students very well in this questions. They obviously understood where poles of a system came from and how the coefficients in the denominator polynomial are related to the physical characteristics (such as damping factor and resonant frequency).

SOLUTION to Q5:

This question tests student's ability to convert a differential equation to an algebraic equation in the form of a transfer function.

$$(s^2 + 6s + 9)\,Y(s) = (2s + 9)X(s)$$

Therefore, $H(s) = \frac{2s+9}{s^2+6s+9}$

[6]

Feedback comments:

This is the easiest question of the whole paper and nearly everyone got full marks for this question. Maybe I should have made this question a little bit harder! Only a couple of students did not get the right answer.

SOLUTION to Q6:

This question tests students understanding of discrete time system specified as a difference equation.

(i)

i	0	1	2	3	4	5	6	7	8	9
x[i]	0	1	1	0	0	0	0	0	0	0
y[i]	0	1	1.5	0.75	0.375	0.188	0.093	0.047	0.023	0.012

[6]

(ii)

$$y[n] - 0.5y[n-1] = x[n]$$

Take z-transform on both sides:

$$Y(z) - \ 0.5z^{-1}Y(z) = X(z)$$
$$\Longrightarrow (1 - 0.5)Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(Z)} = \frac{1}{1 - 0.5Z^{-1}} = \frac{Z}{Z - 0.5}$$

[4]

To find the frequency response of this discrete-time system, let $z = e^{j\Omega}$:

$$|H(e^{j\Omega})|^2 = H(e^{j\Omega})H(e^{-j\Omega})$$
$$= \frac{e^{j\Omega}e^{j\Omega}}{(e^{j\Omega} - 0.5)(e^{-j\Omega} - 0.5)}$$
$$= \frac{1}{(1 + 0.25 - 0.5(e^{j\Omega} + e^{-j\Omega})}$$
$$= \frac{1}{(1.25 - \cos(\Omega))}$$

Therefore

$$|H(e^{j\Omega})| \quad = \sqrt{\frac{1}{(1.25 - \cos(\Omega))}}$$

[6]

Feedback comments:

Most student can do part (i) and first bit of (ii), but all except a handful could do the frequency response.

SOLUTION to Q7:

This question tests student's understanding of IMU and how to process signals from accelerometer and gyroscope.

(i) Angle measurements from an accelerometer is inherently noisy because the angle position is derived from acceleration due to gravity. However, if the device is in motion, this angle measurement will also include the acceleration due to the motion. There is no way to tell the two apart. In contrast, the gyroscope measure the rate of change in angle. To obtain angle measurement from the gyroscope reading, we need to integrate the measurement over time. This gives a good angle measurement, but accumulates offset errors. This error manifest itself as a large offset value, also known as drift.

[6]

(ii) The difference equation is:

$$p[n] = p[n-1] + \alpha\, g[n] + (1-\alpha)\, a[n]$$

[6]

Feedback comments:

Part (i) is quite easy – mostly bookwork from the Laboratory experiment. Many just stated that the accelerometer is noisy, without saying where the noise came from. Some also just stated that the gyroscope has high drift, but not explaining what is the cause of this drift voltage.

Most can do part (ii), but a few could not derive the different equation from the system block diagram.

SOLUTION to Q8:

Each student was part of a 4-people team in designing a controller to balance a 2-wheel vehicle. This question tests student's understanding of the PID controller used.

(i) (Bookwork)

$$C(s) = K_p + K_d s + K_i \frac{1}{s}$$

[4]

(ii) The loop transfer function (with Kd=Ki=0) is:

$$L(s) = C(s)P(s) = \frac{K_p}{1+s}$$

Therefore the close-loop transfer function is:

$$H(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{K_p}{1+s}}{1+\frac{K_p}{1+s}} = K_L \frac{1}{1+\tau_L s}$$

$$K_L = K_p/(1+K_p)$$

$$\tau_L = {}^{1}\!/\!(1+K_p)$$

[10]

(iii) The proportional term provides the main feedback control. As Kp is increased, the system response time improves. However, if Kp is too high, the system may become unstable and starts to oscillate. The derivative term provides prediction of the control variable, and therefore improve response time further, but may cause even more stability issue. The integral terms will reduce the error and improve stability.

[7]

Feedback comments:

Some students have no clue about this question. I suspect those are the ones who did not attended any of my feedback control lecture, nor helped to implement the PID controller on the self-balancing Dancing Segway project. Those who worked on the project seriously and attended the lectures had no difficulty in getting (i) – very easy!

Part (ii) is also not difficult. They do not need to cast the answer in the way I have shown here. Many simply gave:

$$H(s) = \frac{K_p}{1+s+K_p}$$

which would have gained full marks for this part.

Most student did not explain (iii) well.