IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2019

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

Engineering Analysis EA 2.3 – Electronics 2

SOLUTIONS

This paper contains EIGHT questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

SOLUTION to Q1

1. A signal $x(t)$ can be modelled mathematically as:

$$x(t) = \left[2.5 + 1.5\cos(31.42t + \tfrac{\pi}{4})\right] + 0.5\,\delta(t-1)$$

(i) Sketch the waveforms $1.5\cos(31.42t + \frac{\pi}{4})$ and $0.5\,\delta(t-1)$ for $0 \le t \le 0.2$. Hence sketch the signal $x(t)$ for $0 \le t \le 0.2$.

[8]

(ii) Rewrite $x(t)$ in exponential form where appropriate. (There is no need to simplify the equation.)

[4]

(iii) Sketch the amplitude spectrum $|X(j\omega)|$ of the signal x(t).

[4]

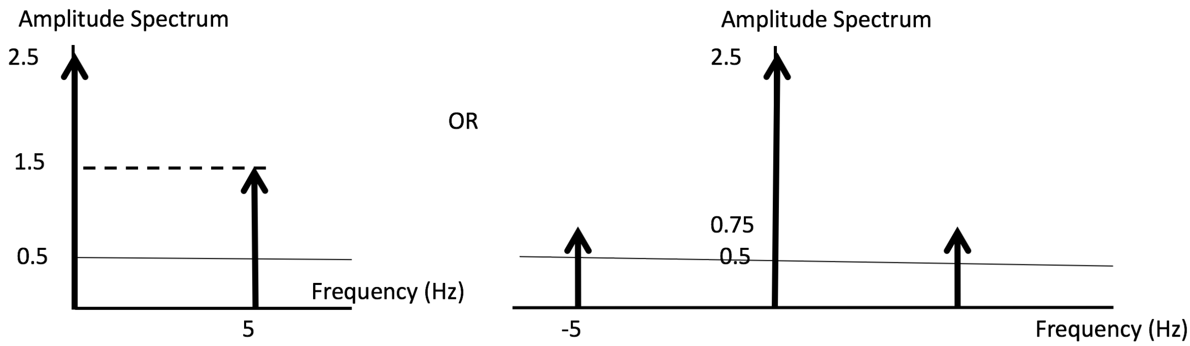
This question tests student's understanding of: 1) using equation to model a signal; 2) sketching waveforms; 3) the Euler formula and the exponential representation of sinusoids; 2) spectral representation of signals.

(i) The signal has three components: a cosine signal at 5Hz with a 45 degrees delay, a 3.5v dc offset, and an impulse of 0.5v at t=1. Students are expected to produce a rough sketch of x(t) of the cosine signal. However, the impulse is outside range of t, so can be omitted.

(ii) Straight forward application of Euler's formula:

$$x(t) = 0.75\left(e^{+j(31.42t+\pi/4)} + e^{-j(31.42jt+\pi/4)}\right) + 2.5 + 0.5\delta(t-1)$$

(iii)



SOLUTION to Q2

2. You are designing a bat detector by directly sampling the echolocation signal emitted by bats. Bats emit ultrasound in the frequency range of 12 kHz to 160 kHz, which is detected by an ultrasound transducer that produces an ac signal in the range of ±10 mV with a 2.5v offset. However, your detector is designed to work up to only 100 kHz since any signal above this frequency is rapidly absorbed in air. Your detector will first directly convert the raw bat signal from analogue to digital form using an A-to-D converter (ADC) with an accuracy of ±0.1%. The ADC operates in the voltage range of 0 to 5V.

   (i) What sampling frequency would you choose for the ADC? Justify your answer. [3]

   (ii) What circuit would you need to connect to the ultrasound detector *before* the ADC? Why? [3]

   (iii) What is the resolution of the ADC required in terms of number of bits? What is its resolution in volts? [3]

   (iv) The detector "converts" the digital bat signal from the ultrasound range of 12 kHz to 160 kHz to the audible frequency range. Describe briefly a method or approach for your instrument that will map the bat's ultrasound to the audible range of 120 Hz to 15 kHz. [6]

This question tests student's understanding of sampling theorem, ADC resolution and problem of aliasing.

(i) Since we need to detect up to 100 kHz, sampling theorem dictates that we need to sample the bat signal at 200 kHz or higher. In practical circuits, we need to choose a sampling rate at least 1.5 times higher than this minimum (say). Accept any sampling rate from 300kHz up to, say, 500 kHz.

(ii) Since the raw signal can be up to 160 kHz, to avoid aliaising, we need to add a lowpass filter that suppresses signal power above 100kHz.

(iii) 0.1% = 1 in 1000. Therefore, minimum number of bits in ADC is 10-bits (2^{10}). The resolution of the ADC in volts would be 5v/1024 = 4.9 mV.

(iv) This last section is open-ended. Mapping from one frequency range to another is not easy, particularly because the audio-range has a bandwidth of only 15kHz, and the bat ultrasound has a bandwidth of nearly 90 kHz. One approach could be to detect zero crossing, count them and then produce a digital signal that changes its state (i.e from low to high or high to low) every 10 transitions of the bat signal.

SOLUTION to Q3:

3. In the Dancing Segway team project, you used the following code segments to implement an interrupt service routine.

```
1  # Interrupt Service Routine
2  def isr_sampling (dummy):    # interrupt happens at 8kHz rate
3      global  ptr              # pointer to buffer
4      global  buffer_full      # buffer status indicator
5
6      s_buf[ptr] = mic.read() # s_buf[] stores samples - pre-allocated
7      ptr = ptr + 1
8      if (ptr == 200):
9          ptr = 0
10         buffer_full = True
11
12 #  Create timer interrupt  - one every 125 usec
13 sampling_timer = pyb.Timer (7, freq = 8000)
14 sampling_timer.callback(isr_sampling)
```

(i) Explain why interrupt is necessary in this application.

[4]

(ii) Brief explain the purpose of each line in this MicroPython code segment

[8]

This question tests student's understanding of interrupt vs polling in a real-time embedded system and their advantages and disadvantages. It also examines student's understanding of the Python code to perform interrupts.

(i) The dancing Segway project requires the embedded system to do two tasks simultaneously: to control the wheels of the Segway so that it will not fall over, and to acquire the musical sound at 8kHz. Using interrupt to sample the music signal allows the main loop to perform control function and to dance without missing any samples.

(ii) Line 2 to 10 is the interrupt service routine.

Line 2 defines the interrupt service routine's name
Line 3 and 4 are the variables used in the ISR that has to be global, meaning visible inside and outside the ISR
Line 6 is where sampling happens – it takes a sample from the microphone and store in the current location in the buffer
Line 7 advances the pointer
Lines 8 to 10 check to see if we gets to the end of the buffer (which is 200 long here), and reset the pointer to zero while raise the buffer_full flag

Line 13 initialise a hardware timer to generate an event once every 125 microseconds, hence fixing the sampling rate to be 8000.
Line 14 is to tell the timer that at every interrupt event, execute the (call) the isr_sampling routine.

SOLUTION to Q4:

4. A system H consists of two sub-systems A and B connected in series. System A is a first-order system, while system B is a second-order system. The combined transfer function H(s) is given by the equation:

$$H(s) = \frac{1}{0.01s + 1} \times \frac{100}{s^2 + s + 50}$$

(i) Deduce the transfer functions of A and B separately?

[2]

(ii) It is known that a second-order system has a transfer function G(s) of the general form:

$$G(s) = K \frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

where $K = \text{ dc gain}$
$\omega_0 = \text{ natural frequency}$
$\zeta = \text{ damping factor}$

Derive the DC gain, nature frequency and damping factor of the system H. State any assumption used.

[4]

(iii) What is the over gain of the system at a frequency of 0.32Hz?

[4]

This question tests student's understanding of how cascaded system is described as product of transfer function.

(i) $A(s) = \frac{1}{0.01s+1}$ $B(s) = \frac{100}{s^2+s+50}$

(ii) Since A(s) has a very short time constant of 10ms, its impact on the transient response of the system is minimal. We only need to consider B(s). With the given general form of a 2nd-order system,

$K = 2$
$\omega_0 = 7.07$
$\zeta = 0.0707$

(iii) The overall gain of the system at 0.32Hz or 2 rad/sec can be calculated as:

$$|H(j\omega)|_{\omega=2}| = \left|\frac{1}{0.01x2j + 1}\right| \times \left|\frac{100}{-4 + 2j + 50}\right| = 2.172$$

SOLUTION to Q5:

5. The transfer function H(s) of a system is given by the following equation:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{200 + s}{200 + 10s + s^2}$$

(i) What is the order of this system?

[2]

(ii) Given that y(t) and x(t) are inverse Laplace transform of Y(s) and X(s), write down the differential equation that relates y(t) and x(t) in the time domain.

[5]

This question tests students the relationship between differential equation and Laplace transform and how a system can be described by either form.

(i) This is a second order system.

(ii) Time domain differential equation is:

$$200y + 10\frac{dy}{dt} + \frac{d^2y}{dt^2} = 200x + \frac{dx}{dt}$$

SOLUTION to Q6:

6. A 5-tap moving average filter has discrete output signal y[n] and input signal x[n], and the system is causal. The filter has a difference equation given by:

$$y[n] = \frac{1}{5}(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4])$$

(i) If Y(z) and X(z) are the z-transform of the discrete signals y[n] and x[n] respectively, derive the transfer function Y(z)/X(z).

[4]

(ii) Sketch, not necessary to scale, the frequency response you expect of such a filter.

[4]

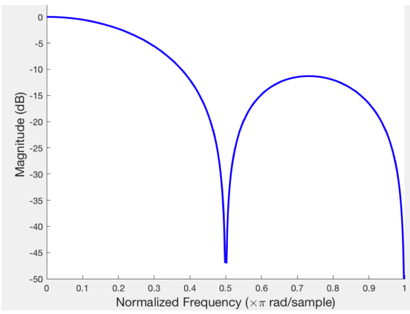
(iii) Given that the sampling frequency of the system is 10 kHz, explain with justifications what you expect this filter will do to a signal at 1 Hz and at 4.5 kHz.

[4]

This question tests students understanding of discrete time system specified as a difference equation, the z-transform representation of such a discrete-time system.

(i) $H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$

(ii)



(ii) 1 Hz is very small compared with the sampling rate of 10 kHz therefore the filter will simply pass the signal through (lowpass). 4.5 kHz is nearly half the sampling frequency; therefore, this filter will attenuate significantly the signal at this frequency.

SOLUTION to Q7:

7. A discrete-time filter is characterised by the z-domain transfer function:

$$H(z) = \frac{Y(z)}{X(Z)} = \frac{1}{1 - 0.2Z^{-1}}$$

where X(z) and Y(z) are the z transforms of the input x[n] and output y[n] respectively.

(i) Derive the difference equation relating x[n] to y[n].

[5]

(ii) Draw a diagram showing how this filter can be implemented using multipliers, adders and delay modules.

[5]

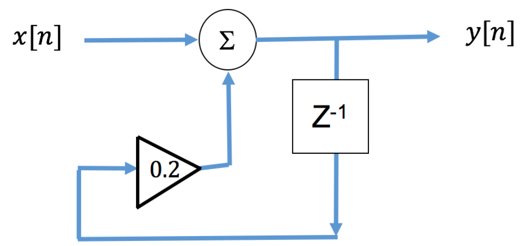
(iii) Assuming that the input is causal and that y[n] = 0 for all n < 0, derive the first 6 samples of the system's impulse response.

[5]

This question tests student's understanding of a simple recursive lowpass digital filter, like part of the complementary filter they used with the IMU in the Group Project.

(i) y[n] = x[n] + 0.2 y[n-1]

(ii)



(iii) Impulse response is the response of this filter an unit impulse, i.e. x[n] = [1,0,0,…]
Therefore, y[n] = [0, 0.2, 0.04, 0.008, 0.0016, 0.00032, …].

SOLUTION to Q8:

8. Figure Q8 show a simple proportional feedback system to control the motor speed y(t) in response to the set-point r(t) in the s-domain. The motor has a system transfer function given by:

$$P(s) = \frac{20}{0.2s + 1}$$

The controller has a constant gain, i.e. C(s) = K_p.

(i) Derive the close-loop transform function of the system Y(s)/R(s).

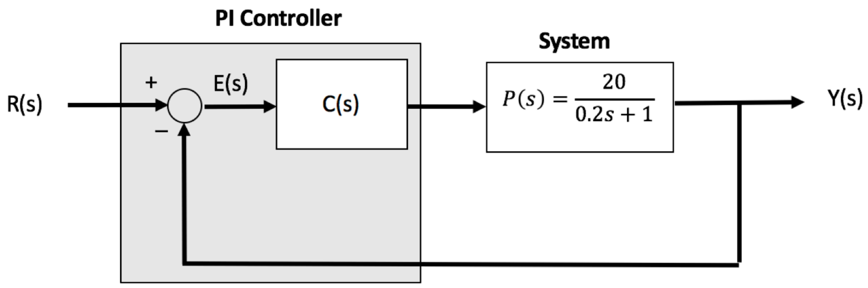
[5]

(ii) Modify the system block diagram in Figure Q8 to include an integral term in the controller and derive the close-loop transform function of the feedback system with this addition.

[5]

(iii) Explain the impact on the dynamic response of the system with the addition of this integral component to controller of this feedback system.

[4]



This question tests student's understanding of the PI controller Part bookwork and part derivation

(i) $H_{CL}(s) = \frac{20K_p}{1+20K_p+0.2s}$

(ii) $C(s) = K_p + \frac{K_i}{s}$

Loop gain $L(s) = 20\frac{K_p s+K_i}{(0.2s+1)s}$

Therefore the close-loop transfer function is:

$$H(s) = \frac{L(s)}{1 + L(s)} = \frac{20(K_p s + K_i)}{0.2s^2 + (1 + 20K_p)s + 20K_i}$$

(i) The integral term eliminates the steady-state error in e(t).