IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2018

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

Engineering Analysis EA 2.3 – Electronics 2

SOLUTIONS

This paper contains EIGHT questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

SOLUTION to Q1

1. You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

   (i) What would you choose as the sampling frequency for the ADC? Why? [4]

   (ii) How many bits must the ADC have as converted values? Why? What is the resolution of the ADC in volts? [4]

   (iii) You were also told that the transducer could pick up interference of unknown frequencies from its surrounding. State with justifications and assumptions how you may avoid your captured signal being corrupted by such interference. [4]

This question tests student's understanding of sampling theorem, ADC resolution and problem of aliasing.

(i) Breathing rate is 40/60 Hz, and we want to capture signals at least 100 times faster. Maximum frequency is about 66.7Hz. Sampling theorem dictates that the sampling rate must be at least 2x of maximum frequency. So, must use sampling rate of 133Hz at least. However, for easy reconstruction, we usually use a practical sampling rate a few times higher than this (say 2.5x) at just over 300Hz. Accept well justified answers.

(ii) 0.05% = 1 in 2000. Therefore, minimum number of bits in ADC is 11-bits (2^{11}). The resolution of the ADC in volts would be 3.3V/2048 = 1.61mV.

(iii) To prevent corrupting our base-band signal due to aliasing effect or frequency folding due to sampling, we need to first filter the signal with an anti-aliasing filter. If we choose a sampling frequency of 1kHz, then we need to use a lowpass filter on the breathing signal before sampling that cut out all signals (at least 40dB attenuation) for signals above 0.5kHz.

Comments: Most students did reasonably well in this question. A small proportion of students did not get (ii) due to lack of understanding between number of bits of a ADC to the accuracy required. Majority of students did not mention anti-aliasing filter requirement in (iii), and lost 2 marks.

SOLUTION to Q2

2. A signal x(t) can be modelled mathematically as:

$$x(t) = \cos(62.8t) + 0.5\sin(125.6t + \frac{\pi}{2})$$

(i) Sketch the waveforms $\cos(62.8t)$ and $0.5\sin(125.6t + \frac{\pi}{2})$ for $0 \leq t \leq 0.2$. Hence sketch the signal $x(t)$ for $0 \leq t \leq 0.2$.

[8]

(ii) Rewrite $x(t)$ in exponential form. (There is no need to simplify the equation.)

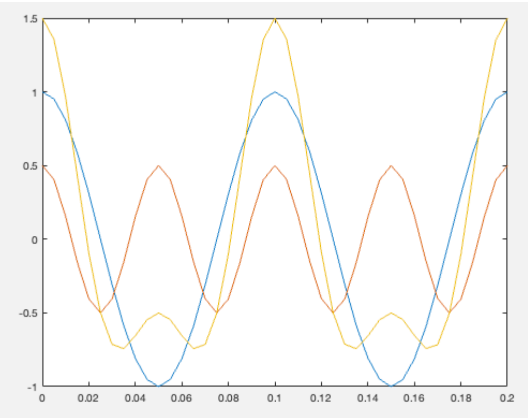
[5]

(iii) Sketch the amplitude spectrum $|X(j\omega)|$ of the signal.

[3]

This question tests student's understanding of: 1) using equation to model a signal; 2) sketching waveforms; 3) the Euler formula and the exponential representation of sinusoids; 2) spectral representation of signals.

(i) There are two signals: cosine signal is 10Hz, and the sinewave is 20Hz. Sine wave has a phase delay of 90 degrees.



(ii) Straight forward application of Euler's formula:

$$x(t) = \frac{1}{2}\left(e^{+62.8jt} + e^{-62.8jt}\right) + \frac{1}{4j}\left(e^{+j(125.6t+\frac{\pi}{2})} - e^{-j(125.6t+\frac{\pi}{2})}\right)$$

Extended explanation:

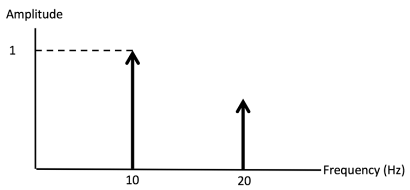
Euler's formula: $e^{j\theta} = \cos\theta + j\ \sin\theta$

From this, we derive: $\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

The answer is simple substitution.

(iii)



Comments: Most students got the phase of the sinewave wrong and lost 2 marks. Some could not add waveforms. Most got (ii), but some got the sign wrong for the sine formula. A small number of students did not have the correct exponents.

SOLUTION to Q3:

3. In designing a real-time electronic system, you are given the choice of using polling or interrupt to handle some external events.

   (i) State the limitations of polling when compared with using interrupts.

   [2]

   (ii) Describe the sequence of action that a microprocessor takes when an interrupt occurs.

   [4]

   (iii) What guidelines must you follow when writing an interrupt service routine?

   [4]

This question tests student's understanding of interrupt vs polling in a real-time embedded system and their advantages and disadvantages. It is mostly BOOKWORK.

(i) Polling is programmed response to external events. Therefore, it may miss the event if the program is not running those code that tests of the event at the time. It is also inefficient because it runs lots of instructions just to test of event. Interrupt is more efficient because it is a hardware-forced function. It will not miss the event and it does not run any code unless the event is detected.

(ii)

A) The processor completes its current instruction
B) The processor stores away the state including the instruction at which it was interrupted
C) The processor jumps to the interrupt service routine for that interrupt event
D) The processor automatically disable further interrupt, but the user may choose to turn interrupt response back on
E) When completing the ISR, the processor restores the state of the processor before the interrupt
F) The process return to the instruction location where it was previously interrupt and continues the running the user code

(iii) Interrupt service routine should be as short as possible. It will not take any parameters (argument) passed to it, and therefore must use global variables to pass information to and from the ISR.

Comments: Most students managed to answer this question to some degree. Those who did not do the Lab on interrupts were obvious in their poor answers. (iii) was the tricky part of this question. Most did not realized the restriction of needing to use global variables.

SOLUTION to Q4:

4. The following differential equation describes the relationship between the output y(t) and the input x(t) of a linear system:

$$7\frac{d^2y}{dt^2} + \frac{dy}{dt}10 = \frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 4y(t)$$

(i) What is the order of this system?

[2]

(ii) Given that Y(s) and X(s) are the Laplace Transforms of y(t) and x(t) respectively, write down the transfer function H(s) = Y(s)/X(s) for the system.

[4]

This question tests students the relationship between differential equation and Laplace transform and how a system can be described by either form.

(i) This is a second order system.

(ii) The transfer function is:

$$H(s) = \frac{s^2 + 2s}{7s^2 + 10s + 4}$$

Comments: Almost ALL students got full marks in this questions. The few who didn't also did poorly in the rest of this paper, indicating that they were genuinely weak in this module. The y(t) term on the right tripped up a couple of students.

SOLUTION to Q5:

5. A system H consists of two circuits A and B connected in series with each other as shown in Figure Q5. The transfer function for circuit A is P(s) and for circuit B is Q(s), and they are known to be:

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

where k is a constant.

(i) Derive the s-domain equation for the transfer function H(s) of the entire system? [5]

(ii) What is the natural or resonant frequency of the system? [3]

(iii) It is known that when k = 10, the system is critically damped. Sketch the step response of the system. Explain your answer. [4]

(iv) If k is 1, sketch the step response of the system. Explain your answer. [4]

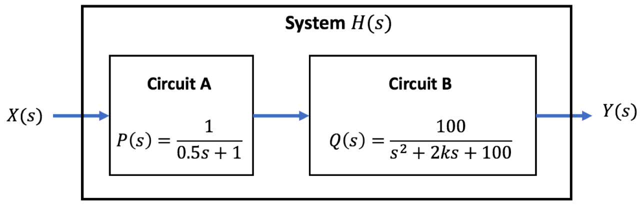
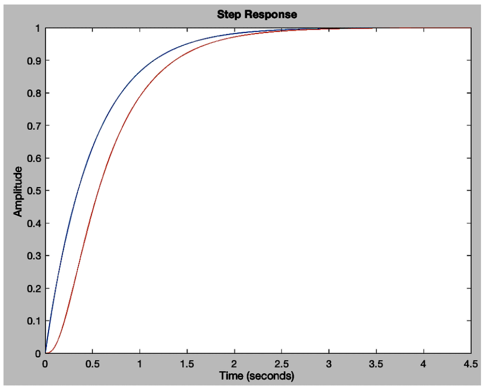


Figure Q5

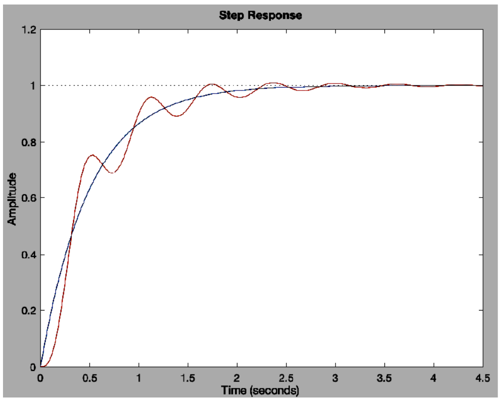
This question tests student's understanding of how cascaded system is described as product of transfer function.



(i) $H(s) = P(s)Q(s) = \frac{100}{0.5s^3 + (k+1)s^2 + (2k+50)s + 100}$

(ii) The resonant frequency is determined by the Q(s) and is $\sqrt{100}$ = 10 rad/sec. [3]

(iii) When the system is critically damped, the step response is dominated by the first-order system P(s), which has a time constant of 0.5sec (shown in blue). Therefore the step response is approximated by an exponential rise, reaching roughly 63% at 0.5s. (Shown in blue).



(iv) When k = 1, the system is underdamped, and therefore you will see oscillation at the natural frequency, which is around 1.6Hz.

Comments: Many students found this question difficult. A few students added the transfer function of A and B instead of multiplying them – showing fundamental misunderstanding of transfer functions. Some did not get (ii) because not realizing that only 2nd stage contributes to the oscillatory response, and not stage A. Many students got (iii) and (iv) types of damping (iii – critically damped and iv – underdamped) but did not get the plot right.

SOLUTION to Q6:

6. The block diagram of a discrete-time shift-invariant system G is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0,1,2,3, \dots$ etc. The system is assumed to be casual, i.e. $x[n] = y[n] = 0$, for $n < 0$.

   (i) Derive the difference equation for the system.

   [4]

   (ii) Derive the output sequence $y[0], y[1], \dots \dots \dots, y[9]$ given that $\alpha = 0.8, \beta = 0.2$, $x[0] = 0$ and $x[n] = 10$ for $n \geq 1$.

   [4]

   (iii) Find the transfer function $G[z]$ of the system for $\alpha = 0.8, \beta = 0.2$.

   [4]

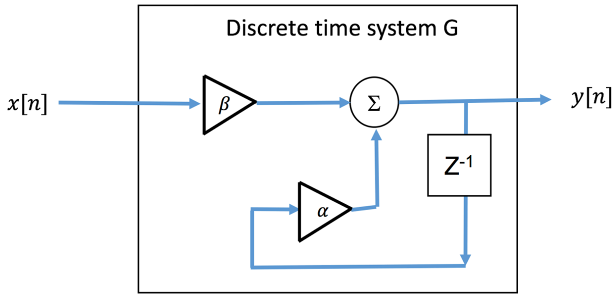


Figure Q6

This question tests students understanding of discrete time system specified as a difference equation.

(i) $y[n] = \alpha y[n-1] + \beta x[n]$

(ii)

n	0	1	2	3	4	5	6	7	8	9
x[n]	0	10	10	10	10	10	10	10	10	10
y[n]	0	2	3.6	4.88	5.9	6.72	7.38	7.9	8.32	8.66

(iii) Take z-transform on both sides of the difference equation:

$$Y(z) = 0.8z^{-1}Y(z) + 0.2X(z)$$
$$\Rightarrow (1 - 0.8z^{-1})Y(z) = 0.2X(z)$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{0.2}{1 - 0.8z^{-1}} = \frac{0.2z}{z - 0.8}$$

Comments: Most students did well in this question. A few who obviously did not attend the lectures on z-transforms could not do this question. Almost all students ignored the fact that values of α and β were given, and did not answer (ii) and (iii) with actual values and lost a few marks.

SOLUTION to Q7:

7. For the group project, you used an IMU to derive the pitch angle of the mini-segway vehicle.

   (i) Explain the principle used by the accelerometer to measure the pitch angle. [4]

   (ii) Why is it not sufficient to rely on the accelerometer alone to provide the pitch angle measurement? [4]

   (iii) What does a gyroscope measure, and how can the gyroscope reading be used to measure the pitch angle? What is its limitations? [4]

   (iv) How did you combine the two types of measurement to provide a better estimate of the pitch angle? [4]

This question tests student's understanding of IMU and how to process signals from accelerometer and gyroscope. Mostly bookwork

(i) Accelerometer is manufactured on silicon with similar technology as that used for making integrated circuits (chips). A fixed mass is levered on an anchor. The lever forms the moving plate of a capacitor. The other plate of the capacitor is attached to the anchor. Different angle of the mass resulted in the moving plate displaced relative to the fixed plate, thus changing the capacitance. By measuring the change in capacitance, the angle of tilt can be derived. (Expect them to draw a diagram.)

(ii) Angle measurements from an accelerometer is inherently noisy because the angle position is derived from acceleration due to gravity. However, if the device is in motion, this angle measurement will also include the acceleration due to the motion. There is no way to tell the two apart.

(iii) A gyroscope measures the rate of change in angle, or angular velocity. To obtain angle measurement from the gyroscope reading, we need to integrate the measurement over time. This gives a good angle measurement, but accumulates offset errors. This error manifest itself as a large offset value, also known as drift.

(iv) To get a good pitch angle measurement, one can lowpass filter the accelerometer angle to reduce the impact of error due to movement, and highpass filter the gyroscope measurements to remove the drift. The digital filter used has a difference equation given by:

$$p[n] = p[n-1] + \alpha\, g[n] + (1-\alpha)\, a[n]$$

where p[n] is the pitch angle, g[n] is the gyro reading and a[n] is the accelerometer reading and α is around 0.9.

Comments: Most students got something from this questions. A few could not answer this question, clearly showing that they did not do the lab nor took part in the project in using the IMU. Many answered (i) poorly, but did well in the rest, demonstrating that they knew how to use IMU but had little knowledge of how the accelerometer works.

SOLUTION to Q8:

8. Figure Q8 shows a proportional -derivative (PD) controller with a transfer function of C(s) controlling the behaviour of a first order system with a transfer function of P(s) in a close-loop feedback arrangement. It is known that:

$$P(s) = \frac{1}{1 + 0.1s}$$

(i) Write down the transfer function of *C(s)* in terms of the proportional gain K_p and the derivative gain K_d of the controller.

[3]

(ii) If K_d = 0, derive the close-loop transfer function *H(s)* of the system where

$$H(s) = Y(s)/R(s).$$

[5]

(iii) Explain the effect of the proportional gain K_p and the derivative gain K_d of the controller on the close-loop response of the system.

[4]

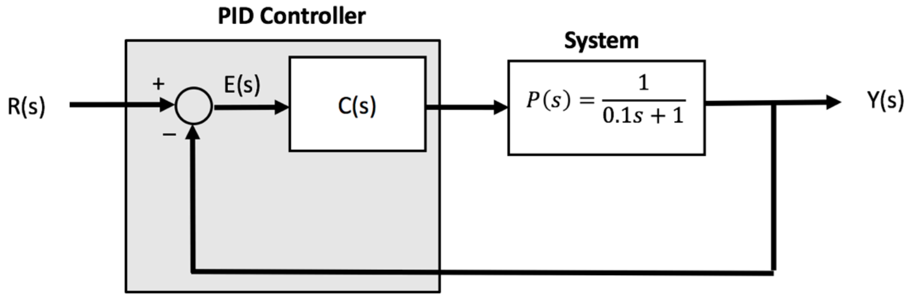


Figure Q8

Each student was part of a 4-people team in designing a controller to balance a 2-wheel vehicle. This question tests student's understanding of the PD controller used. Part bookwork and part derivation

(i) (Bookwork)

$$C(s) = K_p + K_d s$$

(ii) The loop transfer function (with Kd=Ki=0) is:

$$L(s) = C(s)P(s) = \frac{K_p}{1{+}01s}$$

Therefore the close-loop transfer function is:

$$H(s) = \frac{L(s)}{1 + L(s)} = \frac{\frac{K_p}{1 + 0.1s}}{1 + \frac{K_p}{1 + 0.1s}} = \frac{K_p}{1 + K_p + 0.1s}$$

(iii) The proportional term provides the main feedback control. As Kp is increased, the system response time improves. However, if Kp is too high, the system may become unstable and starts to oscillate. The derivative term provides prediction of the control variable, and therefore improve response time further, but may cause even more stability issue.

Comments: Most students got something from this questions. Particularly those who actively did the self-balancing control part of the project answered this question well. A few students obviously did not know any control and were not able to answer this question.

GENERAL COMMENTS

The average marks for this paper is near perfect: 64%. The grade distribution is also reasonable, perhaps with slightly higher A grades than I would like.