IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2020

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

Engineering Analysis DE 2.3 – Electronics 2

SOLUTIONS

This paper contains SIX questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

SOLUTION to Q1

1. A signal $x(t)$ can be modelled mathematically as:

$$x(t) = w(t) + y(t)$$

where

$$w(t) = -\mathrm{j}\left(e^{+0.628jt} - e^{-0.628jt}\right)$$

$$y(t) = 0.5u(t-10) - 0.5$$

(i) Sketch the amplitude spectrum $|W(j\omega)|$ of the signal $w(t)$. [4]

(ii) Sketch the waveforms of $y(t) = 0.5u(t-10) - 0.5$. [4]

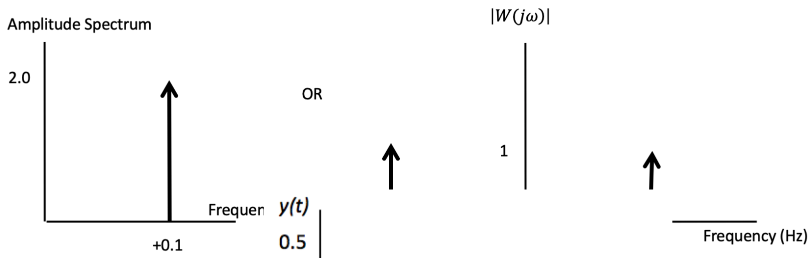
(iii) Hence or otherwise, sketch the signal $x(t)$ for $0 \le t \le 15$. [6]

This question tests student's understanding in: 1) using equation to model a signal; 2) sketching waveforms; 3) the Euler formula and the exponential representation of sinusoids; 2) spectral representation of signals.

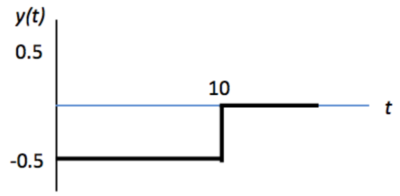
(i) Use Euler's formula, we have:

Therefore:

Hence the spectrum of w(t) is as above.

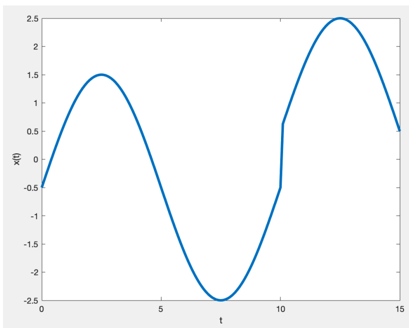


(ii) The waveform for $y(t) = 0.5u(t-10) - 0.5$ is:



(iii) Combine w(t) and y(t), we plot:

$$x(t) = 2\sin(2\pi \times 0.1t) + 0.5u(t-10) - 0.5$$



SOLUTION to Q2

2. The motor used for the mini-Segway project has a 13-poles magnet sensed by two Hall effect sensors, and a gearbox with a step-down ratio of 1:30. The wheel attached to the motor has a diameter of 6.8cm.

   The following code segment was used to measure the speed of the motor using interrupt.

```
import micropython
micropython.alloc_emergency_exception_buf(100)
from pyb import ExtInt

A_speed = 0
A_count = 0

def isr_motorA(dummy):
    global A_count
    A_count += 1

def isr_speed_timer(dummy):
    global A_count
    global A_speed
    A_speed = A_count
    A_count = 0

motorA_int = ExtInt ('Y4', ExtInt.IRQ_RISING, Pin.PULL_NONE,isr_motorA)

speed_timer = pyb.Timer(4, freq=10)
speed_timer.callback(isr_speed_timer)
```

(i) Explain why interrupt is better than polling in detecting the speed of the motor.

[4]

(ii) Briefly explain the purpose of each line in this MicroPython code segment and how this code measures the speed of the motor.

[6]

(iii) Write one line of Python code that calculates the speed of travel of the wheel in cm/sec if the wheel is mounted on a vehicle travelling on a flat surface in a straight line.

[4]

This question tests student's understanding of interrupt vs polling in a real-time embedded system and their advantages and disadvantages. It also examines student's understanding of the Python code to perform interrupts.

(i) (Book work) The speed of the motor is "measured" by counting pulses detected by the hall effect sensor over a time window of 100ms. Using polling to detect rising edge of pulses and to time the 100msec window has the following disadvantages:

a. Waste CPU cycles to test the state of the signal to detect low-to-high transition;

b. Waste CPU cycles to check the elapse time is above 100ms;

c. Run the risk of missing either event, hence less accurate

Advantage is the polling is easier to write, to understand and to debug.

In contrast, using interrupt to detect the rising edges of the sensor signal and to time the 100ms time window is more efficient on CPU cycles and guarantee not to miss any events. This provides better accuracy, but interrupt driven programmes are harder to debug because the interrupt service function may happen at anytime and therefore unpredictable.

(ii) Lines 1 & 2: required by the MicroPython system for interrupt. It reserves some buffer space to handle interrupt related errors.

Line 3: important the Pyboard class to handle external interrupts.

Line 5 & 6: Initialise speed and count variables which respectively store the speed measurement in the previous 100ms window, and the current pulse count.

Line 8: Declaration statement of the motor interrupt service function. This function is run whenever a rising edge is detected on the motorA hall effect sensor output which is connected to a pin on the Pyboard.

Line 9: Declare that the scope of the variable A_count is a global variable visible throughout the entire programme.

Line 10: Count up by 1 as a response to a rising edge which invoke this interrupt service function.

Line 12: Declaration statement for the timer function. This ISR is invoked whenever the timer completed timing for a 100ms period. Timer is set up later in the code.

Line 13 & 14: Declare that the scope of A_count and A_speed is global, hence visible everywhere in the programme.

Line 15: Remember the count value in the previous 100msec window in A_speed.

Line 16: Reset A_count for the next measurement in the following 100ms window.

Line 18: Tells the system that pin Y4 is to generate an interrupt on the rising edge, the pin is an input pin (with no pullup resistors) and finally the interrupt service function is isr_motorA.

Line 19: Set up Timer 4 to time a regular period of 100msec (frequency = 10Hz).

Line 20: Tell the timer that when 100msec is elapsed, invoke the interrupt (or callback) function isr_speed_timer.

(iii) Each revolution of the motor will produce 13 rising edges. Motor will need to rotate 30 times for the wheel to turn once. Therefore one rotation of the wheel will produce 390 pulses.

motor_speed = (A_speed * pi * 6.8)*10/390

This solution is fuller than what I expect students would produce under examination condition.

SOLUTION to Q3:

3. It is known that a second-order system has a transfer function H(s) of the general form:

$$H(s) = K\frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

where $K =$ dc gain
$\omega_0 =$ natural frequency
$\zeta =$ damping factor

A 2nd order mechanical system has an input-output relationship that obeys the following 2nd order differential equation:

$$2\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 80y(t) = 40x(t)$$

(i) Derive the transfer function H(s) of the system.

[4]

(ii) Derive the DC gain, nature frequency and damping factor of the system H(s). State any assumption used.

[4]

(iii) What is the gain of the system if the input x(t) is a sinusoid at a frequency of 0.1Hz?

[4]

This question tests student's understanding of differential equation description of a system and its relationship to Laplace Transform and transfer function, and how to relate the coefficient of the transfer function to physical property of the system.

(i) This is straight forward by knowing that the Laplace transform of dy(t)/dt is sY(s) and LT of $d^2y(t)/dt^2$ is s^2 Y(s).

$$2s^2Y(s) + 2sY(s) + 80\,Y(s) = 40\,X(s)$$

$$(2s^2 + 2s + 80\,)Y(s) = 40\,X(s)$$

Therefore

$$H(s) = \frac{Y(s)}{X(s)} = \frac{40}{(2s^2 + 2s + 80\,)} = 0.5\,\frac{40}{(s^2 + s + 40\,)}$$

(ii) We equate coefficient for H(s) with the formula given:
The DC gain is therefore 0.5. The natural frequency ω_0 in rad/sec is $\sqrt{40} = 6.324$ or 1Hz. The damping ratio is calculated with:

$$2\zeta\omega_0 = 1,\;\; \zeta = \frac{1}{2\omega_0} = 0.079$$

(iii) We can evaluate the frequency response from the transfer function by evaluating H(s) with $s = j\omega = j2\pi \times 0.1 = \;0.628j$, i.e. $|H(s)|_{j\omega}$

$$|H(s)|_{j\omega} = \;0.5\,\frac{40}{(-\omega^2 + j\omega + 40\,)} = \frac{20}{(-0.4 + j0.63 + 40\,)} = \frac{20}{39.6 + j0.63} \approx 0.5$$

SOLUTION to Q4:

4. A digital filter has discrete output signal $y[n]$ and input signal $x[n]$, and the system is causal. The filter has a difference equation given by:

$$y[n] = 0.93\,x[n] - 0.93\,x[n-1] + 0.86\,y[n-1]$$

(i) Given that $x[n]$ is a unit step signal, list the values of $x[n]$ for n = -1, 0, 1, .., 9. [4]

(ii) Derive the transfer function $H[z]$, of this system in the z-transform domain. [4]

(iii) Assuming that $y[-1] = 0$, calculate and sketch the step response of the system for the first ten terms. [4]

(iv) Hence or otherwise, explain with justification the type of filtering the system is performing. [4]

(v) Draw a diagram showing how this filter can be implemented using multipliers, adders and delay modules. [4]

This question tests student's understanding of discrete signals, difference equation, z-transform, simple digital filter, and how digital filter is implemented with the three basic components.

n	x[n]	y[n]
-1	0	0
0	1	0.93
1	1	0.80
2	1	0.69
3	1	0.59
4	1	0.51
5	1	0.44
6	1	0.38
7	1	0.32
8	1	0.28
9	1	0.24

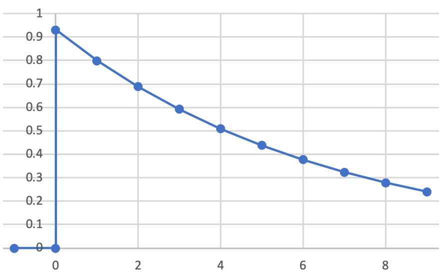
(i) This is slightly tedious but necessary for the rest of the question.

(ii) A sample delay is equivalent to multiply the z-transformed signal by z^{-1}. Therefore, take z-transform of the difference equation::

$$Y[z] = 0.93\,X[z] - 0.93z^{-1}\,X[z] + 0.86\,z^{-1}Y[z]$$

$$(1 - 0.86\,z^{-1})Y[z] = (\,0.93\ - 0.93z^{-1}\,)X[z]$$

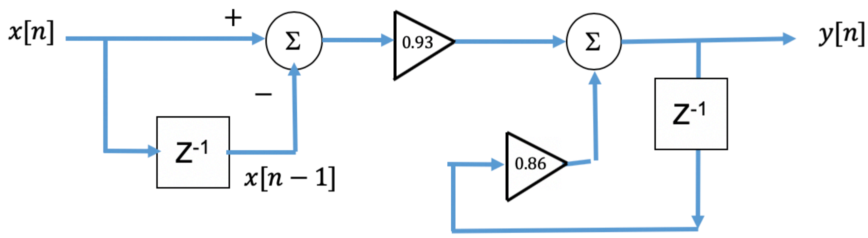
$$(H[z] = \frac{Y[z]}{X[z]} = 0.93 \times (\frac{1 - z^{-1}}{1 - 0.86\,z^{-1}})$$



(iii) Sketching the step response is easy, it is just the values calculated in (i):

(iv) The step response shows this a a first order high-pass filter because it passes the initial step but the output decay exponentially toward zero. It therefore passes high frequency, but blocks the dc component.

(v) The overall gain of the system at 0.32Hz or 2 rad/sec can be calculated as:



SOLUTION to Q5:

5. Your group is designing a device to discover which type of flowers honeybees prefer by sensing the sounds they make. It is known that a honeybee, while hovering to collect honey, has a fundamental wingbeat frequency of 235 ± 15Hz. it also produces detectable vibration up to the fifth harmonic of the wingbeat frequency and each increasing harmonic component has a decreasing power by of a factor of 9 (i.e. the 1st harmonic has one ninth the power of the fundamental; the 2nd harmonic has one ninth of the power of the 1st harmonic etc.).

An A-to-D converter (ADC) is used to convert the sound signal for processing with a microprocessor. Your device is required to detect the presence of honeybee using a microphone as long as the honeybee make a sound that constitutes at least 10% of the full dynamic range of the captured sound signal.

(i) A member of your team suggests that a sampling frequency of 2.5kHz should be used. Explain your opinion on this suggestion and justify your answer.

[5]

(ii) Your team sampled the microphone signal directly after amplification at 2.5kHz. A nearby burglar alarm makes an ear-piercing tone of 1.5kHz. What is the impact of this alarm signal on your captured digital signals in your device? What improvements can you make to your design to mitigate the spurious signal from the alarm?

[5]

(iii) Assuming that microphone amplifier has an automatic gain control that always adjusts the signal voltage to the full voltage range, estimate the resolution of the ADC required in terms of number of bits? State any assumptions used and justify your answer.

[5]

(iv) Your team has decided to use Fourier analysis to detect the presence or absence of honeybee. You are responsible to come up with the recognition algorithm. Explain briefly how you would approach this problem.

[5]

This question tests students ability to apply what they learned in signal processing to a practical scenario. It tests understanding on: sampling theorem, aliasing and frequency folding, anti-aliaising filter's function, signal range and its relationship to ADC bit resolution, spectral components in signals, and signal recognition in frequency domain.

(i) The highest frequency of the relevant signal is 5 x 250Hz = 1250 Hz. Sampling theorem states that the minimum sampling frequency is 2500 Hz. Therefore this is just sufficient. However, I would recommend sampling at higher frequency than this such as 4kHz or even higher.

(ii) The alarm signal is above the Nyquist (or half sampling) frequency. Therefore aliaising will occur. The 1.5kHz signal will appear as 2.5 – 1.5 = 1kHz, corrupting the sampled signal. The mitigate this, one should introduce an anti-aliasing filter to get rid of signal components above 1.25kHz.

(iii) This question requires the assumption that the size of the fifth harmonic determines the resolution of the ADC. This may or not be valid, but is a reasonable first-order assumption. Each harmonic has its power reduced by a factor of 9, hence the amplitude drops by a factor square root of 9, i.e. 3. Therefore the 5^{th} harmonic is a factor 3^5 , i.e. 243 times smaller than the fundamental, which could be 0.1 (10%) of the full dynamic range of the signal. Hence, to detect the 5^{th} harmonic, one would need to differentiate 1 in 2430, or at least 12 bits.

Note that this part of the question is actually quite hard. It demands good understanding of various concepts and therefore I will mark this quite leniently.

(iv) Assuming that only one honeybee visit the flower at a time, then it would be easy to take find the Fourier transform of the audio signal and look for the harmonic pattern. This also assumes that the ambient signals are insufficient to confuse the honeybee harmonic patterns.

SOLUTION to Q6:

6. (a) With the help of an example explain the difference between a closed-loop and an open-loop control system. What are the advantages of closed-loop control systems over an open-loop control system?

[4]

(b) A lighting system is controlled using pulse-width modulation with duty cycle of x(t) (in percent). The brightness of the light y(t) is measured in lumen, and the transfer function G(s) of the lighting system is given by:

$$G(s) = \frac{K_L}{0.1s + 1}$$

where K_L is a constant with a value of 1 if the system is ideal. However, manufacturing process causes this value to vary by ±20%.

(i) If the lighting system is controlled directly as an open-loop system with x(t) set to 50 (i.e. 50% duty cycle), calculate the maximum and minimum steady-state brightness of the system.

[4]

(ii) Sketch the response of the open-loop system if x(t) is a step function $50\,u(t)$. What is the time constant of the system?

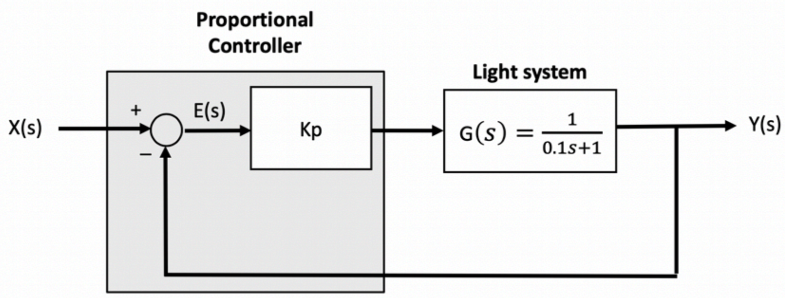
[4]

(iii) Figure Q6 shows a proportional close-loop control system to control the lighting system described above. The proportional gain K_P is 10. Estimate the maximum and minimum value of the output intensity if x(t) is 50.

[4]

(iv) Derive the closed-loop transfer function of the feedback system. Hence state the time constant of the feedback system.

[4]



This question tests students understanding of feedback control systems. It particular tests for understanding of proportional control and its ability to reduce the impact of the error in the forward gain of a system.

(a) (Book work) One possible example is the temperature control systems. An open-loop system would set the temperature which is map to, say the electric heater voltage which drives the heating element through some mapping function (quadratic since energy is square of voltage). However, a close-loop system measures the actual temperature and compares this with the set temperature. It then increases the driving voltage until the desired temperature is reach. Therefore a closed-loop system can achieve a much more accurate output than an open-loop system.

(b)

(i) Nominal brightness is 50, but due to variation of K_L, this may vary by ±0.2. Therefore the maximum and minimum brightness is 40 and 60 lumen.

(ii) This requires students to know that: 1) the transfer function indicates a first order system; 2) its step response is an exponential rise with a time-constant equals to the coefficient value of the s term. Therefore the time-constant is 0.1 second. Sketching is now very easy – the exponential signal reaches 0.63% of 50, or 31.5 at 0.1 second, and eventually reaches 50 in steady state.

(iii) The steady-state error is reduced by the same factor as the loop-gain at DC. Therefore the maximum and minimum is reduced to 50 ± 2%, or 49 and 51 lumen.

(iv) The close-loop transfer function is derived as follow:

$$E(s) = X(s) - Y(s) = Y(s)/K_pG(s)$$

$$Therefore, \quad Y(s)\left[\frac{1}{K_pG(s)} + 1\right] = X(s)$$

$$Y(s)\left(\frac{1 + K_pG(s)}{K_pG(s)}\right) = X(s)$$

Hence,

$$H(s) = \frac{Y(s)}{X(s)} = \left(\frac{K_pG(s)}{1 + K_pG(s)}\right)$$

Therefore

$$H(s) = \frac{\frac{10}{0.1s+1}}{1 + \frac{10}{0.1s+1}} = \frac{10}{0.1s + 11} = \frac{10/11}{\frac{0.1}{11}s + 1} = 0.91 \times \frac{1}{0.0091s + 1}$$

The time-constant is reduced by a factor 11, and is now 9.1msec instead of 100msec.